B-mesons decay constants and form factors from HQET on the lattice

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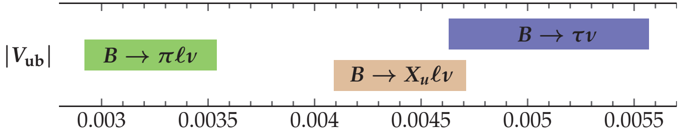
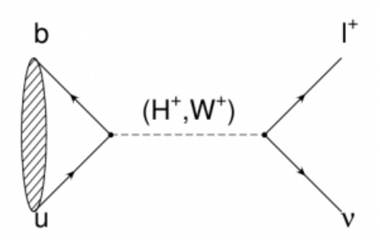
Theoretical input:

- Inclusive $B \to X_u l\nu$. From heavy quark and α_s expansions.
- Exclusive semileptonic $B \to \pi l\nu$. From lattice estimates of the form factor $f_+(q^2)$.
- Inclusive $B \to \tau\nu$. From lattice estimates of the decay constant F_{B}.

Summer 2012: [PDG'12]



Basically the same diagram appears in the two exclusive decays



However, in sensible extensions of the SM the coupling of a possible charged Higgs to $l\nu$ is $\propto m_l$ and therefore $B \to \tau\nu$ can receive much larger NP contributions.

Several scales in the game

$$m_h,\ a,\ m_l(m_\pi), L$$

FSE effects mainly introduced by the low-lying states

$$m_\pi L \geq 4$$

to have exponentially small FSE

discretization effects $\propto (am_h)^n$

$$a \ll \frac{1}{m_h}$$

to accurately describe on the lattice the propagation of a heavy quark

$\Rightarrow$ Approaching the physical situation requires large volumes and fine lattice spacings. Barely doable for charm-physics, need to resort to effective theories for B-physics.

$$S_{HQET} = a^4 \sum_x \left\{ \bar\psi_h (D_0 + \delta m)\psi_h + \omega_{spin} \bar\psi_h(-\sigma \mathbf{B})\psi_h + \omega_{kin} \bar\psi_h \left(-\frac{1}{2}\mathbf{D}^2\right) \psi_h \right.$$

We also consider the currents

$$\begin{aligned}
A_0^{\rm HQET}(x) &= Z_{\rm A}^{\rm HQET}[A_0^{\rm stat}(x) + \sum_{i=1}^{2} c_{\rm A}^{(i)} A_0^{(i)}(x)] \,, \\
A_0^{(1)}(x) &= \overline{\psi}_{\rm l} \tfrac{1}{2} \gamma_5 \gamma_i (\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S}) \psi_{\rm h}(x) \,, \\
A_0^{(2)}(x) &= -\tilde\partial_i A_i^{\rm stat}(x)/2 \,, \quad A_i^{\rm stat}(x) = \overline{\psi}_{\rm l}(x) \gamma_i \gamma_5 \psi_{\rm h}(x) \,,
\end{aligned}$$

$$A_k^{\rm HQET}(x) = Z_{{\rm A}_k}^{\rm HQET} [A_k^{\rm stat}(x) + \sum_{i=3}^{6} c_{\rm A}^{(i)} A_k^{(i)}(x)] \,,$$

$$\begin{aligned}
A_k^{(3)}(x) &= \overline{\psi}_{\rm l}(x) \frac{1}{2} (\nabla_i^{\rm S} - \overleftarrow{\nabla}_i^{\rm S}) \gamma_i \gamma_5 \gamma_k \psi_{\rm h}(x) \,, \quad A_k^{(4)}(x) = \overline{\psi}_{\rm l}(x) \frac{1}{2} (\nabla_k^{\rm S} - \overleftarrow{\nabla}_k^{\rm S}) \gamma_5 \psi_{\rm h}(x) \\
A_k^{(5)}(x) &= \widetilde{\partial}_i \left(\overline{\psi}_{\rm l}(x) \gamma_i \gamma_5 \gamma_k \psi_{\rm h}(x) \right) /2 \,, \quad A_k^{(6)}(x) = \widetilde{\partial_k}\, A_0^{\rm stat}/2
\end{aligned}$$

and analogous expressions for the vector current, 19 coeffs in total.

We will fix all these coeffs non-perturbatively.

ALPHA Collaboration

Why do we like HQET

- Theoretically very sound. In particular, the continuum limit is well defined and can be reached numerically [ALPHA, '03].
- Can be treated non-perturbatively including renormalization and $\mathrm{O}(1/m_b)$ [Heitger and Sommer, '03].
- It is self-consistent. The validity of the $1/m_b$ expansion can be tested down to the charm mass, as opposed to what is done within other approaches, where results are extrapolated from m_c to m_b assuming HQET.
- Numerically is as expensive as other approaches, the matching between QCD and HQET is perfomed in small volumes and it is very cheap concerning CPU-time. The costy part is the large volume, as for everybody.

Let us consider the example

$$m_{B^*}^2 - m_B^2 = C_{mag}(m_b/\Lambda_{\rm QCD})\ \langle B|\bar{\psi}_h \sigma \mathbf{B} \psi_h|B^*\rangle \times (1 + \mathrm{O}(1/m_b))$$

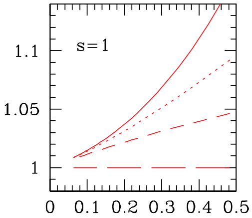
$C_{mag}(m_b/\Lambda_{\rm QCD})$ has a perturbative expansion. The tunctaion at O(n-1)

$$\simeq \alpha(m_b)^n \simeq \left\{ \frac{1}{2b_0 \ln(m_b/\Lambda_{\rm QCD})} \right\}^n >> \frac{\Lambda_{\rm QCD}}{m_b} \quad \text{as } m_b \to \infty$$

The PT corrections to the leading term are larger than the $1/m_b$ ones !

In addition the perturbative series is not always well behaved [R. Sommer, 2010].

C_{PS}/C_V vs $1/\ln(\Lambda/m_b)$



[Chetyrkin and Grozin 2003, Broadhurst and Grozin '91, '95, Bekavac et al. 2010]

Goal: Non-perturbatively fix all parameters (including Wilson coeffs.) appearing at $O(1/m_b)$ in the HQET expansion of the action, the vector and the axial currents.

Strategy: Simulate HQET and QCD (with a relativistic b-quark) on the lattice using a finite volume scheme (Schrödinger Functional) and fix the coeffs by requiring

$$\Phi_i^{\mathrm{HQET,1/m}} = \Phi_i^{\mathrm{QCD}} , \quad i = 1 \ldots N$$

with N large enough.

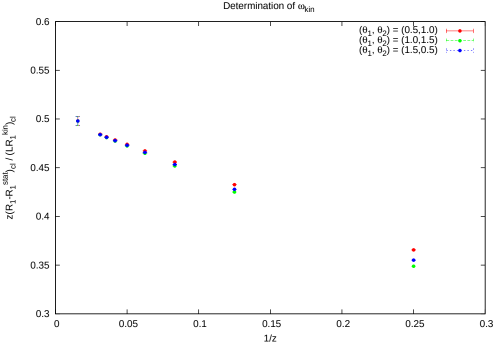
- the spacing a has to be small $\rightarrow$ lattices of small size L ($\simeq 0.5$ fm).
- minimize higher $1/mL = 1/z$ corrections
  $\Rightarrow$ tree level study presented here [in collaboration with S. Dooling and J. Heitger]
- the parameters depend on a. The evolution to values suited for large volume simulations can be done in HQET [ALPHA, JHEP 1209 (2012) 132].

Applications: b-quark mass [DM, Garron, Papinutto, Sommer JHEP 0701 (2007) 007], decay constants and form factors, e.g. $B \rightarrow \pi l \nu$ at large q^2.

The classical values of the coefficients are known:
$c_{\rm A}^{(1)} = c_{\rm A}^{(2)} = -c_{\rm A}^{(3)} = -c_{\rm A}^{(5)} = -\frac{1}{2m_b}$ and $c_{\rm A}^{(4)} = -c_{\rm A}^{(6)} = \frac{1}{m_b}$.

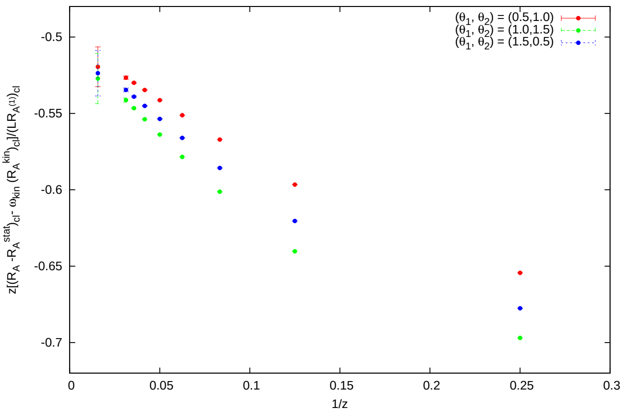
The single terms in the HQET expansion of a correlator are finite and have a continuum limit. Example ω_{kin}

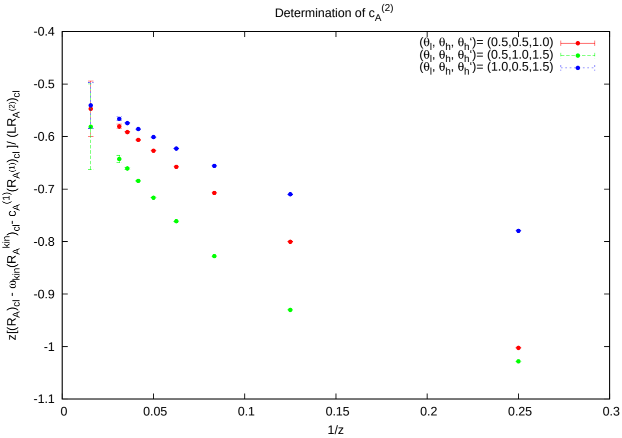
$$\frac{1}{4}(R_1^P + 3R_1^V) - R_1^{\rm stat} = \omega_{kin} R_1^{kin}\,, \quad (T = L/2)$$

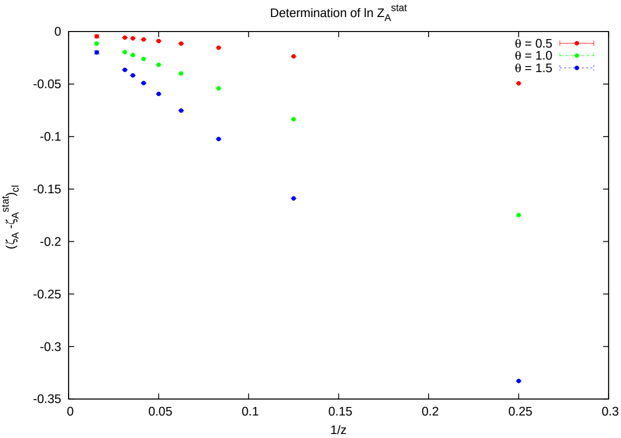


in the non-perturbative study we will have $z_b \simeq 13$.

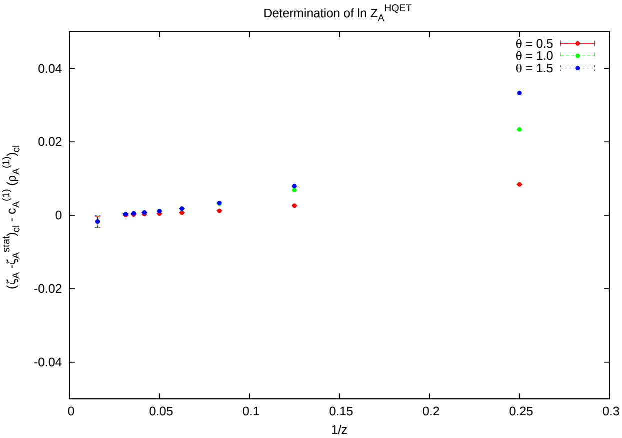
Determination of $c_A^{(1)}$

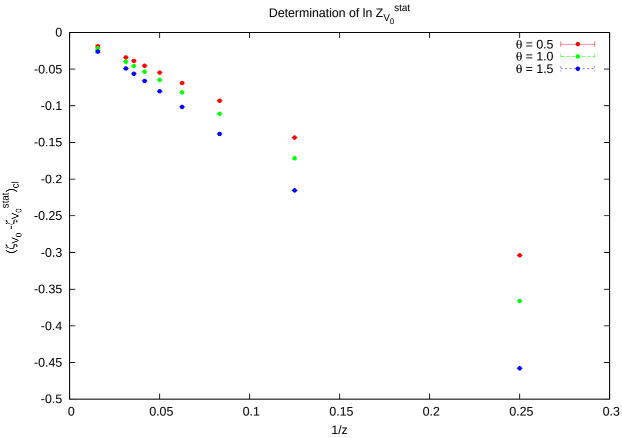


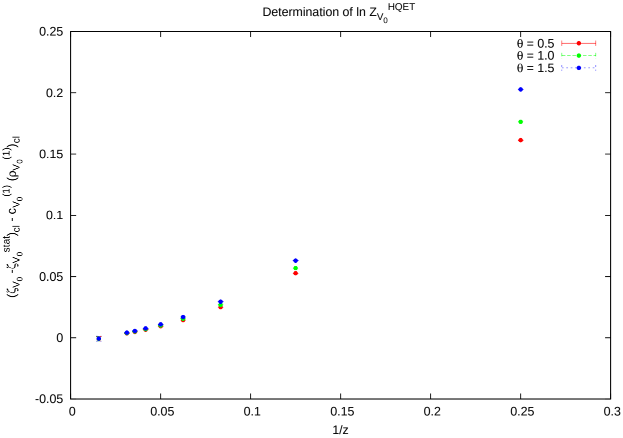

Determination of $c_A^{(2)}$
$(\theta_l, \theta_h, \theta_h')$= (0.5,0.5,1.0)
$(\theta_l, \theta_h, \theta_h')$= (0.5,1.0,1.5)
$(\theta_l, \theta_h, \theta_h')$= (1.0,0.5,1.5)
$z[(R_A)_{cl} - \omega_{kin}(R_A^{kin})_{cl} - c_A^{(1)}(R_A^{(1)})_{cl}] / (LR_A^{(2)})_{cl}$
1/z
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
0
0.05
0.1
0.15
0.2
0.25
0.3



Determination of ln Z_A^{stat}
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
0
0.05
0.1
0.15
0.2
0.25
0.3
1/z
$(\zeta_A - \zeta_A^{stat})_{cl}$
θ = 0.5
θ = 1.0
θ = 1.5


Determination of ln Z_A^{HQET}
$(\zeta_A - \zeta_A^{stat})_{cl} - c_A^{(1)} (\rho_A^{(1)})_{cl}$
1/z
θ = 0.5
θ = 1.0
θ = 1.5
0.04
0.02
0
-0.02
-0.04
0
0.05
0.1
0.15
0.2
0.25
0.3


Determination of ln $Z_{V_0}^{stat}$
$(\zeta_{V_0} - \zeta_{V_0}^{stat})_{cl}$
1/z
θ = 0.5
θ = 1.0
θ = 1.5
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
0
0.05
0.1
0.15
0.2
0.25
0.3



Determination of ln $Z_{V_0}^{HQET}$
$(\zeta_{V_0} - \zeta_{V_0}^{stat})_{cl} - c_{V_0}^{(1)} (\rho_{V_0}^{(1)})_{cl}$
1/z
θ = 0.5
θ = 1.0
θ = 1.5
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3

Results for $N_f = 2$: Decay constants with non-perturbative matching at $O(1/m_b)$

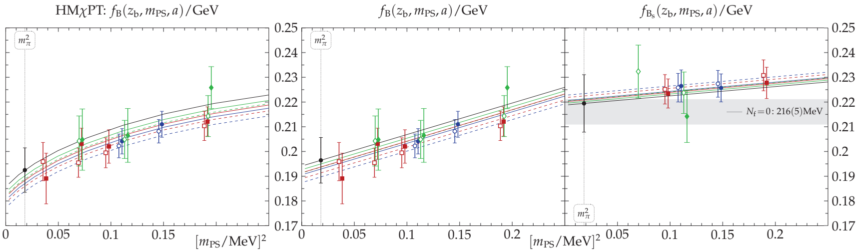
β	$a\,[\mathrm{fm}]$	$L^3 \times T$	$m_\pi\,[\mathrm{MeV}]$	#
5.2	0.075	$32^3 \times 64$	380	1000
		$32^3 \times 64$	330	500
5.3	0.065	$32^3 \times 64$	440	1000
		$48^3 \times 96$	310	500
		$48^3 \times 96$	270	600
		$64^3 \times 128$	**190**	**600**
5.5	0.048	$48^3 \times 96$	440	400
		$48^3 \times 96$	**340**	**900**
		$64^3 \times 128$	270	900

Fit formulae:

$$f_{B_s}(m_{\mathrm{PS}}^2, a^2) = b + c m_{\mathrm{PS}}^2 + d a^2$$

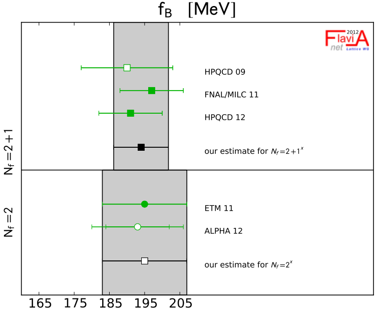
$$f_B(m_{\mathrm{PS}}^2, a^2) = b' \left[1 - \frac{3}{4} \frac{1+3\hat{g}^2}{(4\pi f_\pi)^2} m_{\mathrm{PS}}^2 \ln(m_{\mathrm{PS}}^2)\right] + c' m_{\mathrm{PS}}^2 + d' a^2$$

with f_π from exp. and $\hat{g} = 0.51(2)$ [Bulava et al. PoS LAT10]



$$f_B = 193(9)(4)\ \mathrm{MeV} \qquad f_{B_s} = 219(12)\ \mathrm{MeV}$$

[ALPHA, LAT12]



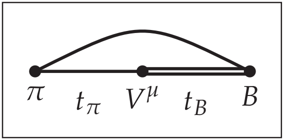
Differential decay rate in $B \to \pi l \nu$

$$\frac{d\Gamma}{dq^2} = \frac{G_{\rm F}^2}{24\pi^3} p_\pi^3 |V_{\rm ub}|^2 |f_+(q^2)|^2 \ ,$$

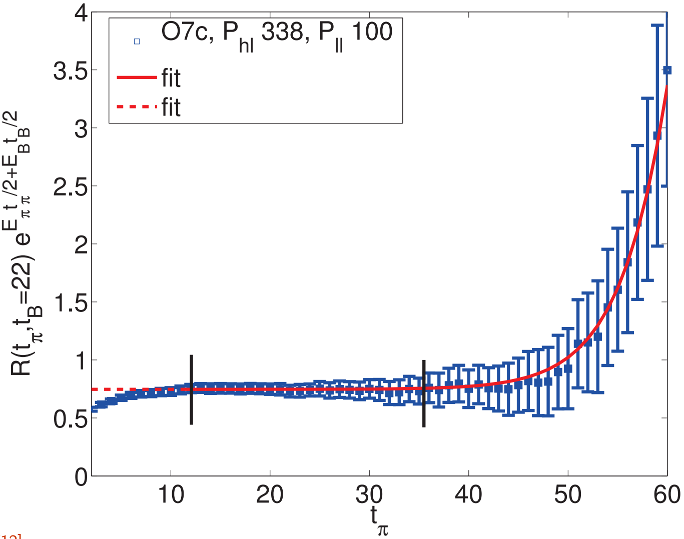
where q is the lepton pair momentum. The form factor $f_+(q^2)$ can be extracted from the matrix element of the vector current

$$\langle \pi(p_\pi)|V^\mu|B(p_{\rm B})\rangle = f_+(q^2)(p_\pi + p_{\rm B} + q\Delta_{m^2})^\mu + f_0(q^2)q^\mu\Delta_{m^2} \ ,$$

Setting $\vec{p}_{\rm B} = \vec{0}$, for each $\vec{p}_\pi$, one has to study a ratio of 3 over 2 -point functions on the lattice looking for a plateau in the insertion time of the current.



$\vec{p}_\pi = 1, 0, 0 \times \frac{2\pi}{L}$



[ALPHA, LAT12]

Summary and Conclusions

- Constraining the CKM matrix requires lattice inputs (eg V_{ub} tension).
- b-quark cannot be directly simulated on the lattice.
- HQET at NLO with non-perturbative matching.
- Tree level study of 19 matching conditions.
- Results: decay constants and form factors (ongoing, just started) with two dynamical flavors.
- The lattice determination of decay constants seems vey solid.