

Combining QED and QCD transverse-momentum resummation for electroweak boson production at hadron colliders

Speaker: **Andrea Autieri**

Based on: **arXiv:2302.05403**

Authors: **Andrea Autieri, Leandro Cieri, Giancarlo Ferrera, German Sborlini**

Thursday Apr 27, 2023: HALC LHC 23, IFIC: Valencia



VNIVERSITAT
D VALÈNCIA



CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

CSIC



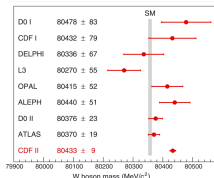
Introductions & Motivations

- Drell-Yan (DY) mechanism¹ is a **paramount** at hadron colliders
 - SM and BSM physics
 - detector calibration
 - SM parameters extraction (m_W , $\sin \theta_W$, ...)
 - ...
- DY process is measured nowadays with an **astonishing experimental precision** (per-thousand level)

⇒ Need of **competitive theoretical predictions** ⇒ **higher-order radiative corrections**

- Distributions in q_T of weak bosons are particularly relevant:
 - $q_T(Z)$ spectrum → W boson production mechanism
 - $q_T(W)$ at low- and intermediate- q_T region → crucial for m_W extraction²
- In the framework of QCD q_T resummation, predictions are known at percent, or even higher, level of precision ⇒ **EW corrections must be taken into account** :

$$\alpha \sim \alpha_S^2$$



¹S.D. Drell and T.-M. Yan, 1970

²ATLAS collaboration, 2018; CDF Collaboration 2022; CDF, D0 Collaboration, 2013, LHC_B collaboration, 2022

State-of-the-art of q_T resummation: N3LL accuracy

Disclaimer: a short review, far-from being exhaustive

- The procedure to implement resummation of q_T logarithmic-enhanced terms is known since long time ([Parisi,Petronzio('79)], [Kodaira,Trentadue('82)], [Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]
- Nowadays, different procedures to perform resummation have been developed in the aim of **N3LL** and beyond accuracy
- Resummation in direct q_T space $\rightarrow$ DY predictions up to **N3LL+N3LO** ([W. Bizo, X. Chen, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, A. Huss, P. F. Monni, E. Re, L. Rottoli, P. Torrielli, D. M. Walker ('18, '19, '21, '22)])
- Resummation in the framework of Effective Theories $\rightarrow$ some relevant kinematic distributions known up to **N3LL** ([M. A. Ebert, J.K.L. Michel, I. W. Stewart, F. J. Tackmann ('21)]), **N3LL+NNLO** ([T. Becher, T. Neumann ('20)]), **N4LL_p + N3LO** ([T. Neumann, J. Campbell ('22)])
- Studies within Transverse-Momentum dependent (TMD) factorization and TMD parton densities $\rightarrow$ TMD parton distributions up to **N3LL** ([A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza, M. Radici ('20)])

State-of-the-art of q_T resummation: N4LL+N4LO approximate accuracy and QED effects

Resummation in b-space and Mellin moments

- In this paper we apply for Drell-Yan transverse-momentum distribution the resummation formalism developed by ([Catani, de Florian, Grazzini ('01)]) and firstly applied for the case of Higgs boson production ([Bozzi, Catani, de Florian, Grazzini ('03, '06, '08)])
 - Resummation performed in the conjugate **b-space**
 - Introduction of Mellin moments
- State of the art q_T distributions can be computed at N3LL+N3LO ([Camarda et al. ('20)], [S. Camarda, L.Cieri, G.Ferrera ('21)]) and even at **approximate N4LL+N4LO** [S. Camarda, L.Cieri, G.Ferrera ('23)] [hep-ph:2303.12781]
- QED corrections at **NLL+NLO** were computed only for **on-shell Z** production (L. Cieri, G. Ferrera, G.F.R. Sborlini, 2018) → we extend the formalism to reach **NLL_{QED} + NLO_{EW}** accuracy for both charged and neutral current on-shell Drell-Yan processes

Drell-Yan q_T distribution

$$\frac{d\sigma_V}{dq_T^2}(q_T, M, s) \stackrel{\text{factorization theorem}}{=} \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_{a/h_1}(x_1, \mu_F^2) f_{b/h_2}(x_2, \mu_F^2) \times \frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2}\left(q_T, M, \hat{s}, \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2\right)$$

- In the region $q_T \gtrsim M_V$ the perturbative fixed-order expansion is reliable:

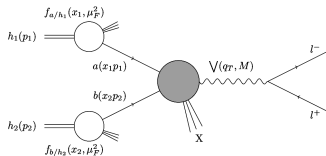
$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{d\hat{\sigma}_{V_{ab}}^{(0)}}{dq_T^2} + \frac{\alpha_S}{\pi} \frac{d\hat{\sigma}_{V_{ab}}^{(1)}}{dq_T^2} + \left(\frac{\alpha_S}{\pi}\right)^2 \frac{d\hat{\sigma}_{V_{ab}}^{(2)}}{dq_T^2} + \mathcal{O}\left[\left(\frac{\alpha_S}{\pi}\right)^3\right]$$

- In the region $q_T \ll M_V$ (bulk of the events) **large logarithmic corrections** of the type $\alpha_S^n \ln^m(M_V^2/q_T^2)$, due to soft and/or collinear parton radiations, spoil the convergence



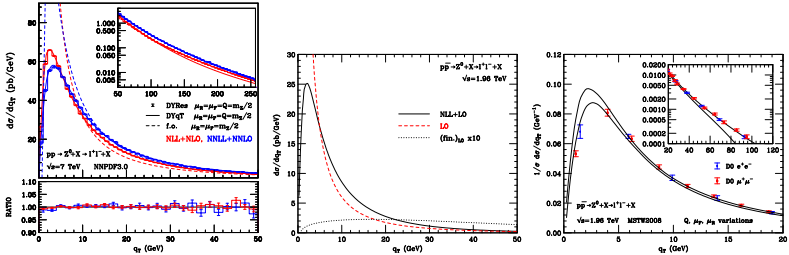
Resummation at all perturbative orders is mandatory:

$$\frac{d\hat{\sigma}_{V_{ab}}}{dq_T^2} = \frac{\hat{\sigma}_V^{(0)}}{q_T^2} \sum_{n=1}^{+\infty} \sum_{m=0}^{2n-1} A_{n,m}^V \ln^m\left(\frac{M^2}{q_T^2}\right) \alpha_S^n(M^2), \quad \alpha_S^n \ln^m(M_V^2/q_T^2) \gg 1$$



Analytic Resummation formalism in q_T

G. Bozzi, S. Catani, D. de Florian, M. Grazzini hep-ph/0508068



G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini [1007.2351[hep-ph]], [1507.06937[hep-ph]]

- Partonic cross section is explicitly splitted as:

$$\frac{d\hat{\sigma}_{a_1 a_2 \rightarrow V}}{dq_T^2} = \frac{d\hat{\sigma}_{a_1 a_2 \rightarrow V}^{res}}{dq_T^2} + \frac{d\hat{\sigma}_{a_1 a_2 \rightarrow V}^{fin}}{dq_T^2}, \quad \text{with} \quad \lim_{Q_T \rightarrow 0} \int_0^{Q_T} dq_T^2 \frac{d\hat{\sigma}_{a_1 a_2 \rightarrow V}^{fin}}{dq_T^2} = 0$$

- Resummation is performed in impact parameter ($\mathbf{b}$) space

$$\frac{d\hat{\sigma}_{a_1 a_2 \rightarrow V}^{res}}{dq_T^2}(q_T; M, \hat{s}; \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}_{a_1 a_2}^V(b; M, \hat{s}, \alpha_S(\mu_R^2), \mu_F^2, \mu_R^2).$$

$\mathcal{W}^V$ can be expressed in an **exponential** and **factorized** form in the Mellin space $\rightarrow$
 $z = M_V^2/\hat{s}$, $f_N = \int_0^1 dz z^{N-1} f(z)$:³

$$\mathcal{W}_N^V = \mathcal{H}_N^V(M, \alpha_S(\mu_R^2)) \times \exp \mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2),$$

$$L = \ln\left(\frac{Q^2 b^2}{b_0^2} + 1\right), \quad b_0 = 2\exp(-\gamma_E), \quad \gamma_E = 0.5772\dots$$

$$\mathcal{H}_N^V\left(M, \alpha_S, \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right) = \hat{\sigma}_0^V(M) \left[1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}_N^{V(n)}\left(\frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right) \right],$$

$$\begin{aligned} \mathcal{G}_N\left(\alpha_S(\mu_R^2), L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) &= - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} \left(A(\alpha_S(q^2)) \log\left(\frac{M^2}{q^2}\right) + \tilde{B}_N(\alpha_S(q^2)) \right) = \\ &= L g_N^{(1)}(\alpha_S(\mu_R^2) L) + g_N^{(2)}\left(\alpha_S(\mu_R^2) L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-2} g_N^{(n)}\left(\alpha_S(\mu_R^2) L; \frac{M^2}{\mu_R^2}, \frac{M^2}{Q^2}\right) \end{aligned}$$

• $\mathcal{H}_N^V$, $A(\alpha_S)$ and $B(\alpha_S)$ have a customary α_S -expansion:

$\Downarrow$

Perturbative structure of the resummed component:

LL accuracy ($\sim \alpha_S^n L^{n+1}$): $g_N^{(1)}$; **NLL** accuracy ($\sim \alpha_S^N L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; **NNLL** accuracy:
($\sim \alpha_S^N L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$; **N3LL** accuracy ($\sim \alpha_S^N L^{n-2}$): $g_N^{(4)}$, $\mathcal{H}_N^{(3)}$

³Flavour indices are understood

On-shell Z boson production

- QED corrections at $\text{NLL} + \text{NLO}$ known
 - [L. Cieri, G. Ferrera, G. F. R. Sborlini 1805.11948[hep-ph]]
 - Direct abelianization of QCD resummation formalism for a colourless final state
- We incorporated also **weak corrections at one loop** within the hard factor $\mathcal{H}$
 - We consider one-loop renormalized form factor
 - Modifications only in the hard factor $\mathcal{H}$ (massive loop corrections)
 - Accuracy at $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$

On-shell W boson production (NEW)

- Charged final state $\rightarrow$ a "naive abelianization" of the QCD formulation for DY process is not suitable
 - We use the formalism of $t\bar{t}$ production S. Catani, M. Grazzini, A. Torre 1408.4564[hep-ph])
- Replacement: $t\bar{t} \rightarrow W$ (colour charged $\rightarrow$ **electrically** charged)
 - Abelianization of QCD result
 - absence of colour correlations involving initial and final state (abelian limit)

On shell W boson production at $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$

- We start from the QCD resummation program generalized to a colourful final state:

$$W_N^V(b, M) = \sum_{c a_1 a_2} \sigma_{c\bar{c}, V}^{(0)}(\alpha_S(M^2)) f_{a_1/h_1, N}(b_0^2/b^2) f_{a_2/h_2, N}(b_0^2/b^2) \times S_c(M, b) \times [(\mathbf{H}^V \Delta C_1 C_2)]_{c\bar{c}, a_1, a_2; N}(M^2, b_0^2/b^2)$$

- $[(\mathbf{H}^V \Delta C_1 C_2)]$ hard factor; S_c Sudakov form factor
- Δ related to soft (non-collinear) wide-angle radiation from final state and from initial-final state interferences ($\Delta = 1$ for neutral final states)
- Applying the abelianization procedure to Eqs. (15-18) of Ref [1408.4564], we obtain:

$$\Delta(\alpha; Q, b) = \exp \left\{ - \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2} D'(\alpha(q^2)) \right\}$$

$$D'(\alpha) = \frac{\alpha}{\pi} D'(1) + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi} \right)^n D'(n)$$

- Resummation of **additional logarithmic-enhanced** contributions of the type:
 $\alpha^n \log(Qb)^k$
 - $D'(\alpha)$ starts to contribute at NLL accuracy (as B'_N)

Sudakov form factor

- The coefficient D can be absorbed in the colourless Sudakov form factor
 - Exponentiation of single-logarithmic enhanced terms due to a charged final state
- In combined QCD-QED resummation formalism, we finally obtain the following generalization:

$$\begin{aligned} G'_N(\alpha, L) &= - \int_{b_0^2}^{b^2} \frac{dq^2}{q^2} \left(A'(\alpha(q^2)) \log\left(\frac{M^2}{Q^2}\right) + \tilde{B}'_N(\alpha(q^2)) + D'(\alpha(q^2)) \right) = \\ &= L g'^{(1)}(\alpha L) + g'^{(2)}_N(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g'^{(n)}_N(\alpha L) \end{aligned}$$

- $A'(\alpha)$ and $\tilde{B}'_N(\alpha)$ are related to QED initial-state radiation $\rightarrow$ direct abelianization of QCD analogous through the replacement:

$$2 C_F \rightarrow (e_q^2 + e_{\bar{q}'}^2)$$

- $D'(\alpha)$ is instead an additional term, due to a charged final state, **characteristic of final-state massive radiation**
 - it can be obtained by a suitable abelianization of the soft anomalous dimension matrix of Eqs. (15)-(17) of Ref. ([1408.4564[hep-ph]])

$$D^{(1)} = \frac{-e_W^2}{2}$$

- Observation: the additional resummed contribution **implies** the replacement $B_1 \rightarrow B_1 + D_1$ in all the parts of the original formalism $(\Sigma, \mathcal{H}, \tilde{S})$

Hard collinear coefficient function

- We started from $t\bar{t}$ subtraction operator of 1408.4564[hep-ph], transforming it properly
- We obtained:

$$\tilde{T}'_V(\epsilon, M^2) = \frac{\alpha(\mu_R)}{2\pi} \tilde{T}'^{(1)}_V(\epsilon, M^2/\mu_R^2) + \sum_{n=2}^{+\infty} \left(\frac{\alpha(\mu_R)}{2\pi} \right)^n \tilde{T}'^{(n)}_V(\epsilon, M^2/\mu_R^2)$$

with

$$\tilde{T}'^{(1)}_V(\epsilon, M^2/\mu_R^2) = - \left(\frac{M^2}{\mu_R^2} \right)^{-\epsilon} \left\{ \left(\frac{1}{\epsilon^2} + \frac{i\pi}{\epsilon} - \frac{\pi^2}{12} \right) \frac{e_{qf}^2 + e_{\bar{q}f'}^2}{2} + \frac{\gamma'_{qf} + \gamma'_{\bar{q}f'}}{2} \frac{1}{\epsilon} + \frac{e_V^2}{2} (1-i\pi) \frac{1}{\epsilon} \right\}$$

- $\gamma'_q = 3e_q^2/2$: from hard-collinear initial-state radiation
- Term proportional to e_V^2 : from soft-wide angle final-state radiation
- We followed the method developed for the QCD case ([1311.1654[hep-ph]])

$$\tilde{\mathcal{M}}_V = (1 - \tilde{T}'_V(\epsilon, M^2)) \mathcal{M}_V$$

$$H'^V = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha}{\pi} \right) H'^{V(n)} = \frac{|\tilde{\mathcal{M}}_V|^2}{|\mathcal{M}_V^{(0)}|^2}$$

- H'^V contains the process dependent part of $\mathcal{H}_N'^V$
- We used the known formulas of one-loop renormalized amplitudes in case of $q_f + \bar{q}_{f'} \rightarrow W$ ([A. Behring, F. Buccioni, F. Caola, M. Delto, M. Jaquier et al. 2009.10386[hep-ph]]) and $q\bar{q} \rightarrow Z$ ([R. Bonciani, F. Buccioni, N. Rana, A. Vicini 2111.12694[hep-ph]])

- The explicit results for the NLO hard-collinear functions $\mathcal{H}'^{V(1)}_{a_1 a_2, N}$ are:

$$\mathcal{H}'^{V(1)}_{q_f \bar{q}_{f'} \leftarrow q_f \bar{q}_{f'}, N} = \frac{e_{q_f}^2 + e_{\bar{q}_{f'}}^2}{2} \left(\frac{1}{N(N+1)} + H'^{V(1)} \right)$$

$$\mathcal{H}'^{V(1)}_{q_f \bar{q}_{f'} \leftarrow \gamma \bar{q}_{f'}, N} = \left(\frac{3e_{q_f}^2}{(N+1)(N+2)} \right)$$

$$\mathcal{H}'^{V(1)}_{q_f \bar{q}_{f'} \leftarrow q_f \gamma, N} = \left(\frac{3e_{\bar{q}_{f'}}^2}{(N+1)(N+2)} \right)$$

Matching at large q_T

- We obtained the finite part of the cross-section from:

$$\frac{d\hat{\sigma}^{(fin.)}}{dq_T^2} = \left[\frac{d\hat{\sigma}}{dq_T^2} \right]_{(f.o.)} - \left[\frac{d\hat{\sigma}^{(res.)}}{dq_T^2} \right]_{(f.o.)}$$

- This contribution is relevant for intermediate/large q_T values where logarithmic resummation is no-longer justified

Combined QCD-QED resummation formalism

- As widely discussed we consistently included QED effects in the well-known QCD resummation formalism
- The generalized expressions are thus double perturbative expansion in α and α_S :

$$\mathcal{G}'_N(\alpha_S, \alpha, L) = \mathcal{G}_N(\alpha_S L) + L g'^{(1)}(\alpha L) + g_N'^{(2)}(\alpha L) + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{(n-2)} g_N'^{(n)}(\alpha L) +$$

$$+ g'^{(1,1)}(\alpha_S L, \alpha L) + \sum_{n,m=1; n+m \neq 2}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g_N'^{(n,m)}(\alpha_S L, \alpha L)$$

and:

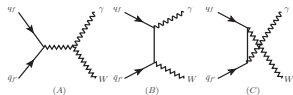
$$\mathcal{H}_N'^V(\alpha_S, \alpha) = \mathcal{H}_N^V(\alpha_S) + \frac{\alpha}{\pi} \mathcal{H}_N'^{V(1)} + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}_N'^{V(n)} + \sum_{n,m=1}^{+\infty} \mathcal{H}_N'^{V(n,m)}$$

- We also considered the mixed QCD-QED contributions at LL, by including $g'^{(1,1)}(\alpha_S L, \alpha L)$ (1805.11948)
- For the sake of completeness, $f_{\gamma/h}(x, \mu_F^2)$ and QED effects in PDF evolution were included in the factorization formula

$\Downarrow$

$(\text{NNLL} + \text{NNLO})_{\text{QCD}} + (\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}})$

Small q_T expansion of real cross sections at NLO: photon emission



- This calculation reproduces the logarithmic structure of the fixed-order expansion of the Sudakov form factor

- Cross-check of our formulas
- Confirms the validity of the replacement $t\bar{t} \rightarrow W$ and abelianization procedures

$$\text{Hadronic cross section : } \sigma = \sum_{ab} \tau \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{ab} \left(\frac{\tau}{z} \right) \frac{1}{z} \int dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2}$$

with: $\tau = Q^2/S$

$$\text{Partonic inclusive cross section : } \hat{\sigma}_{ab}(z) = \int_{(q_T^{cut})^2}^{(q_T^{max})^2} dq_T^2 \frac{d\hat{\sigma}_{ab}(q_T, z)}{dq_T^2},$$

Inclusive hadronic cross section, introducing $f(a) = 2\sqrt{a}(\sqrt{1+a}-\sqrt{a})$, $a \equiv \frac{(q_T^{cut})^2}{Q^2}$:

$$\sigma_{q_f \bar{q}_{f'}}^{>(1)} = \tau \int_0^{1-f(a)} \frac{dz}{z} \mathcal{L}_{q_f \bar{q}_{f'}} \left(\frac{\tau}{z} \right) \frac{1}{z} \hat{\sigma}_{q_f \bar{q}_{f'}}^{(1)}(z) = \tau \int_{\tau}^1 \frac{dz}{z} \mathcal{L}_{q_f \bar{q}_{f'}} \left(\frac{\tau}{z} \right) \hat{\sigma}^{(0)} \hat{G}_{q_f \bar{q}_{f'}}^{(1)}(z), \text{ with :}$$

$$\hat{G}_{q_f \bar{q}_{f'}} = \sum_{m,r} \log^m(a) a^{\frac{r}{2}} \hat{G}_{q_f \bar{q}_{f'}}^{(1,m,r)}(z), \text{ power series in the cutoff}$$

- Final expression obtained:

$$\hat{G}_{q_f \bar{q}_{f'}}^1 = \log(a) \left(\frac{3}{2} \delta(1-z) \frac{(e_{q_f}^2 + e_{q_{f'}}^2)}{2} - \frac{1}{2} \left((P^{\text{QED}})_{q_f q_f} + e_W^2 \delta(1-z) - (P^{\text{QED}})_{\bar{q}_{f'} \bar{q}_{f'}} \right) \right) +$$

$$+ \frac{1}{2} \log^2(a) \delta(1-z) \frac{(e_{q_f}^2 + e_{q_{f'}}^2)}{2} + \sqrt{a} \frac{1}{2} e_W^2 (2\pi \delta'(1-z) - 3\pi \delta(1-z))$$

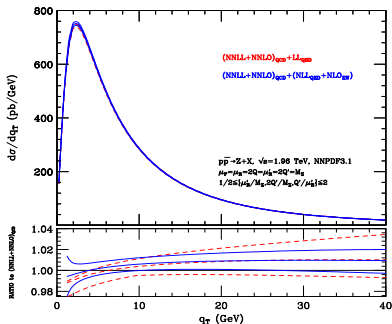
+ finite terms + higher order terms

- $P_{q_f \bar{q}_{f'}}^{\text{QED}}$ AP splitting functions in QED (D. de Florian, G. Rodrigo, G. F. R. Sborlini: 1611.04785[hep-ph], 1512.00612[hep-ph], 1606.02887[hep-ph])
- We reproduce the known A and B perturbative coefficient of the QCD resummation formalism, modulo $C_F \rightarrow \frac{(e_{q_f}^2 + e_{q_{f'}}^2)}{2}$
- Additional logarithmic divergence from the charged final state $\sim D'_1 \log(a)$,
 $D'_1 = -\frac{e_W^2}{2}$
- A linear power correction in the cutoff ($\sqrt{a}$) and proportional to the charged final state is present
 - Accordingly with L. Buonocore, M. Grazzini, F. Tramontano: 1911.10166 [hep-ph] (massive leg emission $\rightarrow$ linear power correction)

- On-shell W and Z boson production
- Resummation formalism together with a consistent matching procedure is implemented in the FORTRAN numerical program DYQT G. Bozzi, S. Catani, G. Ferrera, D. de Florian, M. Grazzini, [1007.2351 [hep-ph]], [0812.2862 [hep-ph]]
- We reached the accuracy: $(\text{NNLL}+\text{NNLO})_{\text{QCD}} + (\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}})$
- Input parameters $\alpha(m_Z^2)$, m_W , m_Z , $|V_{CKM}|$ (PARTICLE DATA GROUP collaboration, 2022)
- To compute EW corrections at NLO we took as input values also m_H and m_t (PARTICLE DATA GROUP collaboration, 2022)
- PDF: NNPDF3.1.luxQED at NNLO in QCD (NNPDF Collaboration, 2109.02653[hep-ph])
 - inclusion of photon PDF and LO QED in the PDFs evolution (LHAPDF framework: A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht et al., 1412.7420)
- Perturbative uncertainty: scale variation method
 - QCD scales: $\mu_F = \mu_R = 2Q = m_V$
 - QED scales: simultaneous variations of Q' and μ'_R , according to:

$$m_V/2 \leq \{\mu'_R, 2Q'\} \leq 2m_V, \quad 1/2 \leq \{\mu'_R/Q'\} \leq 2, \quad \mu'_F = m_V$$

Z Boson production at Tevatron, $\sqrt{s} = 1.96\text{TeV}$



• LL_{QED} :

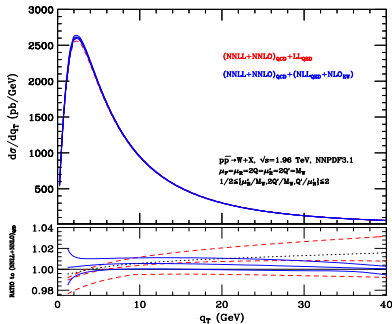
- Spectrum slightly harder (effects of $\mathcal{O}(1\%)$)
- Scale variation band: $\mathcal{O}(2 - 4\%)$

• $NLL_{QED} + NLO_{EW}$:

- Effects of $\mathcal{O}(0.5\%)$
- Scale variation band: reduction by roughly a factor 2

- Analogy with ([L. Cieri, G. Ferrera, G. F. R. Sborlini, 1805.11948[hep-ph]), QED effects up to $(NLL+NLO)_{QED}$

W Boson production at Tevatron, $\sqrt{s} = 1.96\text{TeV}$



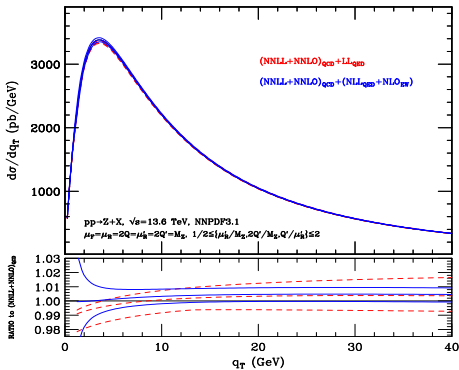
• LL_{QED} :

- Spectrum slightly harder (effects bit less than $\mathcal{O}(1\%)$)
- Scale variation band: $\mathcal{O}(2 - 3\%)$

• $NLL_{QED} + NLO_{EW}$:

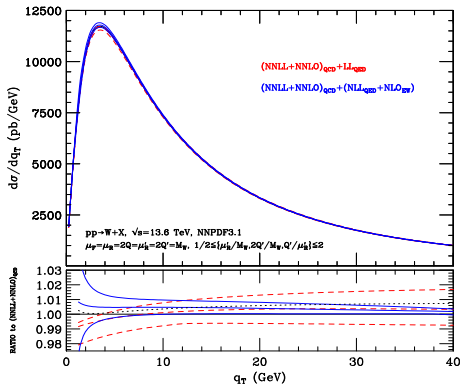
- Effects: $+(-)\mathcal{O}(1\%)$
- Soft wide angle radiation makes the spectrum softer (D'_1 is negative as B'_1) (analogy with S. Catani, M. Grazzini, H. Sargsyan 1806.01601[hep-ph], QCD resummation for $t\bar{t}$)
- Scale variation band: reduction of a factor 1.5-2 (up to 3)

● Z boson production at LHC, $\sqrt{s} = 13.6\text{TeV}$



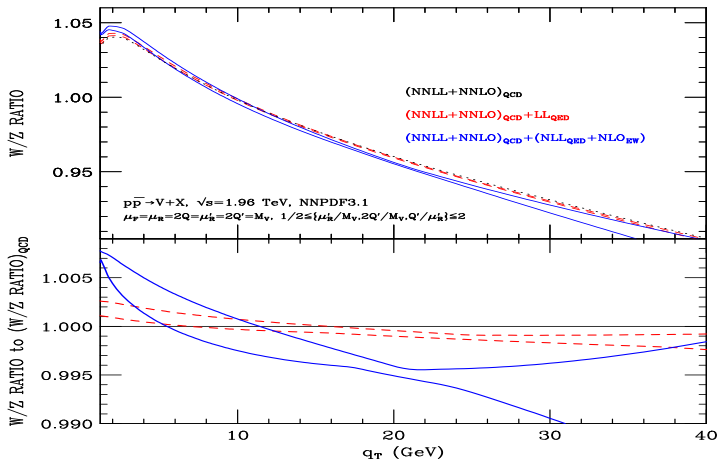
- Impact of QED corrections less than Tevatron case (enhancement of gluon PDF)
- Similar qualitative behaviour of QED effects
- LL_{QED} : $-\mathcal{O}(1\%)$ ($+\mathcal{O}(0.5\%)$) (harder spectrum); scale variation band: $\mathcal{O}(2\%)$
- $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$: effects $+\mathcal{O}(0.5\%)$; scale variation band: reduction by roughly a factor 1.5-2
- Analogy with ([L. Cieri, G. Ferrera, G. F. R. Sborlini, 1805.11948[hep-ph]])

● W boson production at LHC, $\sqrt{s} = 13.6\text{TeV}$



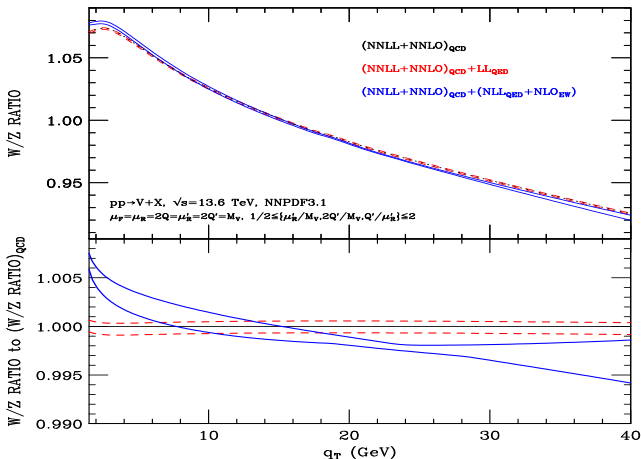
- Impact of QED radiation less than Tevatron case (enhancement of gluon PDF)
- Similar qualitative behaviour of QED effects
- LL_{QED} : spectrum harder; scale variation band $\mathcal{O}(2\%)$
- $\text{NLL}_{\text{QED}} + \text{NLO}_{\text{EW}}$: Soft wide angle radiation makes the spectrum softer; scale variation band: reduction of factor 1.5 – 2 (up to 4)

- $$R(q_T) = \frac{\frac{1}{\sigma_W} \frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z} \frac{d\sigma_Z}{dq_T}}, \text{ Tevatron } \sqrt{s} = 1.96 \text{ TeV}$$



- Relevant for m_W extraction $\boxed{q_T(Z)^{meas.} + R(q_T)^{theo.}} \rightarrow \boxed{q_T(W)}$
- LL_{QED}: slightly softer spectrum; per-thousand level effects and scale variation band
- NLL_{QED} + NLO_{EW}: QED contributions not suppressed; softer spectrum ($\mathcal{O}(0.5 - 1\%)$); scale variation band: $\underline{\mathcal{O}(0.1\%) - \mathcal{O}(1\%)}$

$$R(q_T) = \frac{\frac{1}{\sigma_W} \frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z} \frac{d\sigma_Z}{dq_T}}, \text{ LHC } \sqrt{s} = 13.6 \text{ TeV}$$



- Less impact of QED radiation (enhancement of gluon luminosity)
- LL_{QED}: less than per-thousand level effects and scale variation band
- NLL_{QED} + NLO_{EW}: QED contributions not suppressed; softer spectrum ($\mathcal{O}(0.5\%)$); scale variation band: $\mathcal{O}(0.1\%) - \mathcal{O}(0.5\%)$

an overlap of the bands is not observed (Tevatron, LHC)

- To fully exploit the potential of LHC measurements accurate theoretical predictions are required $\rightarrow$ precise determination of SM parameters (m_W)
- We considered QED corrections to resummation formalism in QCD, to properly include photon radiation, focusing on on-shell W boson production, which has to be treated carefully due to a charged final state
- Final state radiation is fully included by extending the resummation formalism for a coloured final state ($t\bar{t}$)
- Expansion at small q_T of the real inclusive cross section has confirmed the validity of the replacement $t\bar{t} \rightarrow W$
- Through the use of the numerical code DYQT we presented numerical predictions at $(\text{NNLL}+\text{NNLO})_{\text{QCD}} + (\text{NLL})_{\text{QED}} + (\text{NLO})_{\text{EW}}$, finding QED effects from **per thousand** to **percent** level
- We considered also the ratio distribution $p_T(W)/p_T(Z)$, in the direction of m_W extraction $\rightarrow$ a sizeable reduction of scale-variation band is observed at **LL**, while the predictions at **NLL+NLO** do not benefit from the cancellation of common uncertainties

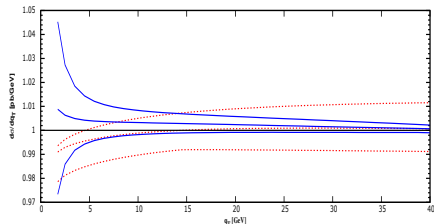
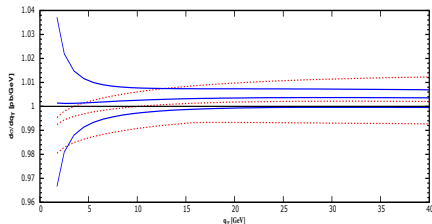
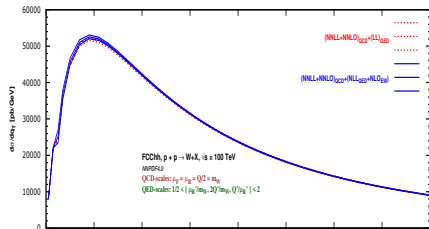
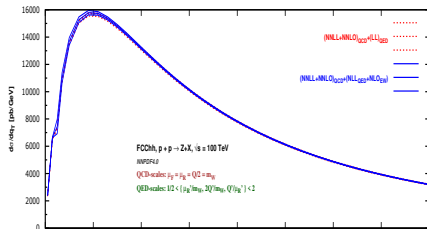
- A natural extension of this work is the inclusion of the decay of weak boson and the radiation from leptonic final state
- **On-going work:** inclusion of these effects in DYTURBO
(fast and precise numerical predictions)
 - Code optimisation (starting from DYQT, DYRES, DYNNOLO)
 - Factorization into production and decay variables
 - Numerical integration based on interpolating functions
 - ...
- Correction to **production** (along the line of DYQT code) and **decay** (NEW)

...

Thank you for the attention!!

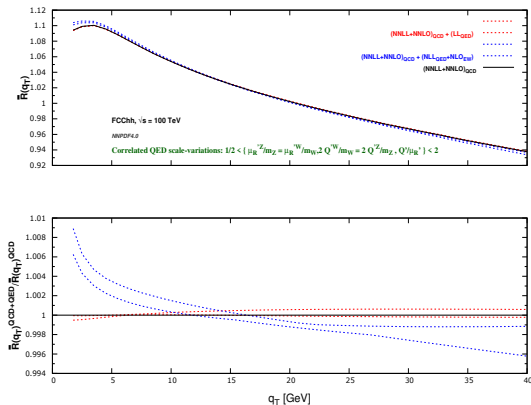
BACKUP SLIDE

Predictions at FCChh, $\sqrt{s} = 100\text{TeV}$, q_T -spectra



- Qualitative features of QED effects analogous to Tevatron and LHC cases (**LL**: spectrum harder; **NLL+NLO**: different functional behaviour between Z and W; reduction of bandwidths at **NLL+NLO** ...
- Strong suppression of quark-induced reactions)
- Slightly worse overlapping of the band with respect to Tevatron and LHC

Predictions at FCChh, $\sqrt{s} = 100\text{TeV}$, $q_T(W)/q_T(Z)$ -spectra



- **LL_{QED}**: less than per-thousand effects and scale-variation band
- **NLL_{QED} + NLO_{EW}**: QED contributions and scale variation band at per-thousand level
- Non-overlapping of the bands

At FCC-hh PDF extrapolation is challenging (out of experimental accessible range)

Matching procedure

- For intermediate q_T values, the resummed component should be properly combined with fixed order expansion $\rightarrow$ **matching procedure**:
 - recover fixed-order series for $q_T \lesssim M_V$ where $\left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{res.} \rightarrow 0$
 - avoid double counting of logarithmic terms $\rightarrow$ counterterm $\left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{asym}$:
- The finite part is $\left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{fin.} = \left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{f.o.} - \left[\frac{d\sigma_{ab}^{res}}{dq_T^2} \right]_{f.o.} = \left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{f.o.} - \left[\frac{d\sigma_{ab}}{dq_T^2} \right]_{asym}$
- The counterterm, in impact parameter and Mellin space, is given by the expansion:

$$\begin{aligned}
 \left[\mathcal{H}_{a_1 a_2, N}^V \times \exp(\mathcal{G}_{a_1 a_2, N}) \right] &\approx \sigma_{c\bar{c}, V}^{(0)}(\alpha_S, M) \left[\delta_{ca_1} \delta_{c\bar{a}_2} \delta(1-z) \right] \\
 &+ \sum_k \left(\frac{\alpha_S}{\pi} \right)^k \Sigma_{c\bar{c} \leftarrow a_1 a_2}^{V, (k)}(z, L; M^2/\mu_R^2, M^2/\mu_F^2, m M^2/Q^2) \\
 &+ \sum_k \left(\frac{\alpha_S}{\pi} \right)^k \mathcal{H}_{c\bar{c} \leftarrow a_1 a_2}(z; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2)
 \end{aligned}$$

Coloured (charged) final state S. Catani, M. Grazzini, A. Torre, [1408.4564 [hep-ph]]

- Drell-Yan final state is colourless
- In case of charged final state (e.g. $t\bar{t}$) production the hadronic resummed contribution is generalized according to:

$$W_N^V(b, M) = \sum_{c a_1 a_2} \sigma_{c\bar{c}, V}^{(0)}(\alpha_S(M^2)) f_{a_1/h_1, N}(b_0^2/b^2) f_{a_2/h_2, N}(b_0^2/b^2) \\ \times S_c(M, b) \times [(\mathbf{H}^V \Delta C_1 C_2)]_{c\bar{c}, a_1, a_2; N}(M^2, b_0^2/b^2)]$$

- $[(\mathbf{H}^V \Delta C_1 C_2)]$ hard factor; S_c Sudakov form factor
- Δ related to soft wide-angle radiation from final state ($\Delta = 1$ for colourless final states)
- We extend this formalism for electrically charged emission in the proceeding of the talk

Higgs production at the LHC using q_T subtraction formalism at N^3LO QCD

- L. Cieri (a,b) , X. Chen (b) , T. Gehrmann (b) , E.W.N. Glover (c) and A. Huss 1807.11501[hep-ph]
- the central prediction at N^3LO almost coincides with the upper edge of the band

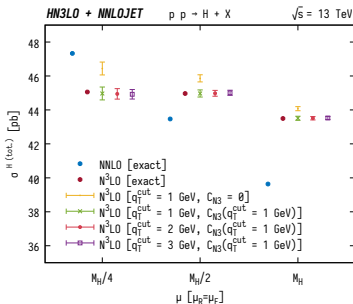
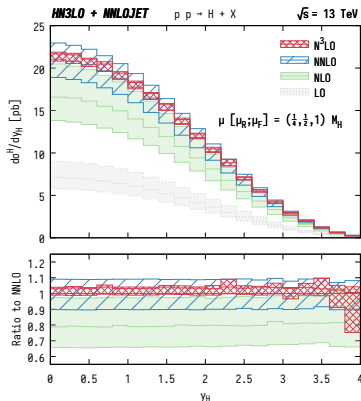


Figure: Rapidity distribution of the Higgs boson computed using the q_T subtraction formalism up to N^3LO (left panel) and the total cross section of the same process.