Elements of a Local Subtraction Method

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@HALC LHC 23—Highest Accuracy perturbative predictions at the Lowest Consumption for LHC phenomenology

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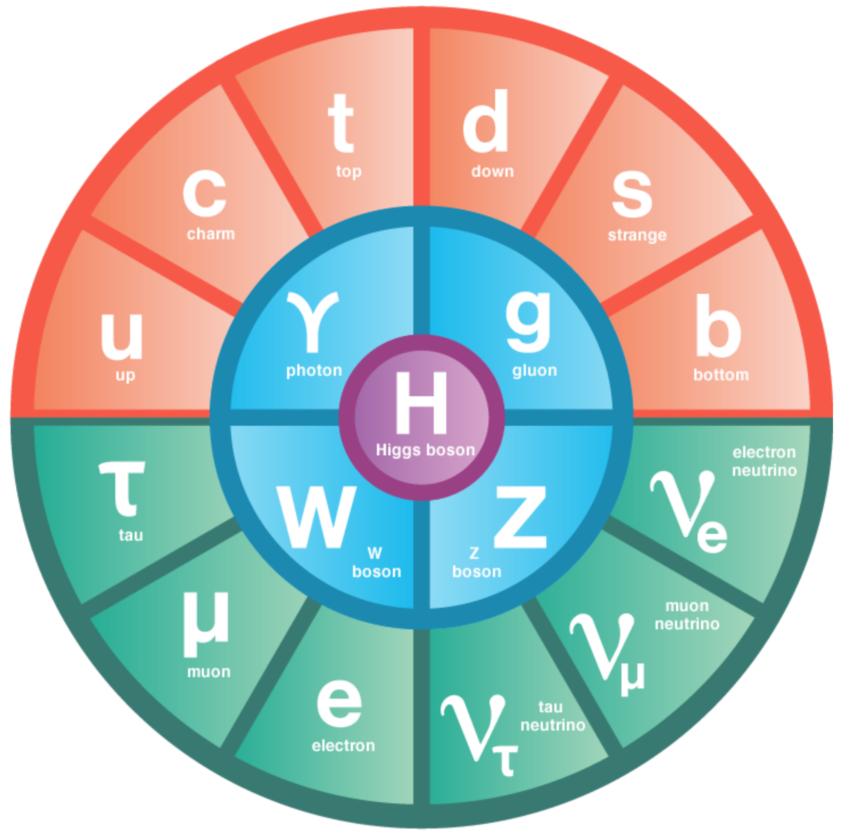
Outline

- Introduction and Motivation
- Computation of Jet Observables
- Phase Space Slicing Vs Local Subtraction Method
- Soft and Collinear Factorisation in QCD
- Future Outlook

The Standard Model of Particle Physics



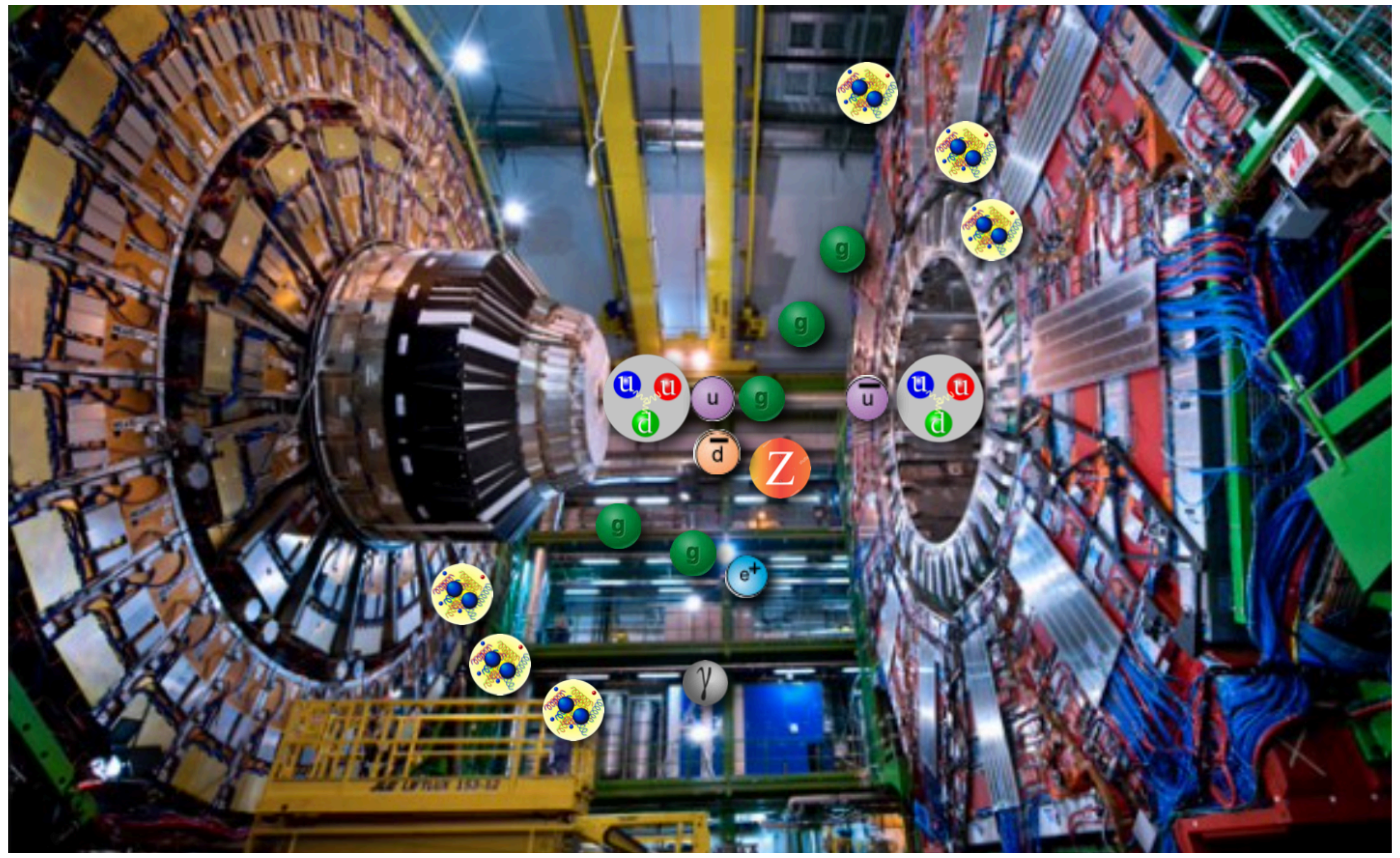
- The SM of particle physics has been instrumental in accurately predicting properties of the fundamental particles including its Masses and Couplings to other fundamental particles

- The most recent is being the discovery of the Higgs Boson at the LHC which lead to 2013 Nobel Prize to Peter Higgs and Francois Englert

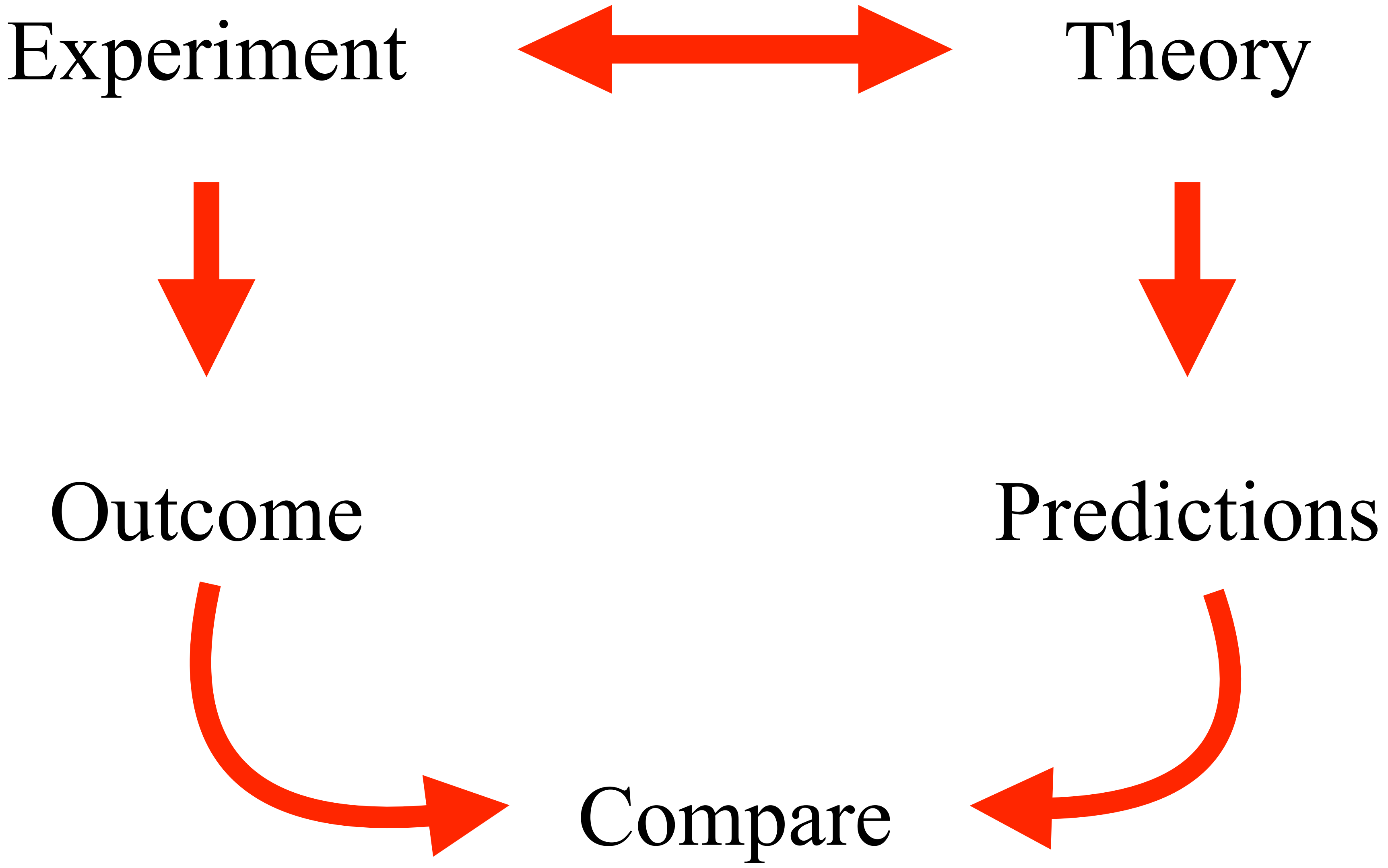
- However, the SM is not the ultimate theory; it has limitations…

What to do? Experiment $\Rightarrow$ Theory

The Large Hadron Collider



Thumb rule for a theoretical physicist



Experiment ⟷ Theory

Outcome

Predictions

Compare

- One approach: study beyond the SM to explain the New Physics
- Other approach: precision study within the SM

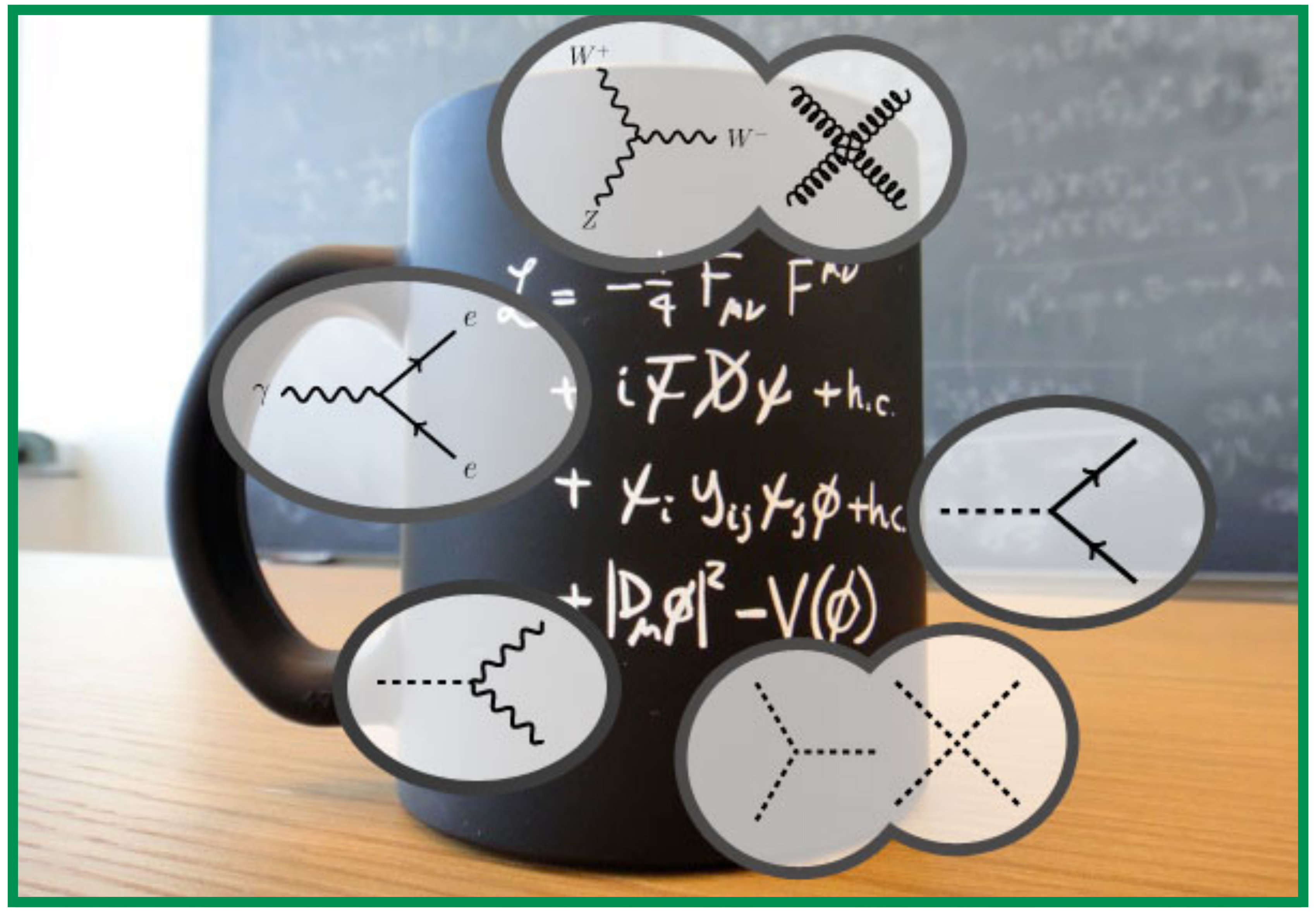
Precision Era of Particle Physics

- Observable of interest

$$\mathcal{O} = \kappa^p \sum_{n=0}^{+\infty} \kappa^n C_n$$

Perturbative contributions

Coupling



C_0 : Leading Order (LO)

C_1 : Next-to-LO (NLO)

C_2 : Next-to-NLO (NNLO)

- Perturbative QCD:

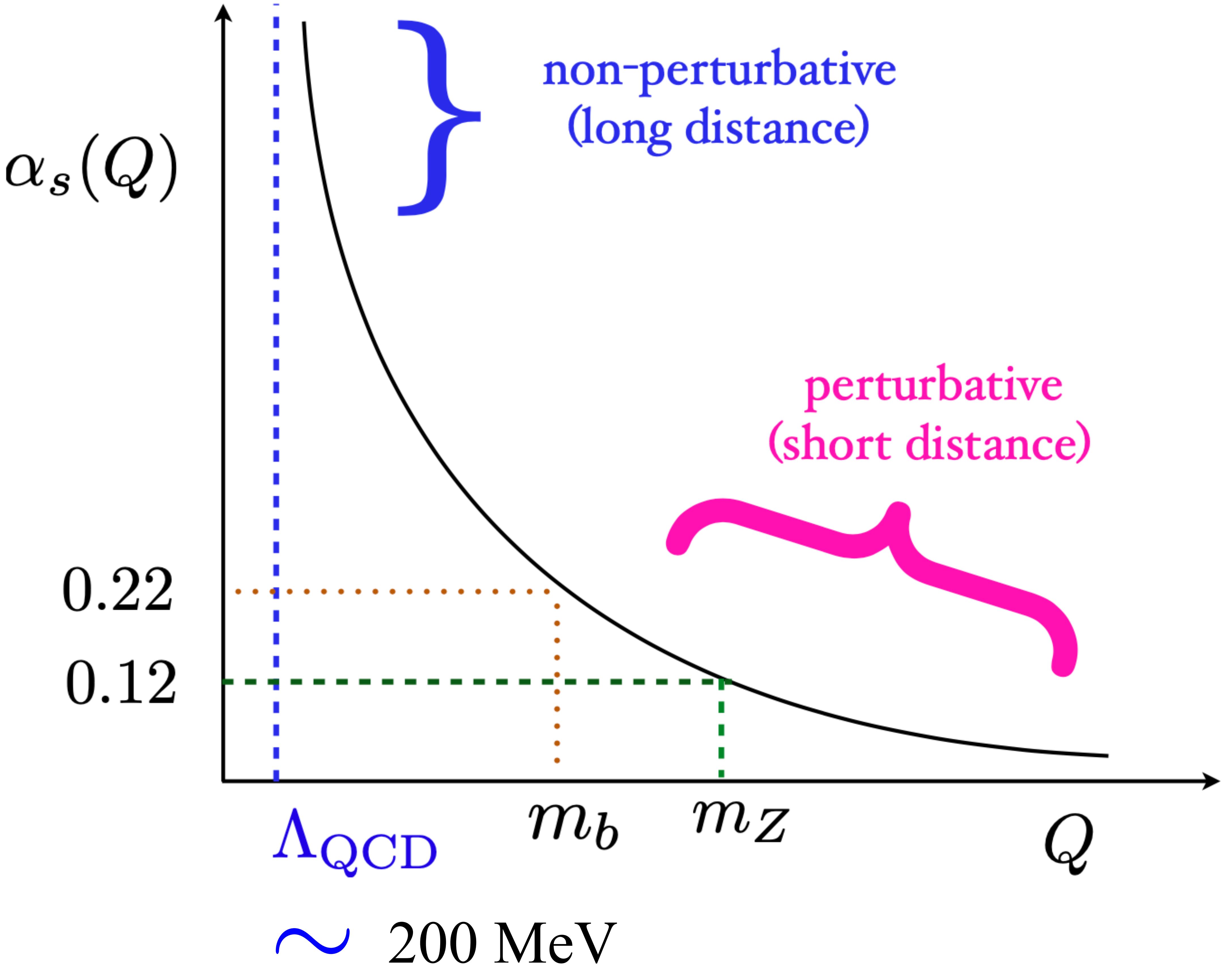
$$\kappa = \alpha_{\rm S} = \frac{g_{\rm S}^2}{4\pi}$$

Strong coupling

Asymptotic Freedom Saves Us!

- Strong coupling runs: large values at low energies and small values at high energies

Confinement They can't be treated as independent states



Asymptotic Freedom

Quarks and gluons
behave as free particles

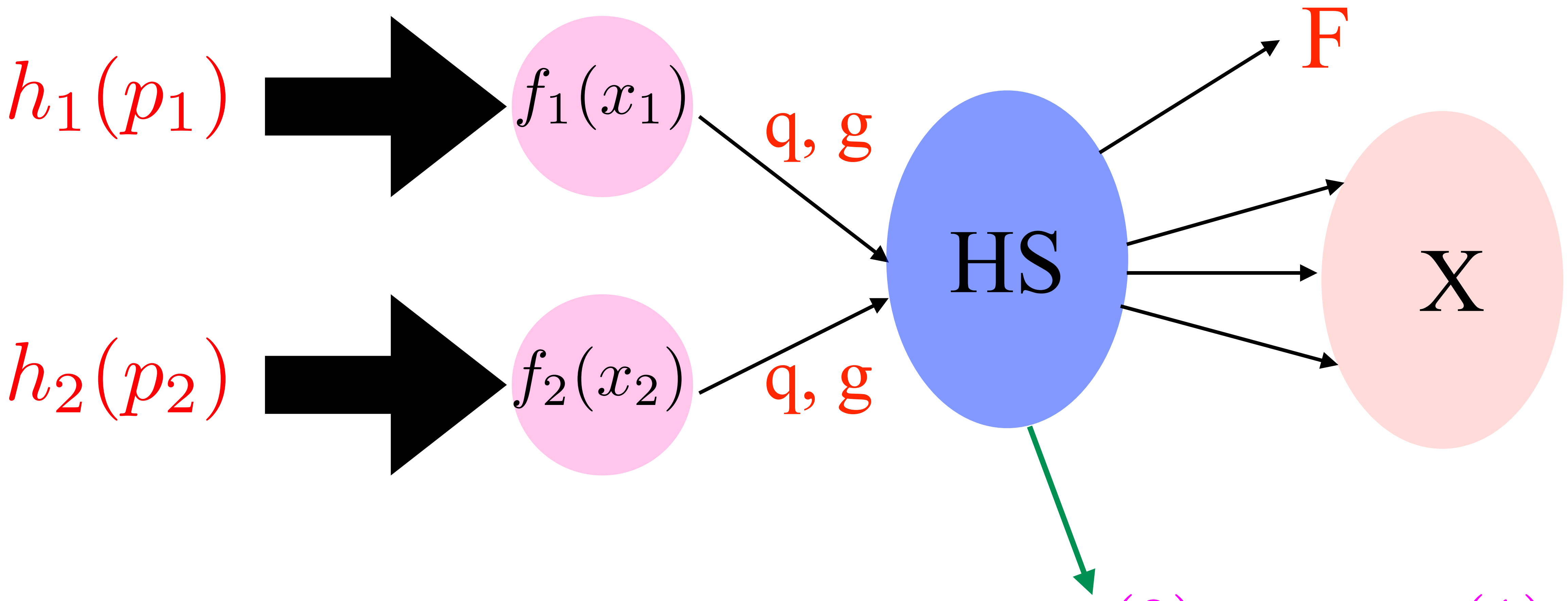
Parton Model: Short Range and Long Range

- For a generic production process

$$h_1(p_1) + h_2(p_2) \to F(\{q_i\}) + X$$

additional radiation

$$c(x_1 p_1) + \bar{c}(x_2 p_2) \to F(q \equiv \sum_i q_i)\Big| q_\mu q^\mu = M^2 = Q^2$$

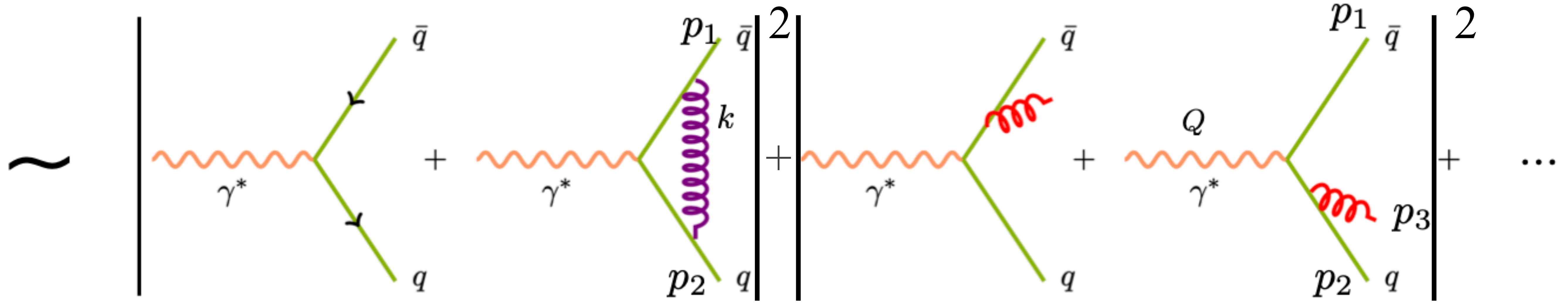


$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0)} + \alpha_S \hat{\sigma}_{ij}^{(1)} + \dots$$

$$\sigma_{h_1,h_2} = \sum_{i,j} \int dx_1 dx_2 \overset{\text{Long distance}}{f_i(x_1) f_j(x_2)} \underset{\text{Short distance}}{\hat{\sigma}_{ij}} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)$$

Fixed Order Perturbation Theory

$$\hat{\sigma} = \hat{\sigma}_0 \quad + \quad \alpha_s \hat{\sigma}_1 \quad + \quad \alpha_s^2 \hat{\sigma}_2 \quad + \quad \mathcal{O}(\alpha_s^3)$$



tree

virtual

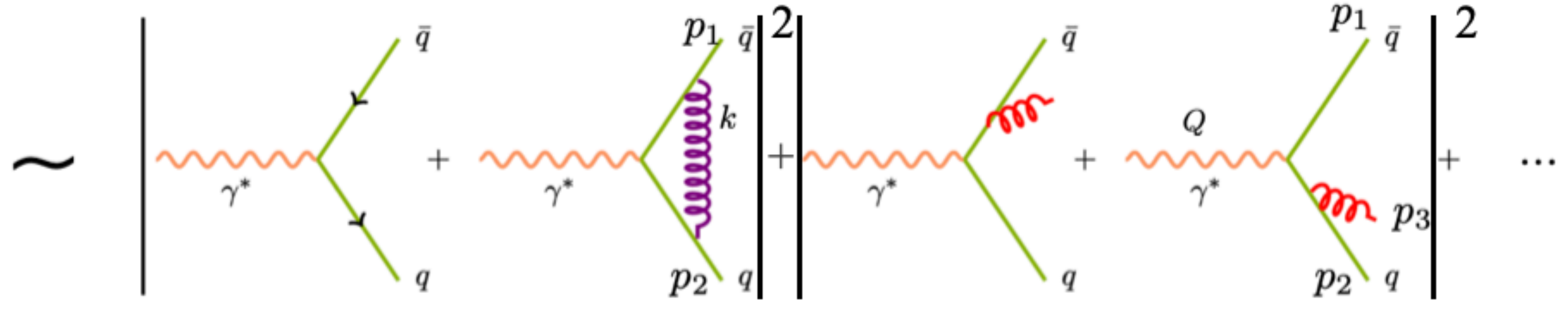
real emission

$$\sim \hat{\sigma}_0 \qquad \underbrace{\sim \hat{\sigma}_1^V \qquad \sim \hat{\sigma}_1^R}_{\sim \hat{\sigma}_1}$$

- real emission: $p_3 \to 0$ soft gluon → soft divergence
  $p_3 || p_1$ or $p_3 || p_2$ collinear → collinear divergence
  infrared
- virtual:
  $k \to 0$ soft div
  $k || p_1$ or $k || p_2$ collinear div
  $k \to \infty$ ultraviolet div

Divergent Terms Cancel !

$$\hat{\sigma} = \hat{\sigma}_0 \quad + \quad \alpha_s \hat{\sigma}_1 \quad + \quad \alpha_s^2 \hat{\sigma}_2 \quad + \quad \mathcal{O}(\alpha_s^3)$$



tree virtual real emission

$$\sim \hat{\sigma}_0 \qquad \underbrace{\sim \hat{\sigma}_1^V \qquad \sim \hat{\sigma}_1^R}_{\sim \hat{\sigma}_1}$$

- real emission: $p_3 \rightarrow 0$ soft gluon → soft divergence
  $p_3 || p_1$ or $p_3 || p_2$ collinear → collinear divergence

  infrared

- virtual: $k \rightarrow 0$ soft div
  $k || p_1$ or $k || p_2$ collinear div
  $k \rightarrow \infty$ ultraviolet div ← UV renormalisation

Cancels: KLN Theorem

Identified partons

↓

Collinear divergence remains!

↓

Renormalise bare PDFs and FFs

Computations of Jet Observables

- Jet cross section up to NNLO:

$$\sigma = \sigma^{LO} + \sigma^{NLO} + \sigma^{NNLO}$$

$$= \underset{\text{LO}}{\boxed{\int_m d\sigma^B}} + \underset{\text{NLO}}{\boxed{\int_{m+1} d\sigma^R + \int_m d\sigma^V}} + \underset{\text{NNLO} \quad \text{Non-trivial in d-dimensions}}{\boxed{\int_{m+2} d\sigma^{RR} + \int_{m+1} d\sigma^{RV} + \int_m d\sigma^{VV}}}$$

- The algorithms that are used to calculate more differential cross sections can be broadly divided into two types:

1. Phase space slicing method
2. Subtraction method

Example: Phase Space Slicing Method

- Let’s consider the integral of the form

Real type contribution

Virtual type contribution

$$I = \lim_{\epsilon \to 0} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

DR like parameter

Observable: $x \to 0$(Soft/collinear)

$$= \lim_{\epsilon \to 0} \left\{ \int_0^\delta \frac{dx}{x} x^\epsilon F(x) + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

$$\sim \lim_{\epsilon \to 0} \left\{ F(0) \int_0^\delta \frac{dx}{x} x^\epsilon + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

$$= F(0) \ln \delta + \int_\delta^1 \frac{dx}{x} F(x)$$

Smaller δ $\Longrightarrow$ Less dependence on δ

Example: Subtraction Method

- Consider again the integral from the previous slide

$$I = \lim_{\epsilon \to 0} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right\}$$

$$= \lim_{\epsilon \to 0} \left\{ \int_0^1 \frac{dx}{x} x^\epsilon \left[F(x) - F(0) \right] + F(0) \int_0^1 \frac{dx}{x} x^\epsilon - \frac{1}{\epsilon} F(0) \right\}$$

$$= \int_0^1 \frac{dx}{x} \left[F(x) - F(0) \right]$$

- It is advantageous to phase space slicing method

Application: Computation of Jet Cross Section

- For a generic observable

$$\boxed{\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V}$$

- We want to find a local counter term such that

$$d\sigma^{NLO} = \left[d\sigma^R - d\sigma^A\right] + d\sigma^A + d\sigma^V$$

it cancels all the divergences coming from the virtual Feynman diagrams

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R - d\sigma^A\right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V$$

in particular, the master formula will look like

$$\boxed{\sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^R\right)_{\epsilon=0} - \left(d\sigma^A\right)_{\epsilon=0}\right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A\right]_{\epsilon=0}}$$

Not easy always!

QCD Factorisation in the Soft Limit

- For a generic scattering amplitude having a soft gluon of momentum q

$$\boxed{\langle c|\mathcal{M}^{(0)}(q,\{p\})\rangle = g_{\mathrm{S}}\mu_0^{\epsilon}\varepsilon^{\mu}(q)J_{\mu}^{c(0)}(q)|\mathcal{M}^{(0)}(\{p\})\rangle + \ldots}$$

where

$$\boldsymbol{J}^{\mu(0)}(q) = \sum_{i=1}^{n}\boldsymbol{T}_i\frac{p_i^{\mu}}{p_i\cdot q}$$

At the squared amplitude level

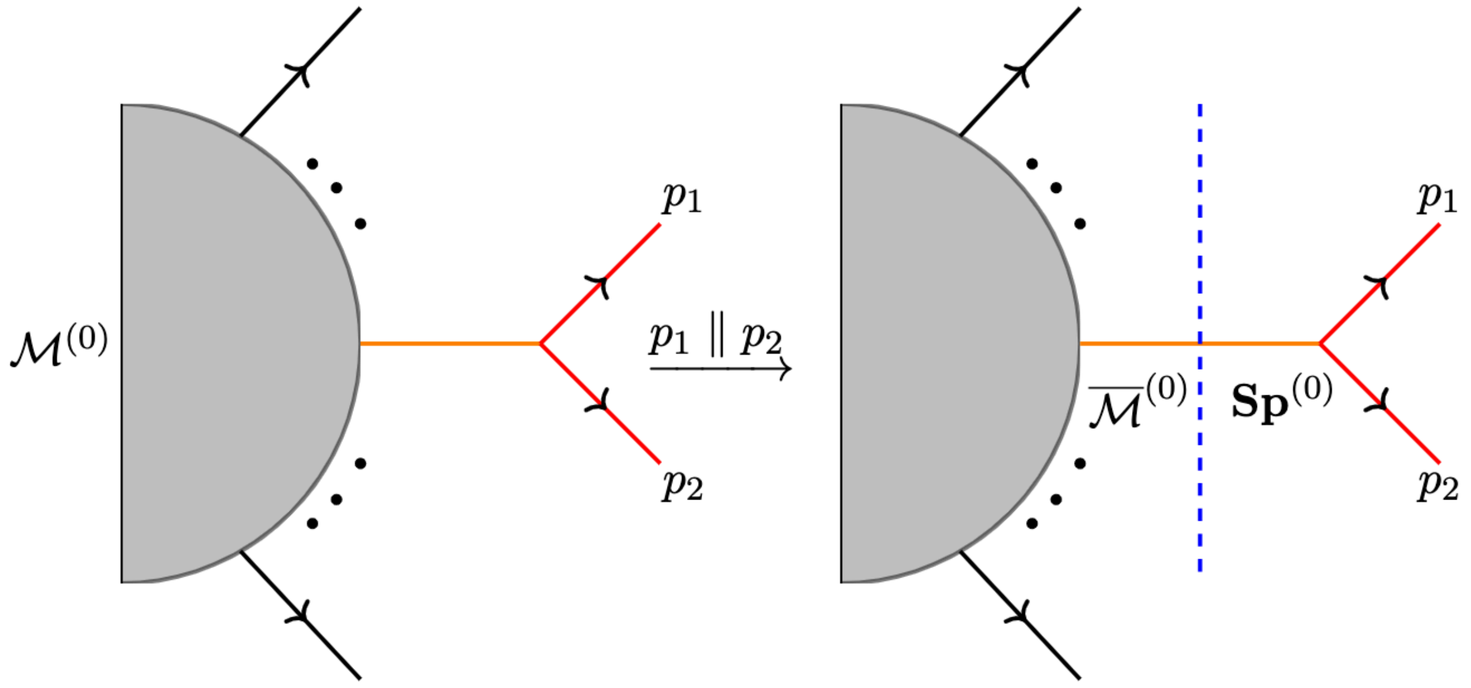
$$|\mathcal{M}^{(0)}(q,\{p\})|^2 \approx -8\pi\alpha_{\mathrm{S}}^{u}\mu_0^{2\epsilon}\sum_{i,j=1}^{n}\mathcal{S}_{ij}(q)|\mathcal{M}_{(i,j)}^{(0)}(\{p\})|^2$$

where

$$\boxed{\mathcal{S}_{ij} = \frac{p_i\cdot p_j}{2\,p_i\cdot q\,p_j\cdot q} = \frac{s_{ij}}{s_{iq}\,s_{jq}}\,.} \qquad |\mathcal{M}_{(i,j)}^{(0)}(\{p\})|^2 \equiv \langle\mathcal{M}^{(0)}(\{p\})|\boldsymbol{T}_i\cdot\boldsymbol{T}_j|\mathcal{M}^{(0)}(\{p\})\rangle$$

QCD Factorisation in the Collinear Limit

- Double parton collinear limit at the tree level



Splitting matrix

Reduced ME

$$\boxed{|\mathcal{M}^{(0)}(\{p\})\rangle\Big|_{p_1\parallel p_2} \simeq \mathbf{Sp}^{(0)}(p_1,p_2;\tilde{P})|\overline{\mathcal{M}}^{(0)}(\tilde{P},\{\bar{p}\})\rangle}$$

At the squared amplitude level

$$\mathbf{P}^{(0)}(p_1,p_2;\tilde{P}) \equiv \mathbf{Sp}^{(0)\dagger}(p_1,p_2;\tilde{P})\mathbf{Sp}^{(0)}(p_1,p_2;\tilde{P})$$

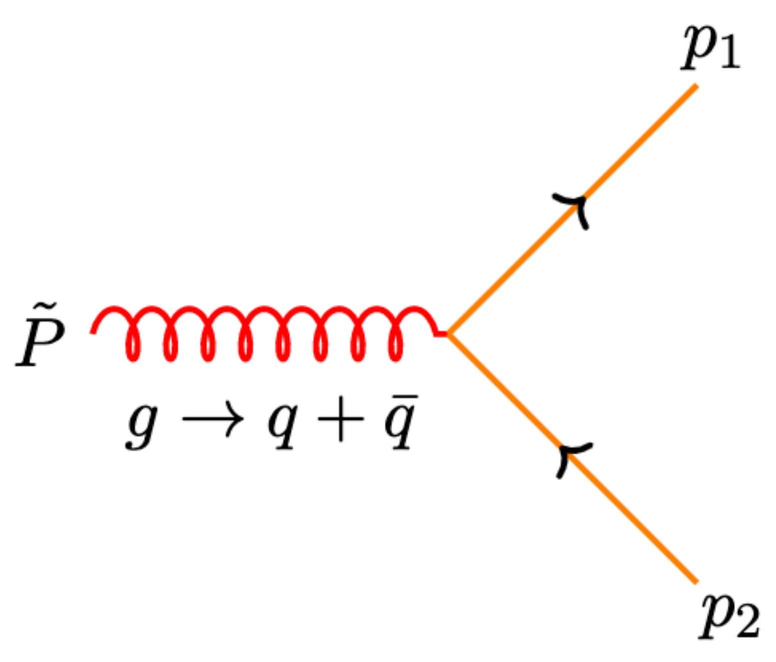
$$\left|\mathcal{M}^{(0)}(\{p\})\right|^2_{p_1\parallel p_2} \simeq \langle\overline{\mathcal{M}}(\{\bar{p}\})|\mathbf{P}^{(0)}(p_1,p_2;\tilde{P})|\overline{\mathcal{M}}(\{\bar{p}\})\rangle$$

$$= \boxed{\frac{8\pi\alpha_S^u\mu_0^{2\epsilon}}{s_{12}}\mathcal{I}^{ss'}_{a,\dots}(\tilde{P},\dots)\widehat{P}^{ss'}_{a_1a_2}}$$

Splitting kernel

LO Splitting Kernels

- Leading order collinear splitting kernels are

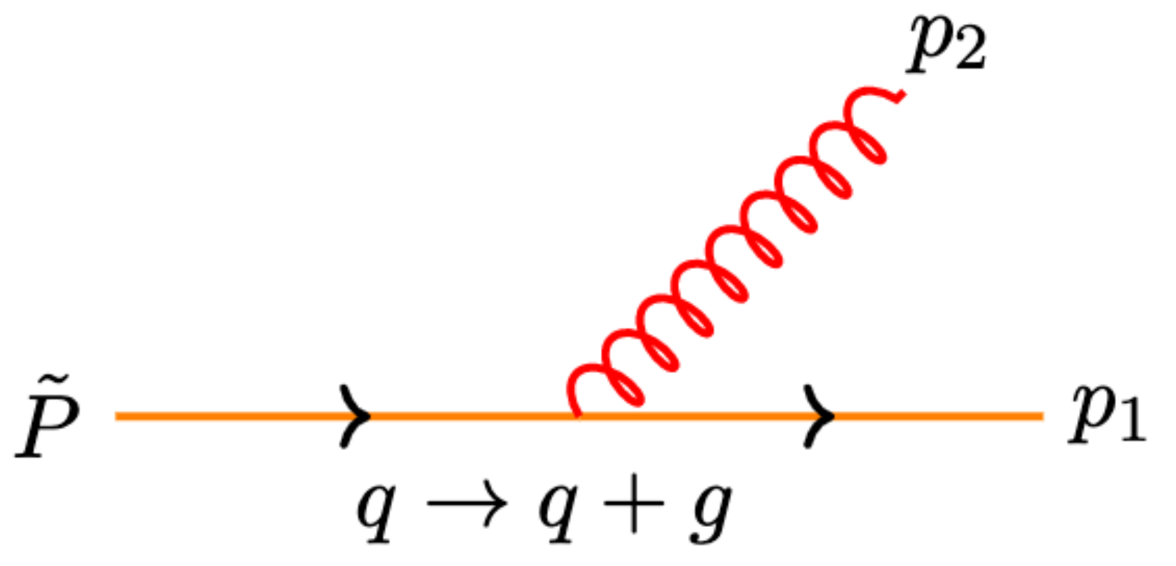


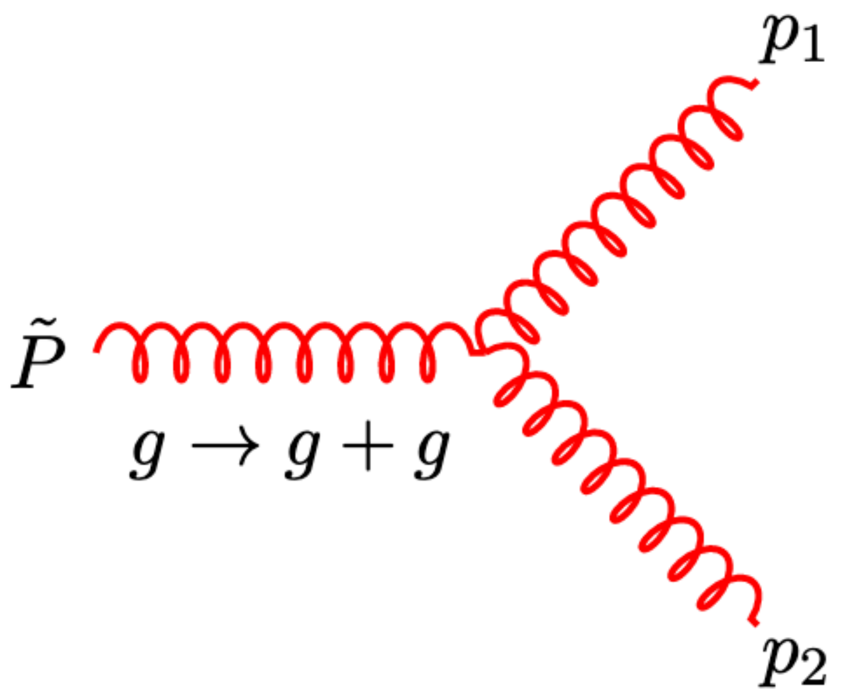
$$\widehat{P}_{qq}^{\mathrm{TL}} = C_F \left[\frac{1+z^2}{1-z} - \epsilon(1-z)\right] ,$$

$$\widehat{P}_{qg}^{\mathrm{TL}} = C_F \left[\frac{1+(1-z)^2}{z} - \epsilon z\right] ,$$

$$\widehat{P}_{gq}^{\mathrm{TL}} = T_R \left[1 - \frac{2z(1-z)}{1-\epsilon}\right] ,$$

$$\widehat{P}_{gg}^{\mathrm{TL}} = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right] .$$





- Note that the splittings kernels depend only on the momenta and quantum numbers of partons those took part in the collinear splitting process.

Future Outlook

- We need to go hand-in-hand with the experiment i.e. we need very precise theory predictions
- Higher order perturbative predictions are non-trivial both analytically and numerically due to presence of divergences
- For efficient automation, one not only needs to find counter terms but also needs to integrate them analytically over unresolved (soft and collinear) phase space to cancel divergences coming from the virtual diagrams (Loop-Tree-Duality can help us here!)
- Since the divergences are of IR origin, the study of Soft and Collinear factorisation of QCD matrix elements to higher orders can help
- Collinear splitting kernels in general can depend on quantum numbers of non-collinear partons, which poses additional difficulties

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Thank You!