Radiative corrections and threshold resummed predictions to pseudoscalar Higgs boson production

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Plan of Talk

Works to be discussed

Project 1 - **Two loop QCD amplitudes for di-pseudoscalar production in gluon fusion**

- JHEP 02 (2020) 121

In collaboration with M. Mahakhud, Prakash Mathews and V. Ravindran.

Project 2 - **Next-to-soft-virtual resummed prediction for pseudoscalar Higgs boson production at NNLO + $\overline{\text{NNLL}}$**

- Phys. Rev. D 105, 116015

In collaboration with M. C. Kumar, Prakash Mathews and V. Ravindran.

Relevance of Pseudoscalar studies

- Will prove beneficial if/when the pseudoscalar Higgs is discovered.

Solution for the problems in the SM $\xrightarrow{\text{may lead to}}$ Possible new physics.

Requirement from theoretical physicists:

Precision calculations of the relevant observable corresponding to both scalar and pseudoscalar production processes to the same order of precision.

- Contribute towards establishing the CP properties of the discovered Higgs boson.

Speculations: The observed Higgs boson at the LHC can be an admixture of scalar-pseudoscalar states.

Exploring such possibilities had already started some time back:

- Y. Gao, A. V. Gritsan *et al.* (2010),
- P. Artoisenet *et al.* (2013),
- F. Maltoni, K. Mawatari, and M. Zaro (2014),
- M. Jaquier and R. Röntsch (2019).

Motivation

pQCD relies on the idea of an order-by-order expansion in the strong coupling, α_s, of a given observable, f: $\mathbf{f} = \mathbf{f_1}\alpha_s + \mathbf{f_2}\alpha_s^2 + \mathbf{f_3}\alpha_s^3 + \cdots$

pQCD calculations at LO give only the **order of magnitude** of "f".

- FO QCD predictions experience divergences of UV and collinear origin.
  - Soft gluon emissions $\rightarrow$ soft divergences $\xrightarrow{\text{cancel with}}$ those of virtual gluons.
- Significant soft-gluon-effects $\xrightarrow{\text{observed in}}$ kinematic configurations $\xrightarrow{\text{where}}$ high imbalance persists between real and virtual contributions $\Rightarrow$ **Threshold Region**.

- **gg→H:** NSV logs contribute $\approx$ 25% of the Born & SV contributes -2.25% at a_s^3 . - **Anastasiou, Duhr, Dulat et al. (2014)**
- **DY:** NSV logs contribute 1.49% of the Born & SV terms contribute 0.02% at a_s^3. - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)**

m_A (GeV)	NNLL/NNLO (%)	$\overline{\text{NNLL}}$/NNLO (%)
125	11.8189	17.0234
700	12.8902	15.8511
1000	13.2377	16.2727
1500	14.8419	18.4658
2000	16.5992	21.0971
2500	18.5168	24.1535

Solution: Systematically sum these logs up to all orders $\Rightarrow$ **Resummation**.

Developments of Work

- Anastasiou, Duhr, Dulat et al. (2015)$\Rightarrow$ completed N^3LO prediction for scalar Higgs boson production *via* gluon fusion in the large top mass limit.
  The corrections to the cross-section were found to be $\approx 1\%$ at NNLO, and $\approx 2\%$ at N^3LO.
- FO cross-section for pseudoscalar Higgs boson production to NNLO accuracy:
  - R. V. Harlander and W. B. Kilgore (2002), & C. Anastasiou and K. Melnikov (2003) - similar but independent works.
  - V. Ravindran, J. Smith and W. van Neerven (2003) - alternative method.
- Development of the resummation formalism:
  - G. F. Sterman(1987),
  - S. Catani and L. Trentadue (1989),
  - V. Ravindran (2005, 2006),
  - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020).**
- **T. Ahmed, M. Bonvini, M. C. Kumar, P. Mathews, N. Rana, V. Ravindran, and L. Rottoli (2016)** $\Rightarrow$ FO computation at NNLO & approx. N^3LO + all-order threshold resummation.
- **T. Ahmed, M.C. Kumar, P. Mathews, N. Rana and V. Ravindran (2015)** $\Rightarrow$ N^3LO SV corrections to pseudoscalar Higgs boson production through gluon fusion.

Success of EFT

Calculations become simpler in the infinite quark mass limit ($m_X \ll 2m_t$) with increasing complexities at higher orders in the perturbation theory.

- In the case of scalar Higgs boson production, the difference between the exact and EFT results at NNLO were found to be within 1% - **A Success!**

  - R. V. Harlander, K. J. Ozeren (2009),
  A. Pak, M. Rogal, M. Steinhauser (2009),
  M. Czakon, R. V. Harlander et al. (2021).

- Eventual observation $\Rightarrow$ the EFT approach, when rescaled with the exact LO results, provides a reasonably good approximation even at masses outside the region of formal validity.

  - M. Spira, A. Djouadi, et al. (1995),
  R. Bonciani, G. Degrassi, A. Vicini (2007),
  C. Anastasiou, C. Duhr, F. Dulat, et al. (2016).

- The difference between the exact and EFT results at NLO reaches $\approx 10\%$ for $m_A = 500$ GeV, but does not increase much as m_A gets larger.

  - R.V. Harlander, S. Liebler, H. Mantler (2013),
  R.V. Harlander, S. Liebler, H. Mantler (2016).

The process: $\mathbf{g}(p_1) + \mathbf{g}(p_2) \rightarrow \mathbf{A}(p_3) + \mathbf{A}(p_4)$

The effective Lagrangian is as follows:

$$\mathcal{L}^A_{eff} = \Phi^A \left[-\frac{1}{8} C_G O_G(x) - \frac{1}{2} C_J O_J(x) \right]$$

- **Nucl. Phys. B535 (1998) 3**

The corresponding Mandelstam variables are defined as below:

$$s = (p_1 + p_2)^2 \, , t = (p_1 - p_3)^2 \, , u = (p_2 - p_3)^2 \, ;$$

where $s + t + u = 2m_a^2$ with $\mathrm{m}_a \Rightarrow$ pseudoscalar mass.

For convenience, we express the amplitude in terms of the dimensionless variables x, y and z:

$$s = m_a^2 \frac{(1+x)^2}{x}, \;\; t = -m_a^2 y, \;\; u = -m_a^2 z,$$

with $\frac{(1+x)^2}{x} - y - z = 2.$

Feynman Diagrams

AIM:

Higher Order QCD corrections to the g+g $\rightarrow$ A+A amplitude in EFT.

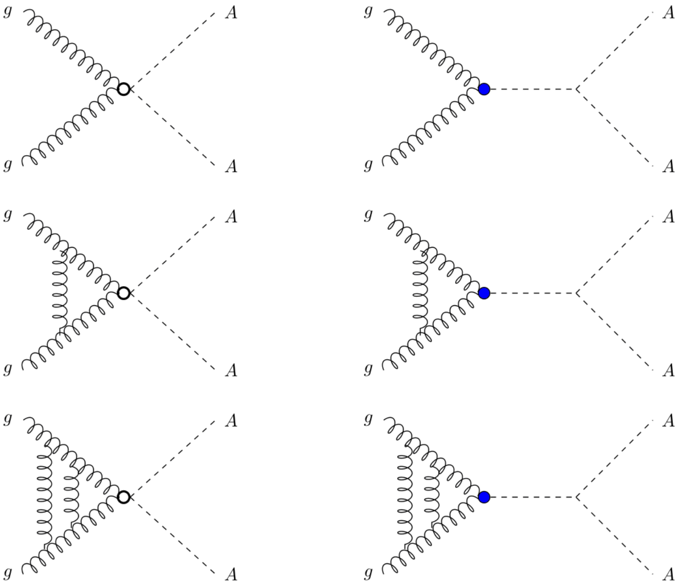
NOTE:

Two topologically distinct classes of contributing diagrams: **Class-A & Class-B (Type-IIa & IIb)**.

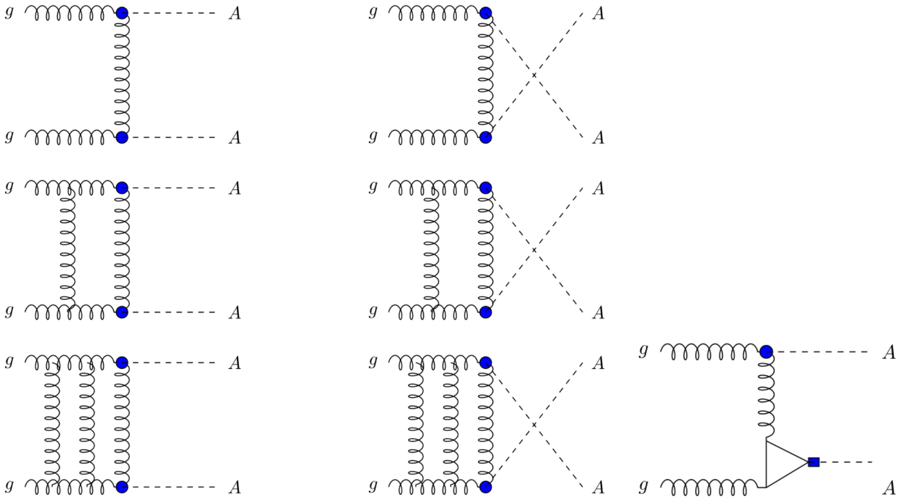
Computation of the Class-B Diagrams:

- FIRST TYPE: In this case, there are 2 LO diagrams, 35 NLO diagrams and 789 NNLO diagrams.
- SECOND TYPE: In this case, there are no LO diagrams and 8 NLO diagrams.

Feynman Diagrams



Feynman Diagrams



The Projector Method

We write the amplitude as

$$\mathcal{M}_{ab}^{\mu\nu} = \text{Colour struct. x } (\textbf{Tensorial struct.} \text{ x scalar funcs.})$$
$$= \delta^{ab}\left(\mathcal{T}_1^{\mu\nu}\mathcal{M}_1 + \mathcal{T}_2^{\mu\nu}\mathcal{M}_2\right)$$

To extract each of the **two Lorentz- and gauge-invariant** structure functions $\mathcal{M}_i$'s, we use **two d-dimensional Projectors**:

$$P_1^{\mu\nu} = \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_1^{\mu\nu} - \frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_2^{\mu\nu},$$
$$P_2^{\mu\nu} = -\frac{1}{4}\frac{d-4}{d-3}\mathcal{T}_1^{\mu\nu} + \frac{1}{4}\frac{d-2}{d-3}\mathcal{T}_2^{\mu\nu}.$$

- JHEP 11 (2018) 130

UV Renormalization

The bare strong coupling constant, $\hat{a}_s$, is related to the renormalised coupling constant, a_s, as below:

$$\frac{\hat{a}_s S_\varepsilon}{\mu_o^\varepsilon} = \frac{a_s}{\mu_R^\varepsilon}\left[1 + a_s\left(\frac{2\beta_0}{\varepsilon}\right) + a_s^2\left(\frac{4\beta_0^2}{\varepsilon^2} + \frac{\beta_1}{\varepsilon}\right)\right]$$

where $S_\varepsilon = \exp\left[(\gamma_E - \ln 4\pi)\,\frac{\varepsilon}{2}\right]$ with $\gamma_E \approx 0.5772...$ being the Euler-Mascheroni constant.

- Phys. Lett. 93B (1980) 429

Operator mixing

Operator Renormalization

- Composite operators O_G and O_J of the effective Lagrangian.
- The bare pseudoscalar gluon operator O_G mixes with the fermionic operator O_J under renormalization.
- In matrix form, the mixing is given by

$$\begin{pmatrix} [O_G] \\ [O_J] \end{pmatrix} = \begin{pmatrix} Z_{GG} & Z_{GJ} \\ Z_{JG} & Z_{JJ} \end{pmatrix} \begin{pmatrix} O_G \\ O_J \end{pmatrix}$$

- The renormalization constants are given by

$$\begin{aligned} Z_{GG} &= 1 + a_s \left[\frac{1}{\varepsilon} \cdots \right] + a_s^2 \left[\frac{1}{\varepsilon^2} \left\{ \cdots \right\} + \frac{1}{\varepsilon} \{\cdots\} \right], \\ Z_{GJ} &= a_s \left[-\frac{24}{\varepsilon} C_F\right] + a_s^2 \left[\frac{1}{\varepsilon^2} \{\cdots\} + \frac{1}{\varepsilon} \{\cdots\} \right], \\ Z_{JJ} &= 1 + a_s \left[-4 C_F\right] + a_s^2 \left[\{\cdots\} + \frac{1}{\varepsilon} \{\cdots\} \right], \\ Z_{JG} &= 0. \end{aligned}$$

- **Phys. Lett. 107B (1981) 115, Nucl. Phys. B 198 (1982) 508**

The UV Renormalized Amplitude

We need the following

$$[O_G O_G] = Z_{GG}^2 \ O_G O_G + 2 Z_{GG} Z_{GJ} \ O_G O_J + Z_{GJ}^2 \ O_J O_J \,,$$
$$[O_G O_J] = Z_{GG} Z_{JJ} \ O_G O_J + Z_{GJ} Z_{JJ} \ O_J O_J \,.$$

The final UV renormalized amplitude is given by

$$\begin{aligned}\mathcal{M}_{GG,g}^{\mathrm{II}} &= Z_{GG}^2 \left(\hat{\mathcal{M}}_{GG,g}^{\mathrm{II}(0)} + \hat{a}_s \hat{\mathcal{M}}_{GG,g}^{\mathrm{II},(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{GG,g}^{\mathrm{II}(2)} \right) \\ &+ 2 Z_{GG} Z_{GJ} \left(\hat{a}_s \hat{\mathcal{M}}_{GJ,g}^{\mathrm{II}(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{GJ,g}^{\mathrm{II}(2)} \right) \\ &+ Z_{GJ}^2 \left(\hat{a}_s \hat{\mathcal{M}}_{JJ,g}^{\mathrm{II}(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{JJ,g}^{\mathrm{II}(2)} \right) ,\end{aligned}$$

$$\begin{aligned}\mathcal{M}_{GJ,g}^{\mathrm{II}} &= Z_{GG} Z_{JJ} \left(\hat{a}_s \hat{\mathcal{M}}_{GJ,g}^{II(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{GJ,g}^{II(2)} \right) \\ &+ Z_{GJ} Z_{JJ} \left(\hat{a}_s \hat{\mathcal{M}}_{JJ,g}^{II(1)} + \hat{a}_s^2 \hat{\mathcal{M}}_{JJ,g}^{II(2)} \right)\end{aligned}$$

$\implies$ **$\hat{\mathcal{M}}_{GJ,g}^{\mathrm{II}(2)}$, $\hat{\mathcal{M}}_{JJ,g}^{\mathrm{II}(1)}$ and $\hat{\mathcal{M}}_{JJ,g}^{\mathrm{II}(2)}$ do not contribute in our case as they are of order higher than a_s^4 when combined with their respective Wilson coefficients.**

IR Factorization

- Stefano Catani

We are left with only divergences of infrared origin.

$$
\begin{aligned}
\mathcal{M}_i^{II,(0)} &= \mathcal{M}_i^{II,(0),fin} \\
\mathcal{M}_i^{II,(1)} &= 2\boldsymbol{I}_g^{(1)}(\varepsilon)\,\mathcal{M}_i^{II,(0)} + \mathcal{M}_i^{II,(1),fin} \\
\mathcal{M}_i^{II,(2)} &= 4\boldsymbol{I}_g^{(2)}(\varepsilon)\,\mathcal{M}_i^{II,(0)} + 2\boldsymbol{I}_g^{(1)}(\varepsilon)\,\mathcal{M}_i^{II,(1)} + \mathcal{M}_i^{II,(2),fin}
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{I}_g^{(1)}(\varepsilon) &= -\frac{e^{-\frac{\varepsilon}{2}\gamma_E}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(\frac{4C_A}{\varepsilon^2} - \frac{\beta_0}{\varepsilon}\right)\left(-\frac{s}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}, \\
\boldsymbol{I}_g^{(2)}(\varepsilon) &= -\frac{1}{2}\boldsymbol{I}_g^{(1)}(\varepsilon)\left[\boldsymbol{I}_g^{(1)}(\varepsilon) - \frac{2\beta_0}{\varepsilon}\right] \\
&\quad + \frac{e^{\frac{\varepsilon}{2}\gamma_E}\Gamma(1+\varepsilon)}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left[-\frac{\beta_0}{\varepsilon} + K\right]\boldsymbol{I}_g^{(1)}(2\varepsilon) \\
&\quad + 2\boldsymbol{H}_g^{(2)}(\varepsilon)\,.
\end{aligned}
$$

- **Phys. Lett. B427 (1998) 161**

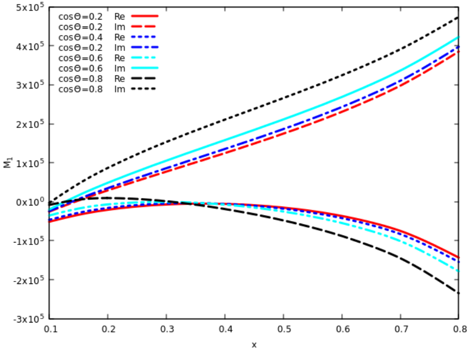
Results

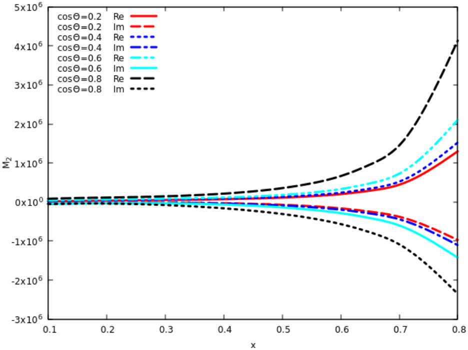
1. We found our results to be in full agreement with the predictions of Catani for all the IR poles.
2. We could completely renormalize the amplitude at the NLO level.
3. At two-loop level, the $\frac{1}{\varepsilon^4}, \frac{1}{\varepsilon^3}$ and $\frac{1}{\varepsilon^2}$ poles are renormalized analytically.
4. The $\frac{1}{\varepsilon}$ pole renormalization is numerically computed to be very close to zero.

$\frac{1}{\varepsilon}$ pole coefficient for Projector 1

x	$\cos\theta$	ε^{-1}
$\frac{1}{13}$	0.2	$(-2.90366 \times 10^{-12} + i\ 8.99075 \times 10^{-14})N^2$
$\frac{8}{17}$	0.2	$(-4.71671 \times 10^{-13} + i\ 1.52335 \times 10^{-14})N^2$
$\frac{1}{13}$	0.8	$(-1.09321 \times 10^{-12} - i\ 4.62596 \times 10^{-11})N^2$
$\frac{8}{17}$	0.8	$(-7.50942 \times 10^{-13} + 1.0692 \times 10^{-14})N^2$

Finite part Plots at NNLO





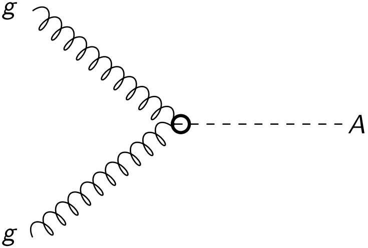
Amplitude for Projector 1: $\mathcal{M}_1 = \frac{\mathcal{M}^{(2),fin}_{1,GG}}{\mathcal{M}^{(0)}_{1,GG}}$.

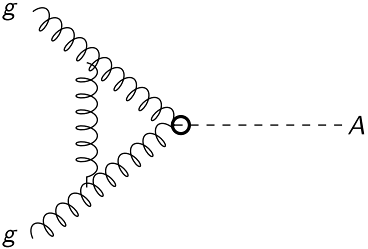
The upper lines represent the Imaginary (Im) values and the lower lines the Real (Re) values.

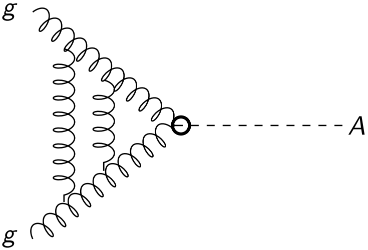
Amplitude for Projector 2: $\mathcal{M}2 = \frac{\mathcal{M}^{(2),fin}_{2,GG}}{\mathcal{M}^{(0)}_{2,GG}}$.

Here the line trend is just opposite to that of the other diagram.

Introduction







Theoretical Outline

Inclusive cross-section for pseudoscalar Higgs boson production:

$$\sigma^A\left(\tau, m_A^2\right) = \sigma^{A,(0)}\left(\mu_R^2\right) \sum_{a,b=q,\bar{q},g} \int_\tau^1 dy\ \Phi_{ab}\left(y, \mu_F^2\right) \Delta_{ab}^A\left(\frac{\tau}{y}, m_A^2, \mu_R^2, \mu_F^2\right),$$

$$\text{where } \Phi_{ab}\left(y, \mu_F^2\right) = \int_y^1 \frac{dx}{x} f_a\left(x, \mu_F^2\right) f_b\left(\frac{y}{x}, \mu_F^2\right).$$

Definitions: $\sigma^{A,(0)}\left(\mu_R^2\right)$: Born cross-section, $\Phi_{ab}\left(y, \mu_F^2\right)$: Parton flux, $\Delta_{ab}^A\left(\tau/y, m_A^2, \mu_R^2, \mu_F^2\right)$: Finite Partonic Coefficient Function, a and b: Initial state partons, f_a and f_b : Parton distribution functions (PDFs).

- Partonic Coefficient Function near threshold, $z = \frac{\tau}{y} \to 1$:

$$\Delta_{ab} \sim \underbrace{a_i \left[\frac{\ln^i(1-z)}{(1-z)}\right]_+ + b\delta(1-z)}_{\text{Leading power (LP)/ Soft-Virtual (SV) corrections}} + \underbrace{c_i \ln^i(1-z)}_{\text{Next-to-Leading power (NLP)/ Next-to-soft virtual (NSV) corrections}} + \mathbf{d}.$$

Leading power (LP)/ **Soft-Virtual (SV)** corrections

Next-to-Leading power (NLP)/ **Next-to-soft virtual (NSV)** corrections

SV+NSV partonic CF near threshold - A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)

$$\Delta^X_{ab}\left(z,q^2,\mu_R^2,\mu_F^2\right) = \underbrace{\Delta^{X,SV+NSV}_{ab}(z,q^2,\mu_i^2)}_{\delta(1-z),\ \mathcal{D}_i,\ \log^i(1-z)} + \underbrace{\Delta^{X,hard}_{ab}(z,q^2,\mu_i^2)}_{\text{Regular terms in } z \text{ like } (1-z)^i}$$

Mass factorised SV+NSV coefficient function for diagonal channels (since we will consider terms till NSV):

$$\Delta^{X,SV+NSV}_c\left(z,q^2,\mu_R^2,\mu_F^2\right) = \mathcal{C}\exp\left\{\Psi^X_c\left(z,q^2,\mu_R^2,\mu_F^2,\varepsilon\right)\right\}\Big|_{\varepsilon=0}$$

The finite distribution for $c=g$ channel:

$$\Psi^A_g\left(z,q^2,\mu_R^2,\mu_F^2,\varepsilon\right) = \left(\ln\left[Z^A_g\left(\hat{a}_s,\mu_R^2,\mu^2,\varepsilon\right)\right]^2 + \ln\left|\mathcal{F}^A_g\left(\hat{a}_s,Q^2,\mu^2,\varepsilon\right)\right|\right)\delta(1-z)$$
$$+\ 2\Phi^A_g\left(\hat{a}_s,q^2,\mu^2,z,\varepsilon\right) - 2\mathcal{C}\ln\Gamma_{gg}\left(\hat{a}_s,\mu_F^2,\mu^2,z,\varepsilon\right).$$

$Z^A_g \rightarrow$ overall operator UV renormalization constant, $\mathcal{F}^A_g \rightarrow$ form factors, $\Phi_g \rightarrow$ soft collinear distribution, $\Gamma_{gg} \rightarrow$ mass factorization kernels.

Constituent elements

- $\mathbf{Z_g^A}$: Removes UV divergences
  - Functional form : $$\mu_R^2 \frac{d}{d\mu_R^2} \ln Z_g^A \left(\hat{a}_s, \mu_R^2, \mu^2, \varepsilon\right) = \sum_{i=1}^{\infty} a_s^i \gamma_{g,i}^A$$
- $\mathcal{F}_g^A$: deals with virtual corrections
  - Functional form : $$\ln \mathcal{F}_g^A \left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \hat{\mathcal{L}}_{g,i}^A(\varepsilon)$$

Dependents:

- $\gamma_{g,i} \rightarrow$ UV anomalous dimensions,
- $A_{g,i} \rightarrow$ cusp anomalous dimensions,
- $G_{g,i}^A(\varepsilon) \rightarrow$ resummation functions which decompose into
  1. process dependent $g_{g,j}^{A,i}$'s, and
  2. collinear (B_g), soft (f_g) and UV (γ_g) anomalous dimensions.

Constituent elements

- $\Gamma_{\mathbf{gg}}$: Removes soft and collinear (IR) divergences
  - Functional form: $\boxed{\Gamma_{\mathbf{gg}}\left(\mathbf{z},\mu_{\mathbf{F}}^{2},\varepsilon\right)=\delta\left(\mathbf{1}-\mathbf{z}\right)+\sum_{\mathbf{i}=\mathbf{1}}^{\infty}\hat{\mathbf{a}}_{\mathbf{s}}^{\mathbf{i}}\left(\frac{\mu_{\mathbf{F}}^{2}}{\mu^{2}}\right)\mathbf{S}_{\varepsilon}^{\mathbf{i}}\Gamma_{\mathbf{gg}}^{(\mathbf{i})}\left(\mathbf{z},\varepsilon\right)}$,
- $\Phi_{\mathbf{g}}$: Has pole structure in ε similar to the residual divergences
  - Functional form: $\boxed{\Phi_{\mathbf{g}}=\Phi_{\mathbf{g}}^{\mathbf{SV}}+\Phi_{\mathbf{g}}^{\mathbf{NSV}}}$ where

$$\Phi_{g}^{SV}\left(\hat{a}_{s},q^{2},\mu^{2},z,\varepsilon\right)=\sum_{i=1}^{\infty}\hat{a}_{s}^{i}\left(\frac{q^{2}\left(1-z\right)^{2}}{\mu^{2}}\right)^{i\frac{\varepsilon}{2}}S_{\varepsilon}^{i}\left(\frac{i\varepsilon}{1-z}\right)\hat{\phi}_{g}^{SV,(i)}\left(\varepsilon\right),\text{ and}$$

$$\Phi_{g}^{NSV}\left(\hat{a}_{s},q^{2},\mu^{2},z,\varepsilon\right)=\sum_{i=1}^{\infty}\hat{a}_{s}^{i}\left(\frac{q^{2}\left(1-z\right)^{2}}{\mu^{2}}\right)^{i\frac{\varepsilon}{2}}S_{\varepsilon}^{i}\varphi_{g}^{NSV,(i)}\left(z,\varepsilon\right).$$

☞ $\hat{\phi}_{\mathbf{g}}^{\mathbf{SV},(\mathbf{i})}\left(\varepsilon\right)\Rightarrow$ **cusp $\left(A_{g,i}\right)$ and soft $\left(f_{g}\right)$ anomalous dimensions, z-independent constants, $\overline{\mathcal{C}}_{g,i}^{A}$, and $\overline{\mathcal{G}}_{g,i}^{A,k}$.**

Constituent elements

$$\varphi_{g}^{NSV,(i)}(z,\varepsilon) = \varphi_{s,g}^{NSV,(i)}(z,\varepsilon) + \varphi_{f,g}^{NSV,(i)}(z,\varepsilon)$$

$\varphi_{s,g}^{NSV,(i)}(z,\varepsilon) \rightarrow$ these singular coefficients should acquire a definite structure. For $g + g \rightarrow A$, we evaluated them to be

$$\varphi_{s,g}^{NSV,(1)}(z,\varepsilon) = -\frac{8C_A}{\varepsilon},$$

$$\varphi_{s,g}^{NSV,(2)}(z,\varepsilon) = \frac{8\beta_0 C_A}{\varepsilon^2} + \frac{1}{\varepsilon}\left\{C_A^2\left(8\zeta_2 - \frac{268}{9}\right) + \frac{40C_A n_f}{9} + 16C_A^2 \log(1-z)\right\}.$$

$\varphi_{f,g}^{NSV,(i)}(z,\varepsilon) \xRightarrow[\text{in terms of}]{\text{can be expressed}}$ certain finite coefficients $\mathcal{G}_{L,i}^{g}(z,\varepsilon)$

$$\varphi_{f,g}^{NSV,(1)}(z,\varepsilon) = \frac{1}{\varepsilon}\mathcal{G}_{L,1}^{g}(z,\varepsilon),$$

$$\varphi_{f,g}^{NSV,(2)}(z,\varepsilon) = \frac{1}{\varepsilon^2}\left\{-\beta_0\mathcal{G}_{L,1}^{g}(z,\varepsilon)\right\} + \frac{1}{2\varepsilon}\mathcal{G}_{L,2}^{g}(z,\varepsilon),$$

where $\mathcal{G}_{L,i}^{g,(j)}(z) \xrightarrow{\text{parameterized}} \mathcal{G}_{L,i}^{g,(j,k)}$ and $\log^k(1-z)$, $k = 0, 1, \cdots$.

Expansion coefficients

The parameterized finite coefficients, $\mathcal{G}_{L,i}^{g,(j,k)}$, are related to certain expansion coefficients, $\varphi_{f,g}^{NSV,(i)}$, as below:

$$\varphi_{g,1}^{(k)} = \mathcal{G}_{L,1}^{g,(1,k)}, \qquad k = 0, 1$$
$$\varphi_{g,2}^{(k)} = \frac{1}{2}\mathcal{G}_{L,2}^{g,(1,k)} + \beta_0 \mathcal{G}_{L,1}^{g,(2,k)}, \qquad k = 0, 1, 2.$$

Our Observation: The $\varphi_{g,i}^{(k)}$'s, for the scalar and the pseudoscalar Higgs boson productions in gluon fusion, are identical to each other till the two-loop level.

Earlier Observations: - **A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)**

- Same was noticed for the DY process and scalar Higgs production *via* bottom quark annihilation up to two-loop level.
- This failed for the quark annihilation process at third order for $k = 0, 1$.

Hence, this behaviour at third order for the pseudoscalar Higgs boson production can be checked only when the corresponding explicit N^3LO results are available.

Determining the expansion coefficients

By exploiting the similarity between pseudoscalar and scalar Higgs!

- T. Ahmed, M. Bonvini, M. C. Kumar, P. Mathews, N. Rana, V. Ravindran, L. Rottoli (2016)

Conclusion $\Rightarrow$ The pseudoscalar result can be approximated from the available scalar Higgs results

$\Downarrow$

$$\Delta^A_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)=\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}\left[\Delta^H_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)+\delta\Delta^A_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)\right].$$

- $\delta\Delta^{A,NSV}_{gg}\left(z,q^2,\mu_R^2,\mu_F^2\right)\rightarrow$ correction to the scalar Higgs coefficient functions,
- $g_0\left(a_s\right)$ and $g_0^H\left(a_s\right)\rightarrow$ constant functions of resummation for pseudoscalar and scalar Higgs, respectively.
- **Ratio:**

$$\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}=1+a_s\left(8C_A\right)+a_s^2\left[\frac{1}{3}\left\{-215C_A^2\ldots\right\}\right]+a_s^3\left[\frac{1}{81}\left\{68309C_A^3+\ldots\right\}\right].$$

Borrowing this Idea

- T. Ahmed, M. Bonvini, et. al. *arXiv:1606.00837 [hep-ph]*

A Conjecture to all higher orders.

- $\delta\Delta^A_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections vanish at the one-loop level.
- At two-loop level, these $\delta\Delta^A_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections contain only the next-to-next-to-soft terms.
- These observations $\xrightarrow[\text{conclusion}]{\text{lead to the}}$ $\delta\Delta^A_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right)$ corrections do not contain any NSV terms at $\mathcal{O}(a_s^3)$.

Consequence:

- $\delta\Delta^A_{gg}\left(z, q^2, \mu_R^2, \mu_F^2\right) = 0 \xrightarrow{\text{leads to}}$ the approximate N^3LO cross-sections denoted by N^3LO_A.

Implications on our Analysis

Hence, we simply rescale the Higgs SV+NSV CF to obtain the corresponding one for the pseudoscalar using

$$\Delta_{gg}^{A,NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right)=\frac{g_0\left(a_s\right)}{g_0^H\left(a_s\right)}\left[\Delta_{gg}^{H,NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right)\right].$$

The ratio and the CF's are known up to NNLO $\xrightarrow{\text{leading to}}$ successful computation of $\Delta_{gg}^{A}\left(z,q^2,\mu_R^2,\mu_F^2\right)$ up to two-loop level.

To evaluate the SV+NSV CFs for pseudoscalar higgs boson production from gluon fusion, we follow the following procedure:

1. Using the analytical formalism. - A. H. Ajjath, P. Mukherjee, and V. Ravindran (2020)
2. Using the ratio, $g_0\left(a_s\right)/g_0^H\left(a_s\right)$, and combining it with the available scalar Higgs SV+NSV CFs. - T. Ahmed, M. Bonvini, M. C. Kumar, et. al. (2016)

$\Downarrow$

1 yields the corresponding pseudoscalar Higgs SV+NSV CFs in terms of the $\varphi_{g,i}^{(k)}$'s which are evaluated by comparison with the result from 2 .

Resummation in Mellin space

$\Delta_{gg}^{A,SV+NSV}\left(z,q^2,\mu_R^2,\mu_F^2\right) \xrightarrow{\text{convenience}}$ Mellin (N-moment) space

- Convolutions $\Rightarrow$ Simple products.
  - $z \to 1$ translates to $N \to \infty$ near threshold.
  - Keep $\mathcal{O}(1/N)$ corrections.

$$\Delta_{g,N}(q^2,\mu_R^2,\mu_F^2) = C_0(q^2,\mu_R^2,\mu_F^2)\exp\left(\Psi_N^g(q^2,\mu_F^2)\right)$$

$$\Psi_N^g = \Psi_{\mathrm{SV,N}}^g + \Psi_{\mathrm{NSV,N}}^g$$

$$\Downarrow$$

$$\Psi_{\mathrm{SV,N}}^g = \log(g_0^g(a_s(\mu_R^2))) + g_1^g(\omega)\log(N) + \sum_{i=0}^{\infty} a_s^i(\mu_R^2)g_{i+2}^g(\omega);$$

$$\Psi_{\mathrm{NSV,N}}^g = \frac{1}{N}\sum_{i=0}^{\infty} a_s^i(\mu_R^2)\left(\bar{g}_{i+1}^g(\omega) + h_i^g(\omega,N)\right), \text{ with } h_i^g(\omega,N) = \sum_{k=0}^{i} h_{ik}^g(\omega)\ \log^k(N).$$

Coefficients g_i^g, $\bar{g}_i^g$ and h_i^g are available; $C_0 \to$ process-dependent coefficients.

Predicting higher logs

Significance of this method of computation:

- Obtained the SV+NSV CFs for pseudoscalar production up to $\mathcal{O}(a_s^3)$.
- Predicting coefficients of three highest logarithms of $\Delta_{gg}^{A,SV+NSV}$, from $\mathcal{O}(a_s^4)$ to $\mathcal{O}(a_s^7)$.

a_s^4	$\log^7(1-z)$	$\log^6(1-z)$	$\log^5(1-z)$
	$-4096/3\, C_A^4$	$98560/9\, C_A^4 - 7168/9\, n_f C_a^3$	$-335104/9\, C_a^4 + 174208/27\, n_f C_a^3 - 4096/27\, n_f^2 C_a^2 + 23552\, \zeta_2 C_a^4$
a_s^5	$\log^9(1-z)$	$\log^8(1-z)$	$\log^7(1-z)$
	$-8192/3\, C_a^5$	$96256/3\, C_a^5 - 8192/3\, n_f C_a^4$	$-131685640/81\, C_a^5 + 569216/81\, n_f C_a^4 - 81920/81\, n_f^2 C_a^3 + 262144/3\, \zeta_2 C_a^5$
a_s^6	$\log^{11}(1-z)$	$\log^{10}(1-z)$	$\log^9(1-z)$
	$-65536/15\, C_a^6$	$9490432/135\, C_a^6 - 180224/27\, C_a^5 n_f$	$-4458496/9\, C_a^6 + 8493056/81\, C_a^5 n_f - 327680/81\, C_a^4 n_f^2 + 671744/3\, \zeta_2 C_a^6$
a_s^7	$\log^{13}(1-z)$	$\log^{12}(1-z)$	$\log^{11}(1-z)$
	$-262144/45\, C_a^7$	$3309568/27\, C_a^7 - 1703936/135\, C_a^6 n_f$	$-92717056/81\, C_a^7 + 115835488/45\, C_a^6 n_f - 917504/81\, C_a^5 n_f^2 + 1310720/3\, \zeta_2 C_a^7$

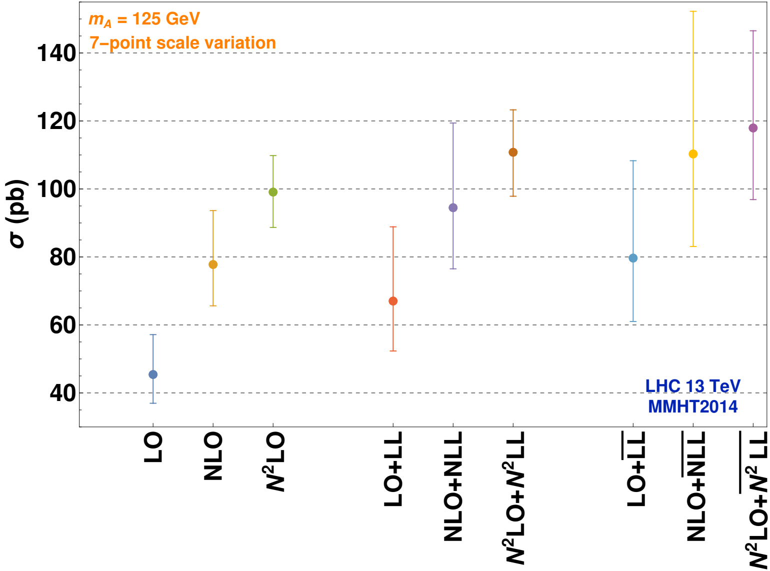
Still we are left with certain other logarithms that cannot be predicted from previous order informations.

Numerical results

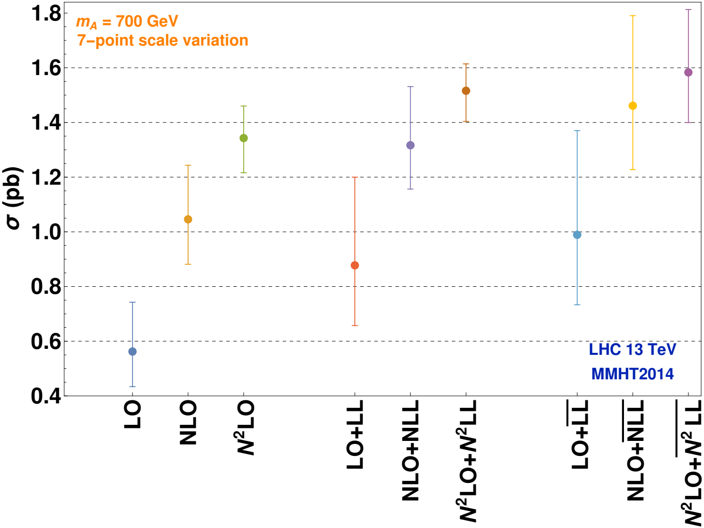
Assumptions:

- Based on EFT.
- 13 TeV C.O.M. energy at the LHC.
- $\cot\beta = 1$ (other values can be obtained by rescaling).
- $C_J^{(2)} = 0$ because of non-availability.
- PDF's: Corresponding MMHT 2014 up to NNLO;
  MMHT 2014 NNLO at N^3LO (for non-availability).
- For NSV resummation $\Rightarrow$ Resum threshold logs only for gluon fusion channel.
- Theoretical uncertainties computed at $m_A = 125$ GeV, 700 GeV for seven point scale uncertainties, and by varying one scale & keeping the other fixed.
  - $\{\mu_R/m_A, \mu_F/m_A\} = (0.5, 0.5), (0.5, 1.0), (1.0, 0.5), (1.0, 1.0), (1.0, 2.0), (2.0, 1.0)$ and $(2.0, 2.0)$.
  - $\{\mu_R/m_A$ or $\mu_F/m_A\} = \{0.5, 1.0, 2.0\}$ and the other scale fixed at m_A.

7-point scale uncertainty plot for $m_A = 125$ GeV



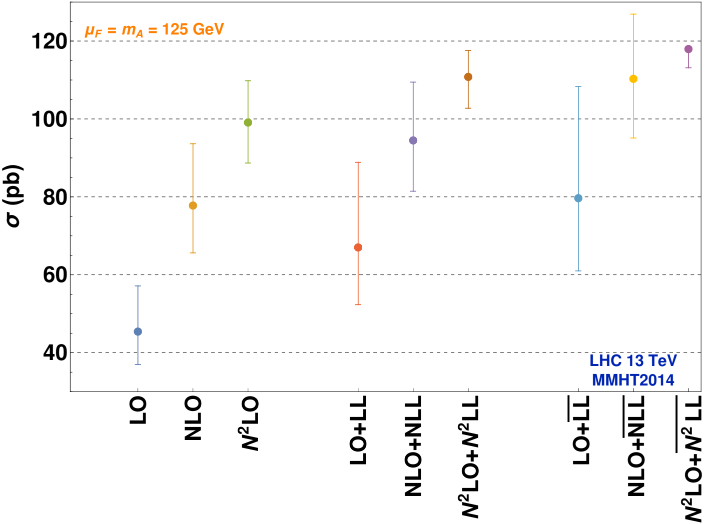
7-point scale uncertainty plot for $m_A = 700$ GeV



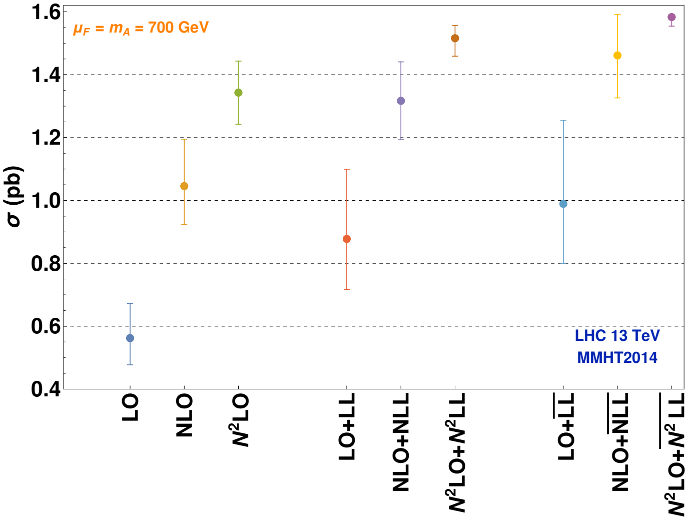
Observation $\Rightarrow$ The uncertainties reduce from NLO to NNLO, NLO+NLL to NNLO+NNLL, and NLO+$\overline{\text{NLL}}$ to NNLO+$\overline{\text{NNLL}}$ $\xrightarrow{\textbf{Problem}}$ *NSV resummation exhibits higher uncertainties than SV.*

Uncertainty plot for μ_F scale fixed at $m_A = 125$ GeV

To comprehend this unexpected behaviour $\xrightarrow{\text{we study}}$ scale variations due to μ_R and μ_F separately by varying one and keeping the other fixed at m_A.

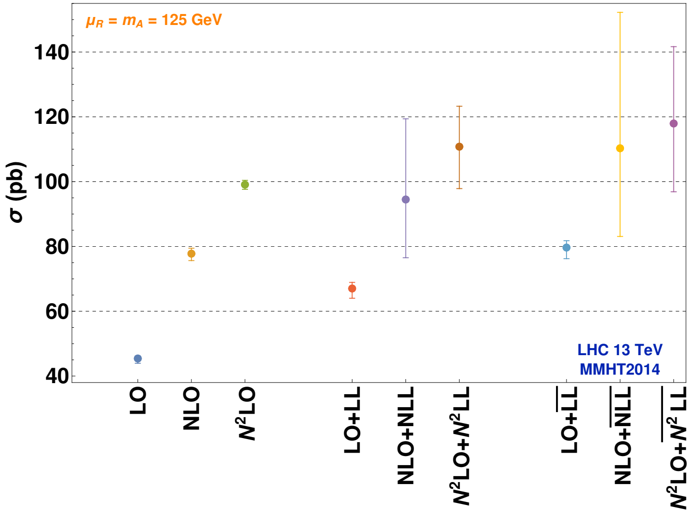


Uncertainty plot for μ_F scale fixed at $m_A = 700$ GeV



Observation: At the 2nd order, uncertainties reduce from FO results to resummed ones with the NSV results being more stable than the SV ones.

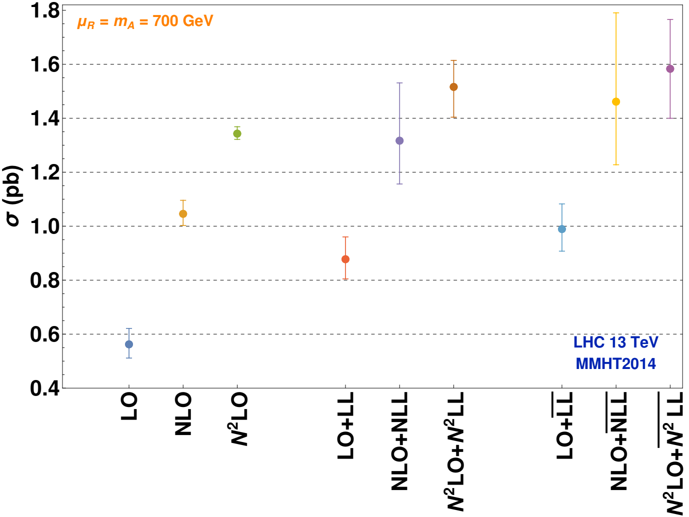
Uncertainty plot for μ_R scale fixed at $m_A = 125$ GeV



Observations: Results are in contrast to the μ_R variation ones, *i.e.*

- **NLO+$\overline{\text{NLL}}$ > NLO+NLL > NLO,**
- **NNLO+$\overline{\text{NNLL}}$ > NNLO+NNLL > NNLO.**

Uncertainty plot for μ_R scale fixed at $m_A = 700$ GeV



Conclusion: Contributions from other partonic channels for resummation because different partonic channels are expected to mix when the μ_F scale varies.

Possibility of scalar-pseudoscalar Higgs boson mixed state

Parameter: Mixing angle α.

- M. Jaquier, R. Röntsch (2019)

Consider a Higgs boson production, while neglecting its decay,

$\xrightarrow[\text{the results up to NNLO}]{\text{for any arbitrary value of } \alpha,}$ may be obtained by the simple rescaling formula below.

$$\sigma = \cos^2 \alpha \cdot \sigma_H + \sin^2 \alpha \cdot \sigma_A$$

K-Factor	$\alpha = 0$ **(pure scalar)**	$\alpha = \pi/2$ **(pure pseudoscalar)**	$\alpha = \pi/4$ **(mixed state)**	$\alpha = \pi/6$ **(mixed state)**
$K_{(1)}$	1.6990	1.7124	1.7083	1.7048
$K_{(2)}$	2.1571	2.1814	2.1741	2.1677
$K^{resum}_{(1)}$	2.0033	2.0803	2.0570	2.0368
$K^{resum}_{(2)}$	2.2785	2.4392	2.3907	2.3485
$\overline{K}^{resum}_{(1)}$	2.3425	2.4284	2.4025	2.3799
$\overline{K}^{resum}_{(2)}$	2.4737	2.5966	2.5595	2.5272

Observation

Changing the mixing angle α modifies the corresponding QCD corrections only by a few percent.

Consequence: Availibility of the pseudoscalar Higgs boson production cross-section to a precision comparable to that of the scalar Higgs

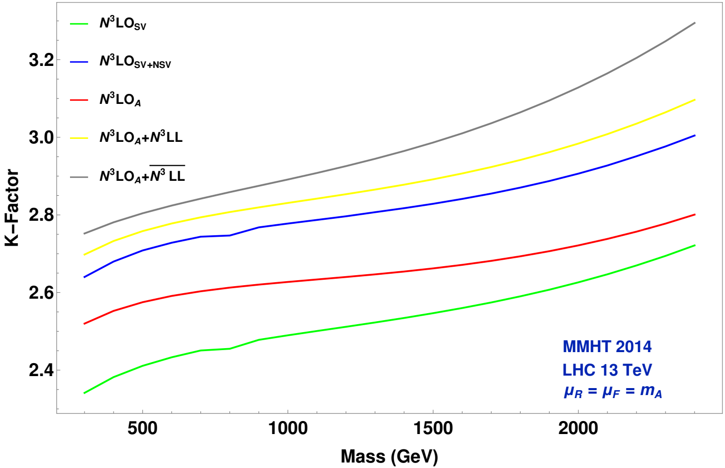
$\Downarrow$

In extracting the mixing angle, α, to a better accuracy.

☞ While studying Higgs decay processes, the simple reweighting formula above fails. Hence, the corresponding K-factors, similar to those given in the above table, get modified slightly $\xRightarrow{\textbf{Why?}}$ Number of angular observables get involved.

- M. Jaquier, R. Röntsch (2019)

N^3LO results : K-factors



Observations:
- $N^3LO_{SV} \xrightarrow{\text{are closer to}} N^3LO_A$ in the high mass region,
- $N^3LO_{SV+NSV} \xrightarrow{\text{are closer to}} N^3LO_A$ in the small mass region.
- Inclusion of NSV corrections substantially increase the cross-sections.

Summary of Project 1

AIM: Two loop virtual amplitudes that are relevant for studying production of a pair of pseudo-scalar Higgs bosons in the gluon fusion subprocess at the LHC

- Computation done in EFT where the top quark degrees of freedom is integrated out ($n_f = 5$).
- The pseudo-scalar Higgs boson directly couples to gluons and light quarks through two local composite operators O_G and O_J, respectively.
- The composite operators mix under UV renormalisation with the corresponding renormalization constants are already known.
- We found that the IR poles are in agreement with the predictions by Catani which is a test on the correctness of our computation.
- **Our results provide one of the important components relevant for studies related to production of a pair of pseudo-scalar Higgs bosons at the LHC up to order $\mathcal{O}\left(a_s^4\right)$.**

Summary of Project 2

Aim: NSV resummation for pseudoscalar Higgs boson production *via* gluon fusion to $\overline{\text{NNLL}}$ accuracy.

1. Compute the NSV corrections up to second order, and compare them with the corresponding FO corrections.
   - **Conclude** These corrections significantly impact the pseudoscalar production cross-section compared to the conventional SV logarithms.
2. Estimate theory uncertainties.
   - The 7-point scale uncertainties do not improve much after NSV resummation.
   - The μ_F scale variations increase the uncertainties.
   - For μ_R scale variations, the uncertainties reduce significantly.

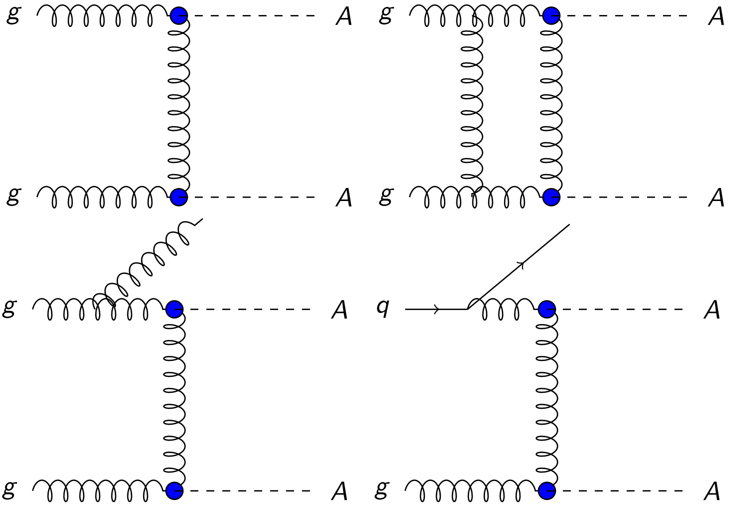
   $\Downarrow$

   Conclude $\rightarrow$ The need of NSV contributions from other parton channels, & beyond NSV contributions in the gluon fusion channel.
3. Evaluate the production cross-sections for mixed scalar-pseudoscalar states.
   - Study their behavior for different values of the mixing angle, α.
   - **Conclude** $\Rightarrow$ QCD corrections change with α by a few percent.

Project 3:

Combine FO computation + Resummation for a $2 \to 2$ process to cover the entire phase space region & develop a general numerical code.

Di-pseudoscalar production from gluon fusion



Plan & Status

Aim: **NLO+$\overline{\text{NLL}}$ correction to di-pseudoscalar production from gluon fusion.**

- ☞ Requirements from Project 1:
  - ✔ We have $\mathcal{M}_1$ and $\mathcal{M}_2$ at LO and NLO.
  - ✔ Calculate the total amplitude, $\mathcal{M}_{ab}^{\mu\nu}$, at LO & NLO.
  - ✔ Compute total cross-section for the virtual diagrams contributing at LO & NLO.
  - ✔ Analytical check of the pole structure of the virtual diagrams. - **2010.02979**
- ☞ New computation:
  - ✔ Total cross-section for the real diagrams.
    - • g+g→A+A+g • g+q→A+A+q
- ☞ Requirement from Project 2:
  - ✔ Knowledge of the resummation formalism & CFs.
- ✖ Numerical analysis -
  - ♦ Problem: Large size.
  - ♦ Solution: Extrapolation
    - ✍ Draft code is ready. Incorporation of the analytical results and execution left.

Thank you...