Beyond Three Neutrino Oscillations

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Outline

1. LSND and Mini-BooNE. A Theorist’s Summary;

2. Possible Solutions;


4. One Concrete Example – Low-Energy Seesaw, and a Warning;

5. Concluding Remarks – What Are We Really After?

I was relieved to hear that several other speakers were also given both very broad and/or surprising (“are you sure you want ME to talk about this?”) assignments from the organizers...
The LSND Anomaly

The LSND experiment looks for $\bar{\nu}_e$ coming from

- $\pi^+ \to \mu^+ \nu_\mu$ decay in flight;
- $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ decay at rest;

produced some 30 meters away from the detector region.

It observes a statistically significant excess of $\bar{\nu}_e$-candidates. The excess can be explained if there is a very small probability that a $\bar{\nu}_\mu$ interacts as a $\bar{\nu}_e$, $P_{\mu e} = (0.26 \pm 0.08)\%$.

However: the LSND anomaly (or any other consequence associated with its resolution) is yet to be observed in another experimental setup including Mini-BooNE(?).
The Mini-BooNE Data:

- No Excess in “LSND region”
- Low Energy Excess (3σ)

Inconsistent with 2 flavor oscillation interpretation of LSND anomaly.

<table>
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<th>reconstructed neutrino energy bin (MeV)</th>
<th>Data</th>
<th>total background</th>
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<td>300-475</td>
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<td>369±19</td>
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<td>380±19</td>
<td>380±19</td>
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\( \nu_e \) intrinsic: 26, 67, 229
\( \nu_\mu \) induced: 258, 207, 129

[Steve Brice at ν08]
Combining $\nu_e$ BDT + $\nu_e$ TBL Samples

The combination of the two $\nu_e$ samples gives an increase in coverage in the region $\Delta m^2 < 1$ eV$^2$.

Differences in the details are due to the specific fluctuations in the data samples and the interplay with correlations among them.

The combination yields a consistent result.

10%-30% improvement in 90% C.L. limit below $\sim 1$eV$^2$. 
Updates to Low Energy $\nu_e$ Prediction

Nearing the end of a comprehensive review of the $\nu_e$ appearance backgrounds and their uncertainties
→ Not Quite Ready for Release Yet

Arrows indicate whether effect is to increase or decrease the low energy data excess
The effects have different magnitudes despite the arrows all being the same size

↓ • Included photonuclear effect
   – Absent from GEANT3 – creates background from $\pi^0$s

← • More comprehensive hadronic errors
   – e.g. uncertainties in final state following photonuclear interaction

↓ • Better handling of beam $\pi^+$ production uncertainties
   – Errors propagated in model-independent way

↑ • Improved measurement of $\nu$ induced $\pi^0$s
   – e.g. finer momentum binning

↑ • Incorporation of MiniBooNE $\pi^0$ coherent/resonant measurement
   – No longer need to rely on more uncertain past results

↓ • Better handling of the radiative decay of the $\Delta$ resonance
   – Comprehensive review of how the $\Delta^{0,+}$ radiative decay rate is inferred from the measured $\pi^0$ rate
Anomaly Mediated Background

• “Anomaly mediated neutrino-photon interactions at finite baryon density.”
  • Standard Model process
    ⇒ Under active investigation, prediction of
    ~140 (g_ω/10)^4 events, where g_ω is 10 to 30.
  • Can use photon energy and angle to check prediction.
    (Harvey, Hill, and Hill, arXiv:0708.1281[hep-ph])

Could explain (part) of low-energy excess. Currently under investigation.

ν + N → ν + N + γ interesting for small liquid argon detector (µBooNE).
Interpreting the LSND Anomaly, A.M.-B.

If oscillations \( \Rightarrow \Delta m^2 \sim 1 \text{ eV}^2 \)

- does not fit into 3 \( \nu \) picture;
- 2 + 2 scheme ruled out (solar, atm);
- 3 + 1 scheme ruled out;
- 3 \( \nu \)'s CPTV ruled out (KamLAND, atm);
- 2+ “heavy” decaying sterile neutrinos;
- 3 + 1 + 1 scheme;
- 3 + 1 + new gauge interactions;
- 3(+ \( \nu \) vs and decoherence;
- 3(+ \( \nu \) vs and Lorentz-invariance violation;
- 4 \( \nu \)'s CPTV;
- something completely different.

LSND: strong evidence for \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \)
\[ \Delta m^2 \text{sol} \rightarrow \Delta m^2 \text{atm} \]

\[ \Delta m^2 \text{LSND} \rightarrow \Delta m^2 \text{atm} \]

\[ \Delta m^2 \text{sol} \rightarrow \Delta m^2 \text{LSND} \]

\[
\Rightarrow 2+2 \text{ requires large sterile effects in either solar or atmospheric oscillations, not observed}
\]
3+1 scheme ruled out

3+1+1 Fits Introduce an Extra $\Delta m^2$ and New Mixing Parameters

| data set                  | $|U_{e4}U_{\mu4}|$ | $\Delta m^2_{41}$ | $|U_{e5}U_{\mu5}|$ | $\Delta m^2_{51}$ |
|---------------------------|--------------------|-------------------|--------------------|-------------------|
| appearance (MB475)        | 0.044              | 0.66              | 0.022              | 1.44              | 1.12$\pi$        |
| appearance (MB300)        | 0.31               | 0.66              | 0.27               | 0.76              | 1.01$\pi$        |
| global data (MB475)       | 0.11               | 0.16              | 0.89               | 0.12              | 6.49              | 1.64$\pi$        |
| global data (MB300)       | 0.12               | 0.18              | 0.87               | 0.11              | 0.089             | 1.91              | 1.44$\pi$        |


Mini-BooNE and LSND fit “perfectly,” including low-energy excess (MB300).

However, severely disfavored by disappearance data, especially if MB300 is included [“$>3\sigma$”].

July 4, 2008

MB and sterile $\nu$
LSND (+ Mini-BooNE Low Energy Excess) – Exotic Solutions

The LSND effect is very small ($P_{\mu e} \sim 0.3\%$). No other experiment has achieved better sensitivity to $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$, at these values of $L$ and $E$ . . .

. . . except for Karmen II $\rightarrow L$ independent effect ruled out

. . . except for Mini-BooNE $\rightarrow L/E$ effect ruled out* (including disappearance)

We need an effect that varies with $E$ and $L$ (not via $L/E$) such that the small LSND effect is explained but all other data are left untouched. It is, then, almost always the case that a small effect at LSND translates into a much larger effects elsewhere (atmospheric/solar neutrinos, short baseline experiments, etc).

One needs to work very hard at it!

Bottom line: we have, so far, failed to find a consensus, “feel good” solution to the LSND anomaly, especially after Mini-BooNE (regardless of whether there is a new physics related low-energy excess)
Issues with energy dependency:

1. \( L/E \) behavior is a consequence of Lorentz invariance \( \rightarrow \) need Lorentz invariance violation of some sort.
   - Matters effects can do the trick or
   - need Lorentz violating physics can do the trick.

2. • LSND Energies are around 30 MeV;
   • Mini-BooNE has energies around 300 MeV (excess) to 1+ GeV (no excess);
   • Solar neutrinos have energies less than 10 MeV;
   • Short Baseline experiments (CHORUS, NOMAD, NuTeV) have multi-GeV energies. Very stringent constraint on \( P_{\mu e} \)!

\( \Rightarrow \) Need (apparent) Lorentz violating effect that “peaks” at LSND energies and does not blow up at solar or short baseline energies.
III. Neutrino decay and decoherence

Explaining LSND with decoherence

- Recent suggestion [54]: $3\nu$ oscillations + decoherence, with $\gamma_{21} = 0$, $\gamma_{31} = \gamma_{32} = \gamma$ and $\gamma(E) = \kappa_{-4}(E_{\nu}/\text{GeV})^{-4}$;
- only 1 new parameter. Probabilities:

$$P_{\mu e}(\gamma, L) = P_{e\mu}(\gamma, L) = 2|U_{\mu 3}|^2|U_{e 3}|^2 \left[1 - e^{-\gamma E_{\nu}} \cos(\Delta_{31} L)\right],$$
$$P_{ee}(\gamma, L) = 1 - 2|U_{e 3}|^2(1 - |U_{e 3}|^2) \left[1 - e^{-\gamma E_{\nu}} \cos(\Delta_{31} L)\right],$$
$$P_{\mu\mu}(\gamma, L) = 1 - 2|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \left[1 - e^{-\gamma E_{\nu}} \cos(\Delta_{31} L)\right];$$
- Best fit: $\kappa_{-4}^{\text{atm}} = 1.7 \times 10^{-23} \text{ GeV};$
- Explicit prediction: $\sin^2 \theta_{13} > (2.6 \pm 0.8) \times 10^{-3};$
- Other possibilities: decay + sterile neutrinos [55], decoherence + CPT-violation [56], decoherence with unusual $L$ dependence [57], …

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Back To Sterile Neutrinos – General Comments

Here I’ll concentrate on LIGHT sterile neutrinos ($M_{\nu_s} < 1$ GeV). Such states only interact with the SM via weak mixing with the active neutrinos we know and love.

Regardless of LSND et al, people often talk about “sterile neutrinos.” Why? There are many theoretical complaints related to light sterile neutrinos:

- Who ordered that? What are sterile neutrinos good for?
- Why awould they be light? Sterile neutrinos are “theoretically expected” to be very heavy...
- If there are sterile neutrinos, can we say anything about their properties? Say, is the sterile–active neutrino mixing angle calculable? Are there preferred regions of the sterile neutrino parameter space?
- ...

July 4, 2008
Why Not?

Sterile neutrinos are gauge singlet fermions, and qualify, along with a gauge singlet scalar, as the most benign, trivial extension of the SM matter sector. “Hidden Sector”

More interesting is the fact that gauge singlets only communicate to the SM (at the renormalizable level) in two ways:

- Scalars couple to the Higgs boson;
- Fermions couple to neutrinos (via Yukawa coupling → mixing).

Active sterile neutrino mixing provides one of only two ways to communicate with gauge singlet fields that may be out there!

Of course, one may ask if there is any evidence for such a hidden sector. The answer is “we don’t know.” . . .
... However:

- Dark matter could be a very weakly coupled “weak-scale” mass particle. And it can certainly be either one of the Hidden sector particles!

- Light sterile neutrinos in particular may be a good warm dark matter candidate.

- It is often speculated that light sterile neutrinos may play an important role in supernova explosions. They may aid on the synthesis of heavy elements and may be the reason behind the large peculiar velocity of neutron stars (pulsar kicks).

- Sterile neutrinos are often a side-effect of active neutrino masses. Remember:
  
  Sterile Neutrino = Right-Handed Neutrino = Gauge Singlet Fermion
Concrete Example: The Seesaw Lagrangian, No Prejudices

A simple\(^a\), renormalizable Lagrangian that allows for neutrino masses is

\[
\mathcal{L}_\nu = \mathcal{L}_{\text{old}} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,
\]

where \(N_i\) (\(i = 1, 2, 3\), for concreteness) are SM gauge singlet fermions. \(\mathcal{L}_\nu\) is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the \(N_i\) fields.

After electroweak symmetry breaking, \(\mathcal{L}_\nu\) describes, besides all other SM degrees of freedom, six Majorana fermions: six neutrinos.

\(^a\)Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.
To be determined from data: $\lambda$ and $M$.

The data can be summarized as follows: there is evidence for three neutrinos, mostly “active” (linear combinations of $\nu_e$, $\nu_\mu$, and $\nu_\tau$). At least two of them are massive and, if there are other neutrinos, they have to be “sterile.”

This provides very little information concerning the magnitude of $M_i$ (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large $M$: $M \gg v$. Popular examples include $M \sim M_{\text{GUT}}$ (GUT scale), or $M \sim 1 \text{ TeV}$ (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14} \text{ GeV}$, while thermal leptogenesis requires the lightest $M_i$ to be around $10^{10} \text{ GeV}$.

*we can impose very, very few experimental constraints on $M$*
What We Know About $M$:

- $M = 0$: the six neutrinos “fuse” into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all $M_i$ vanish. Small $M_i$ values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha \beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ \[ m \propto 1/\Lambda \Rightarrow \Lambda = M/\mu^2 \].

  This the \textbf{seesaw mechanism}. Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though $L$-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).
Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale $M$,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$ 

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or

- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or

- “cancellations” among different contributions render neutrino masses accidentally small (fine-tuning, or horizontal symmetry).
Low-Energy Seesaw \[\text{[AdG PRD72,033005]}\]

The other end of the $M$ spectrum ($M \ll 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light;
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- Sterile–active mixing can be qualitatively predicted – hypothesis is testable!
- Small values of $M$ are natural (in the ‘tHooft sense). In fact, theoretically, no value of $M$ should be discriminated against!
Dark Matter(?)

Pulsar Kicks

Oscillations

Mass (eV)

$\nu_{1}$

$\nu_{2}$

$\nu_{3}$

$\nu_{4}$

$\nu_{5}$

$\nu_{6}$

$\nu_{s1}$

$\nu_{s2}$

$\nu_{s3}$

$\nu_{e}$

$\nu_{\mu}$

$\nu_{\tau}$

Also effects in $0\nu\beta\beta$,

tritium beta-decay,

supernova neutrino oscillations,

non-standard cosmology.

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]

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MB and sterile $\nu$
Active–Sterile Mixing Angles:

\[ |U_\alpha|^2 \sim \frac{m_\nu}{M} \]

\begin{align*}
m_\nu &= 1 \text{ eV} \\
m_\nu &= 0.05 \text{ eV} \\
m_\nu &= 0.009 \text{ eV} \\
m_\nu &= 0.001 \text{ eV}
\end{align*}
Predictions: **Tritium beta-decay**

Heavy neutrinos participate in tritium $\beta$-decay. Their contribution can be parameterized by

$$m^2_\beta = \sum_{i=1}^{6} |U_{ei}|^2 m^2_i \simeq \sum_{i=1}^{3} |U_{ei}|^2 m^2_i + \sum_{i=1}^{3} |U_{ei}|^2 m_i M_i,$$

as long as $M_i$ is not too heavy (above tens of eV). For example, in the case of a 3+2 solution to the LSND anomaly, the heaviest sterile state (with mass $M_1$) contributes the most: $m^2_\beta \simeq 0.7 \text{ eV}^2 \left( \frac{|U_{e1}|^2}{0.7} \right) \left( \frac{m_1}{0.1 \text{ eV}} \right) \left( \frac{M_1}{10 \text{ eV}} \right)$.

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1}) \text{ eV}^2$. 

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MB and sterile $\nu$
sensitivity of tritium beta decay to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]
Predictions: Neutrinoless Double-Beta Decay

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \rightarrow (Z + 2)e^-e^-$. 

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

$$m_{ee} = \left| \sum_{i=1}^{6} U_{ei}^2 m_i \right| \sim \left| \sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \bar{\nu}_{ei}^2 M_i \right|.$$ 

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. The contribution of light and heavy neutrinos exactly cancels! This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \rightarrow m_{ee} \text{ is identically zero!}$$
(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

$M_{ee} = Q^2 \sum U_{ei}^2 \frac{m_i}{Q^2 + m_i^2}$

Region Required to explain Pulsar kicks and warm dark matter

$Q = 50$ MeV

$M_{ee}$: $\nu_{light}$

$M_{ee}$: $\nu_{light} + \nu_{heavy}$
Summary and Concluding Remarks

1. Mini-BooNE results do not confirm the LSND result. Interpretations of all data with sterile neutrinos “ruled out.” We await more data and a detailed analysis of the low-energy excess.

2. There are baroque capable of fitting all the data. All are taylor made, and often require “multiple miracles.” There is no fell good solution to the LSND puzzle with the advent of the Mini-BooNE results.

3. Gauge singlet fermions (sterile neutrinos) are a simple, benign extension of the standard model (Hidden Sector). They will only manifest themselves through mixing with the active neutrinos.

4. Light sterile neutrinos may be the dark matter, may play an important role in supernovae, and may be evidence of the physics responsible for neutrino masses. Testable hypotheses!
What is the “Real” Goal of Neutrino Oscillation Experiments?

- What is the $\nu_e$ component of $\nu_3$? $(\theta_{13} \neq 0?)$
- Is CP-invariance violated in neutrino oscillations? $(\delta \neq 0, \pi?)$
- Is $\nu_3$ mostly $\nu_\mu$ or $\nu_\tau$? $(\theta_{23} > \pi/4,$ $\theta_{23} < \pi/4,$ or $\theta_{23} = \pi/4?)$
- What is the neutrino mass hierarchy? $(\Delta m_{13}^2 > 0?)$

MORE IMPORTANT: test the three neutrino mixing hypothesis. Are we missing anything?

More New $\nu$ Physics? $\rightarrow$ Sterile $\nu$ vs prime candidates!
Backup Slides . . .
### Global Fits to Experiments

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<th>LSND</th>
<th>KARMEN2</th>
<th>MB</th>
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Anti-nue Appearance Sensitivity

\[ \Delta m^2 \]

\[ \sin^2(2\theta) \]

- \( \nu_e \) signal only, all error but dirt
- 2.0 E20 POT, 90% C.L.
- 5.0 E20 POT, 90% C.L.
- 10. E20 POT, 90% C.L.

Region allowed at 90% C.L. by joint analysis of LSND and KARMEN

Preliminary

Only anti-neutrinos allowed to oscillate
On Astrophysical / Cosmological Bounds

[AdG, Jenkins, Vasudevan, hep-ph/0608147]
High-energy seesaw has no observable consequence other than non-zero neutrino masses, except, perhaps,

**Baryogenesis via Leptogenesis**

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the observed baryon asymmetry of the Universe can be obtained from a baryon–antibaryon symmetric initial condition plus well understood dynamics. [Baryogenesis]

This isn’t just for aesthetic reasons. If the early Universe undergoes a period of inflation, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

It turns out the seesaw mechanism contains all necessary ingredients to explain the baryon asymmetry of the Universe as long as the right-handed neutrinos are heavy enough – \( M > 10^9 \) GeV (with some exceptions that I won’t have time to mention, [e.g., see poster P-22]).
More Details, assuming three right-handed neutrinos $N$:

$$m_\nu = \begin{pmatrix} 0 & \lambda v \\ (\lambda v)^t & M \end{pmatrix},$$

$M$ is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where $m_a$ is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of $M$. 
$6 \times 6$ mixing matrix $U \ [U^t m_\nu U = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)]$ is

$$U = \left( \begin{array}{cc} V & \Theta \\ -\Theta^\dagger V & 1_{n \times n} \end{array} \right),$$

where $V$ is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1, m_2, m_3),$$

and the matrix that governs active–sterile mixing is

$$\Theta = (\lambda v)^* M^{-1}.$$

One can solve for the Yukawa couplings and re-express

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},$$

where $R$ is a complex orthogonal matrix $RR^t = 1.$
What if $1 \text{ GeV} < M < 1 \text{ TeV}$?

Naively, one expects

$$\Theta \sim \sqrt{\frac{m_a}{M}} < 10^{-5} \sqrt{\frac{1 \text{ GeV}}{M}},$$

such that, for $M = 1 \text{ GeV}$ and above, sterile neutrino effects are mostly negligible.

However,

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3) R^\dagger M^{-1/2}},$$

and the magnitude of the entries of $R$ can be arbitrarily large $[\cos(ix) = \cosh x \gg 1 \text{ if } x > 1]$.

This is true as long as

- $\lambda v \ll M$ (seesaw approximation holds)
- $\lambda < 4\pi$ (theory is “well-defined”)

This implies that, in principle, $\Theta$ is a quasi-free parameter – independent from light neutrino masses and mixing – as long as $\Theta \ll 1$ and $M < 1 \text{ TeV}$.
What Does $R \gg 1$ Mean?

It is illustrative to consider the case of one active neutrino of mass $m_3$ and two sterile ones, and further assume that $M_1 = M_2 = M$. In this case,

$$ \Theta = \sqrt{\frac{m_3}{M}} \left( \begin{array}{cc} \cos \zeta & \sin \zeta \end{array} \right), $$

$$ \lambda v = \sqrt{m_3 M} \left( \begin{array}{cc} \cos \zeta^* & \sin \zeta^* \end{array} \right) \equiv \left( \begin{array}{cc} \lambda_1 & \lambda_2 \end{array} \right). $$

If $\zeta$ has a large imaginary part $\Rightarrow \Theta$ is (exponentially) larger than $(m_3/M)^{1/2}$, $\lambda_i$ neutrino Yukawa couplings are much larger than $\sqrt{m_3 M}/v$

The reason for this is a strong cancellation between the contribution of the two different Yukawa couplings to the active neutrino mass

$\Rightarrow m_3 = \lambda_1^2 v^2 / M + \lambda_2^2 v^2 / M.$

For example: $m_3 = 0.1$ eV, $M = 100$ GeV, $\zeta = 14i \Rightarrow 
\lambda_1 \sim 0.244, \lambda_2 \sim -0.244i$, while $|y_1| - |y_2| \sim 3.38 \times 10^{-13}$. 
Weak Scale Seesaw, and Accidentally Light Neutrino Masses

What does the seesaw Lagrangian predict for the LHC?

- Nothing much, unless...
  - $M_N \sim 1 - 100$ GeV,
  - Yukawa couplings larger than naive expectations.

$\Leftarrow H \rightarrow \nu N$ as likely as $H \rightarrow b\bar{b}$!

( NOTE: $N \rightarrow \ell q' \bar{q}$ or $\ell\ell'\nu$ (prompt)
  “Weird” Higgs decay signature! )

[+ Lepton Number Violation at Colliders]