Hydrodynamic model for particle beam-driven wakefield in CNT

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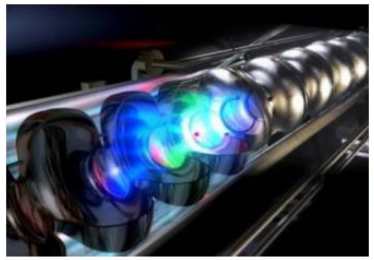
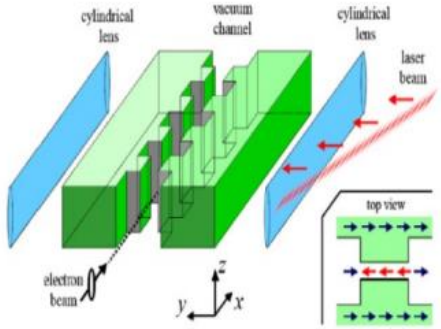
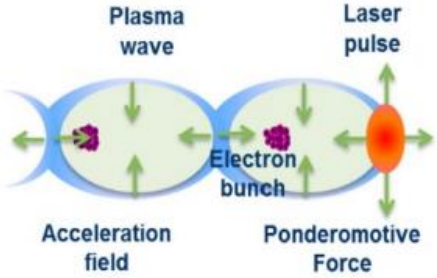
Contents

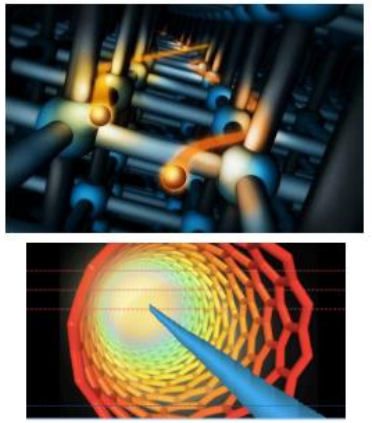
1. Introduction

- The current state-of-the-art of the RF techniques for particle acceleration is limited to gradients on the order of 100 MV/m
- To obtain higher energies, we can increase the length of the accelerators... or use new techniques of acceleration with higher gradients





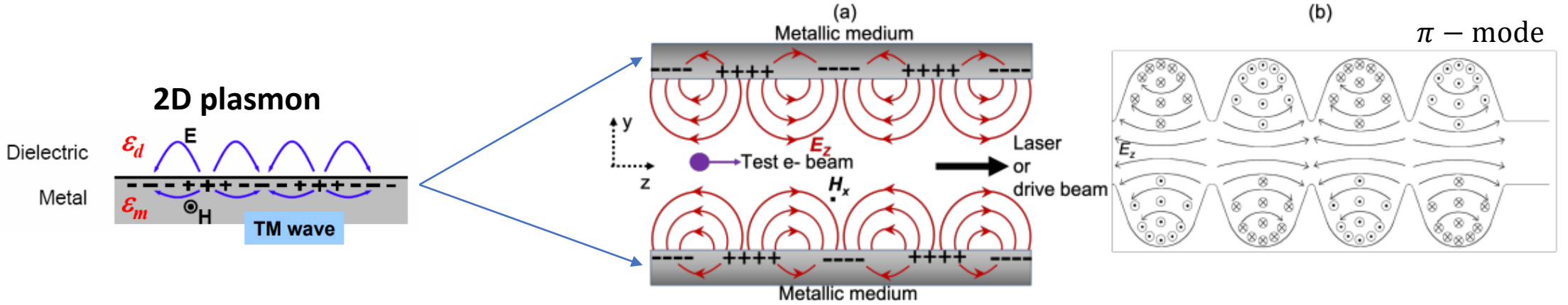




	Conventional RF cavities	Dielectric laser – driven acceleration (DLA)	Plasma / Laser wakefield acceleration (PWFA / LWFA)	Solid-state plasma wakefield acceleration
Based on	Normal / superconducting cavities	Quartz / silicon structure	Gaseous plasma	Crystals, nano-channels, CNTs
Max. longitudinal electric field	$\sim 100\ \mathrm{MV/m}$	$\sim 10\ \mathrm{GV/m}$	$\sim 100\ \mathrm{GV/m}$	$\sim 1 - 100\ \mathrm{TV/m}$ (prediction)
Limitation	Surface breakdown	Damage threshold	Wave breaking	Wave breaking

1. Introduction

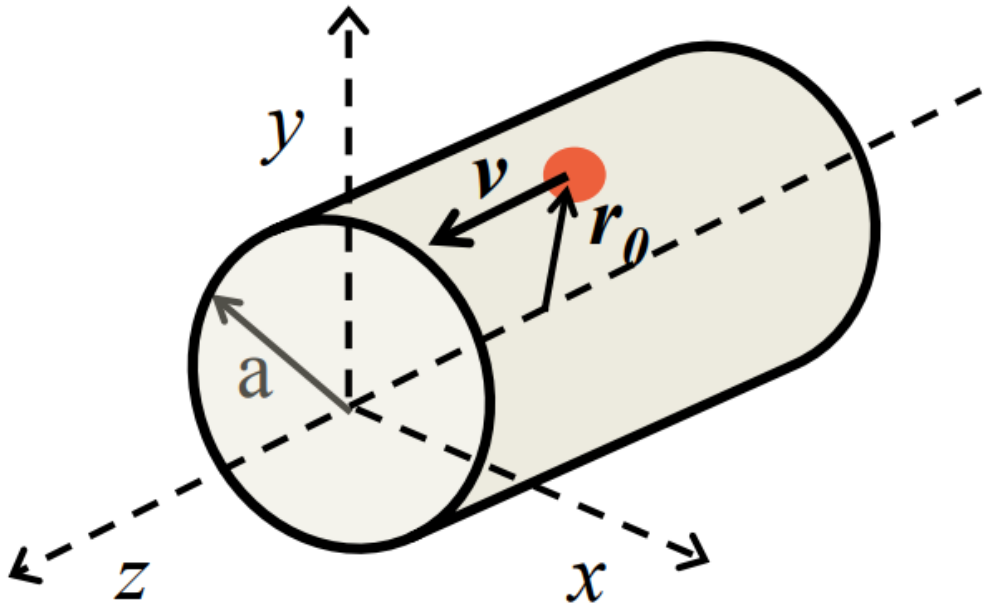
- **Plasmonic acceleration**
  - Excitation of surface plasmonic modes by laser (laser-driven) or charged particle beam (beam-driven)
  - Collective motion of wall electrons acting like a structured plasma
  - To properly excite wakefields laser or beam driving parameters need to be in the time and space scale of the plasmon wave



	Plasmonic acceleration	RF cavities
Aperture size	$\sim\mu$m	$\sim$cm
Length	$\sim$mm	~ 10cm − m
Longitudinal electric field	~ 100 GV/m	~ 100 MV/m
Operation	Travelling wave (TW)	Standing Wave (SW) or TW

2. Theoretical background

- **Geometry**
  - 2D electron gas confined in the cylindrical surface of a nanotube or microtube (made of CNTs, metallic nature)
  - Let us assume a point-like charge q travelling with speed $v \approx c$ inside a single walled tube



Driving charge at position:

$$\mathbf{r}_0 = (r_0, \varphi_0, z_0)$$

Position of electrons excited at the surface:

$$\mathbf{r}_a = (a, \varphi_a, z_a)$$

Y-N. Wang, Z. L. Mišković, Phys. Rev. A 69 (2004) 022901
Z. L. Mišković et al., Phys. Lett. A 329 (2004) 94

2. Theoretical background

- **Electronic excitation on the tube wall:**
  - Continuity equation:

velocity of the plasma

$$\partial_t n_1(\mathbf{r}_a, t) + n_0 \nabla_{\parallel} \cdot \mathbf{u}(\mathbf{r}_a, t) = 0$$

Perturbed surface plasma density

Unperturbed surface plasma density

  - Momentum-balance equation:

$$\partial_t \mathbf{u}(\mathbf{r}_a, t) = \frac{e}{m_e} \nabla_{\parallel} \Phi(\mathbf{r}_a, t) - \underbrace{\frac{\alpha}{n_0} \nabla_{\parallel} n_1(\mathbf{r}_a, t)}_{\text{Thomas-Fermi electron fluid pressure / Acoustic modes}} - \underbrace{\gamma \mathbf{u}(\mathbf{r}_a, t)}_{\text{Frictional force}} + \underbrace{\frac{\beta}{n_0} \nabla_{\parallel} [\nabla_{\parallel}^2 n_1(\mathbf{r}_a, t)]}_{\text{Quantum correction to the semiclassical Thomas-Fermi theory}}$$

2. Theoretical background

- **Electronic excitation on the tube wall:**
  - Poisson's equation:

$$\nabla^2\Phi(\mathbf{r},\mathbf{t}) = \frac{1}{\epsilon_0}\left[\underbrace{en_1(\mathbf{r}_a,t)}_{\text{Contribution from plasma perturbation}} - \underbrace{q\delta(\mathbf{r}-\mathbf{r}_0)}_{\text{Contribution from driving charge}}\right]$$

  - Boundary conditions:
    - Vanishing normal component of the velocity perturbation at the surface of the cylinder
    - Vanishing of the potential as $r \to \infty$
    - The finiteness of the electric potential at the origin

These boundary conditions will allow the expansion of the potential and the charge density disturbance in terms of modified Bessel functions I_m and K_m of integer order m

2. Theoretical background

- The total potential inside the tube: $\Phi_{in} = \Phi_0 + \Phi_{ind}$

  - Coulomb potential from the driving charge:

$$\Phi_0(r,\varphi,z,t) = \frac{1}{4\pi\epsilon_0}\frac{q}{\|\mathbf{r}-\mathbf{r_0}\|}$$

Modified Bessel functions

$$= \frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{d}k e^{ik\zeta+im(\varphi-\varphi_0)} I_m(|k|r_<)K_m(|k|r_>),$$

  - Induced potential from the tube surface electron fluid:

$$\Phi_{ind}(r,\varphi,z,t) = \frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{d}k e^{ik\zeta+im(\varphi-\varphi_0)} I_m(|k|r_0)I_m(|k|r)A_m(k),$$

$\zeta = z - vt$: comoving coordinate; $r_< = \min(r, r_0)$, $r_> = \max(r, r_0)$

2. Theoretical background

- **The total potential outside the tube:**

$$\Phi_{out}(r,\varphi,z,t)=\frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty}\mathrm{d}k e^{ik\zeta+im(\varphi-\varphi_0)}I_m(|k|r_0)K_m(|k|r)B_m(k)$$

Boundary conditions at $r=a$:

$$\Phi_{in}(r,\varphi,z,t)\Big|_{r=a}=\Phi_{out}(r,\varphi,z,t)\Big|_{r=a}$$

$$\frac{\partial\Phi_{out}(r,\varphi,z,t)}{\partial r}\Big|_{r=a}-\frac{\partial\Phi_{in}(r,\varphi,z,t)}{\partial r}\Big|_{r=a}=n_1(\varphi,z,t)$$

$$A_m(k)=\frac{\Omega_p^2a^2(k^2+m^2/a^2)K_m^2(ka)}{kv(kv+i\gamma)-\omega_m^2(k)}$$

$$B_m(k)=\frac{kv(kv+i\gamma)-\alpha(k^2+m^2/a^2)-\beta(k^2+m^2/a^2)^2}{kv(kv+i\gamma)-\omega_m^2(k)}$$

Resonance at $kv=\omega_m(k)$

2. Theoretical background

- **Resonance frequencies:**

$$\omega_m(k) = \left[\alpha \left(k^2 + \frac{m^2}{a^2} \right) + \beta \left(k^2 + \frac{m^2}{a^2} \right)^2 + \Omega_p^2 a^2 \left(k^2 + \frac{m^2}{a^2} \right) K_m(|k|a) I_m(|k|a) \right]^{1/2}$$

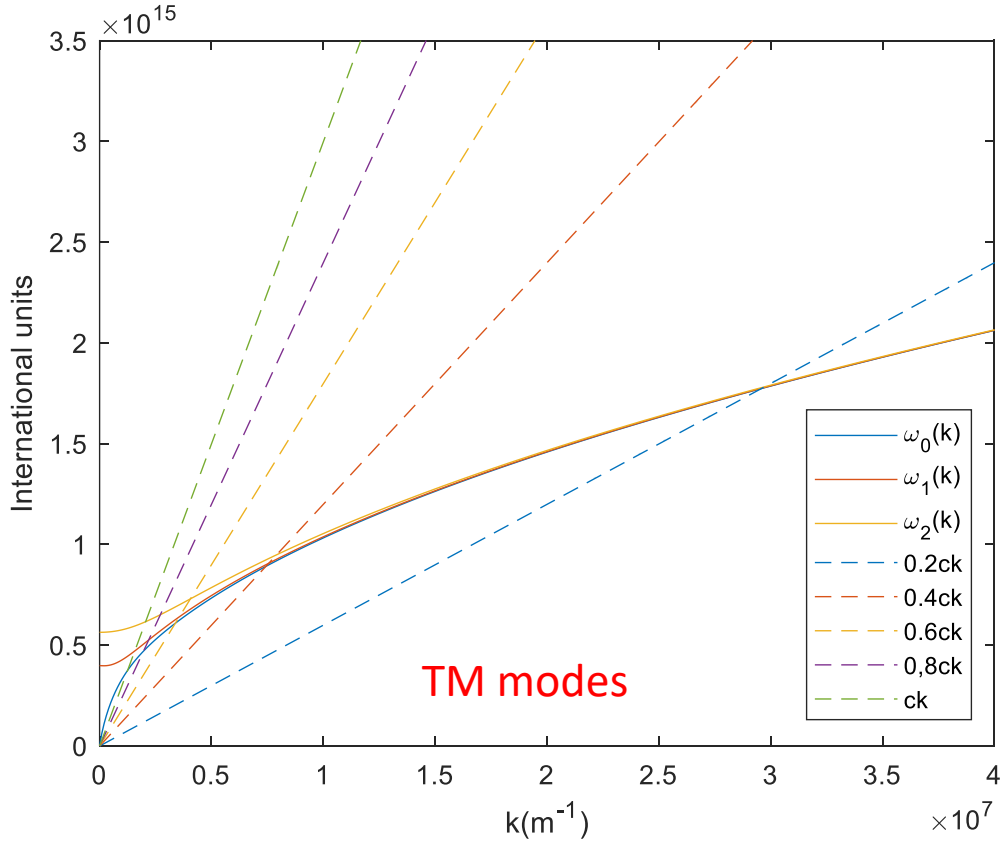
Correction for acoustic modes in the plasma

Quantum correction to the semiclassical Thomas-Fermi theory

$$\Omega_p = \sqrt{\frac{e^2 n_0}{\epsilon_0 m_e a}}$$ Plasma frequency

2. Theoretical background

- **Resonance frequencies:** $$\omega_m(k) = \left[\alpha\left(k^2 + \frac{m^2}{a^2}\right) + \beta\left(k^2 + \frac{m^2}{a^2}\right)^2 + \Omega_p^2 a^2\left(k^2 + \frac{m^2}{a^2}\right) K_m(|k|a) I_m(|k|a)\right]^{1/2}$$



Density of electrons: $n_0/a = 10^{26}\ \mathrm{m}^{-3}$
Nanotube radius: $a = 0.2\lambda_p \approx 0.67\ \mu\mathrm{m}$
Correction parameters:

$$\alpha = \frac{\hbar}{m_e}\left(\frac{3\pi^2 n_0}{a}\right)^{1/3}, \beta = \frac{1}{4}\left(\frac{\hbar}{m_e}\right)^2, \gamma = 0.05\Omega_p$$

- If the radius *a* or the density n_0/a increases, then ω_m increases and the resonance condition moves to higher *k*.
- If the velocity decreases, the resonance condition moves to higher *k*.
- If *v* is non-relativistic all modes have the same resonance *k*.

The **resonance condition** for particle acceleration is achieved whenever the dashed particle velocity lines intercept the dispersion curves

2. Theoretical background

- Longitudinal electric wakefield inside the tube:

$$W_z = -\partial_z \Phi_{in}$$

$$W_z(r,\varphi,z,t) = \frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty}\int_{-\infty}^{+\infty} \mathrm{d}k k e^{im(\varphi-\varphi_0)}$$

$$\times \Big[I_m(|k|r_<) K_m(|k|r_>)\sin(k\zeta) \qquad \text{Coulomb term}$$

$$+ I_m(|k|r_0) I_m(|k|r)\Big(\Re[A_m(k)]\sin(k\zeta) \qquad \text{2nd term}$$

$$+ \Im[A_m(k)]\cos(k\zeta)\Big)\Big]. \qquad \text{3rd term}$$

$\zeta = z - vt$: comoving coordinate; $r_< = \min(r, r_0)$, $r_> = \max(r, r_0)$

3. Results

- **Longitudinal electric wakefield**:

Amplitude at different transverse position r

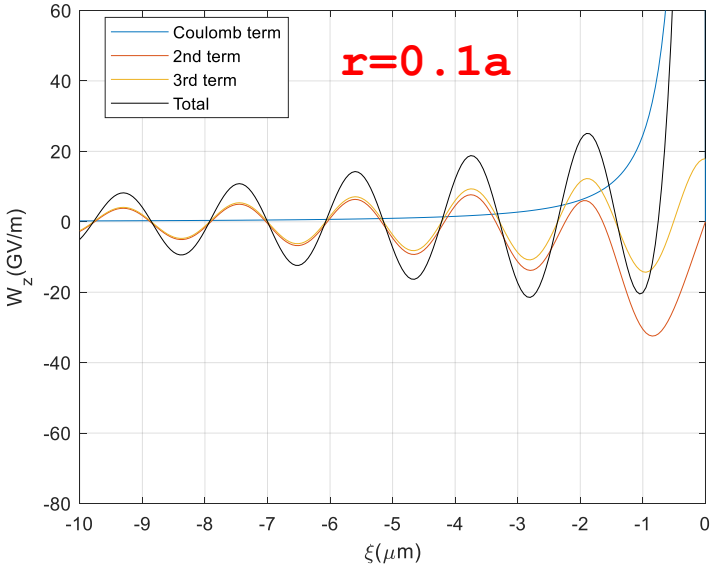
For $m = 0$; nanotube radius $a = 0.2\lambda_p \simeq 0.7\ \mu$m

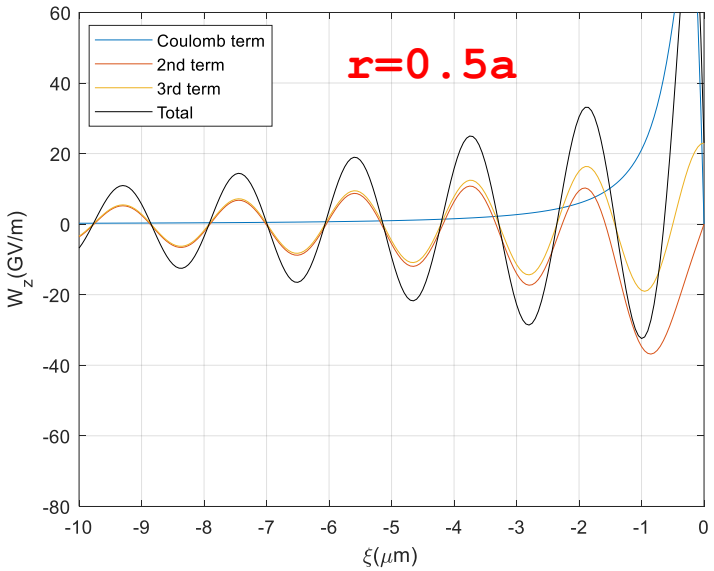
Density of electrons $n_0/a = 1 \times 10^{26}\ \mathrm{m}^{-3}$

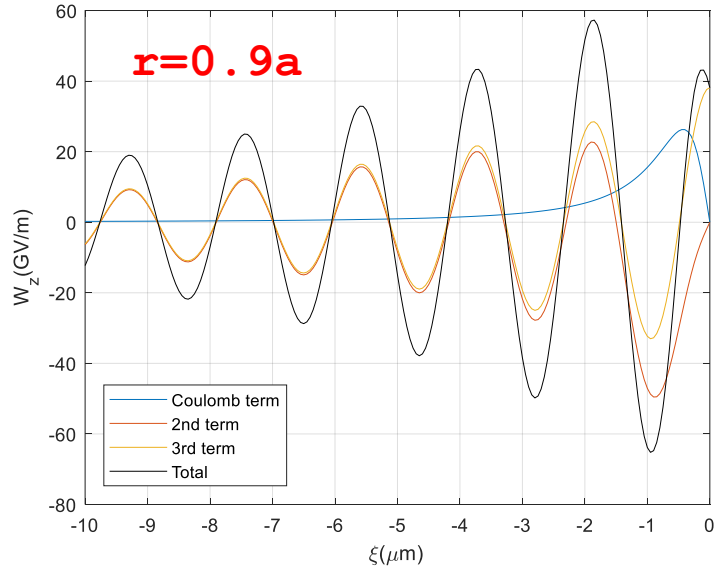
Driver parameters : point-like charge $q = -2.75$ pC; $r_0 = 0$ (particle on axis); $v = 0.6c$

Correction parameters : $\alpha = \frac{\hbar}{m_e}(3\pi^2 n_0/a)^{1/3}$; $\beta = \frac{1}{4}\left(\frac{\hbar}{m_e}\right)^2$; $\gamma = 0.05\Omega_p$

m=0 is the only mode excited







Electric fields increase near to the nanotube surface

3. Results

- **Longitudinal electric wakefield**:

For different driver velocities

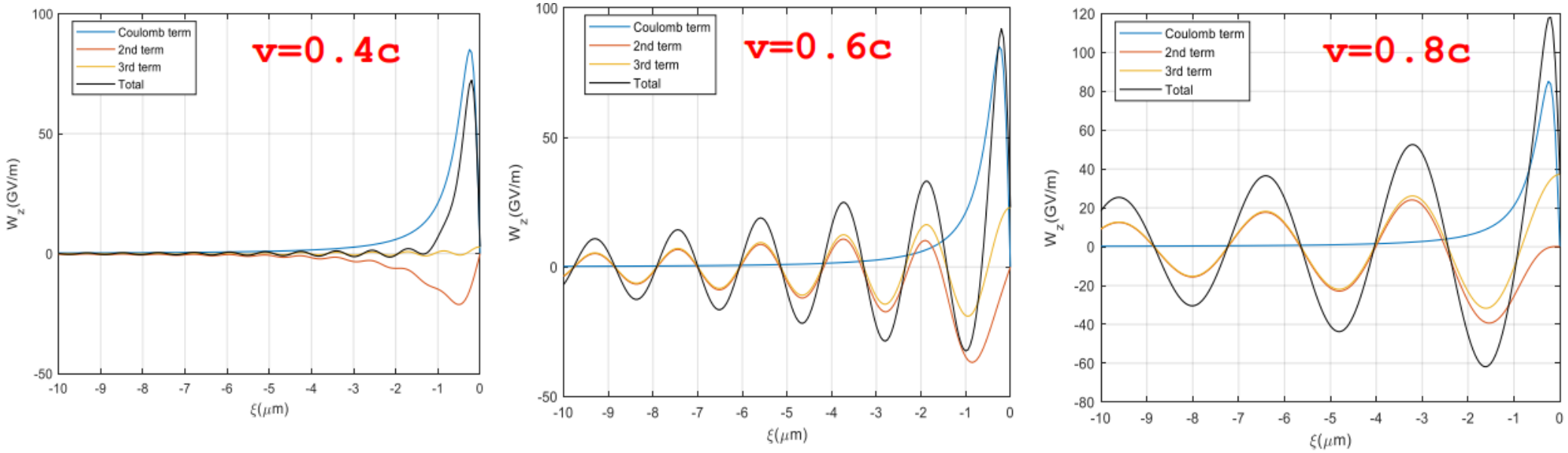
For $m = 0$; nanotube radius $a = 0.2\lambda_p \simeq 0.7\ \mu$m

Density of electrons $n_0/a = 1 \times 10^{26}\ \mathrm{m}^{-3}$

m=0 is the only mode excited

Driver parameters : point-like charge $q = -2.75$ pC; $r_0 = 0$ (particle on axis); $r = 0$

Correction parameters : $\alpha = \frac{\hbar}{m_e}(3\pi^2 n_0/a)^{1/3}$; $\beta = \frac{1}{4}\left(\frac{\hbar}{m_e}\right)^2$; $\gamma = 0.05\Omega_p$



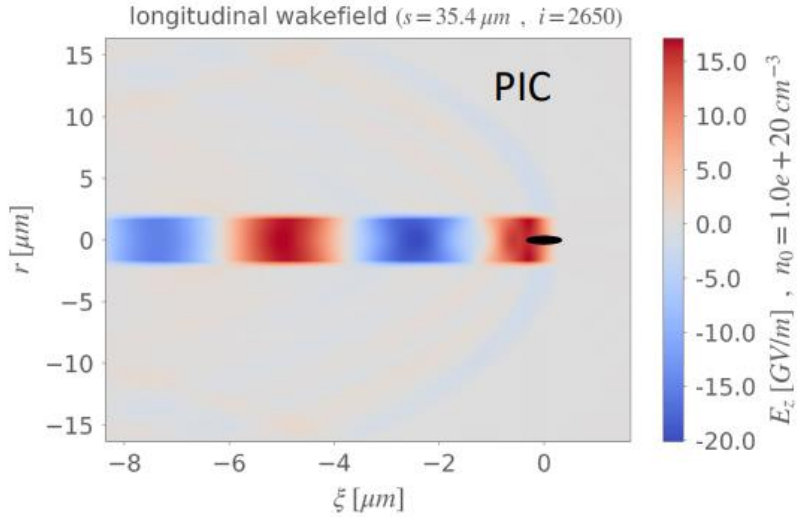
The plasmon wavelength increases as v increases. If v is too small there is no excitation.

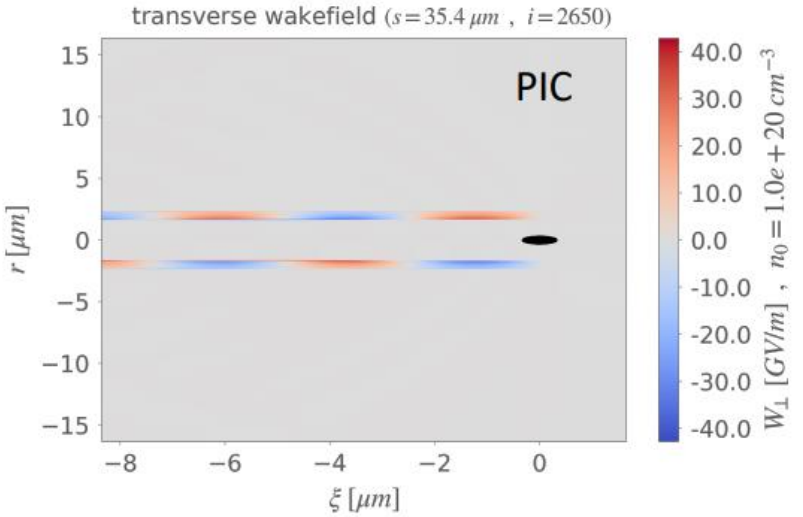
Electric fields increase as v increases, although for v>0.8c there is no big change (for the density used).

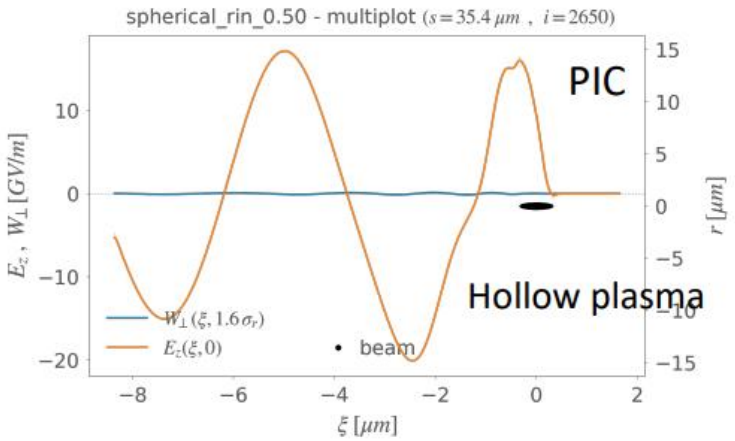
3. Results

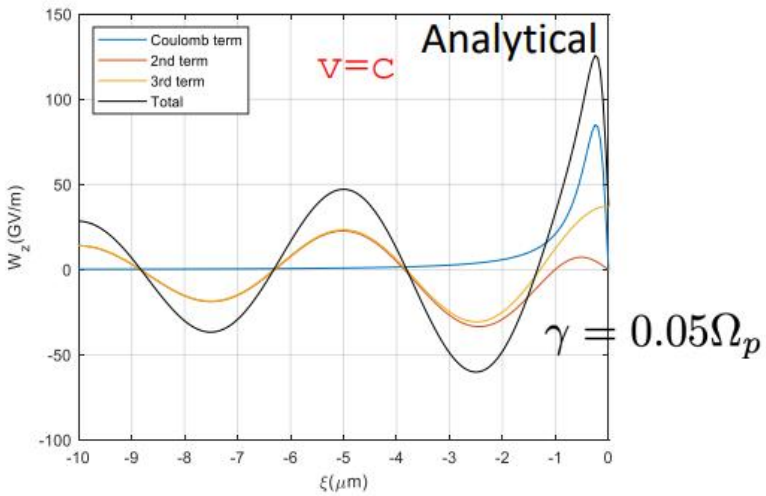
- **Longitudinal electric wakefield**:

Comparison with PIC codes









3. Results

- **Longitudinal electric wakefield**:

Depending on friction parameter γ

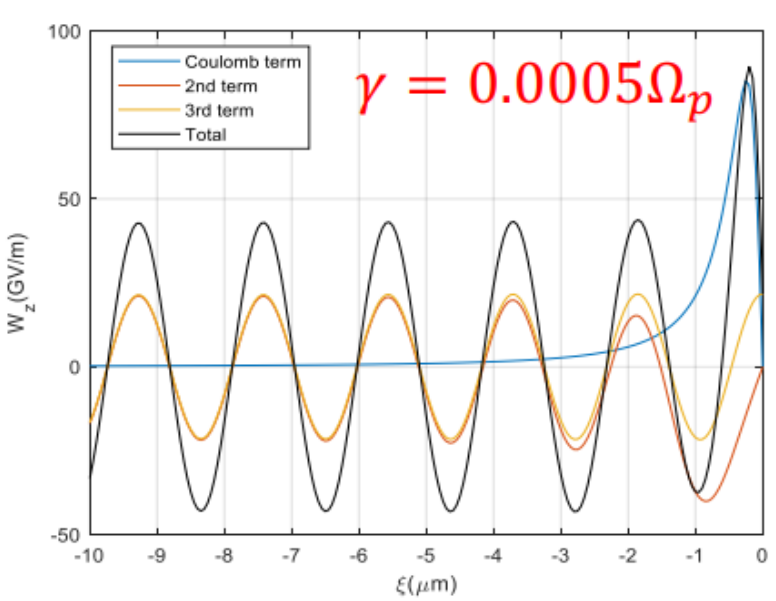
For $m = 0$; nanotube radius $a = 0.2\lambda_p \simeq 0.7\ \mu\text{m}$

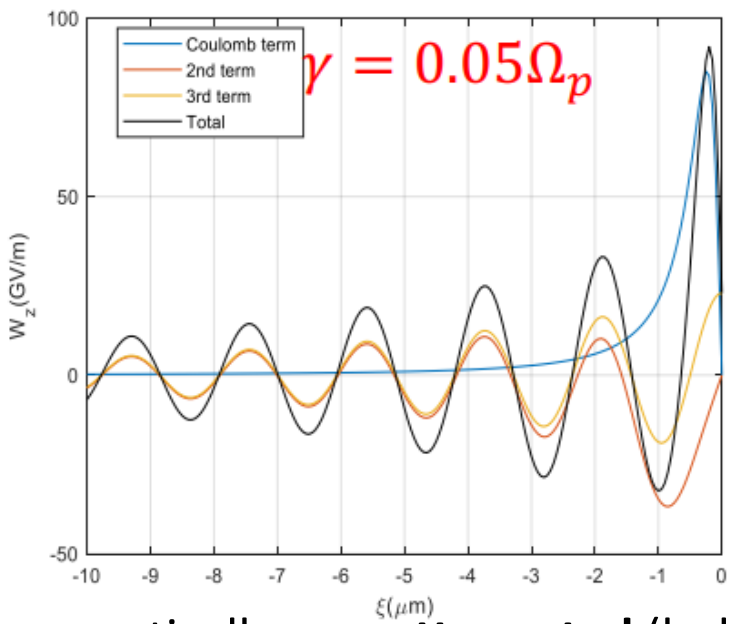
Density of electrons $n_0/a = 1 \times 10^{26}\ \text{m}^{-3}$

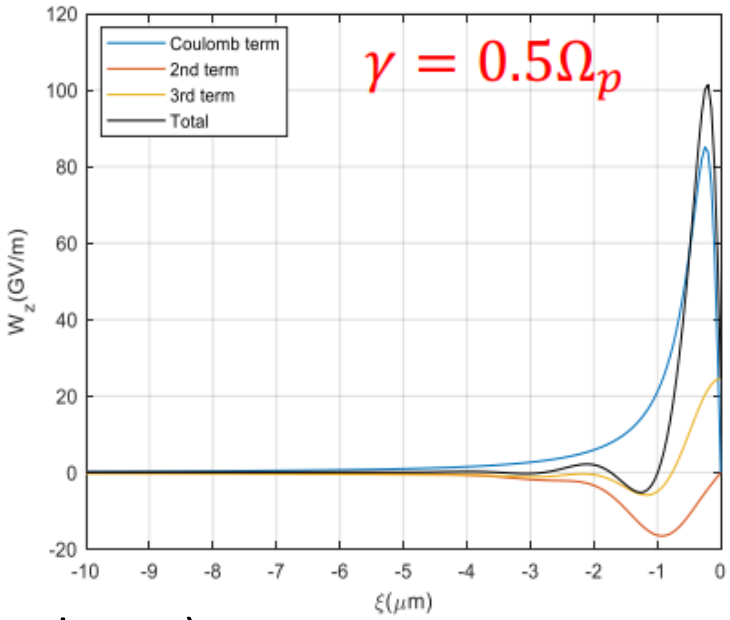
m=0 is the only mode excited

Driver parameters: point-like charge $q = -2.75$ pC; $r_0 = 0$ (particle on axis); $v = 0.6c$; $r = 0$

Correction parameters: $\alpha = \frac{\hbar}{m_e}(3\pi^2 n_0/a)^{1/3}$; $\beta = \frac{1}{4}\left(\frac{\hbar}{m_e}\right)^2$; $\gamma = \cdots$







For $\boldsymbol{\gamma < 0.005\Omega_p}$, electric fields are practically **non-attenuated** (hollow plasma).

If $\boldsymbol{\gamma}$ **increases** there is **no excitation** of plasmons.

3. Results

- **Longitudinal electric wakefield**:

If $\gamma \to 0^+$, the third term of the longitudinal electric wakefield can be calculated via the residue theorem:

$$W_z^{3rd}(r,\varphi,z,t) = -\frac{q}{4\pi^2\epsilon_0}(2\pi)\sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi_0)} k_m I_m(|k_m|r_0) I_m(|k_m|r)\Omega_p^2 a^2 \left(k_m^2+\frac{m^2}{a^2}\right) K_m^2(|k_m|a) \left|\frac{\partial Z_m}{\partial k}\right|_{k=k_m}^{-1} \cos(k_m\zeta),$$

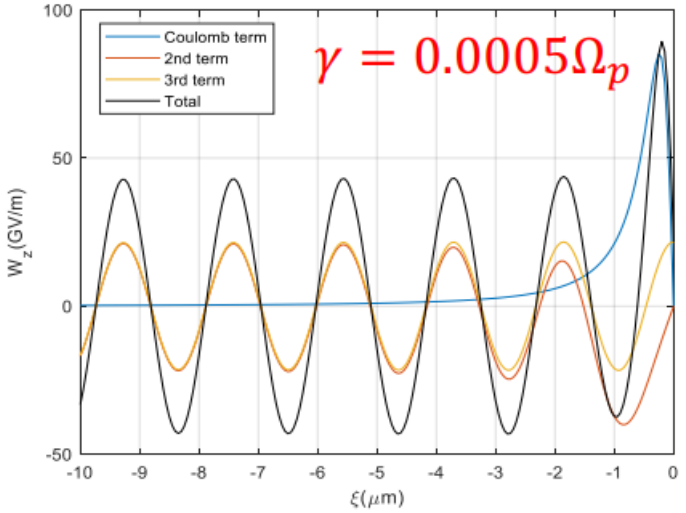
L_m: amplitude of the wakefield due to the third term for $\gamma \to 0^+$
(The **total amplitude** is approximately **twice** because of the ***second term***)

We have defined the quantity:

$$Z_m(k) = (kv)^2 - \omega_m^2(k),$$

and k_m is determined by the condition of the plasma resonance,

$$Z_m(k_m) = (k_m v)^2 - \omega_m^2(k_m) = 0$$

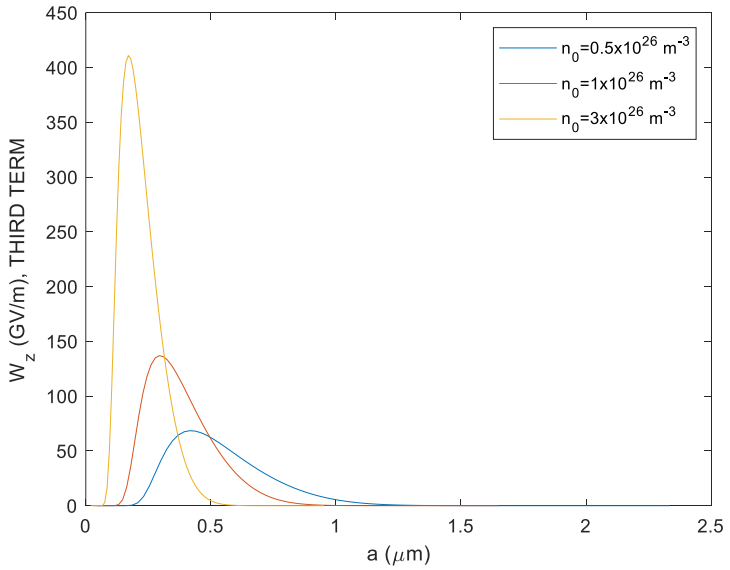


3. Results

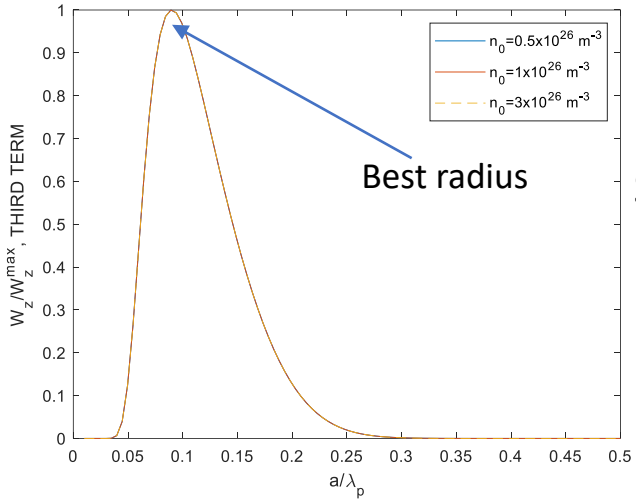
- **Longitudinal electric wakefield**:

If $\gamma \to 0^+$:

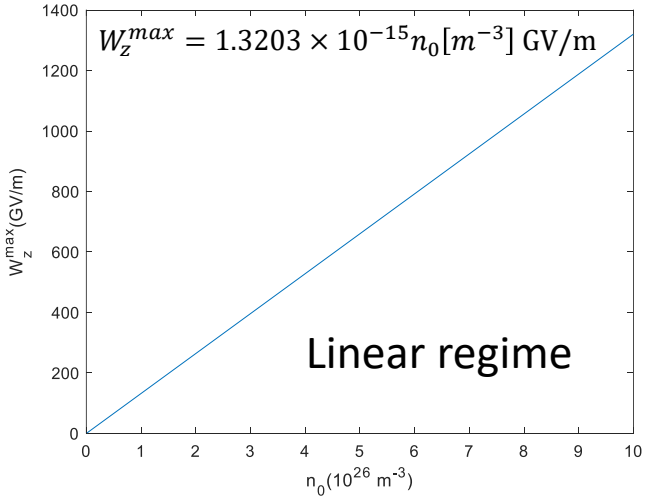
$v = 0.6c, \qquad r = r_0 = 0$



$$\lambda_{\mathrm{plasmon}}[\mathrm{nm}] = 330 \ \left[n_e(10^{22}\mathrm{cm}^{-3})\right]^{-1/2}$$



Same curve scaling axes



Correction parameters:

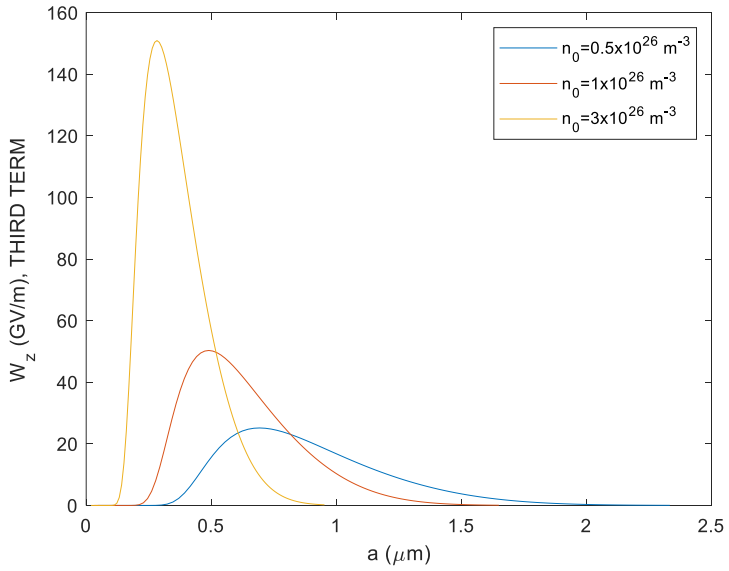
$$\alpha = \frac{\hbar}{m_e}\left(\frac{3\pi^2 n_0}{a}\right)^{1/3}, \beta = \frac{1}{4}\left(\frac{\hbar}{m_e}\right)^2$$

3. Results

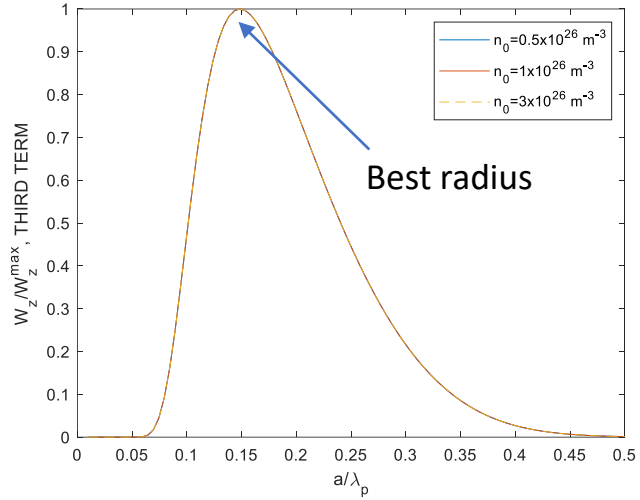
- **Longitudinal electric wakefield**:

If $\gamma \to 0^+$:

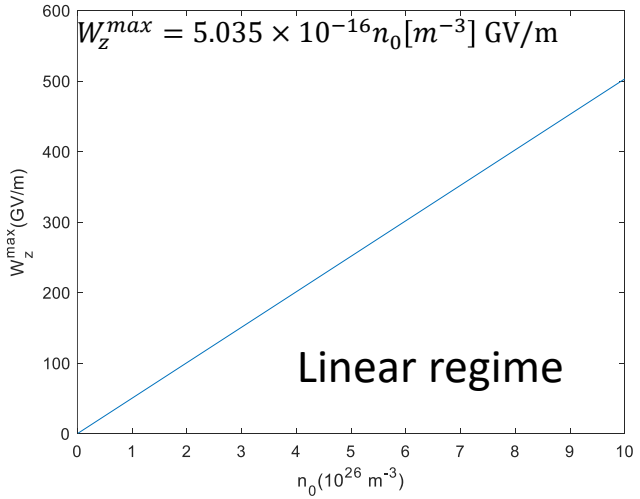
$v = 0.99c, \qquad r = r_0 = 0$



$$\lambda_{\text{plasmon}}[\text{nm}] = 330\ \left[n_e(10^{22}\text{cm}^{-3})\right]^{-1/2}$$



Same curve scaling axes



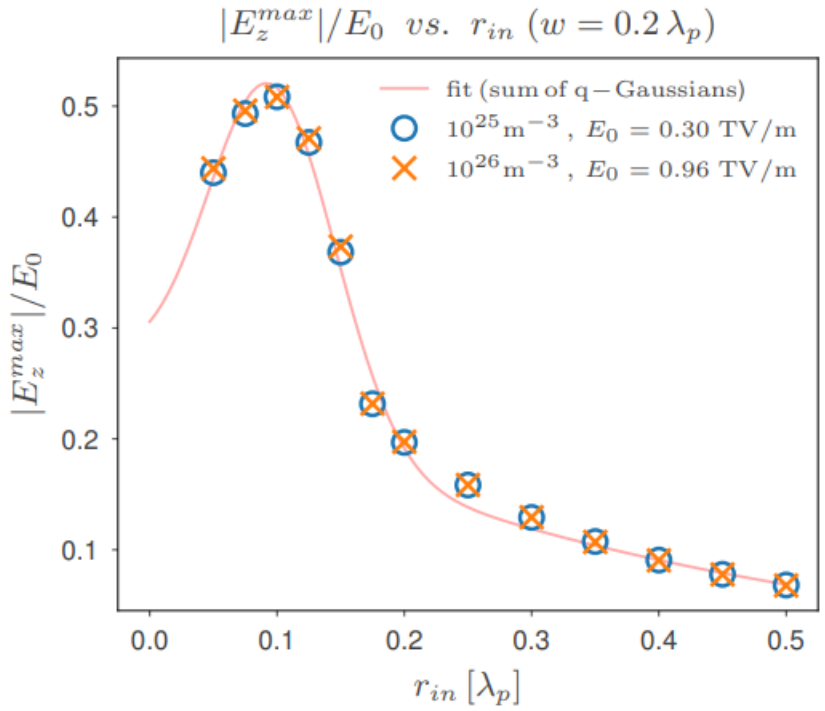
For higher velocities, the maximum wakefield is lower and the best radius increases (in units of the plasmon wavelength)

3. Results

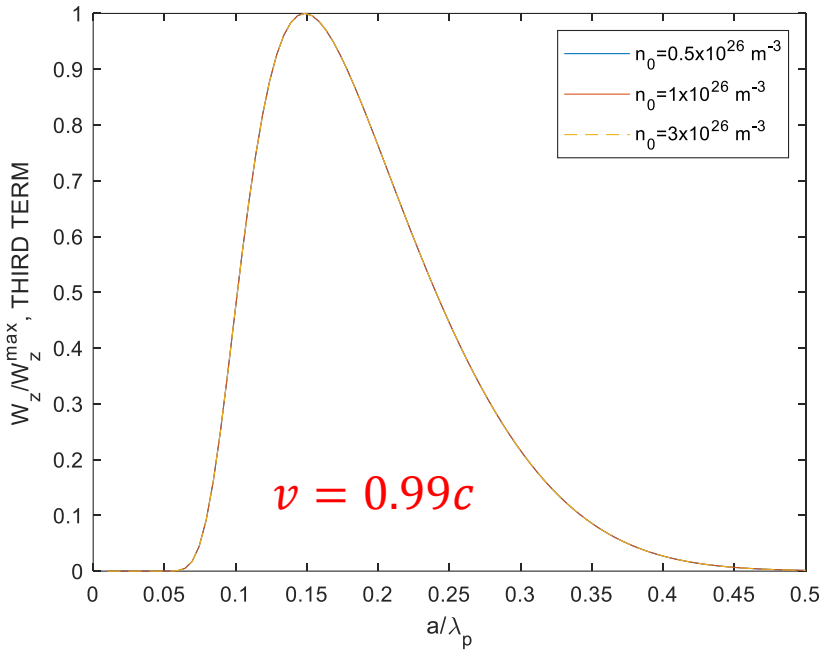
- **Longitudinal electric wakefield**:

If $\gamma \to 0^+$:

PIC simulation using
a Gaussian bunch distribution as a driver



Analytical calculation using
a point-like charge as a driver



Despite the discrepancies, the analytical model allows us to predict an optimal radius

3. Results

- **Transversal electric wakefield**:

$$W_r = -\partial_r \Phi_{in} = -\frac{q}{4\pi\epsilon_0}\partial_r\left(\frac{1}{\|\boldsymbol{r}-\boldsymbol{r_0}\|}\right) +$$

$$-\frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi_0)}\int_{-\infty}^{+\infty} dkk[I_m(|k|r_0)I'_m(|k|r)(\mathrm{Re}[A_m(k)]\cos(k\zeta) - \mathrm{Im}[A_m(k)]\sin(k\zeta))] =$$

$$= W_r^{Coulomb}(r,\varphi,z,t) + W_r^{2nd}(r,\varphi,z,t) + W_r^{3rd}(r,\varphi,z,t)$$

If $\gamma \to 0^+$:

$$W_z^{3rd}(r,\varphi,z,t) = \frac{q}{4\pi^2\epsilon_0}\sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi_0)}\int_{-\infty}^{+\infty} dkkI_m(|k|r_0)I'_m(|k|r)\mathrm{Im}[A_m(k)]\sin(k\zeta) =$$

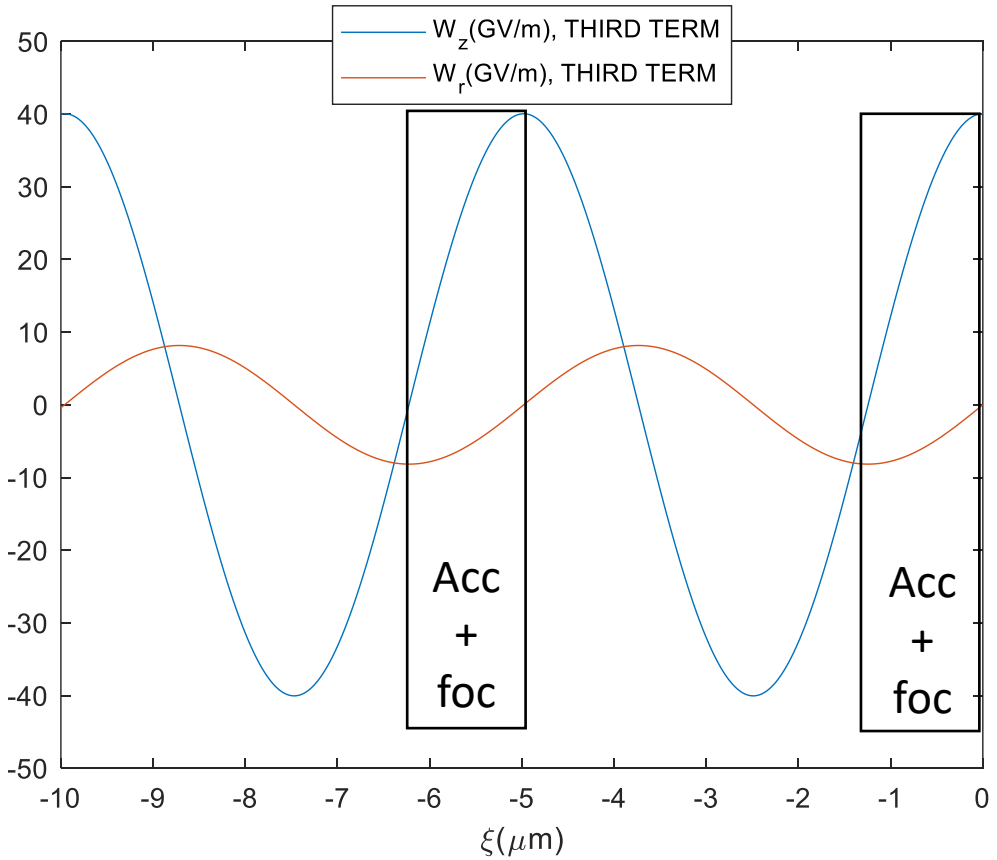
$$= -\frac{q}{4\pi^2\epsilon_0}(2\pi)\sum_{m=-\infty}^{+\infty} e^{im(\varphi-\varphi_0)}k_m I_m(|k_m|r_0)I'_m(|k_m|r)\Omega_p^2 a^2\left(k_m^2+\frac{m^2}{a^2}\right)K_m^2(|k_m|a)\left|\frac{\partial Z_m}{\partial k}\right|^{-1}_{k=k_m}\sin(k_m\zeta),$$

3. Results

- **Transversal electric wakefield**:

If $\gamma \to 0^+$:

$n_0 = 10^{26}\mathrm{m}^{-3}, \quad v = 0.99c, \quad r = 0.5a, \quad r_0 = 0, \quad a = 0.2\lambda_p$



4. Conclusions and outlook

- We have presented a **linear hydrodynamic model for wakefield excitation** by charges in nanotubes and micro-tubes.
- A driver particle can excite **plasmons** in nanotubes.
- The **electric fields** are **higher** if the particle travels **near** to the **CNT surface**. For this reason, it is better to use CNT with **small radius** (about $0.1\lambda_p$).
- The results show **>GV/m fields** and are **qualitatively similar to the PIC simulations** and the order of magnitude is similar. Systematic comparisons with PIC simulations are ongoing.
- Changing the values of $\boldsymbol{\alpha}$ **and** $\boldsymbol{\beta}$ 2-3 orders of magnitude (respect to the typical values of these parameters) **does not affect** to the results.
- **All the calculations** have been made for the **mode m=0** (since if r0=0 is the only mode excited). **Higher modes are less important**, but they are not negligible if the driver is off-axis near to the surface.
- In the **limit** $\boldsymbol{\gamma \rightarrow 0^+}$ we can **calculate easily the excited wakefield** and the dependences on the different parameters.

Acknowledgments

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Thank you for your attention

BACKUP SLIDES

3. Results

- **Longitudinal electric wakefield**:

If $\gamma \to 0^+$:

$$a = 0.2\lambda_p;\ r = 0.5a;\ r_0 = 0.2a$$