

Cross Sections and Observables

Vincent MATHIEU

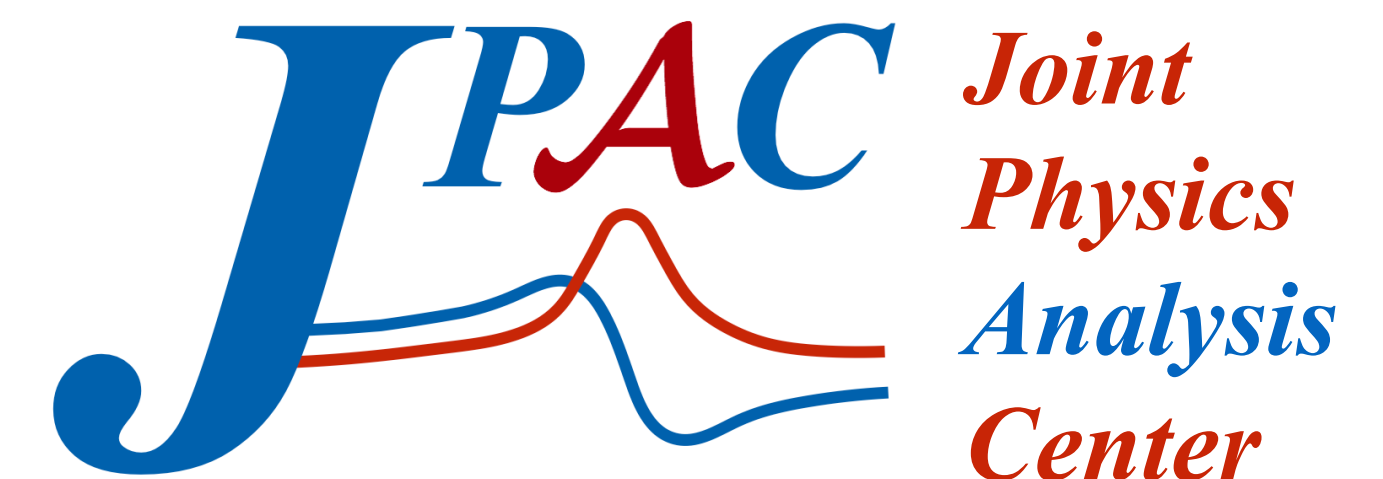
University of Barcelona

Joint Physics Analysis Center
Exotic Hadron Topical Collaboration

Horizon2020 Summer School
Salamanca September 2023



UNIVERSITAT DE
BARCELONA



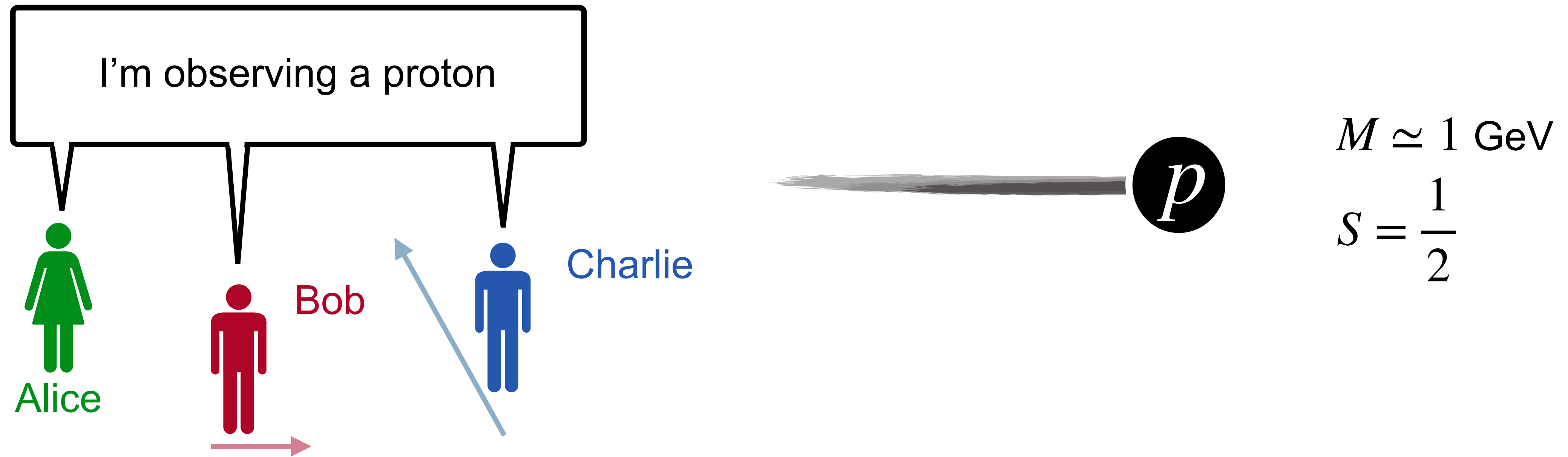
Following SU Chung, Spin formalisms, <https://suchung.web.cern.ch/spinfmt1.pdf>

See also Weinberg, the quantum theory of fields, vol I

What is a particle?

Alice, Bob and Charlie are different observers, in relative motion

The intrinsic properties of a particle must be identical for all observers



Group of symmetries of the space-time is the **Poincaré group**

It has two invariant quantities: **mass** and **spin**

States

Lorentz group include boosts and rotations

$$\begin{pmatrix} p'_x \\ p'_y \\ p'_z \end{pmatrix} = R(\alpha, \beta, \gamma) \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

↘ 3x3 matrix

Spelling out the matrix form

$$p'_{m'} = \sum_{m=-1}^1 R_{m,m'}(\alpha, \beta, \gamma) p_m$$

Lorentz group has other representations

$$|m'\rangle = \sum_{m=-J}^J D_{m,m'}^J(\alpha, \beta, \gamma) |m\rangle$$

A spin J has $2J+1$ component $-J, \dots, J$

Explicit form

$$D_{m,m'}^J(\alpha, \beta, \gamma) = e^{-im\alpha} d_{m,m'}^J(\beta) e^{-im'\gamma}$$

Convention

$$d_{1,0}^1(\beta) = -\frac{\sin \beta}{\sqrt{2}}$$

States at Rest

States are tensorial product

$$|\vec{p}, m\rangle = |\vec{p}\rangle \otimes |m\rangle$$

Implicit dependence on M, s

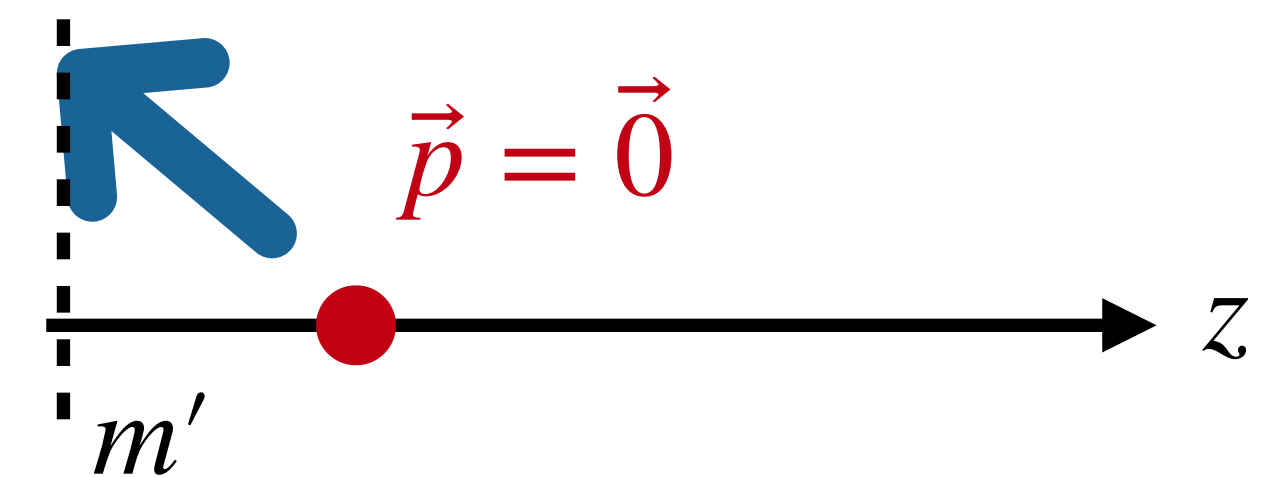
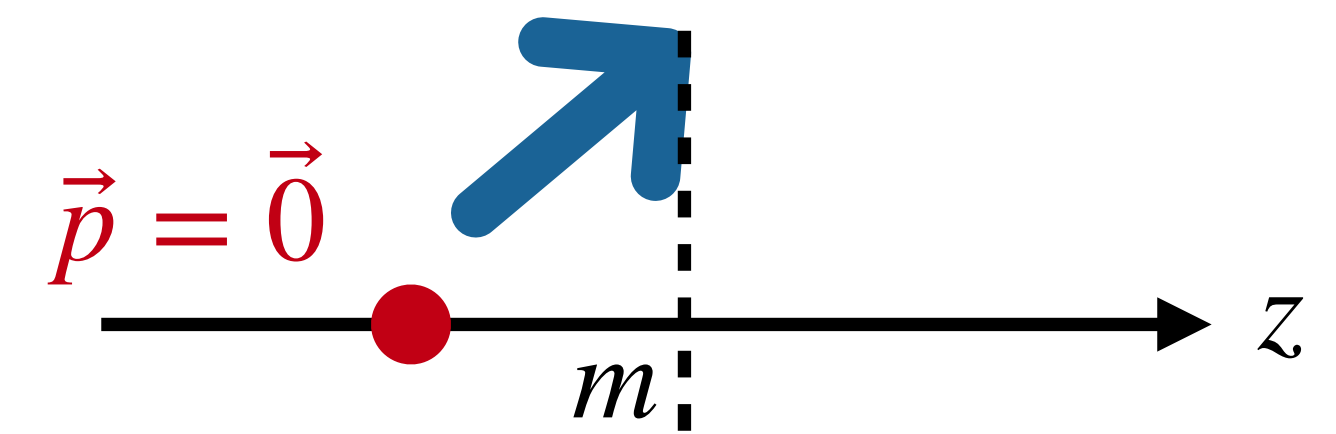
Let's first consider state at rest $\vec{p} = \vec{0}$

The spin projection m is defined by z axis

$$|m'\rangle = \sum_{m=-J}^J D_{m,m'}^J(\alpha, \beta, \gamma) |m\rangle$$

$$|m'\rangle = \sum_{m=-J}^J d_{m,m'}^J(\beta) |m\rangle$$

Under a rotation,
the spin projection m changes

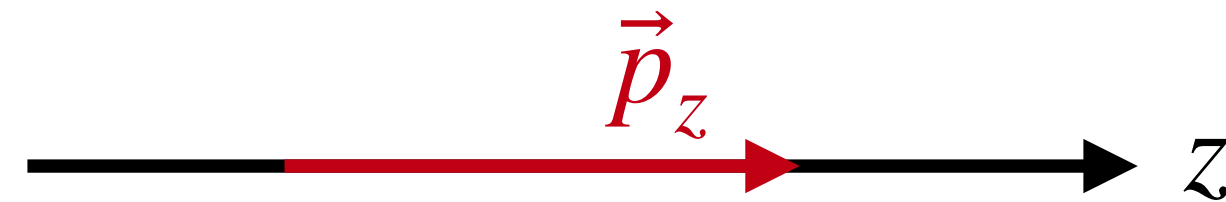


Boosted States

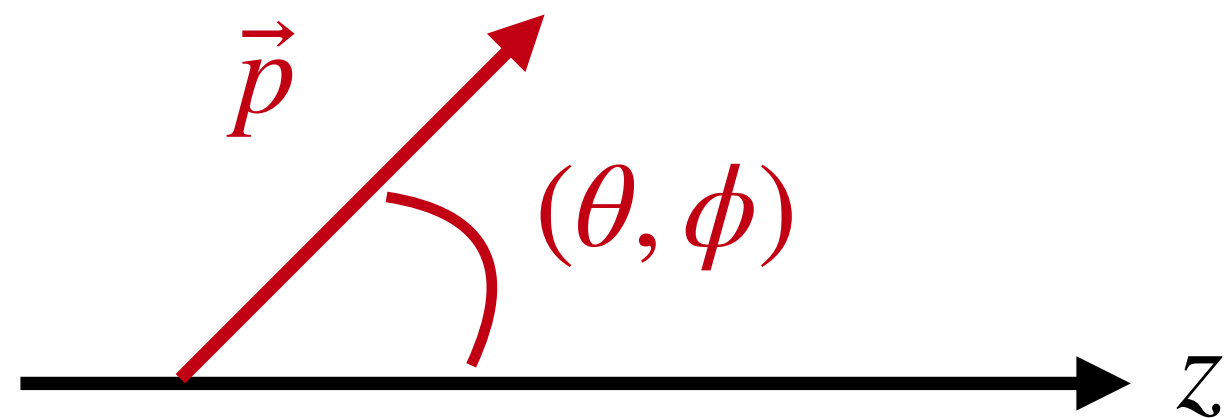
How do we boost from $\vec{0}$ to \vec{p} ?



$$|\vec{p}_z\rangle = L_z(p) |\vec{0}\rangle$$



$$|\vec{p}\rangle = R(\phi, \theta, 0) |\vec{p}_z\rangle$$



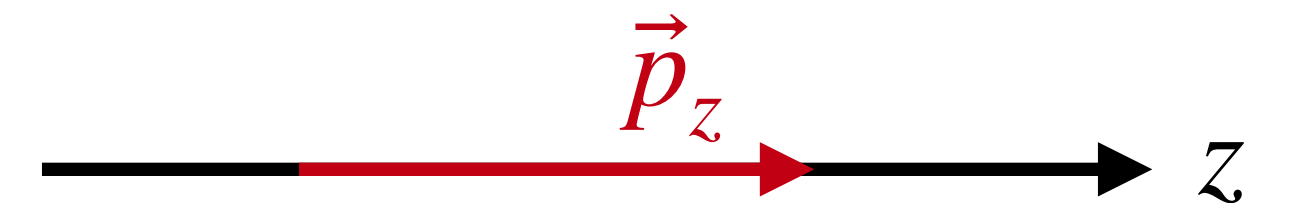
$$|\vec{p}\rangle = R(\phi, \theta, 0) L_z(p) |\vec{0}\rangle$$



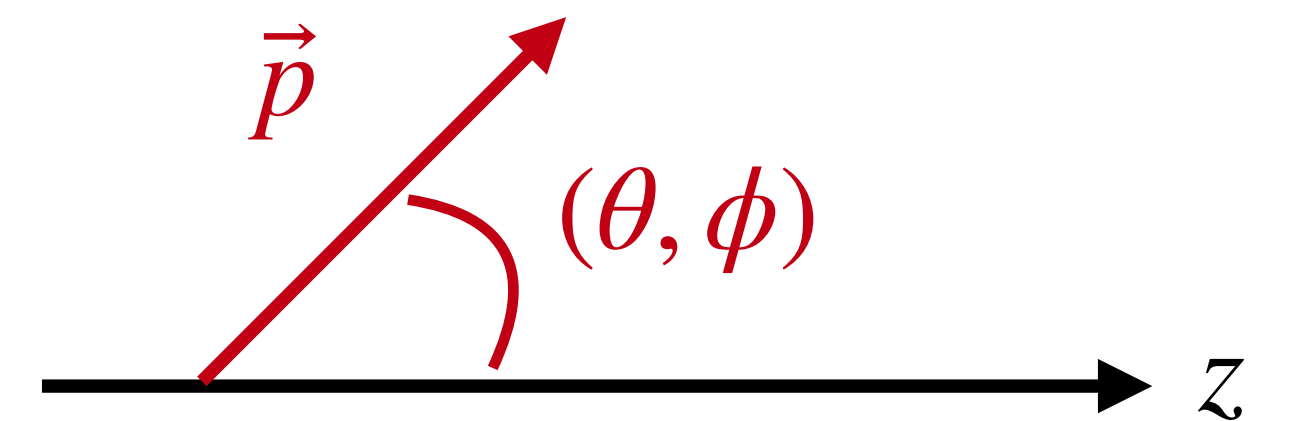
$$|\vec{0}\rangle = R^{-1}(\phi, \theta, 0) |\vec{0}\rangle$$



$$|\vec{p}_z\rangle = L_z(p) |\vec{0}\rangle$$



$$|\vec{p}\rangle = R(\phi, \theta, 0) |\vec{p}_z\rangle$$



$$|\vec{p}\rangle = R(\phi, \theta, 0) L_z(p) R^{-1}(\phi, \theta, 0) |\vec{0}\rangle$$

Rotation of States

Helicity states

$$|\vec{p}, \lambda\rangle = R(\Omega)L_z(p)|\vec{0}, \lambda\rangle$$

$$\begin{aligned}R(\Omega')|\vec{p}, \lambda\rangle &= R(\Omega')R(\Omega)L_z(p)|\vec{0}, \lambda\rangle \\ &= R(\Omega' \cdot \Omega)L_z(p)|\vec{0}, \lambda\rangle \\ &= |\vec{p}', \lambda\rangle \quad \text{With } \vec{p}' = R(\Omega')\vec{p}\end{aligned}$$

Under a rotation, the helicity is conserved

Helicity is the spin projection on \vec{p}

Canonical states

$$|\vec{p}, m\rangle = R(\Omega)L_z(p)R^{-1}(\Omega)|\vec{0}, m\rangle$$

$$\begin{aligned}R(\Omega')|\vec{p}, m\rangle &= R(\Omega')R(\Omega)L_z(p)R^{-1}(\Omega)|\vec{0}, m\rangle \\ &= R(\Omega' \cdot \Omega)L_z(p)R^{-1}(\Omega' \cdot \Omega)R(\Omega')|\vec{0}, m\rangle \\ &= \sum_{m'} D_{m',m}^s(\Omega')|\vec{p}', m'\rangle \\ &\quad \text{With } \vec{p}' = R(\Omega')\vec{p}\end{aligned}$$

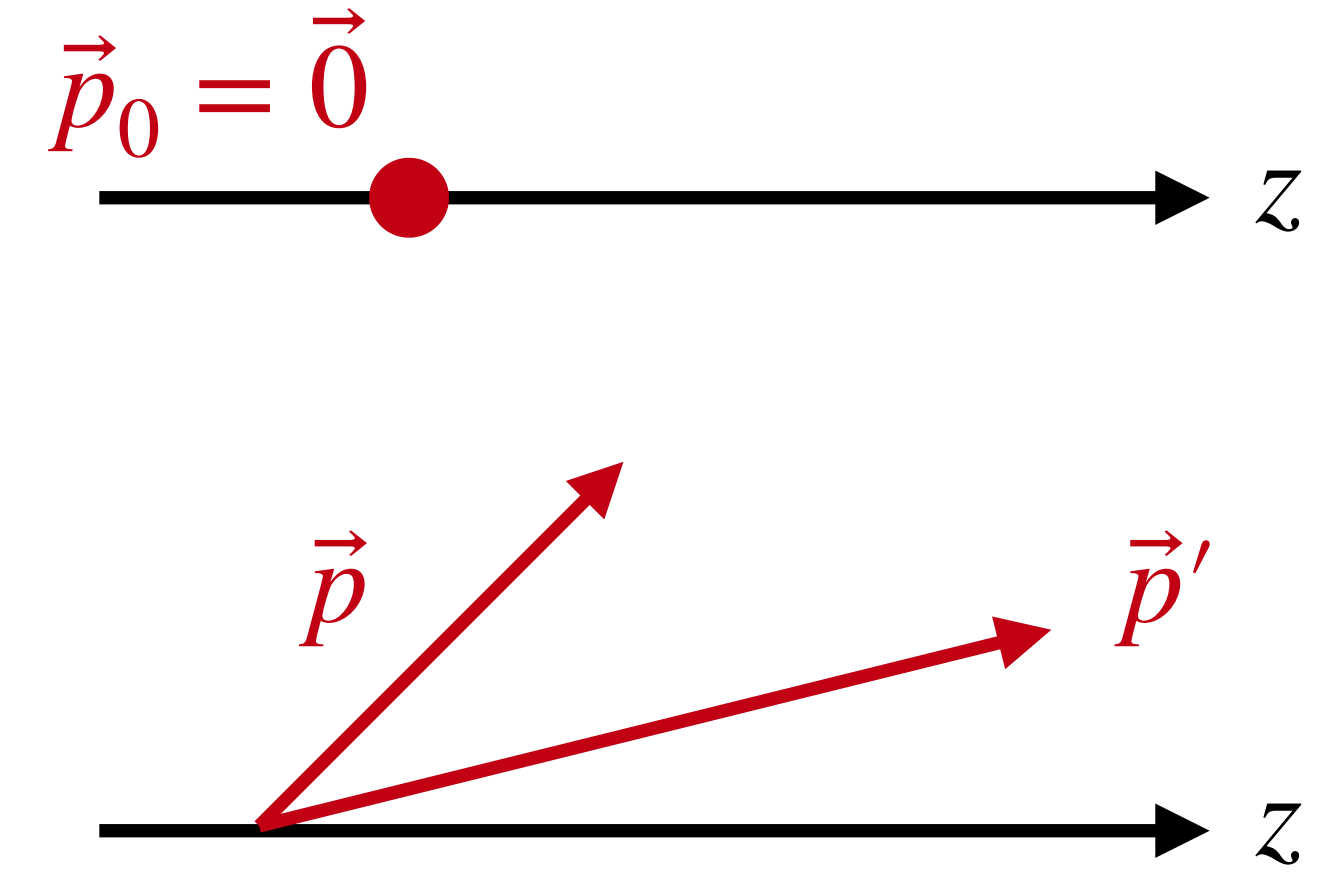
Under a rotation, the spin projection changes

Boosting States

Consider two different frames

$$|\vec{p}\rangle = B(\vec{p} \leftarrow \vec{0}) |\vec{0}\rangle \quad |\vec{p}'\rangle = B(\vec{p}' \leftarrow \vec{0}) |\vec{0}\rangle$$

Related by a Lorentz transf. $\Lambda(\vec{p} \leftarrow \vec{p}') |\vec{p}'\rangle = |\vec{p}\rangle$



Consider the cycle $\vec{0} \rightarrow \vec{p}' \rightarrow \vec{p} \rightarrow \vec{0}$

$$B(\vec{0} \leftarrow \vec{p}) \Lambda(\vec{p} \leftarrow \vec{p}') B(\vec{p}' \leftarrow \vec{0}) = R(\Omega_W)$$

The rest states $|\vec{0}\rangle$ can differ by a rotation!

$$\Lambda(\vec{p} \leftarrow \vec{p}') B(\vec{p}' \leftarrow \vec{0}) = B(\vec{p} \leftarrow \vec{0}) R(\Omega_W)$$

So the spin projection rotates as well!

$$\Lambda(\vec{p} \leftarrow \vec{p}') |\vec{p}', \lambda\rangle = \sum_{\lambda'} D_{\lambda', \lambda}^s(\Omega_W) |\vec{p}, \lambda'\rangle$$

Ω_W is called the Wigner rotation

Wigner Rotations

Under a Lorentz boost, states undergo a Wigner rotation

$$\Lambda(\vec{p} \leftarrow \vec{p}') |\vec{p}', \lambda\rangle = \sum_{\lambda'} D_{\lambda', \lambda}^s(\Omega_W) |\vec{p}, \lambda'\rangle$$

The Wigner rotation is determined by the boosting chain

$$R(\Omega_W) = B(\vec{0} \leftarrow \vec{p}) \Lambda(\vec{p} \leftarrow \vec{p}') B(\vec{p}' \leftarrow \vec{0}) \quad \Omega_W \text{ different for helicity and canonical}$$

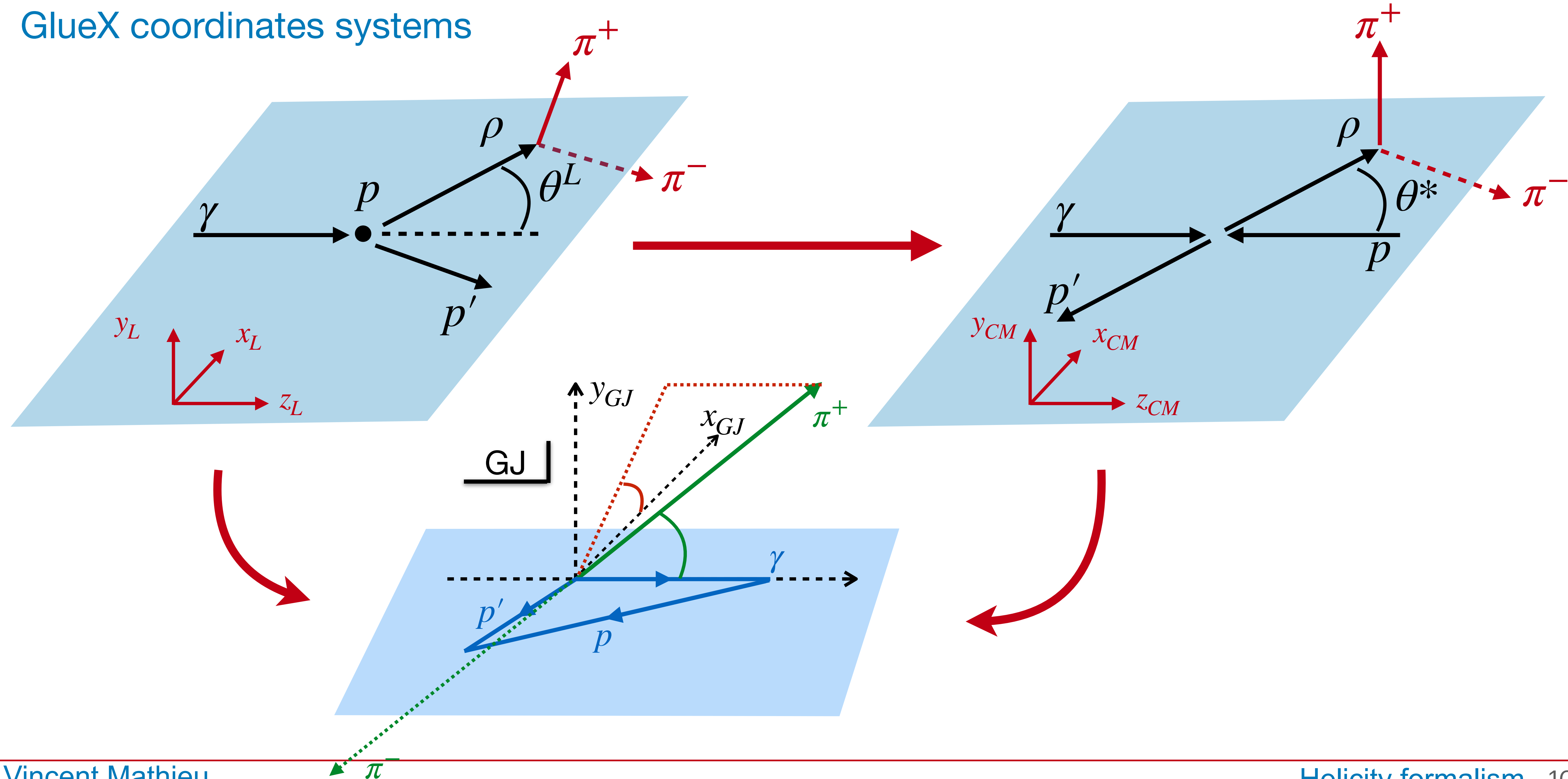
The boosts depend on the quantification!

Helicity states $B(\vec{p} \leftarrow \vec{0}) = R(\Omega_p) L_z(p)$

Canonical states $B(\vec{p} \leftarrow \vec{0}) = R(\Omega_p) L_z(p) R^{-1}(\Omega_p)$

A Concrete Example

GlueX coordinates systems



Transformations of States

Helicity states

$$|\vec{p}, \lambda\rangle = R(\Omega)L_z(p)|\vec{0}, \lambda\rangle$$

$$R(\Omega')|\vec{p}, \lambda\rangle = |\vec{p}', \lambda\rangle \quad \text{With } \vec{p}' = R(\Omega')\vec{p}$$

Under a rotation, the helicity is conserved

$$\Lambda(\vec{p} \leftarrow \vec{p}')|\vec{p}', \lambda\rangle = \sum_{\lambda'} D_{\lambda', \lambda}^s(\Omega_W)|\vec{p}, \lambda'\rangle$$

If \vec{p}' and \vec{p} are parallel, the helicity is conserved

Canonical states

$$|\vec{p}, m\rangle = R(\Omega)L_z(p)R^{-1}(\Omega)|\vec{0}, m\rangle$$

$$R(\Omega')|\vec{p}, m\rangle = \sum_{m'} D_{m', m}^s(\Omega')|\vec{p}', m'\rangle$$

Under a rotation, the spin projection changes

$$\Lambda(\vec{p} \leftarrow \vec{p}')|\vec{p}', m\rangle = \sum_{m'} D_{m', m}^s(\Omega_W^c)|\vec{p}, m'\rangle$$

In the NR limit, the helicity is conserved

Transformations of Helicity States

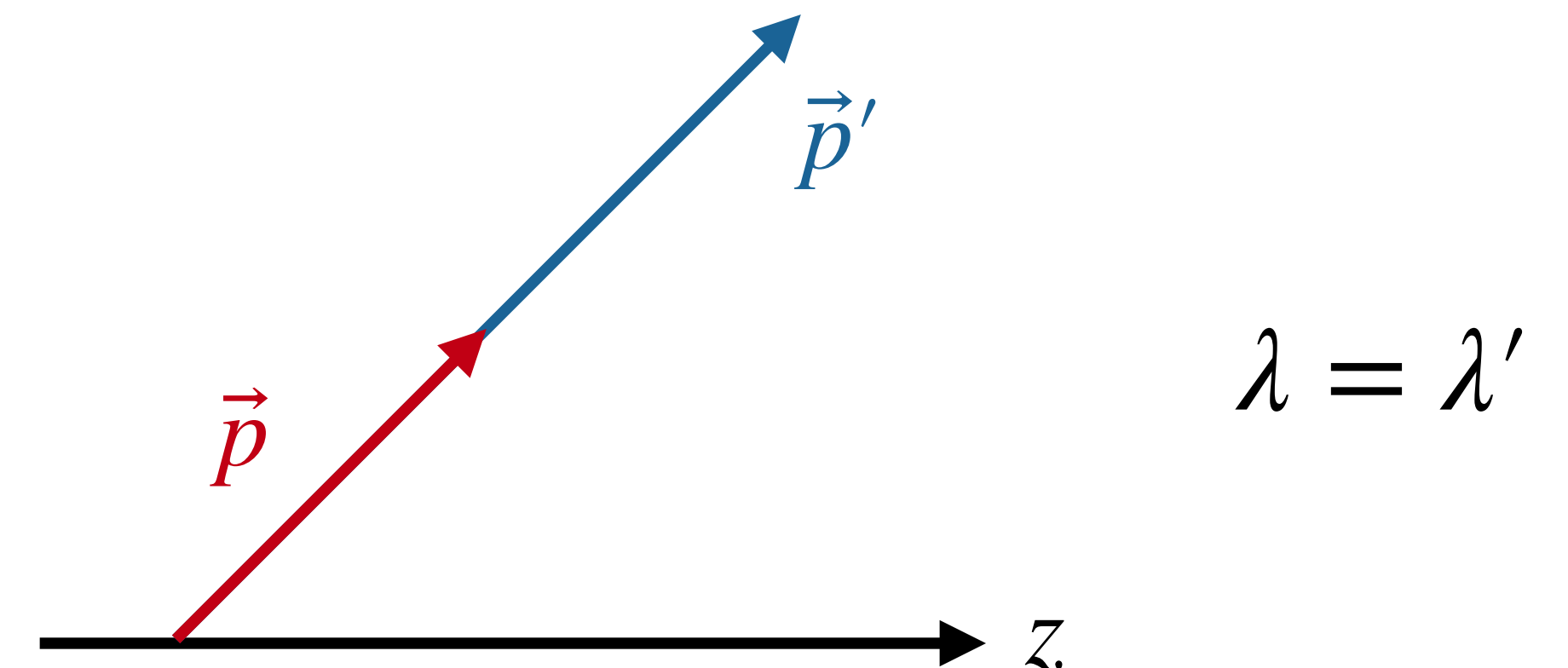
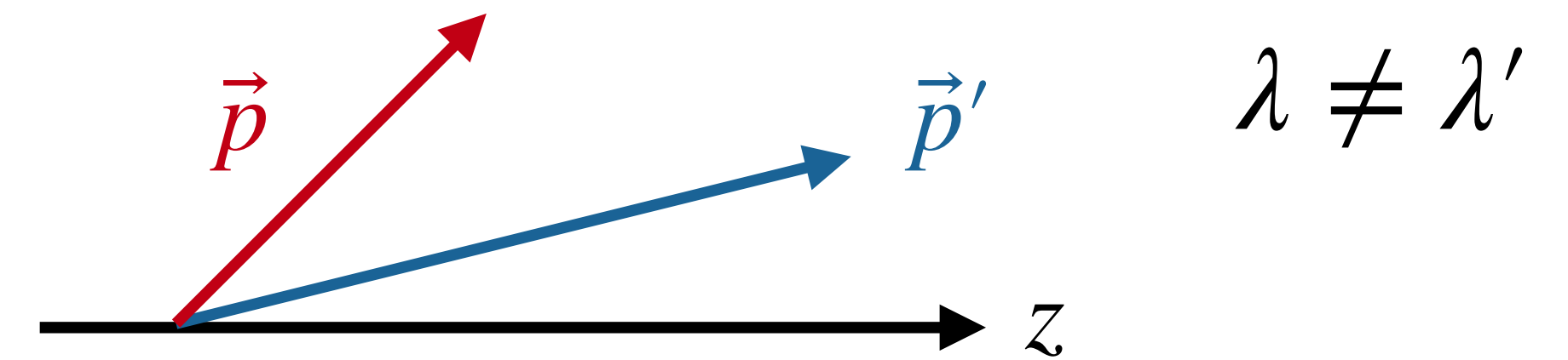
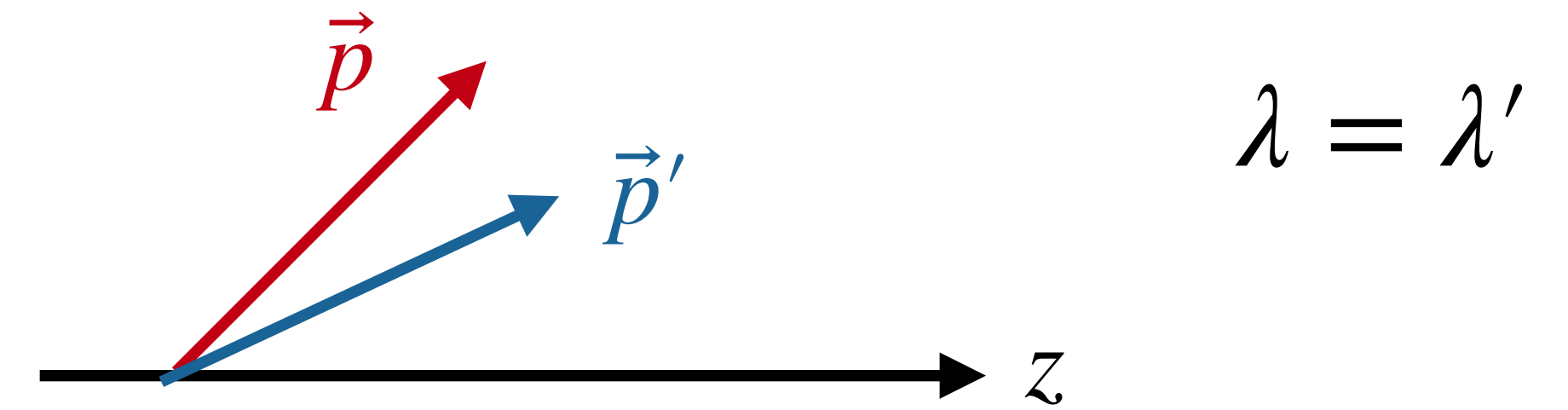
$$|\vec{p}, \lambda\rangle = R(\Omega)L_z(p)|\vec{0}, \lambda\rangle$$

$$R(\Omega')|\vec{p}, \lambda\rangle = |\vec{p}', \lambda\rangle \quad \text{With } \vec{p}' = R(\Omega')\vec{p}$$

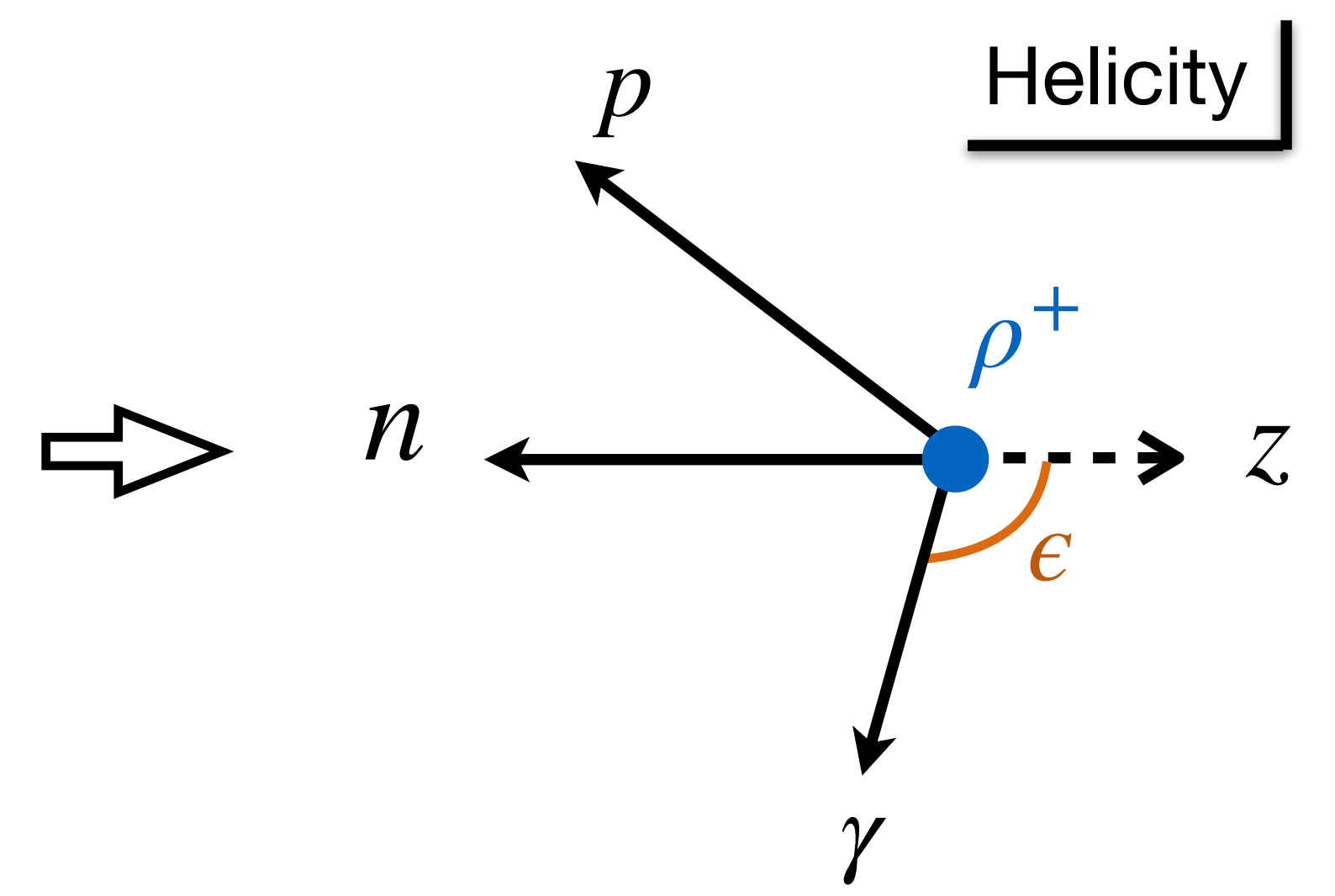
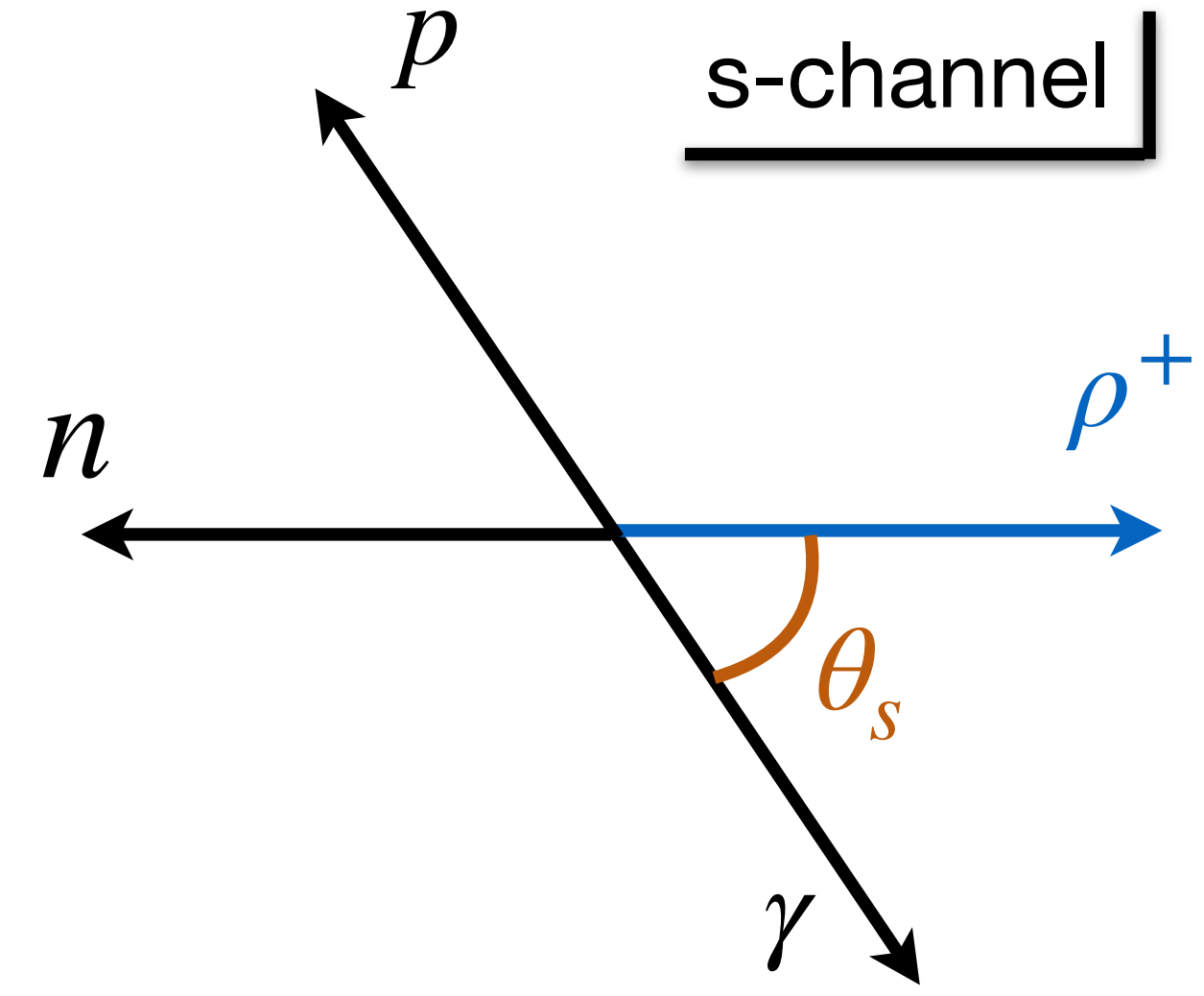
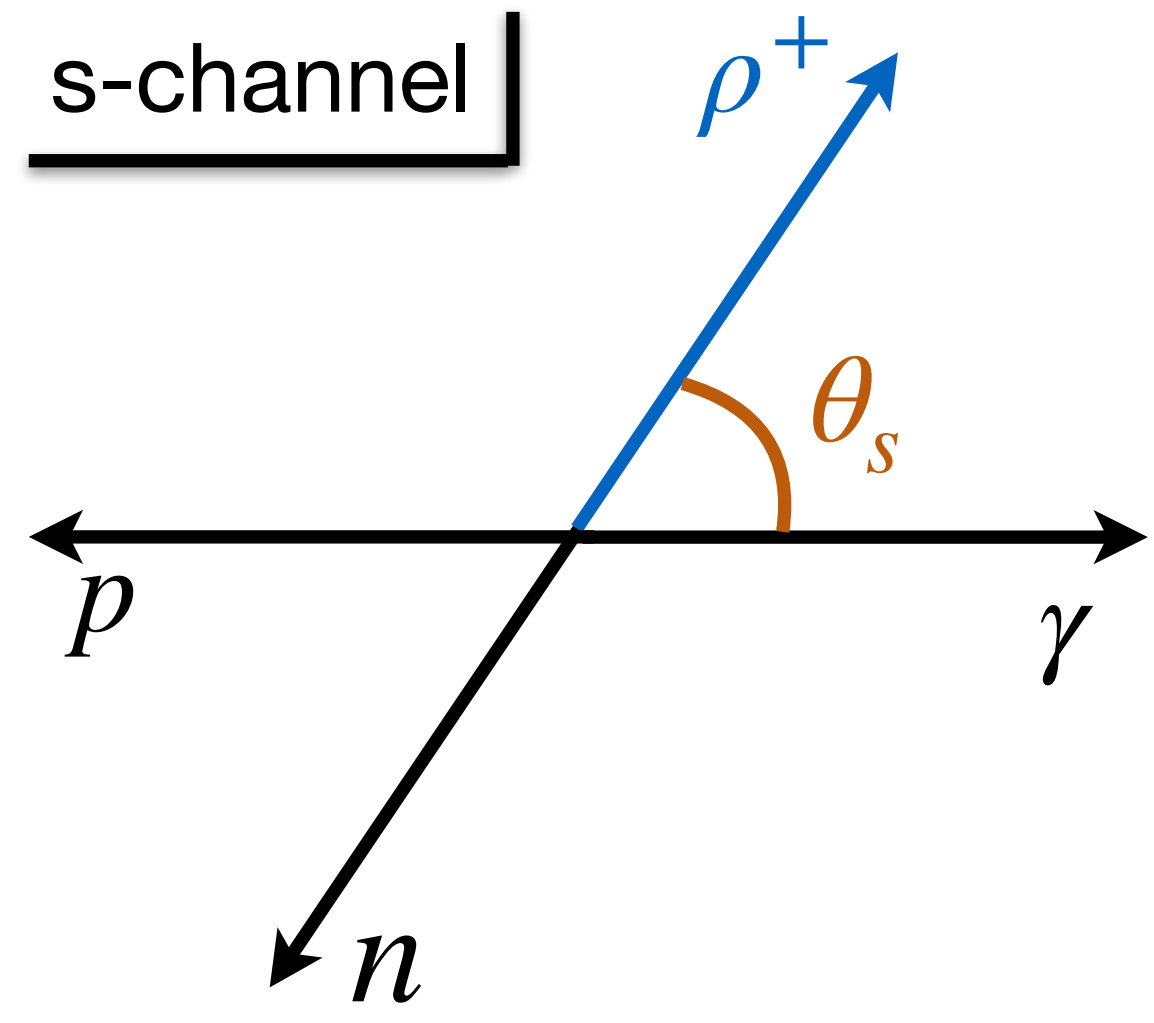
Under a rotation, the helicity is conserved

$$\Lambda(\vec{p} \leftarrow \vec{p}')|\vec{p}', \lambda\rangle = \sum_{\lambda'} D_{\lambda', \lambda}^s(\Omega_W)|\vec{p}, \lambda'\rangle$$

If \vec{p}' and \vec{p} are parallel, the helicity is conserved

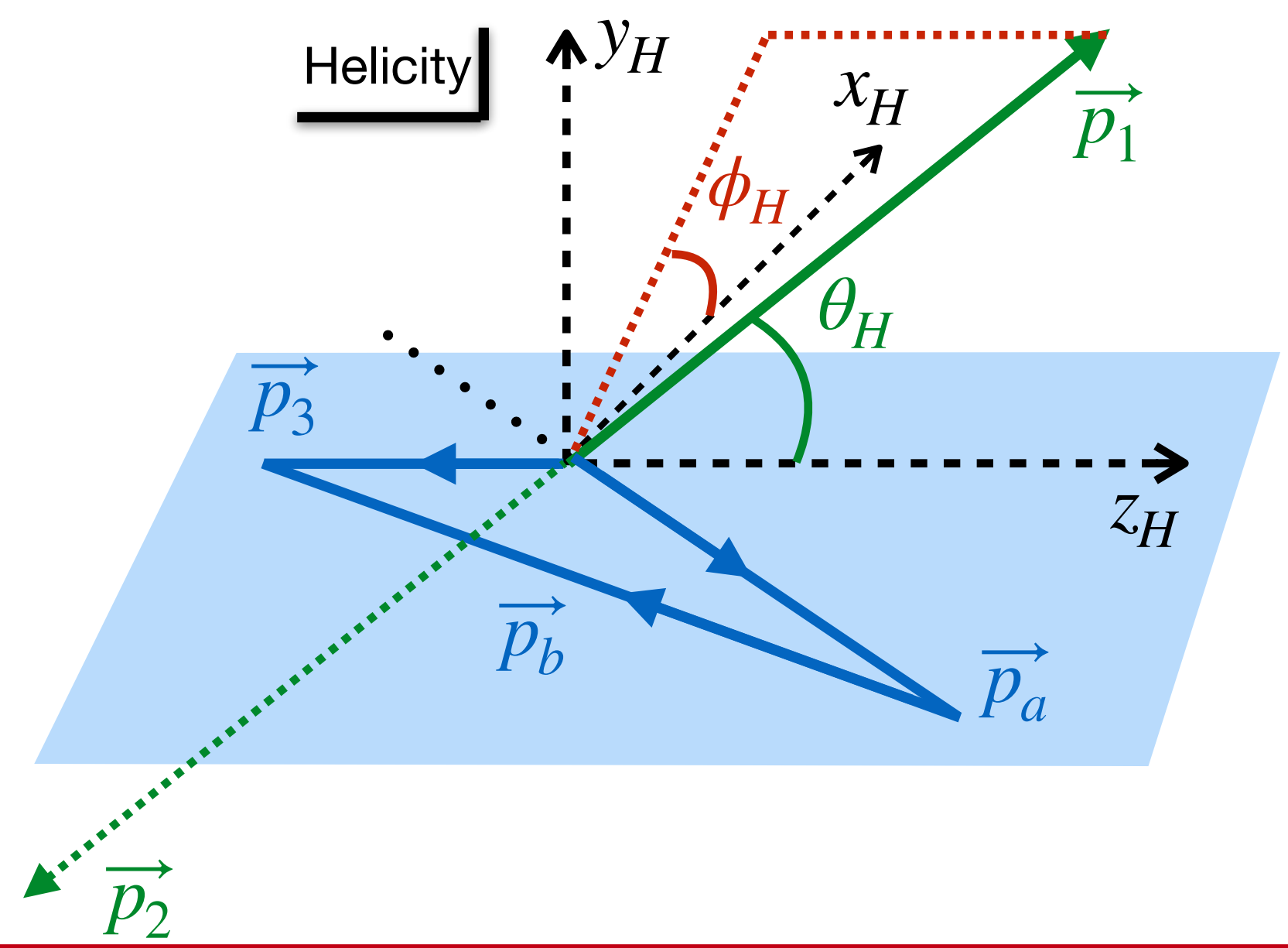


Helicity Frame



The scattering amplitude depends on helicities (which are frame dependent)

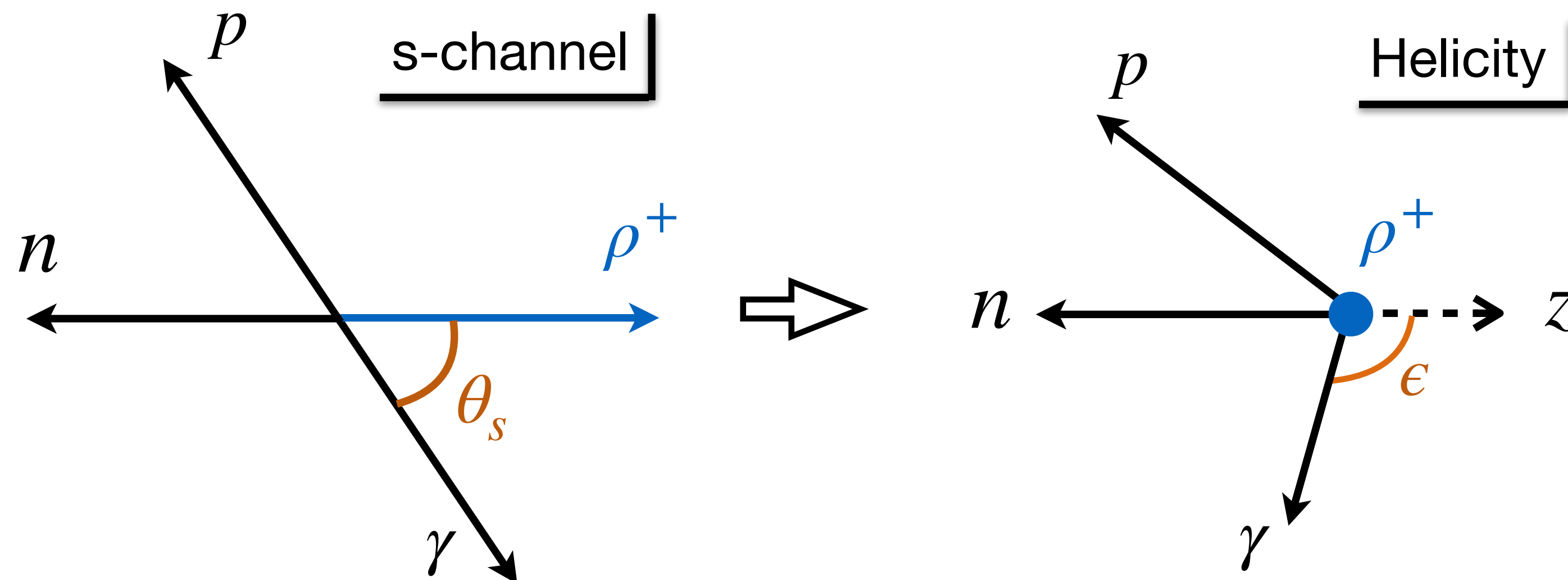
In the helicity frame, the resonance has the same helicity as in the CoM



Helicity Frame

In the helicity frame, the resonance has the same helicity as in the CoM

The scattering amplitude depends on helicity (which are frame dependent)



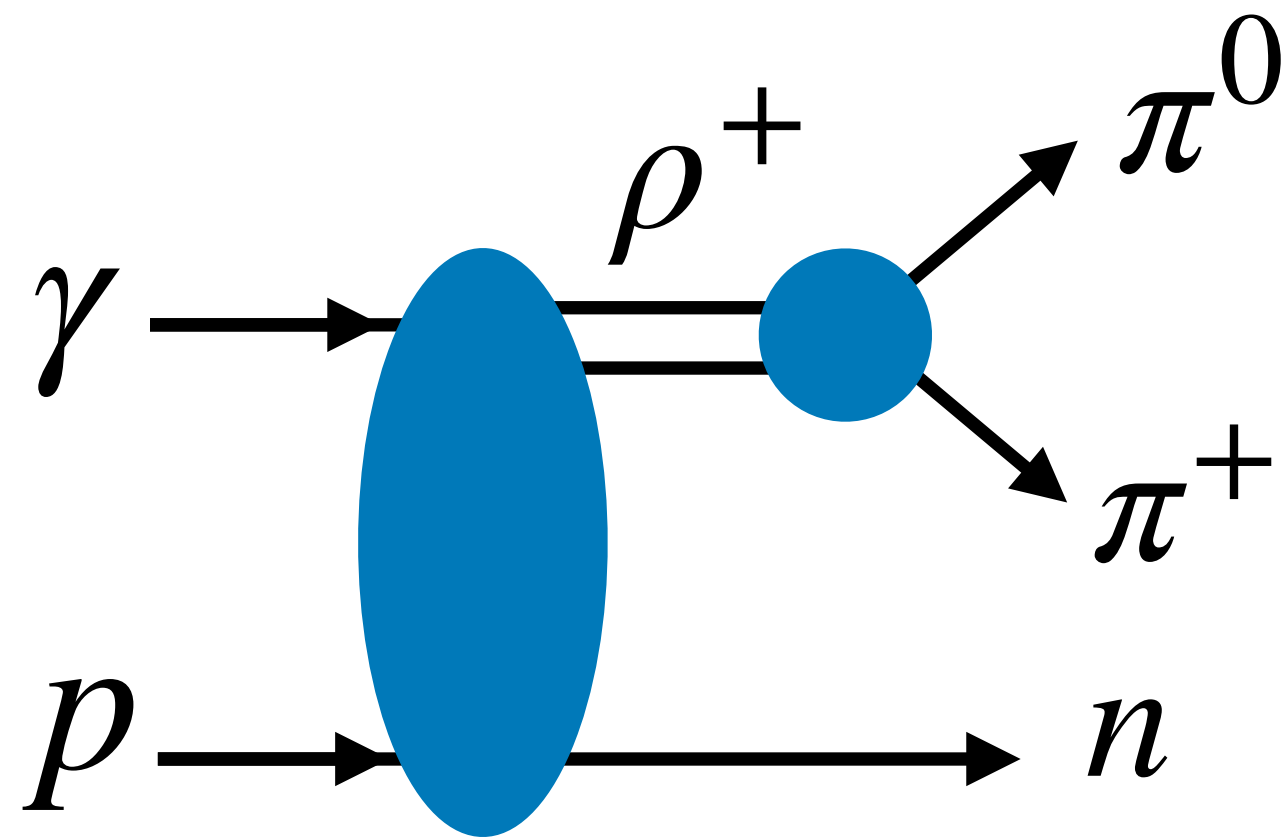
$$\sum_m A_{\lambda_\gamma, \lambda, \lambda', m} Y_m^1(\theta, \phi)$$

$$I(\theta, \phi) \propto \sum_{\lambda_\gamma, \lambda, \lambda'} \left[\sum_m A_{\lambda_\gamma, \lambda, \lambda', m} Y_m^1(\theta, \phi) \right] \left[\sum_{m'} A_{\lambda_\gamma, \lambda, \lambda', m'} Y_{m'}^1(\theta, \phi) \right]^* = \sum_{m, m'} \rho_{m, m'} Y_m^1(\theta, \phi) Y_{m'}^{1*}(\theta, \phi)$$

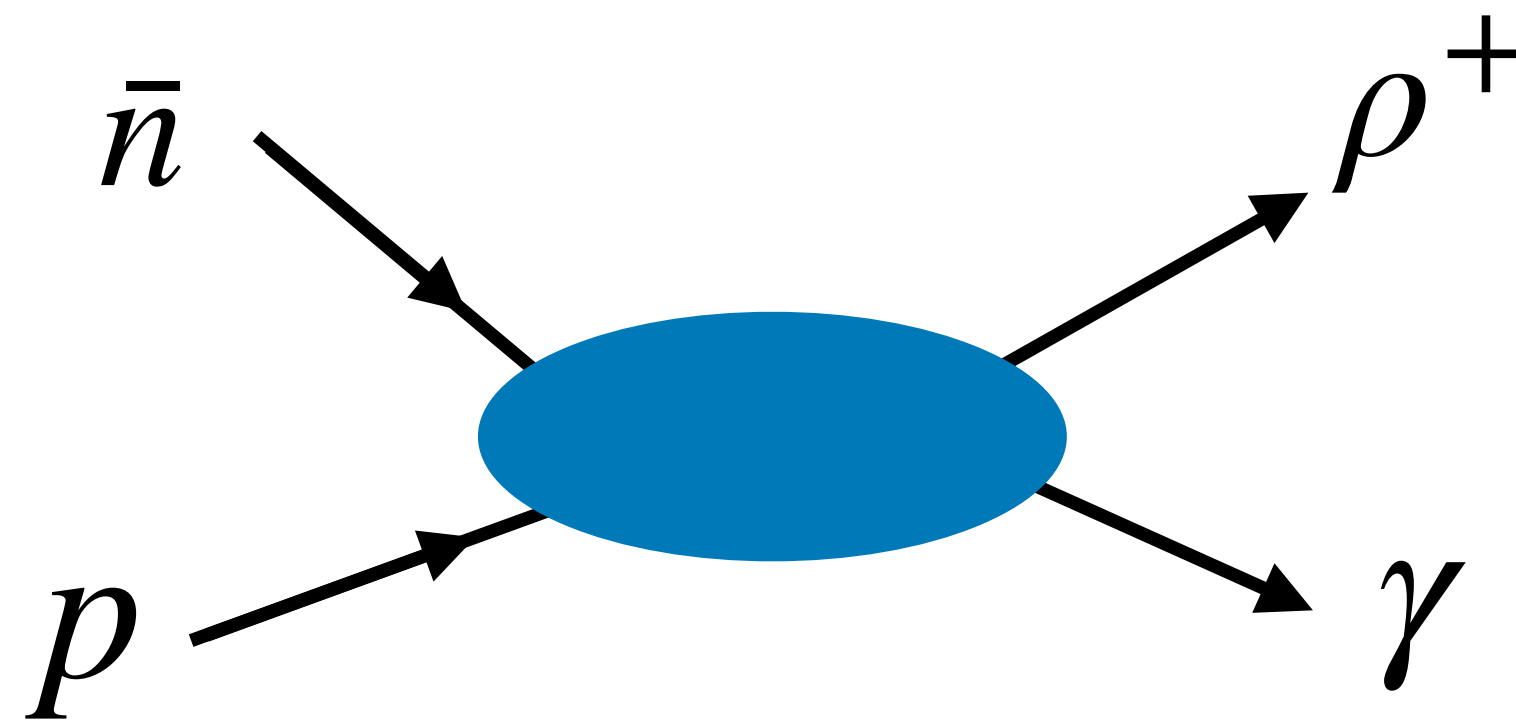
With $\rho_{m, m'} = \sum_{\lambda_\gamma, \lambda, \lambda'} A_{\lambda_\gamma, \lambda, \lambda', m} (A_{\lambda_\gamma, \lambda, \lambda', m'})^*$ $\rho_{m, m'}$ is the same in the helicity frame and s-channel!

Gottfried-Jackson frame

Consider the reaction

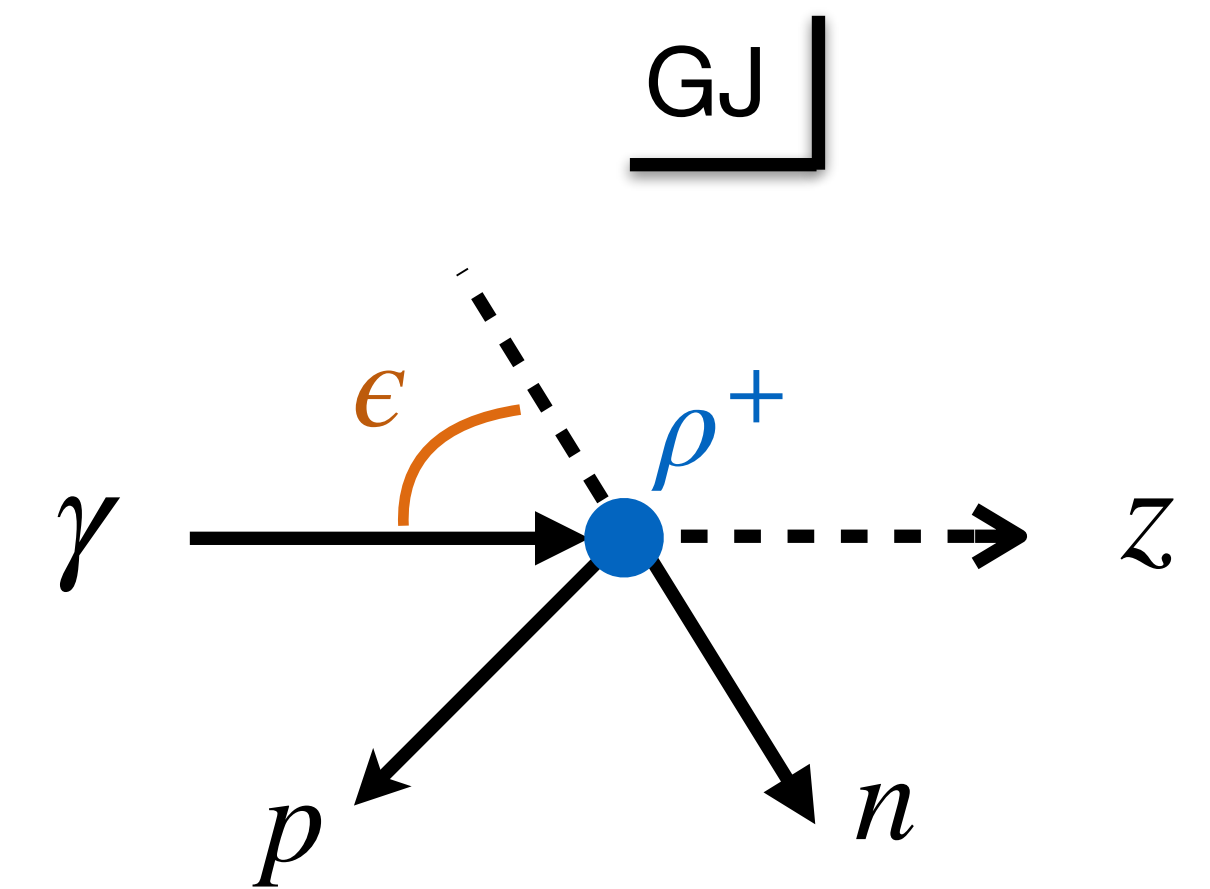
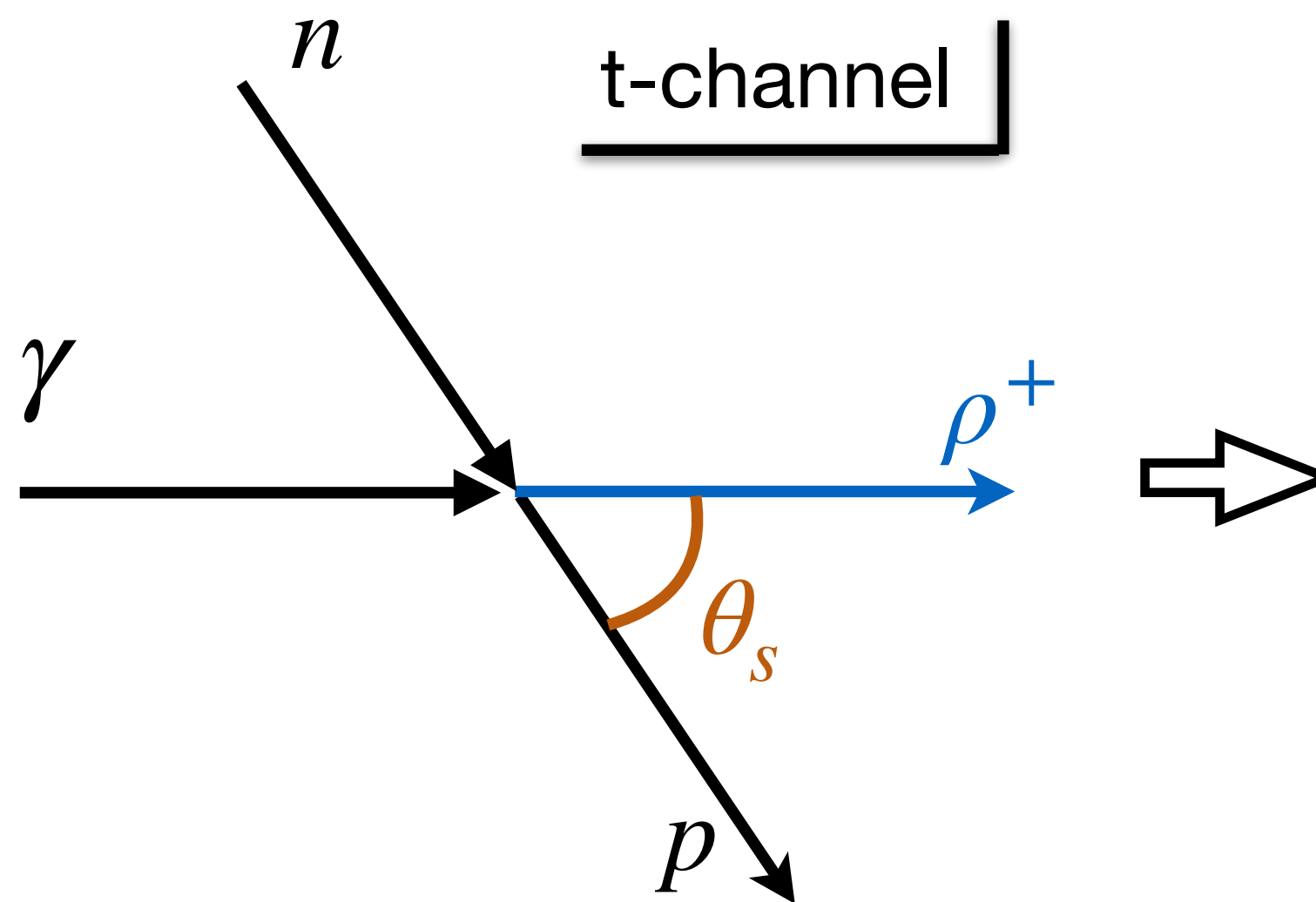


The crossed reaction is



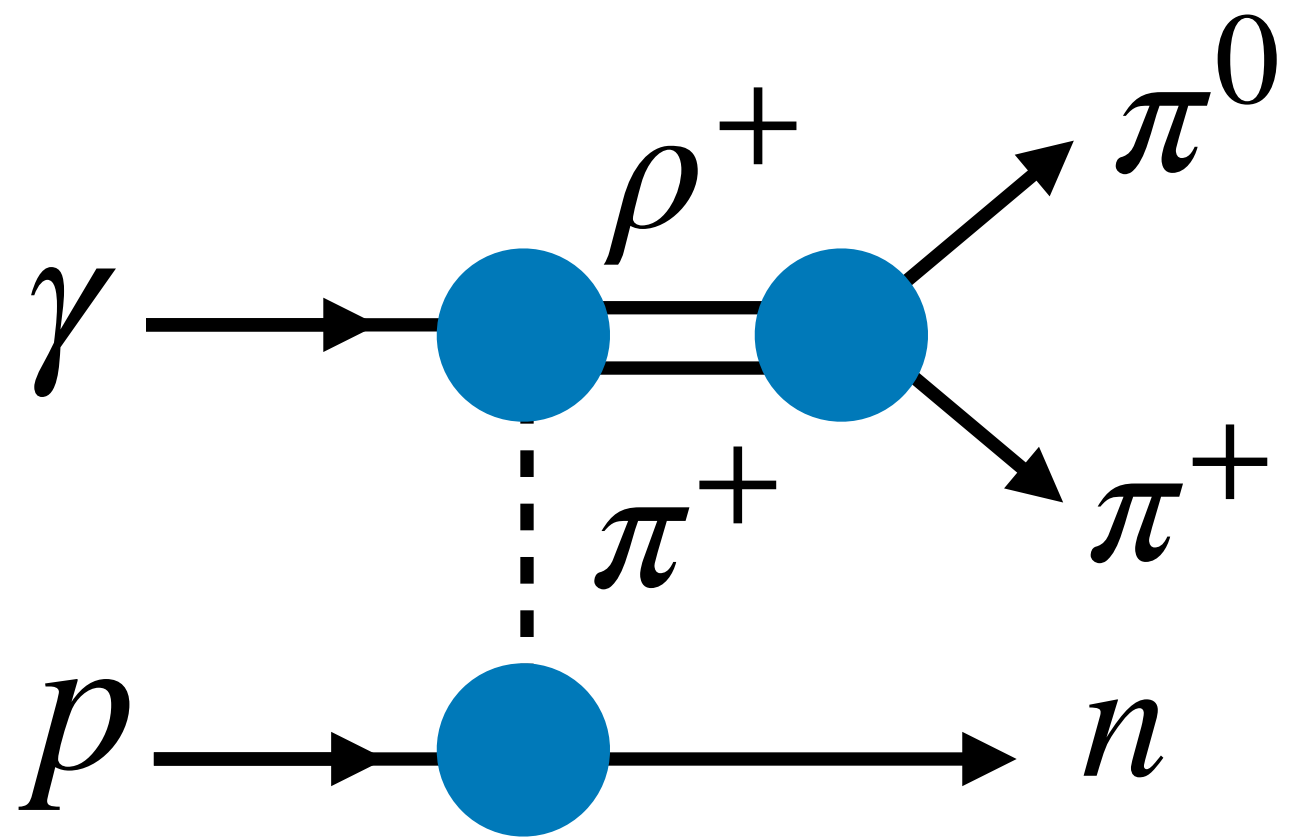
The t-channel is the CoM of the crossed reaction

In the GJ frame, the resonance has the same helicity as in the t-channel

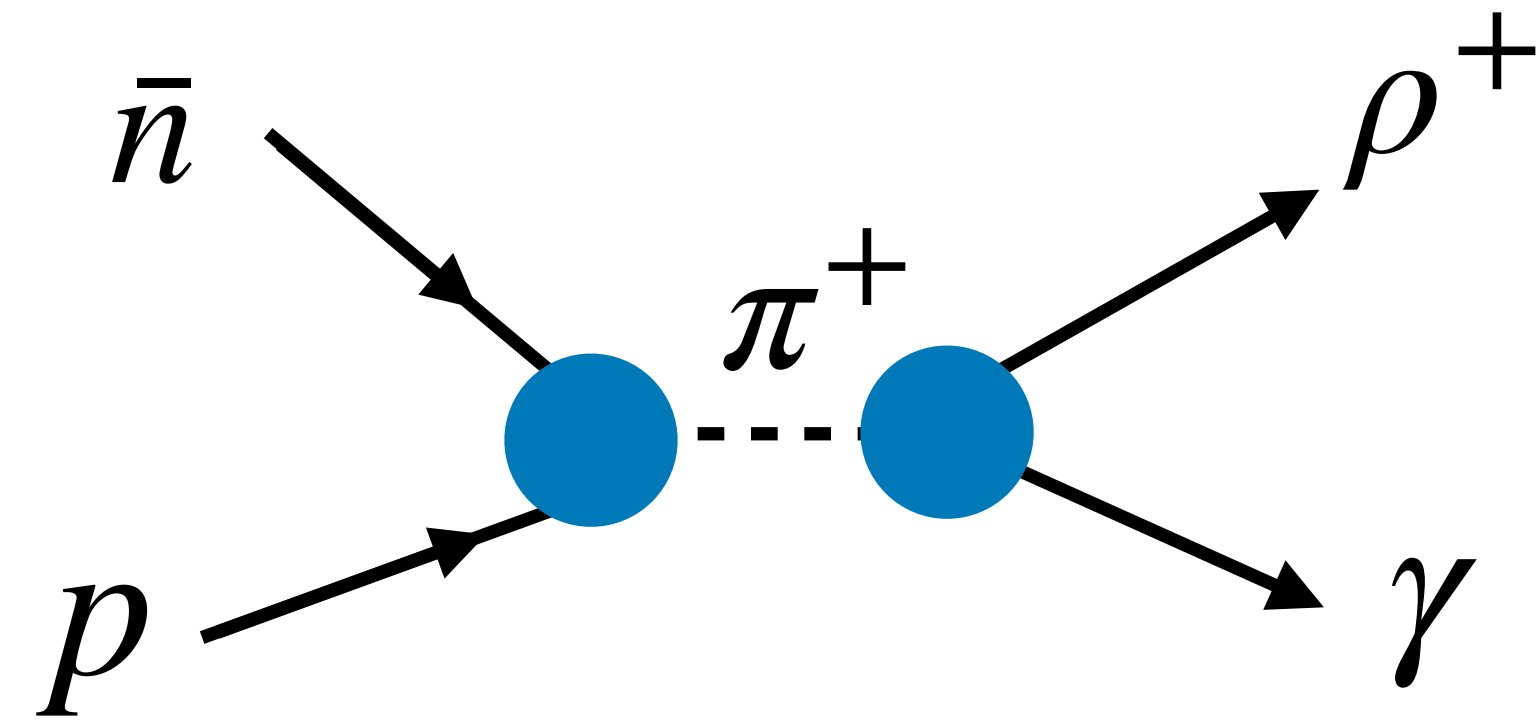


T-channel frame

Consider the reaction



The crossed reaction is

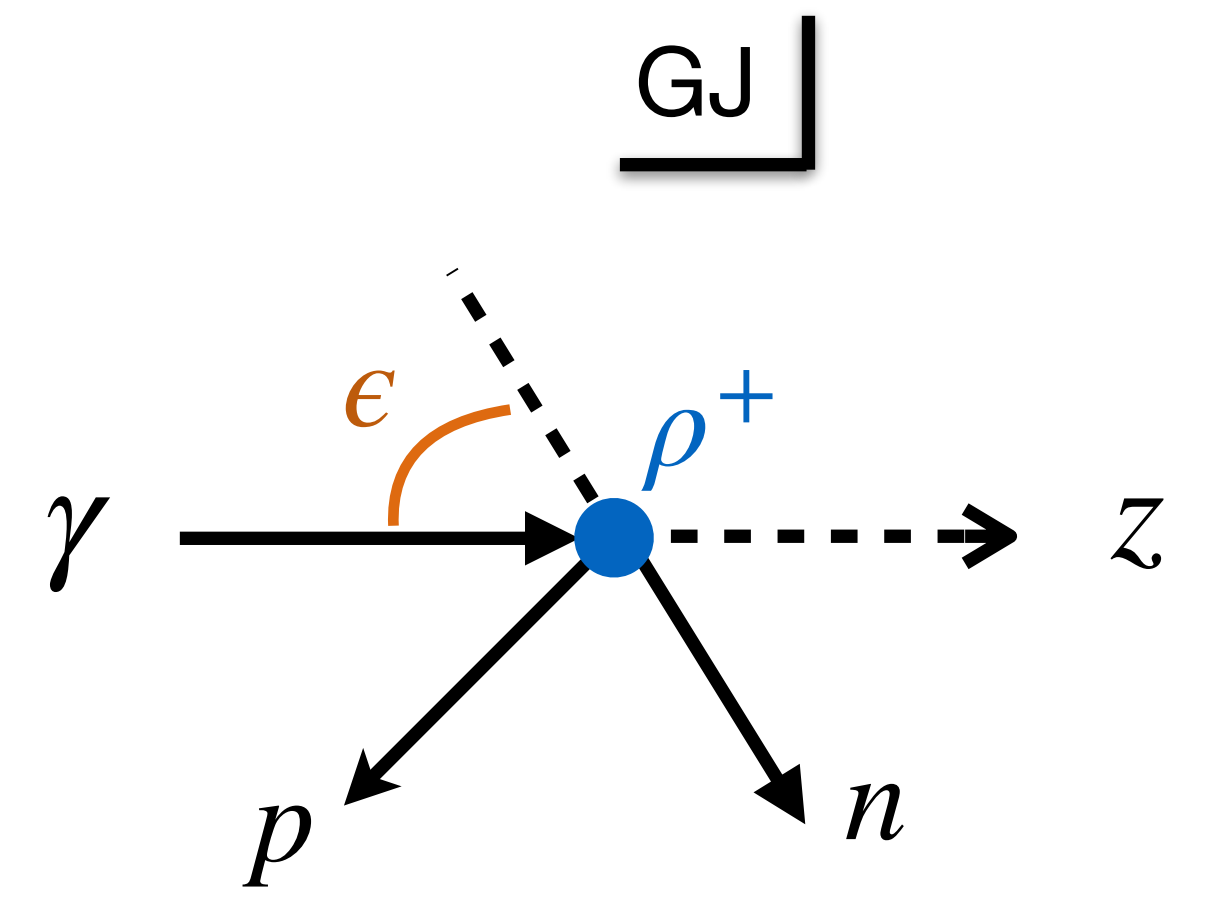
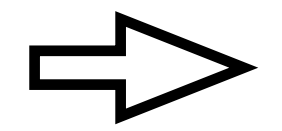
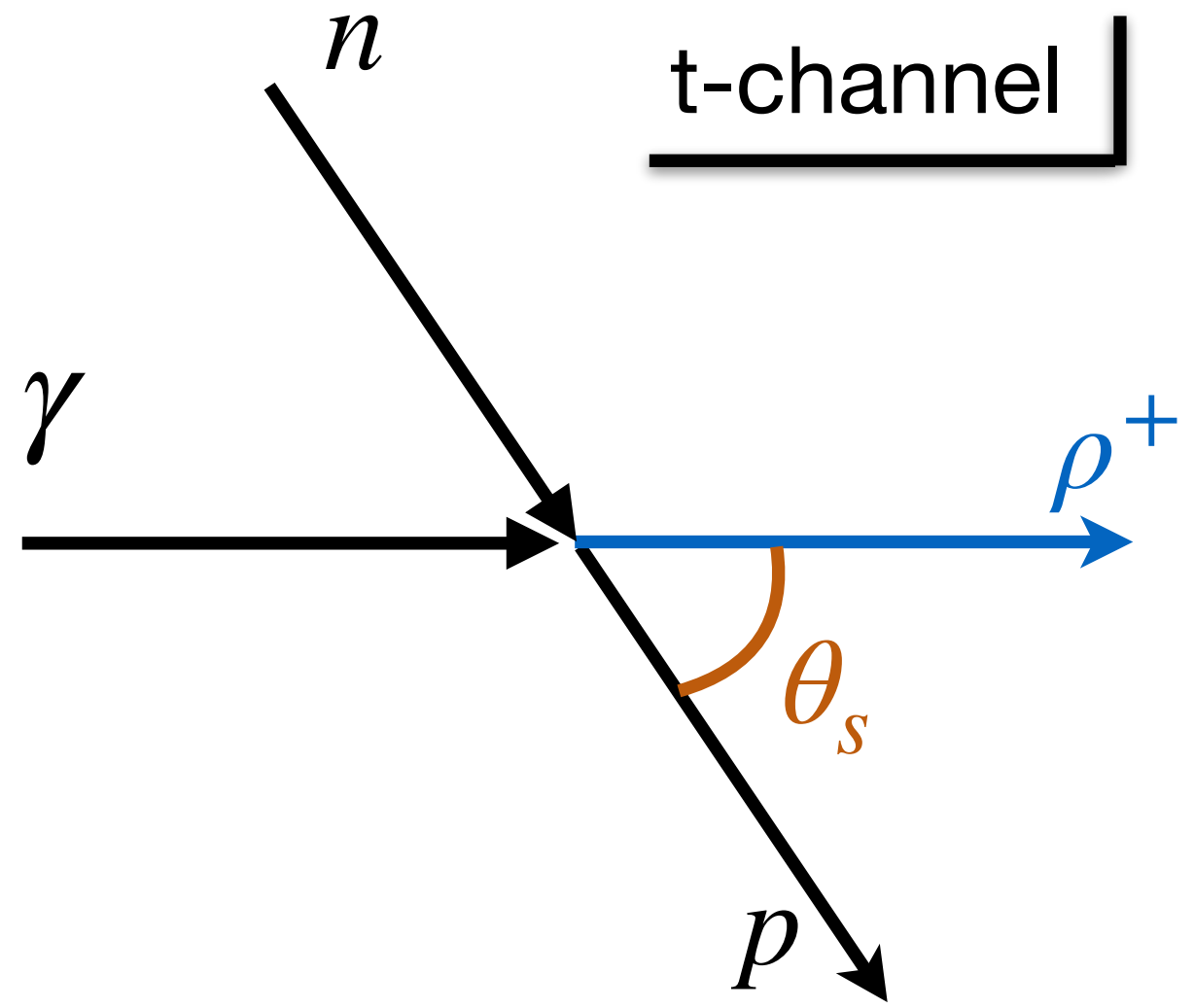


In the GJ frame, the resonance has the same helicity as in the t-channel

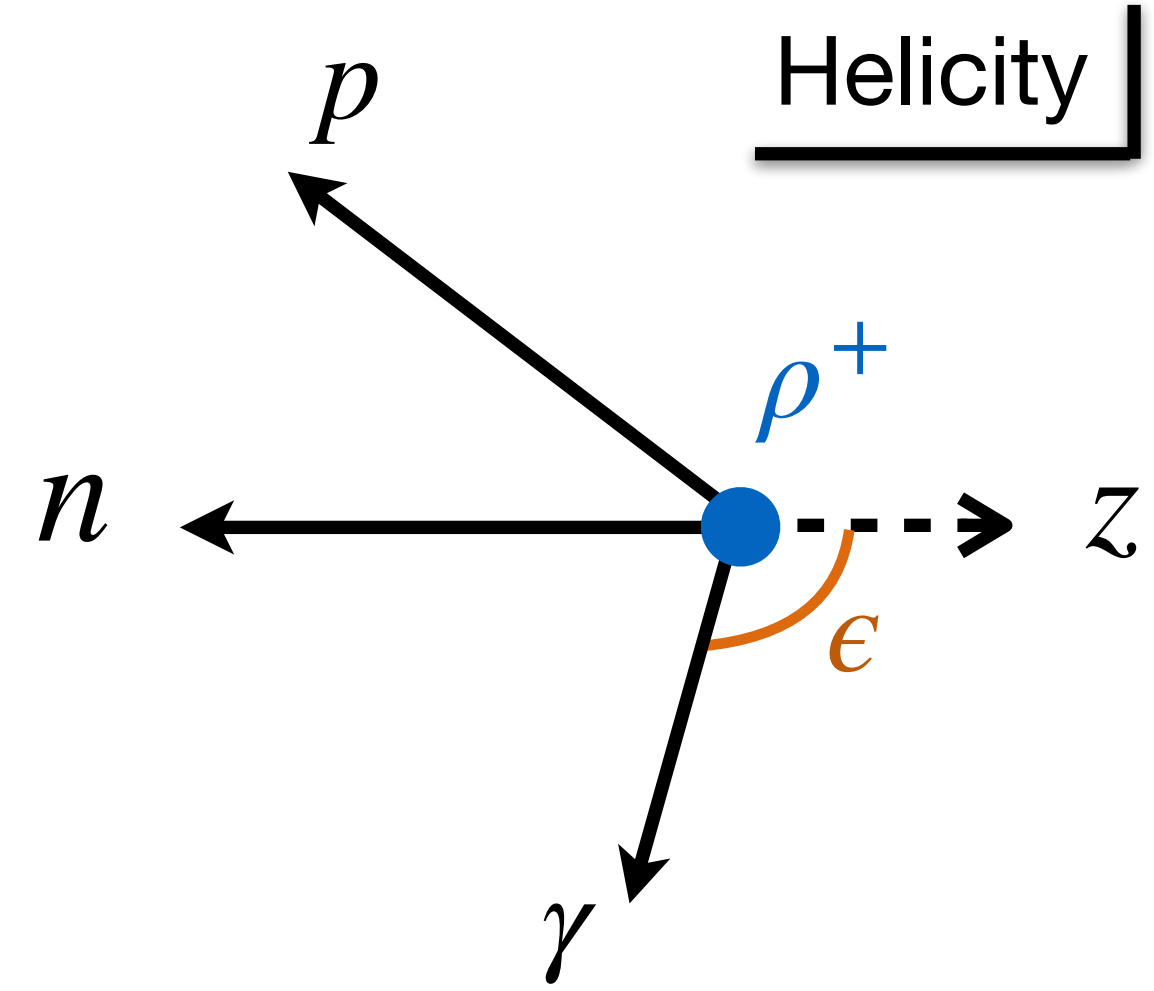
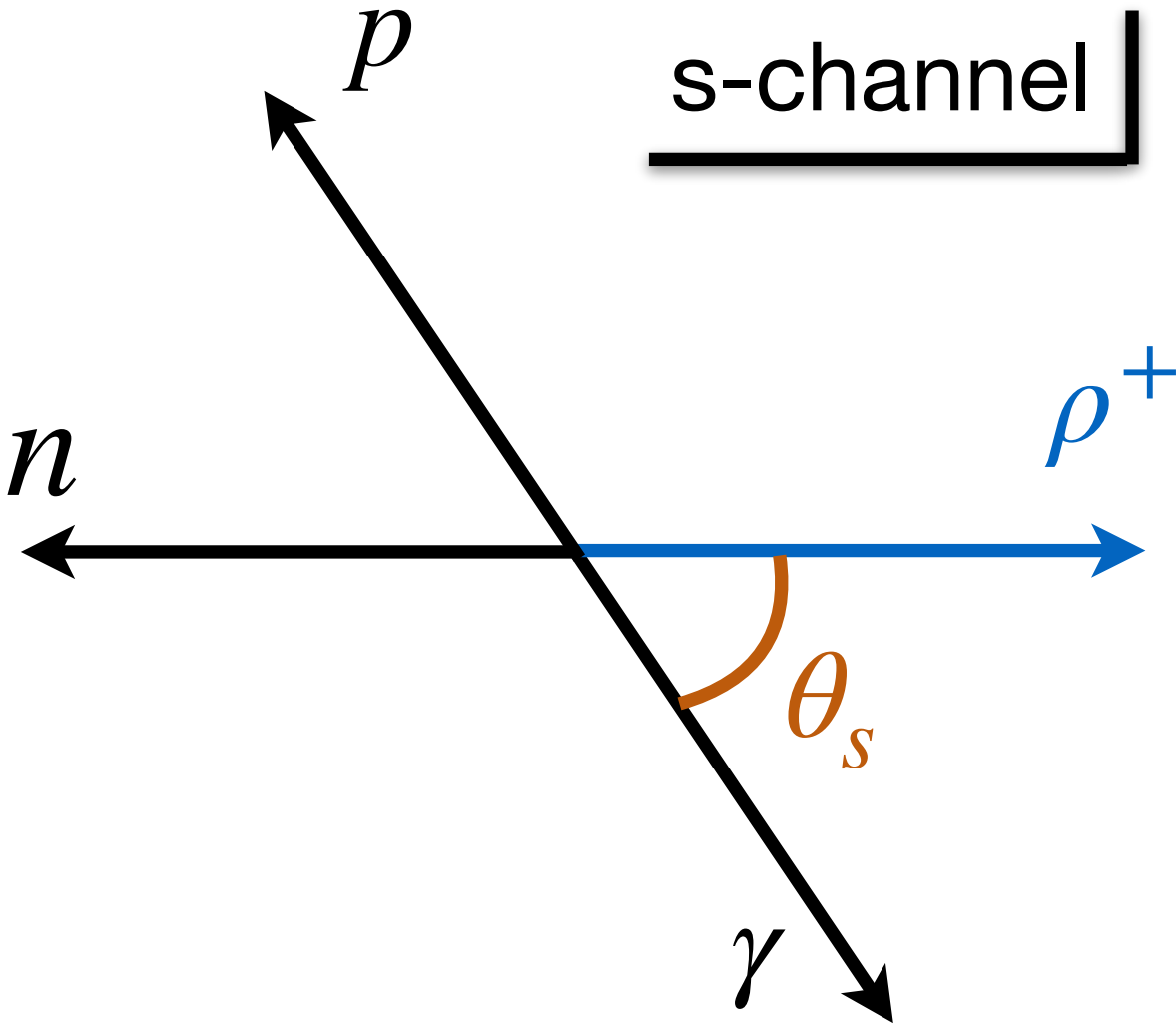
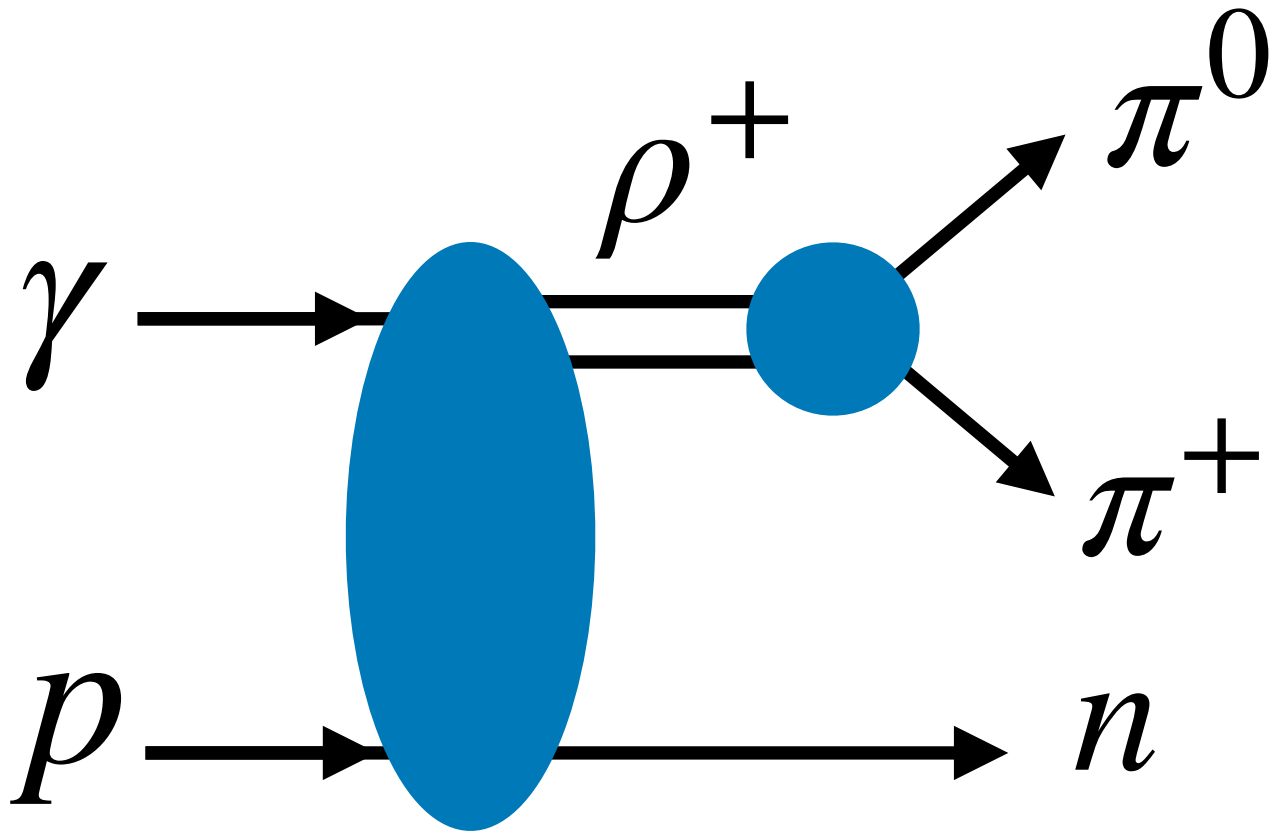
$$\rho_{m,m'}^{GJ} = \sum_{\lambda_\gamma, \lambda, \lambda'} A_{\lambda_\gamma, \lambda, \lambda', m}^{GJ} (A_{\lambda_\gamma, \lambda, \lambda', m'}^{GJ})^*$$

$$A_{\lambda_\gamma, \lambda, \lambda', m}^t \propto \delta_{\lambda, \lambda'} \delta_{\lambda_\gamma, m}$$

Only $\rho_{1,1} = \rho_{-1,-1}$ is non-zero!



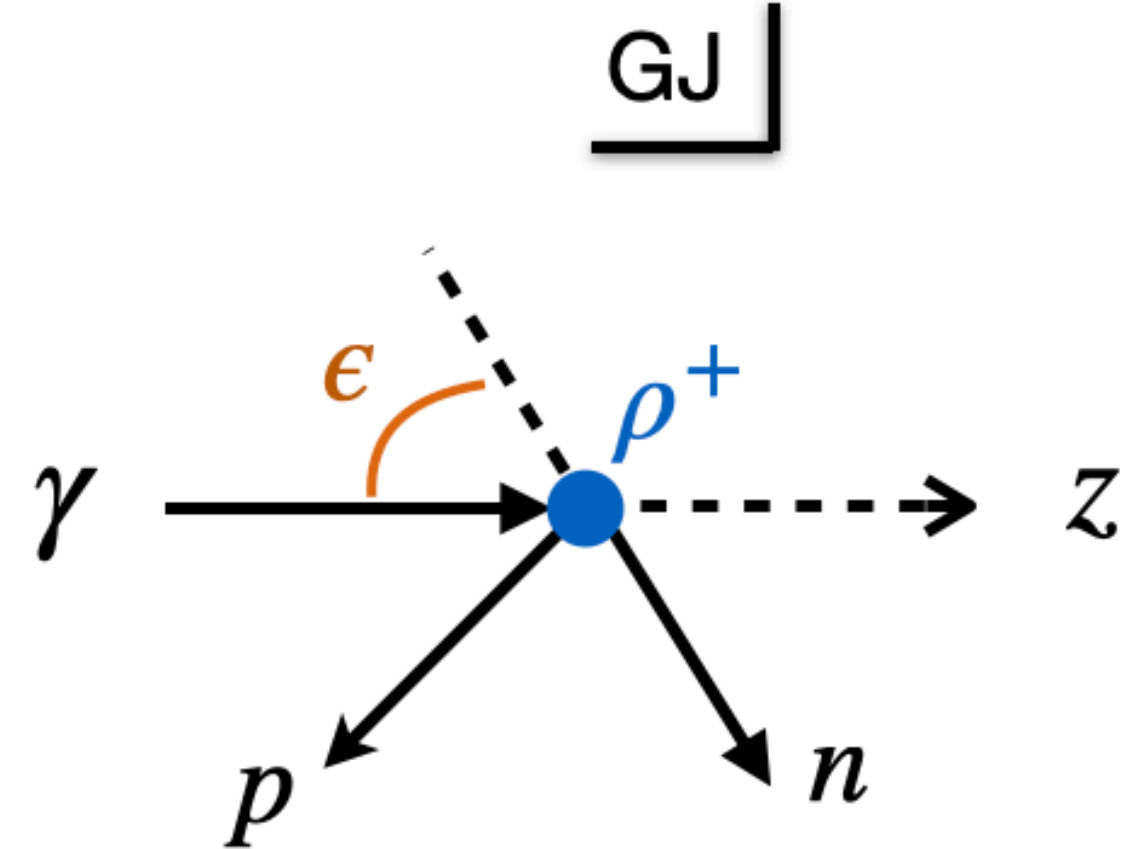
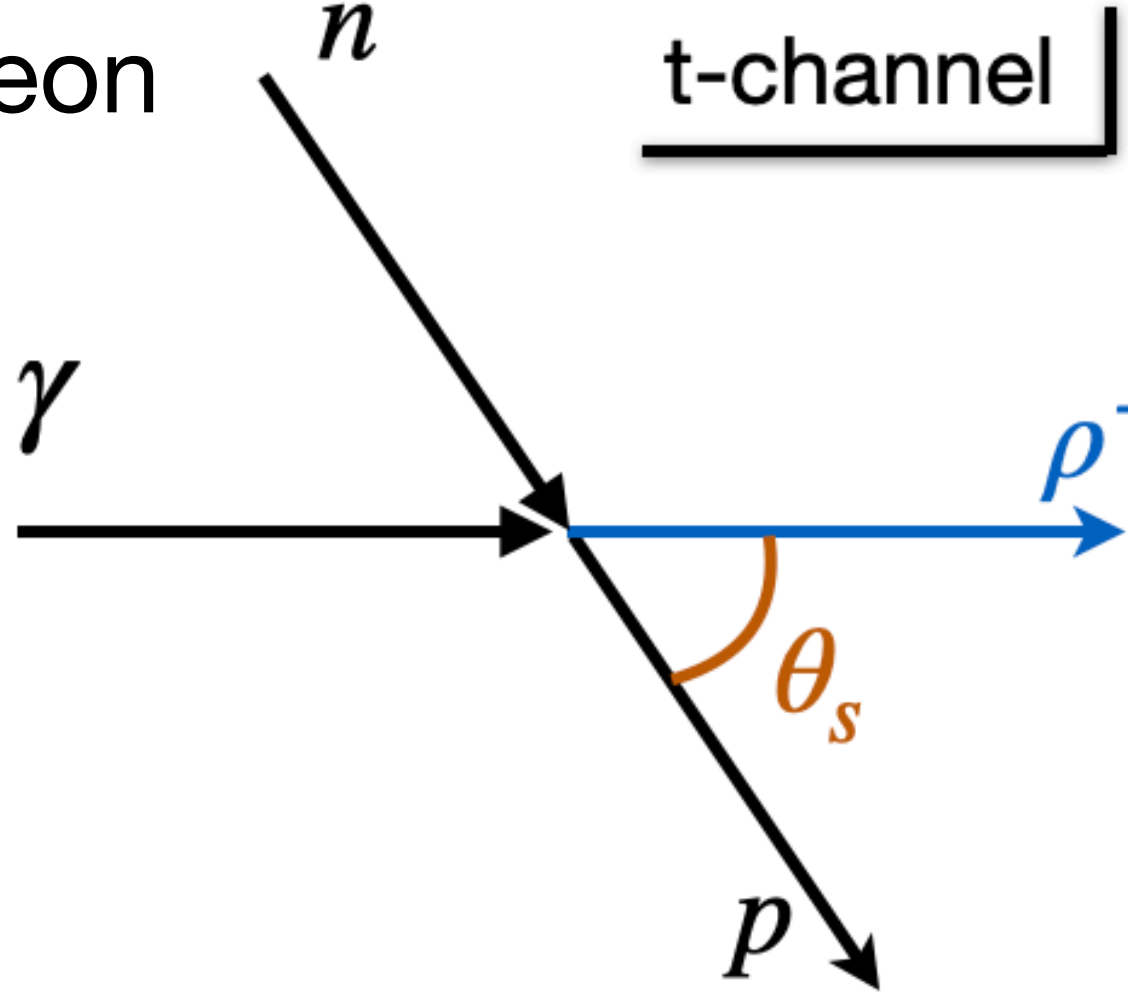
Frames in Meson Production



In beam fragmentation:

Helicity frame: z is opposite to recoil nucleon

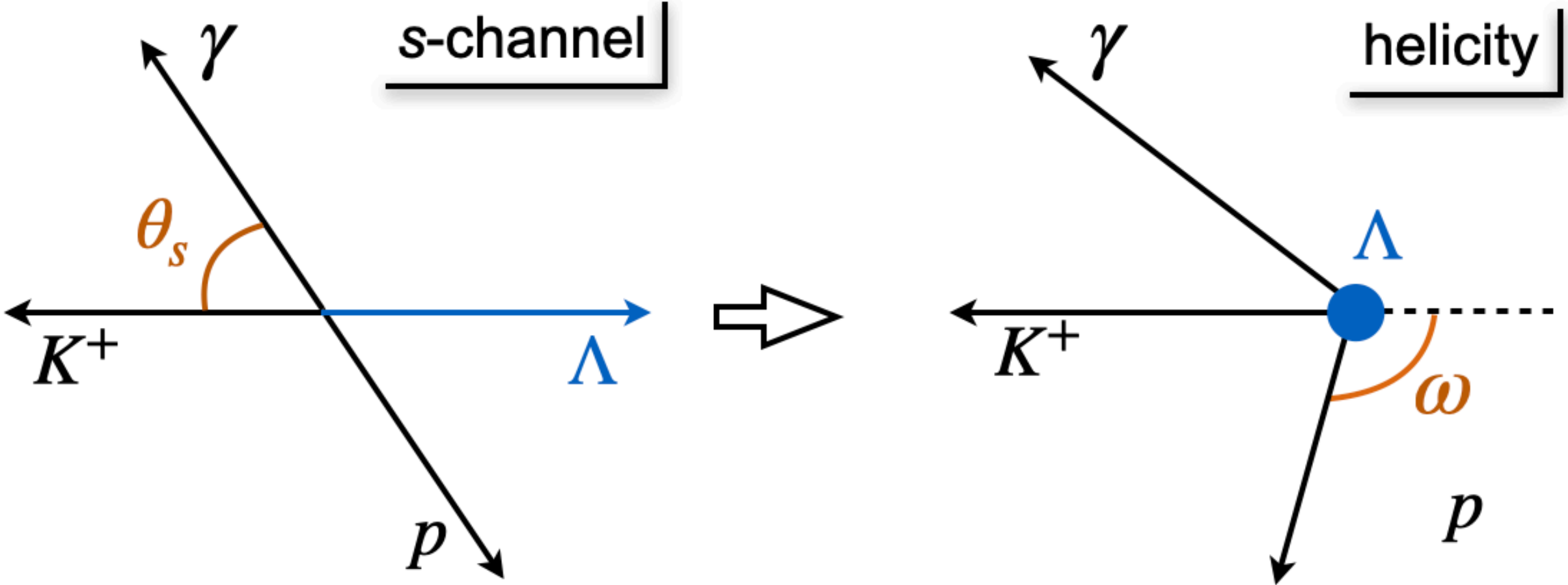
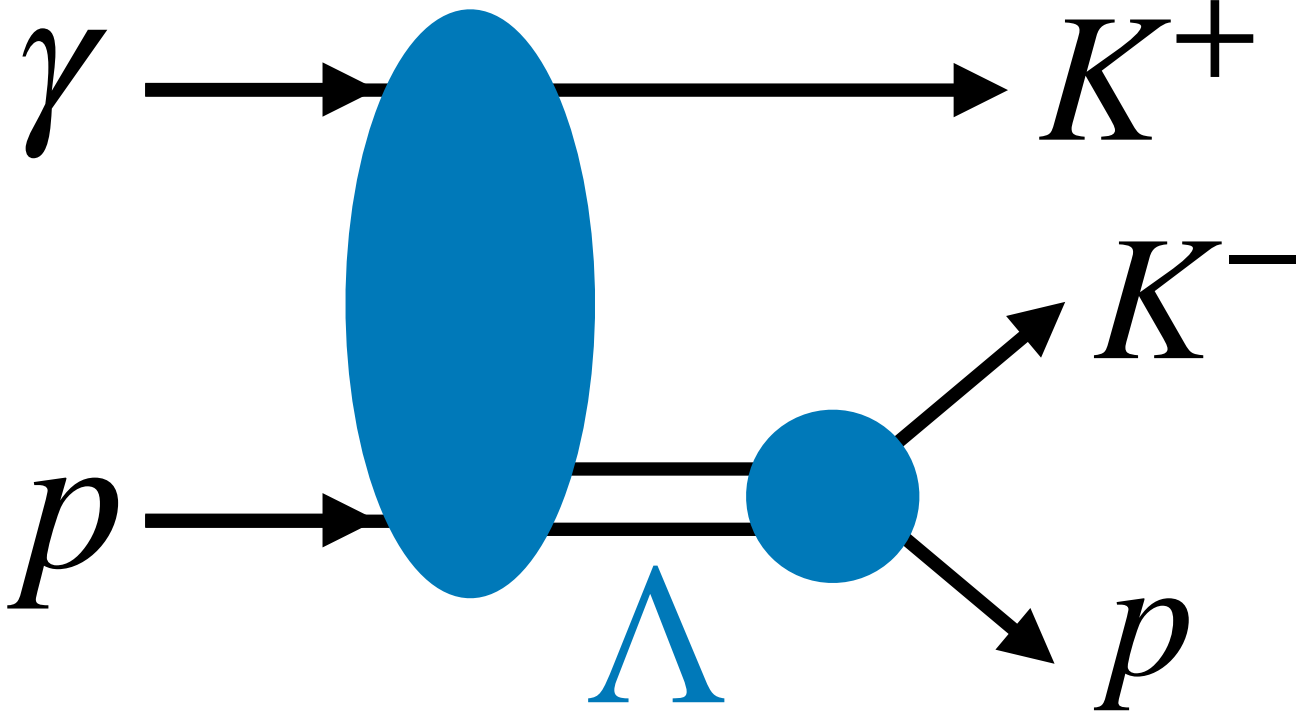
GJ frame: z is parallel to beam



Rotation of ϵ between frames

$$\rho_{m,m'}^H = \sum_{\lambda,\lambda'} d_{\lambda,m}^J(\epsilon) \rho_{m,m'}^{GJ} d_{\lambda',m'}^J(\epsilon)$$

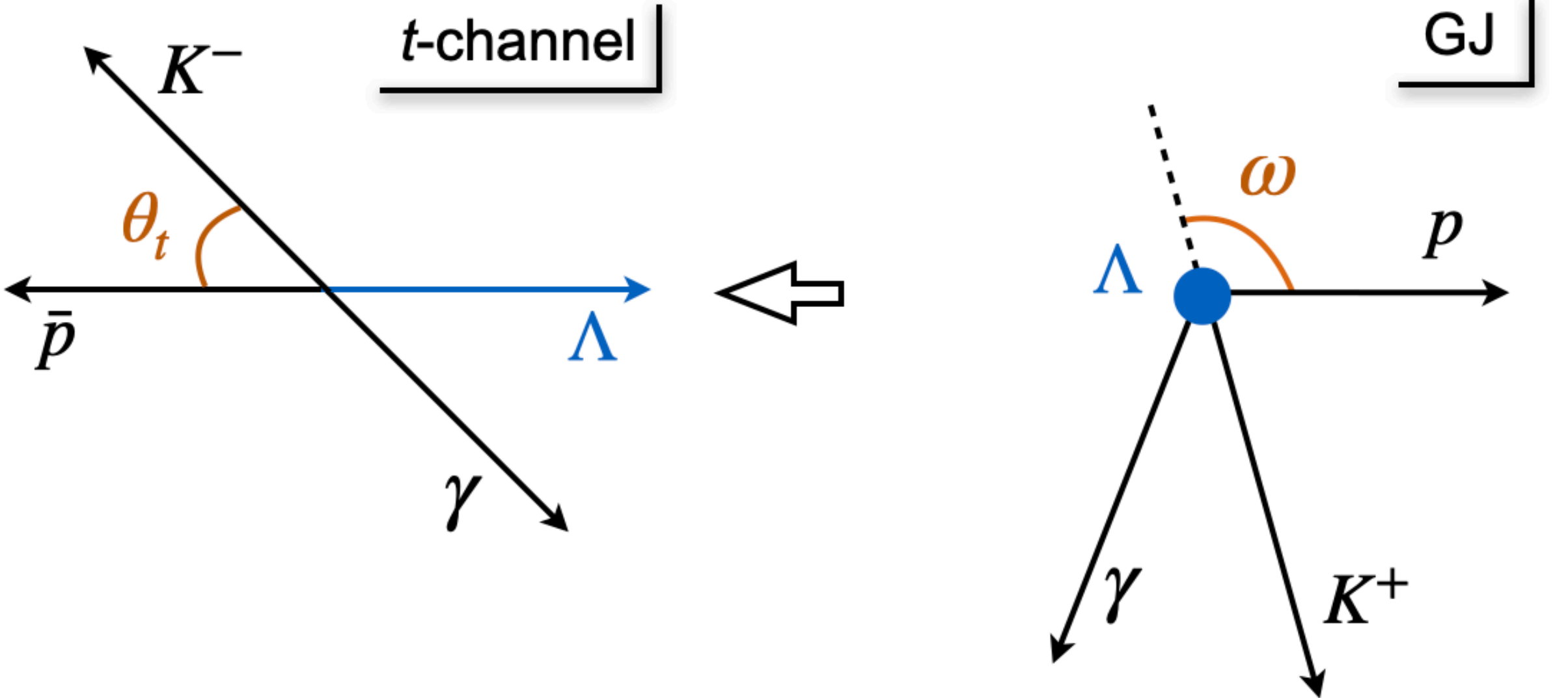
Baryon Production



In target fragmentation:

Helicity frame: z is opposite to recoil meson

Gottfried-Jackson frame: z is parallel to target



Rotation of ω between frames

$$\rho_{m,m'}^H = \sum_{\lambda,\lambda'} d_{\lambda,m}^J(\omega) \rho_{m,m'}^{GJ} d_{\lambda',m'}^J(\omega)$$

Wigner Rotation for Helicity states

Helicity is the spin projection along the momentum

The plan including \vec{p} and \vec{p}' is the $x - z$ plane

$$\Lambda(\vec{p}' \leftarrow \vec{p}) |\vec{p}, \lambda\rangle = \sum_{\lambda'} d_{\lambda', \lambda}^s(\omega) |\vec{p}', \lambda'\rangle$$

First boost to rest frame

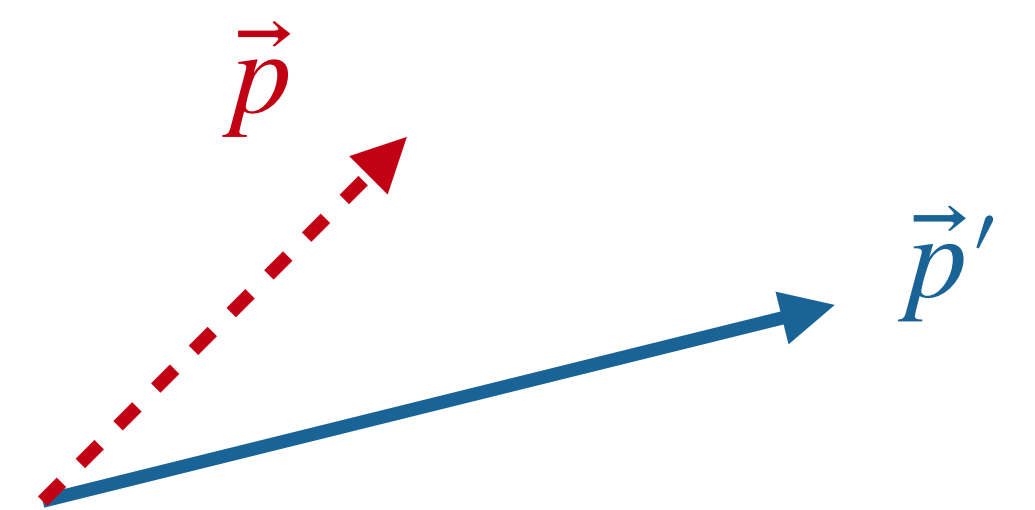
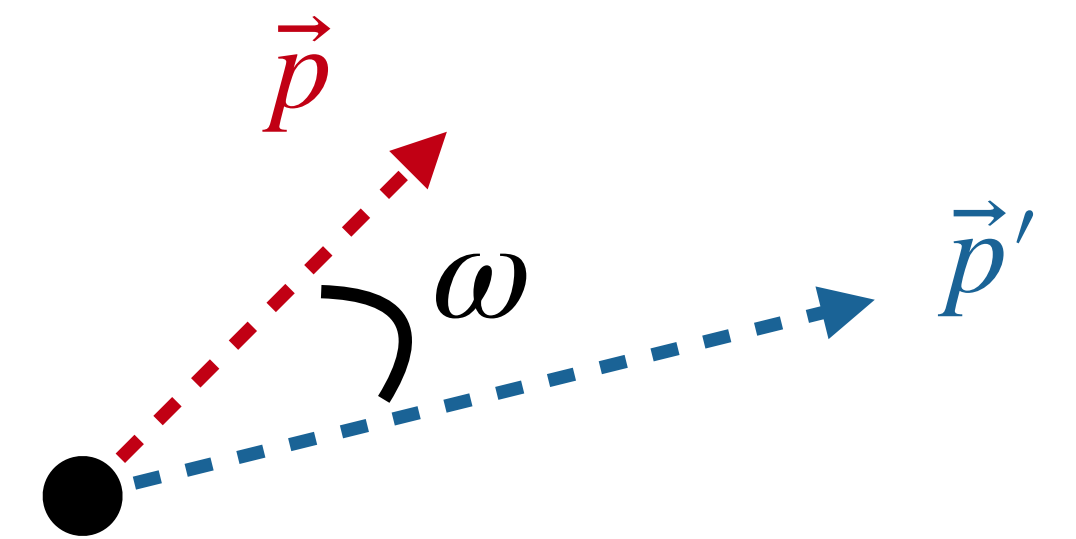
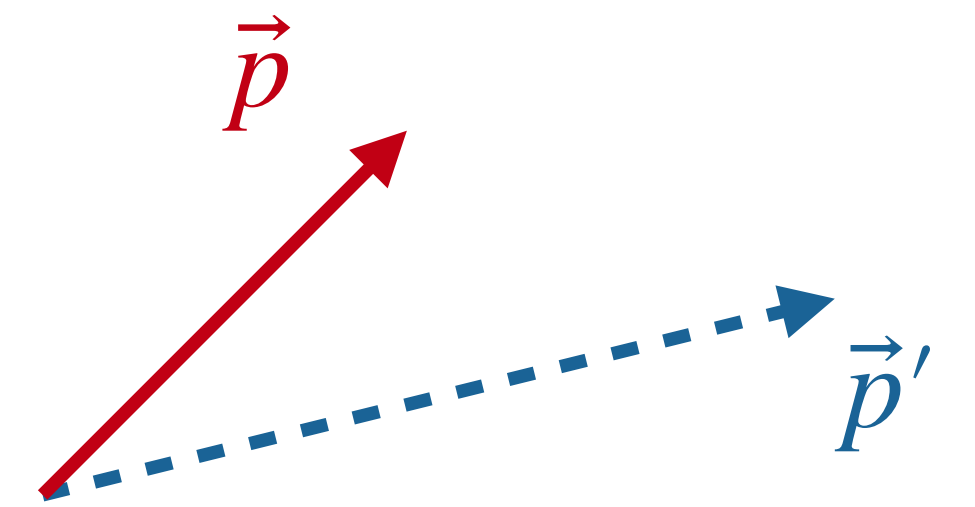
Helicity becomes spin projection along the former momentum

Then rotate to align the z to the future direction

The spin projection changes

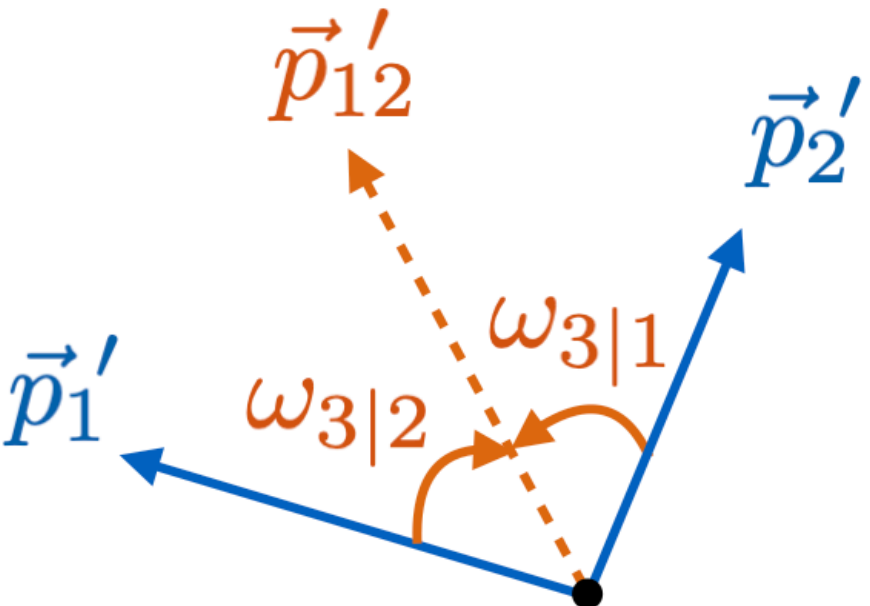
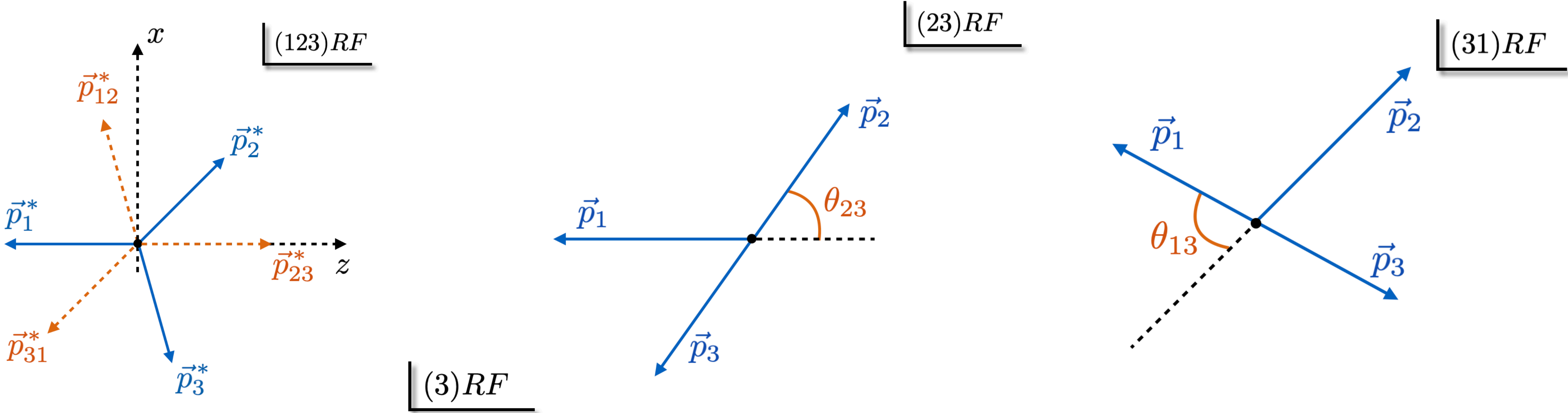
Finally boost to the final momentum

The Wigner rotation is the angle between the two direction,
as seen from the particle rest frame



Computing Wigner Rotation

Boosting from (23)RF to (123)RF will (Wigner) rotate the helicities of 2 and 3 (but not 1)

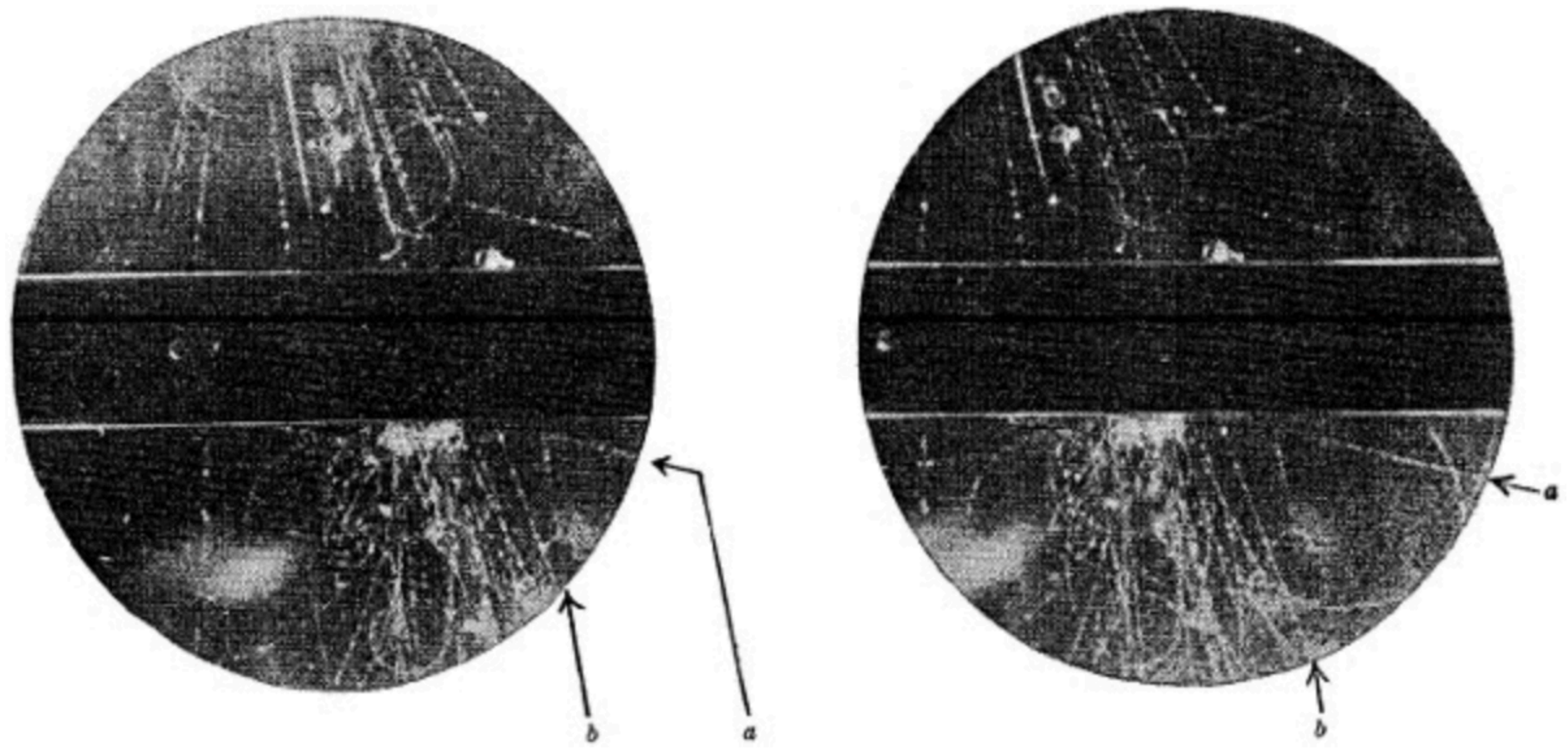


$\omega_{3|1}$ is the Wigner rotation of 3 from (23)RF to (123)RF

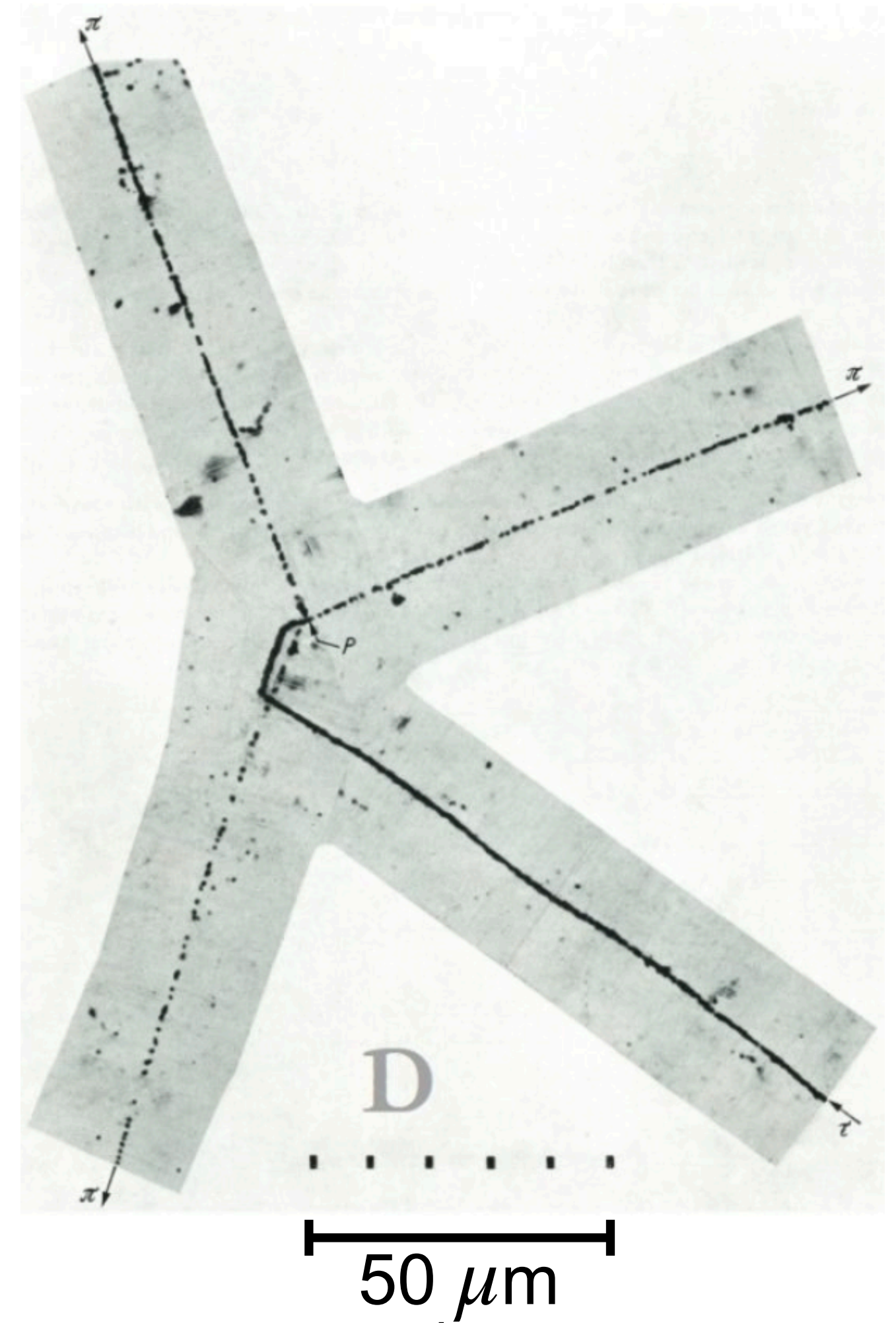
$\omega_{3|2}$ is the Wigner rotation of 3 from (31)RF to (123)RF

Kaons

G.D. Rochester and C.C. Butler (1947)
discovered two “V” tracks in cloud chamber



R. Brown et al (1949)
discovered “K” tracks in cloud chamber



Source: <https://www.cloudylabs.fr/wp/kaoninteractions/>

Λ^0 baryon

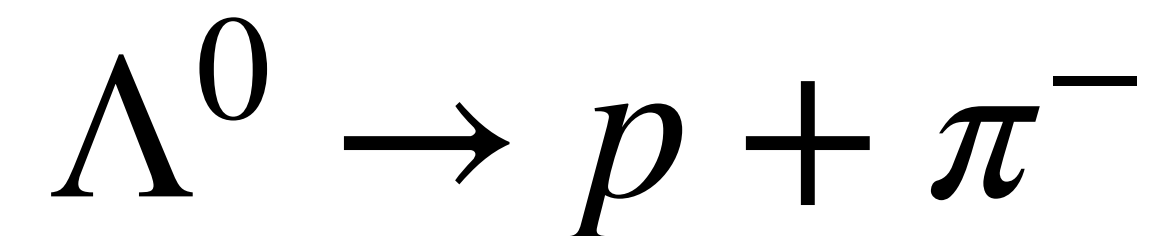
Evidence Concerning the Existence of the New Unstable Elementary Neutral Particle

V. D. HOPPER AND S. BISWAS

Department of Physics, University of Melbourne, Melbourne, Australia

October 30, 1950

Observation from cosmic rays of the decay



Unexpected long life-time $\tau \sim 10^{-10}$ s

Resonance typical life-time $\tau \sim 10^{-23}$ s

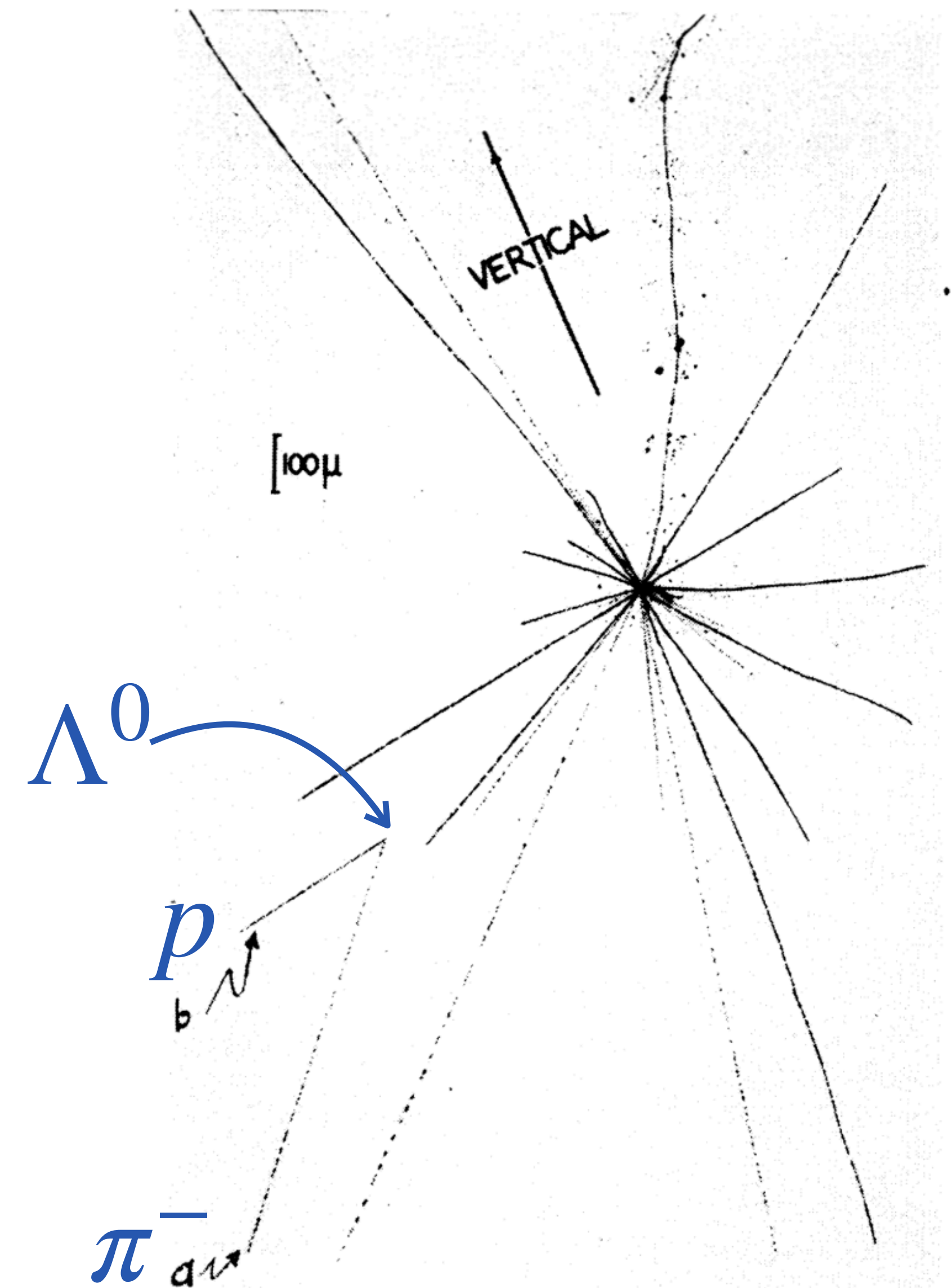


FIG. 1. Facsimile drawing which shows position of the two-pronged star relative to a large star. The plane of the two-pronged star does not coincide with the center of the large star. Track (a) corresponds to a meson and track (b) to a proton.

LETTERS TO THE EDITOR

Isotopic Spin and New Unstable Particles

M. GELL-MANN

*Department of Physics and Institute for Nuclear Studies,
University of Chicago, Chicago, Illinois*

(Received August 21, 1953)

Phys. Rev. 92 (1953) 833

M. Gell-Mann

1929 - 2019



At the end of this section it is worth to recall the reminiscences of Gell-Mann [50]. “Now let me return to the paper that I did sent off in August 1953 Isotopic Spin and the New Unstable Particles. That was not my title, which was: **Isotopic Spin and Curious Particles**. Physical Review rejected “Curious Particles”. I tried **“Strange Particles”** and they rejected that too. They insisted on: “New Unstable Particles”. That was the only phrase sufficiently pompous for the editors of the Physical Review. I should say that I have always hated the Physical Review Letters and almost twenty years ago I decided never again to publish in that journal, but in 1953 I was scarcely in the position to show around.”

Source: <https://www.fuw.edu.pl/~ajduk/hyperakw.pdf>

Long list of new particles with “strange” properties

$$K^\pm, K^0, \bar{K}^0, \Lambda^0, \Sigma^\pm, \Sigma^0, \Xi^-, \Xi^0, \dots$$

19 listed resonances in 1957, 26 in 1963

Long life time, appear in pair, etc

ϕ meson

EXISTENCE AND PROPERTIES OF THE ϕ MESON*

P. L. Connolly, E. L. Hart, K. W. Lai, G. London,[†] G. C. Moneti,[‡] R. R. Rau,
N. P. Samios, I. O. Skillicorn, and S. S. Yamamoto
Brookhaven National Laboratory, Upton, New York

and

M. Goldberg, M. Gundzik, J. Leitner, and S. Lichtman
Syracuse University, Syracuse, New York

(Received 27 March 1963)

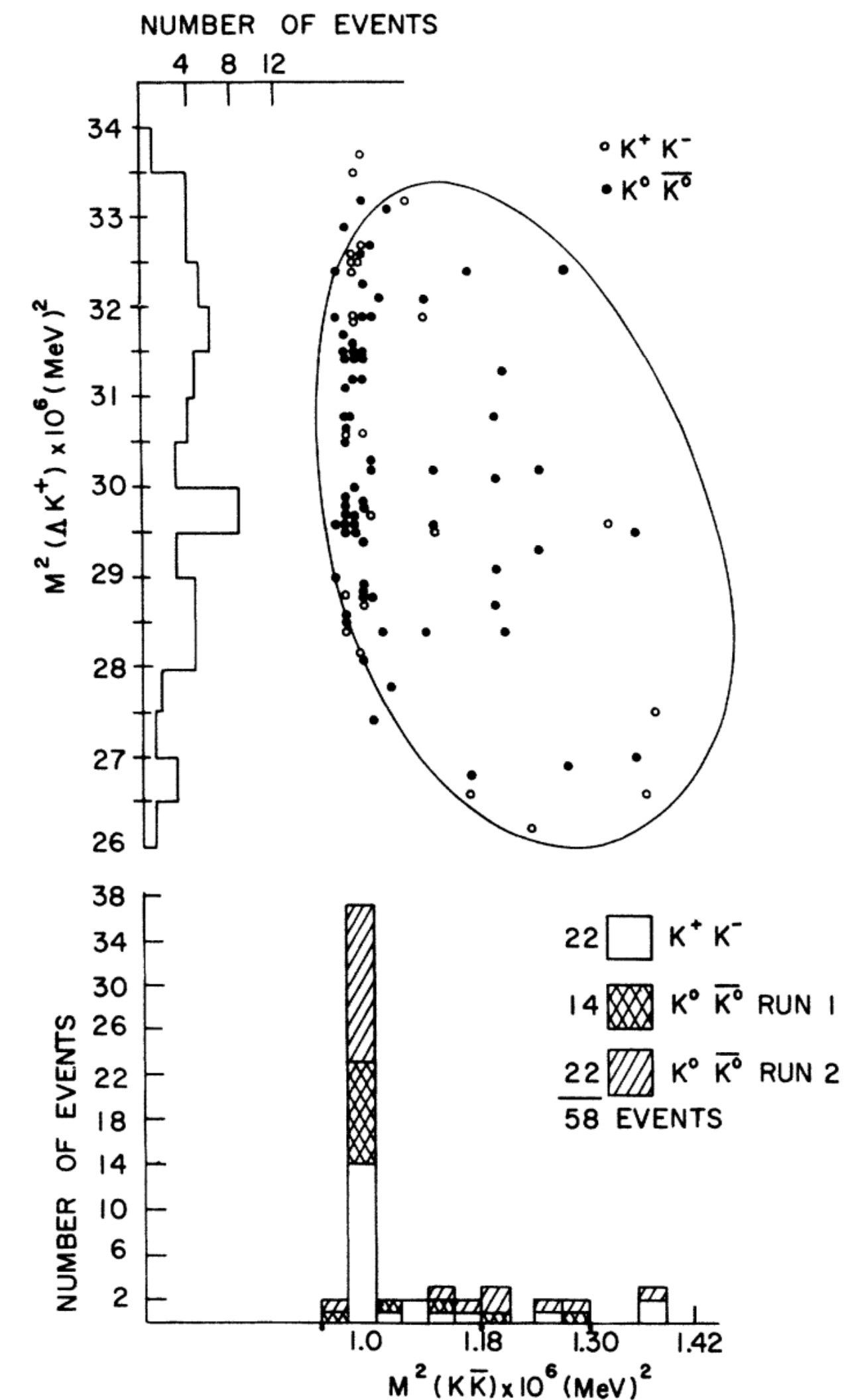
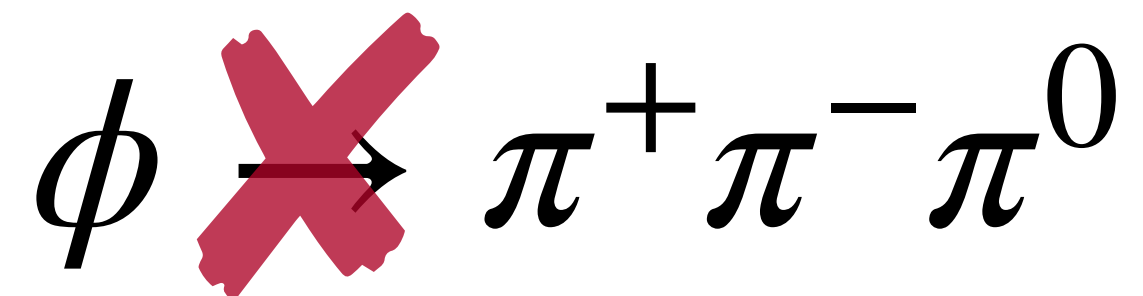
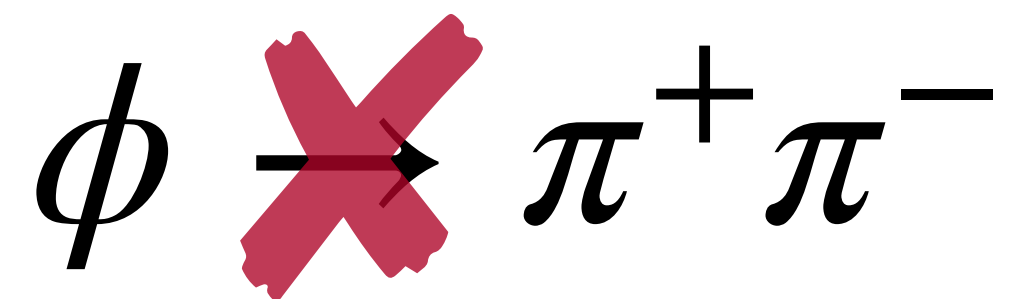
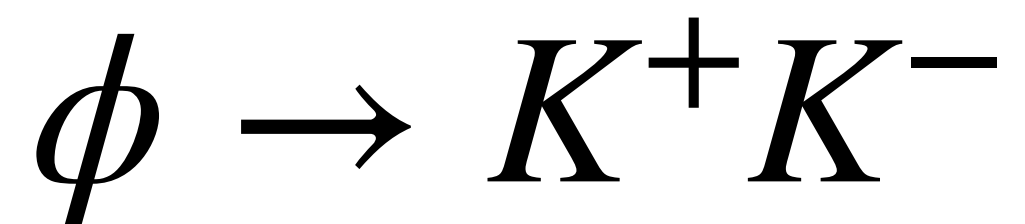
Phys. Rev. Lett. 10 (1963) 371

Exp. collaboration becomes larger

Use of kaon beam

~ 40 events in a single experiment

Don't seem to decay into pions



The eightfold way

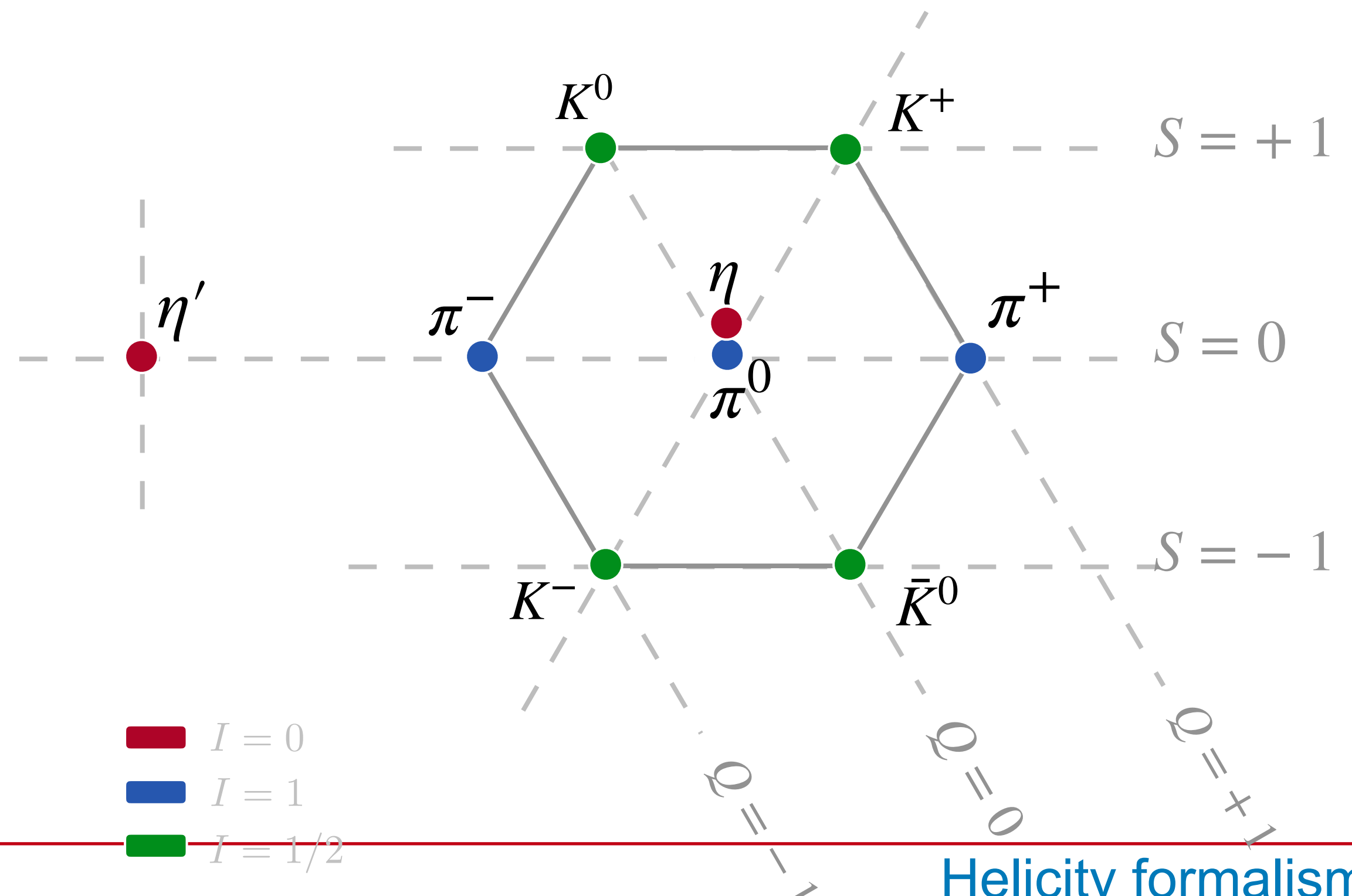
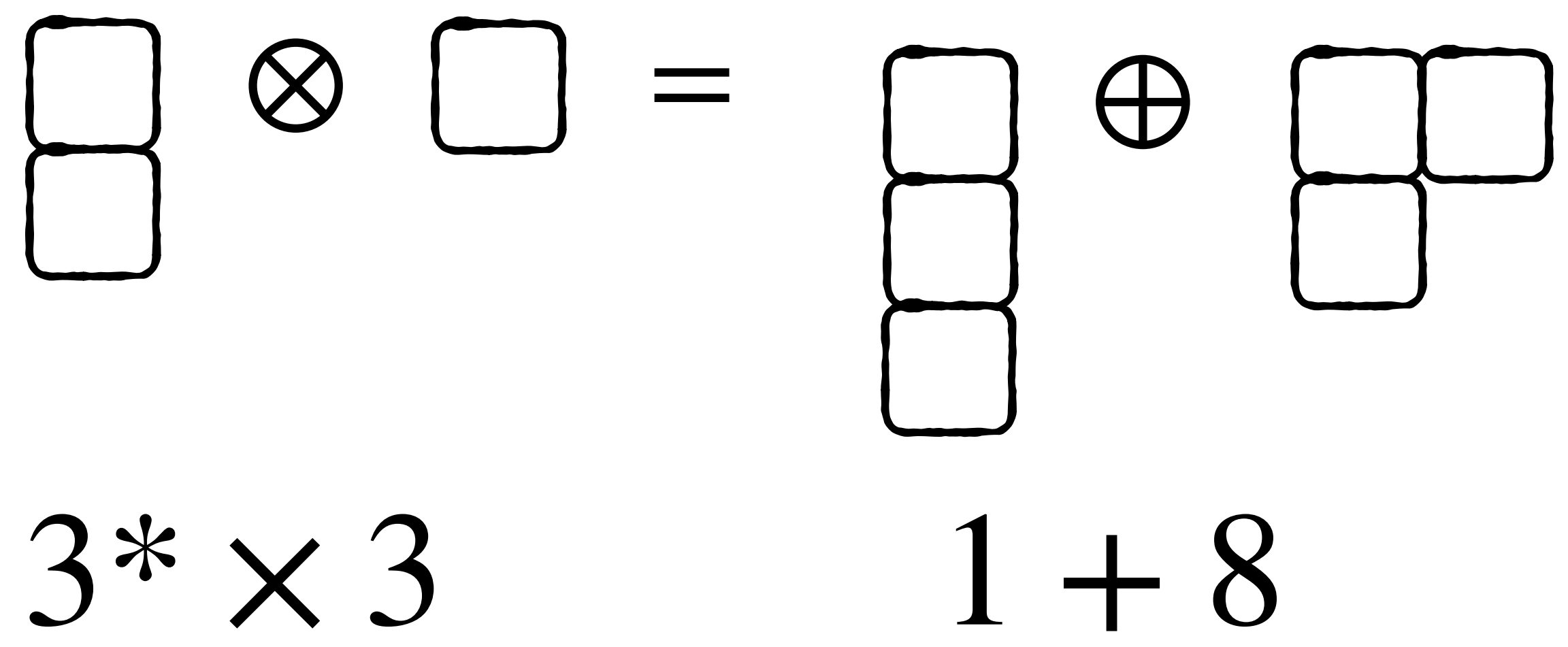
Rotation in 3 real dimensions: angular momentum

Rotation in 2 complex dimensions: isospin

Both are mathematically equivalent!

What about rotation in 3 complex dimensions?

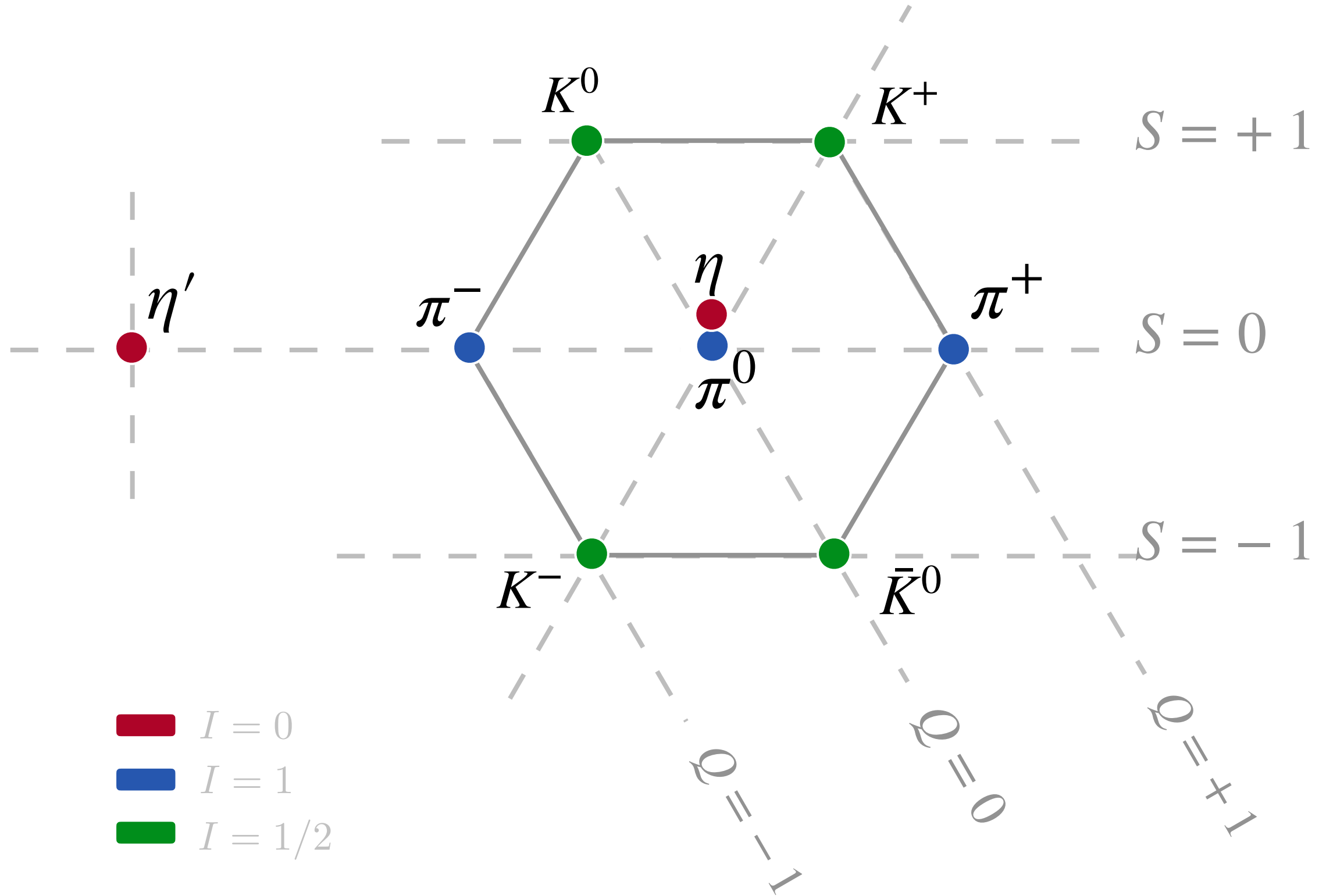
“Flavor” is the generalization of isospin



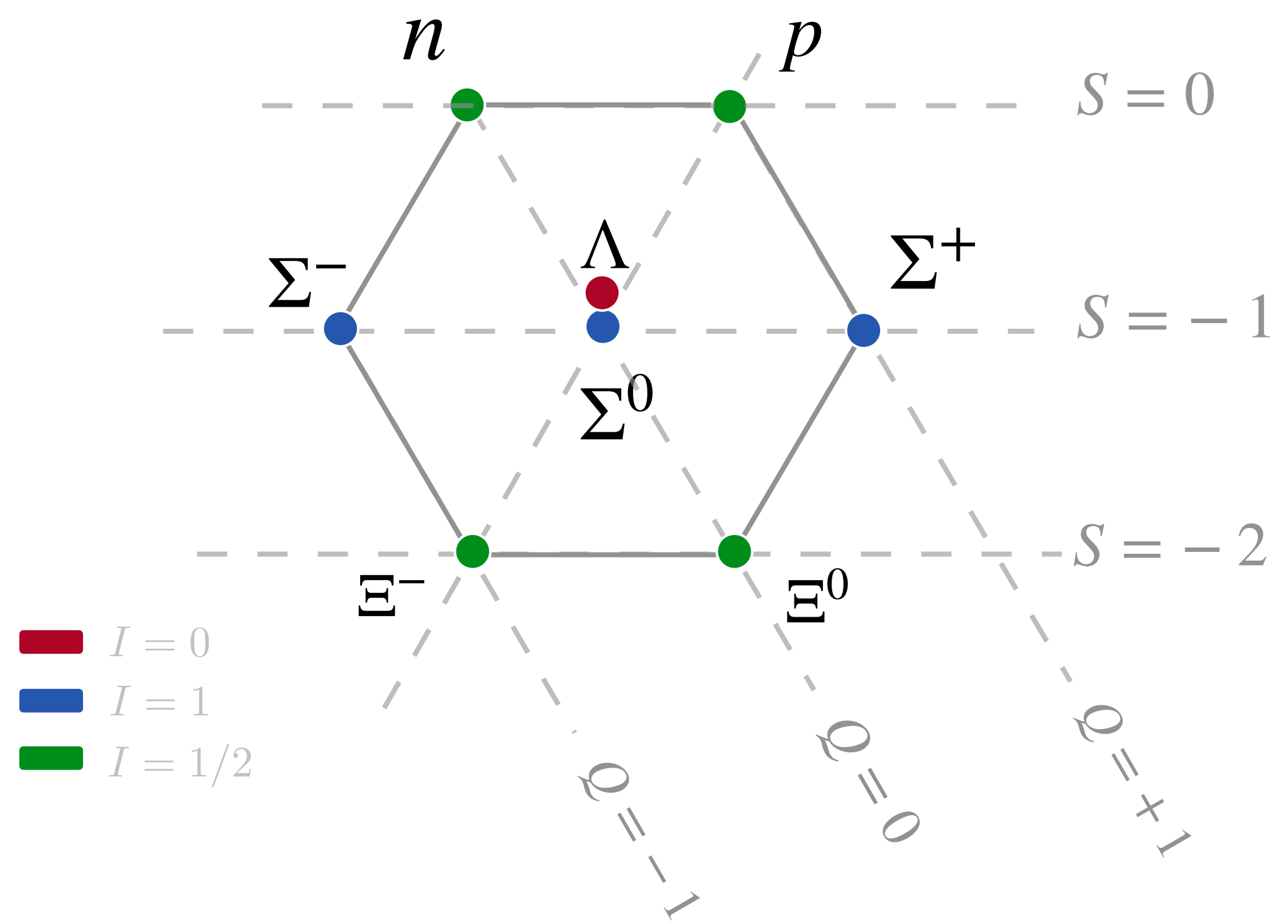
The eightfold way

Mesons split into a singlet and an octet

$$\begin{array}{c}
 \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \\
 3^* \times 3 = 1 + 8
 \end{array}$$



The baryon octet



$$m_N = m_0 + 3m_u$$

$$m_\Sigma = m_0 + 2m_u + m_s$$

$$m_\Xi = m_0 + m_u + 2m_s$$

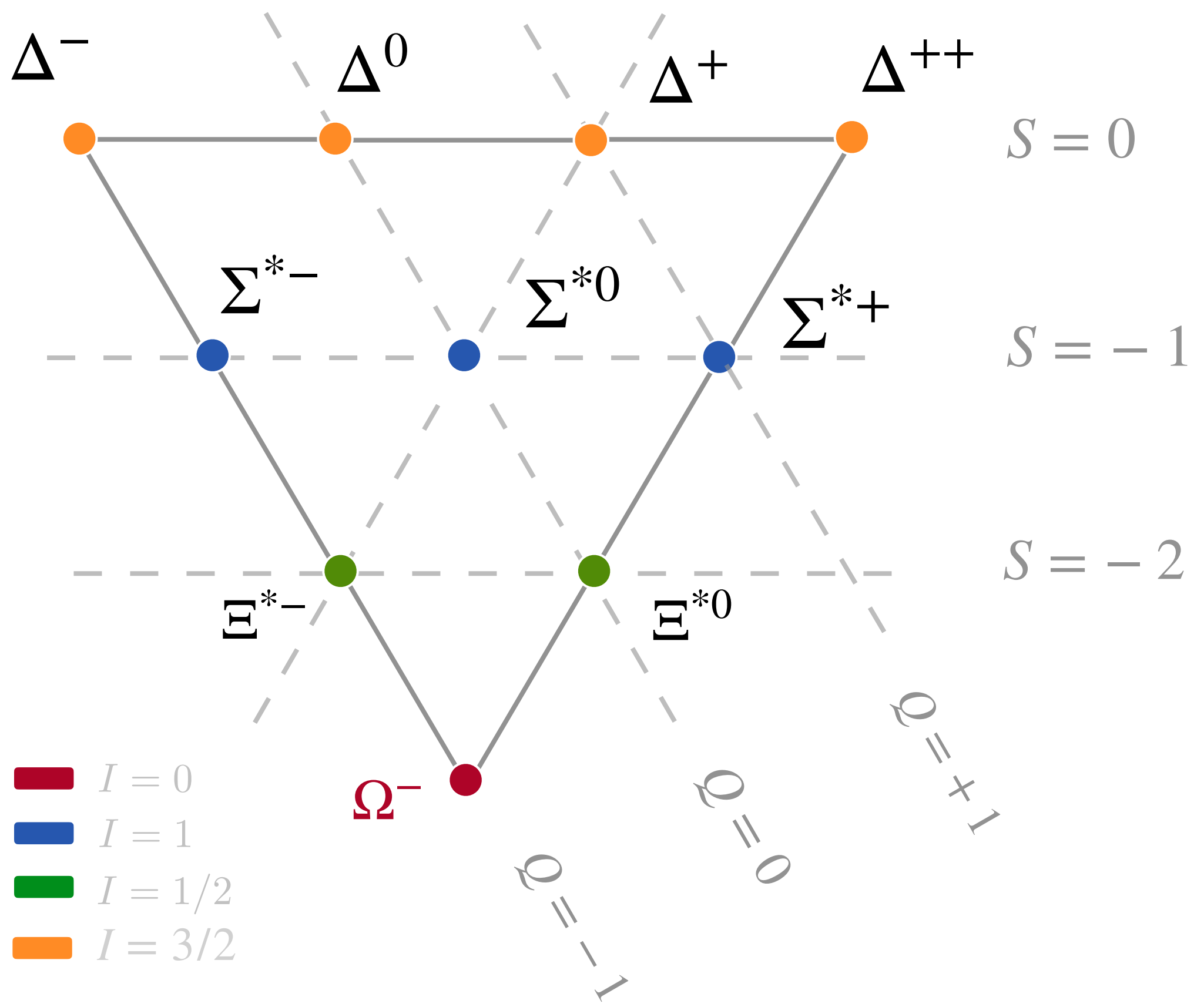
$$m_\Lambda = m_0 + 2m_u + m_s$$

	Name	Symbol	Isospin	Strangeness	Mass (MeV/c ²)
Octet	Nucleons	N	1/2	0	939
	Lambda baryons	Λ	0	-1	1116
	Sigma baryons	Σ	1	-1	1193
	Xi baryons	Ξ	1/2	-2	1318

Gell-Mann - Okubo mass relation

$$m_\Sigma + 3m_\Lambda = 2m_N + 2m_\Xi$$

The baryon decuplet



Mass difference = 153 MeV

Mass difference = 148 MeV

Mass difference ~ 150 MeV

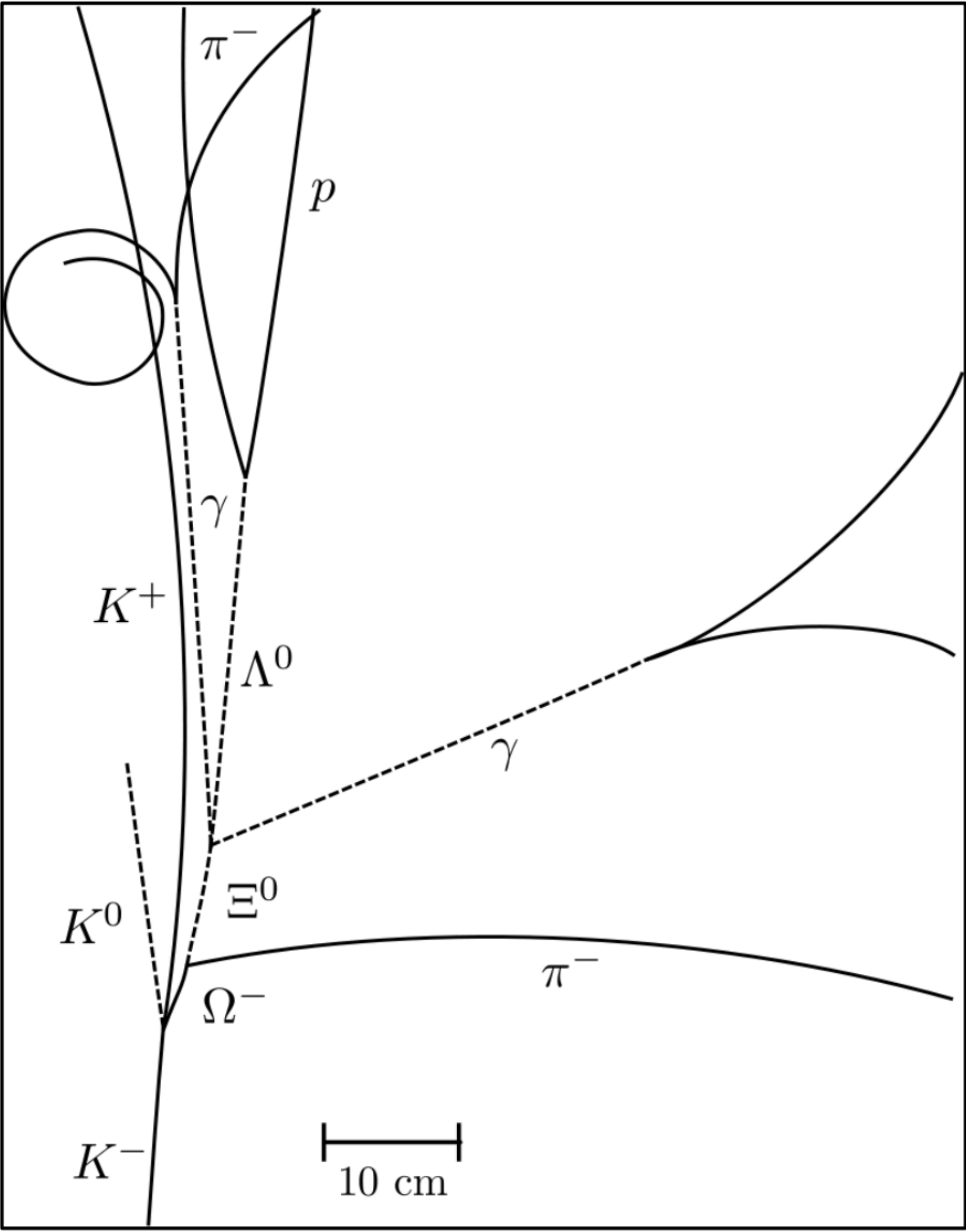
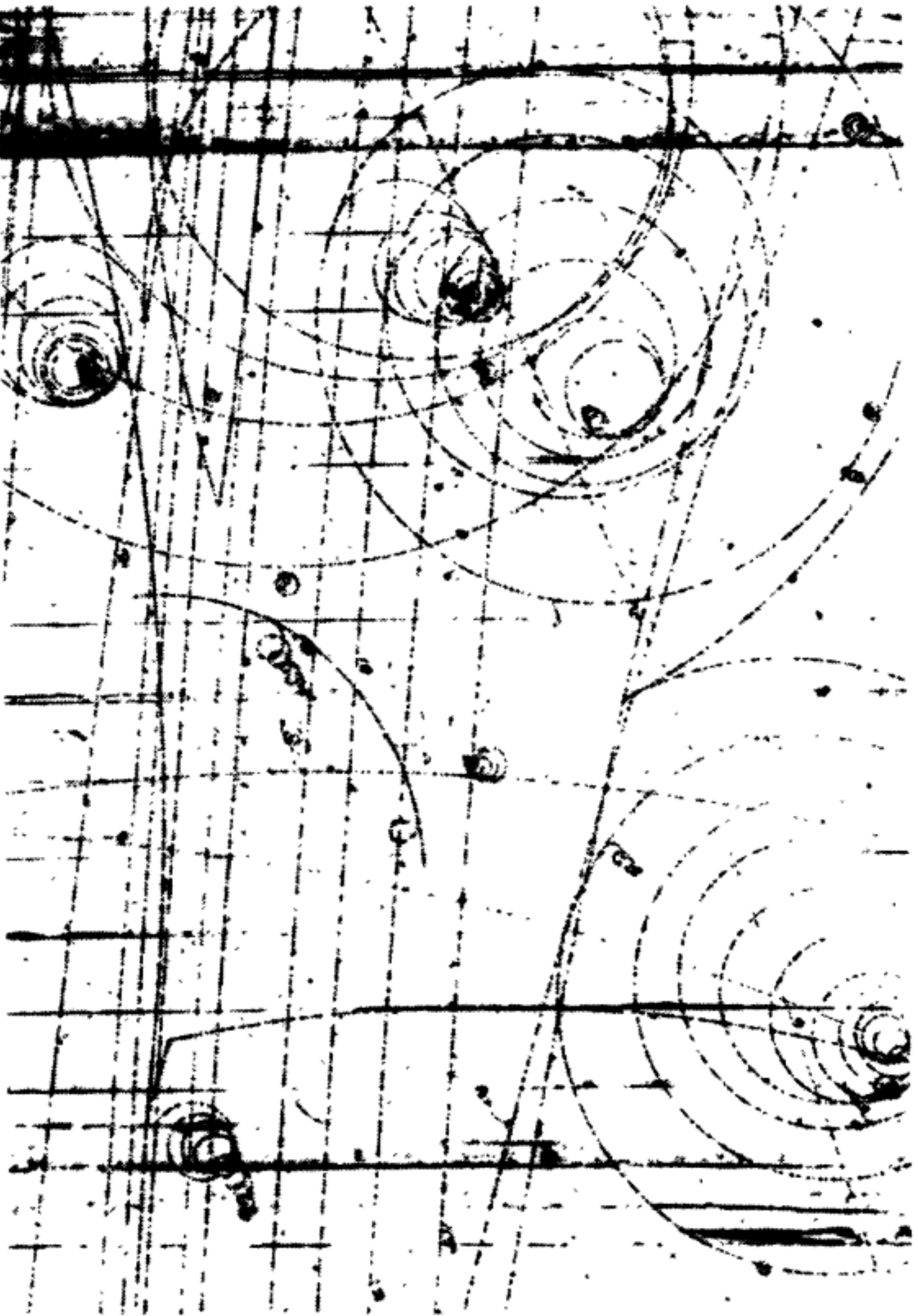
Prediction of a double strange baryon, with a negative electric charge and a mass around 1680 MeV by Gell-Mann in 1962

	Name	Symbol	Isospin	Strangeness	Mass (MeV/c ²)
Decuplet	Delta baryons	Δ	$3/2$	0	1232
	Sigma baryons	Σ^*	1	-1	1385
	Xi baryons	Ξ^*	$1/2$	-2	1533
	Omega baryon	Ω	0	-3	1672

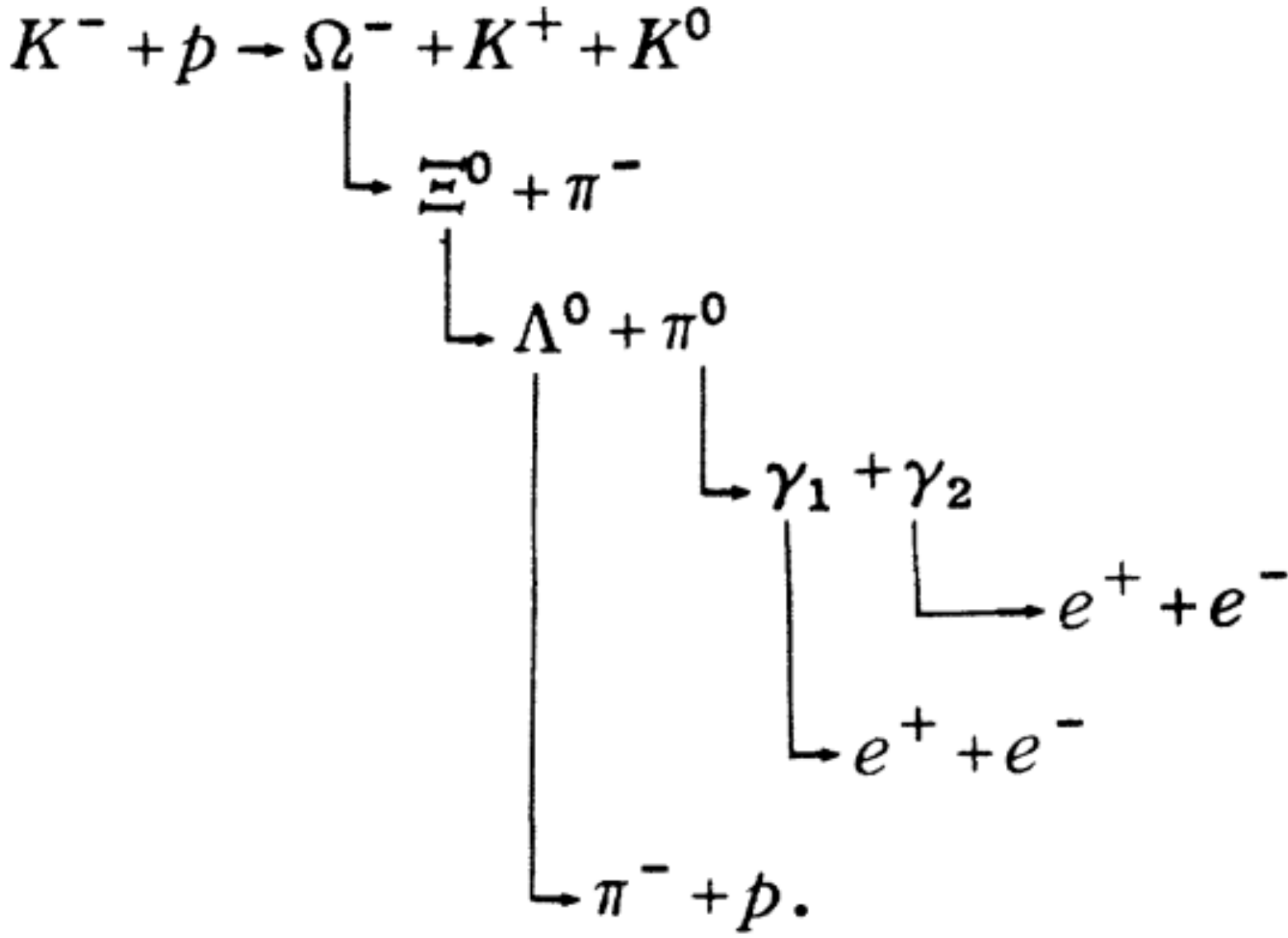
Discovery of the Ω^- baryon

Barnes et al Phys. Rev. Lett. 12 (1964) 204

Can you spot the Ω^- baryon?

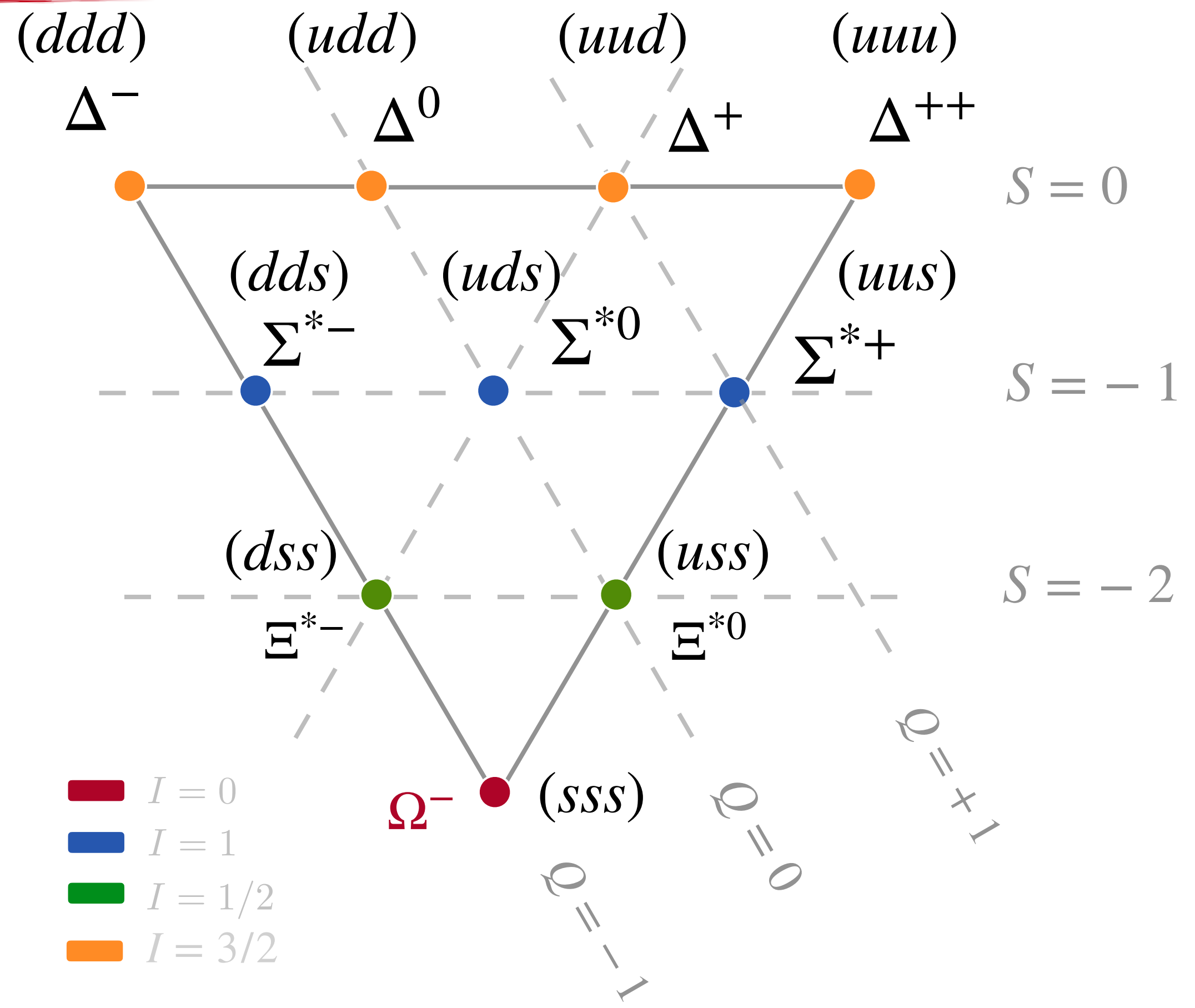
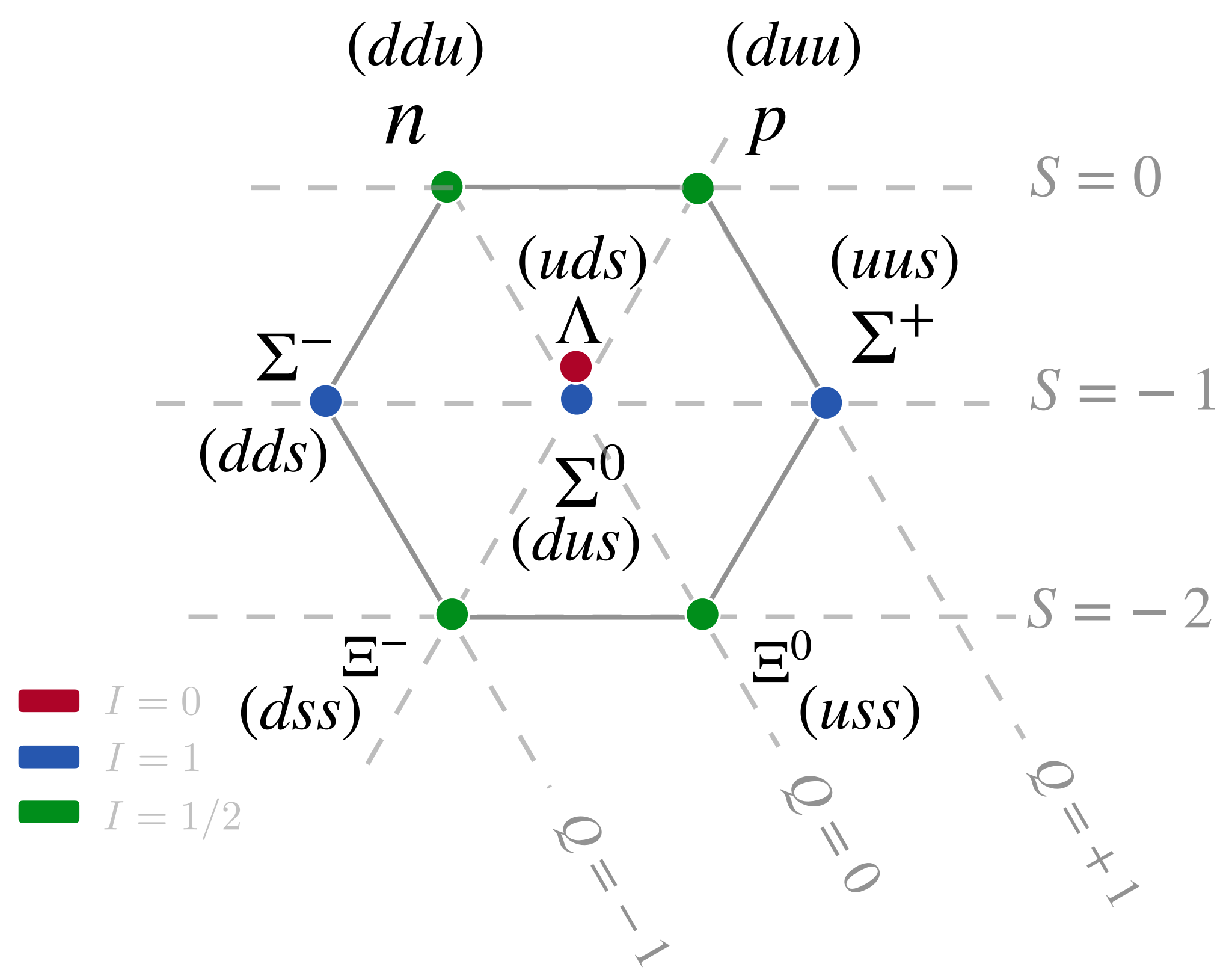


Strange cascade:



Neutral particles (dashed lines) don't leave a track

Three flavor of quarks



	I	S	\mathcal{B}	e_c
u	$1/2$	0	$1/3$	$+2/3$
d	$1/2$	0	$1/3$	$-1/3$
s	0	-1	$1/3$	$-1/3$