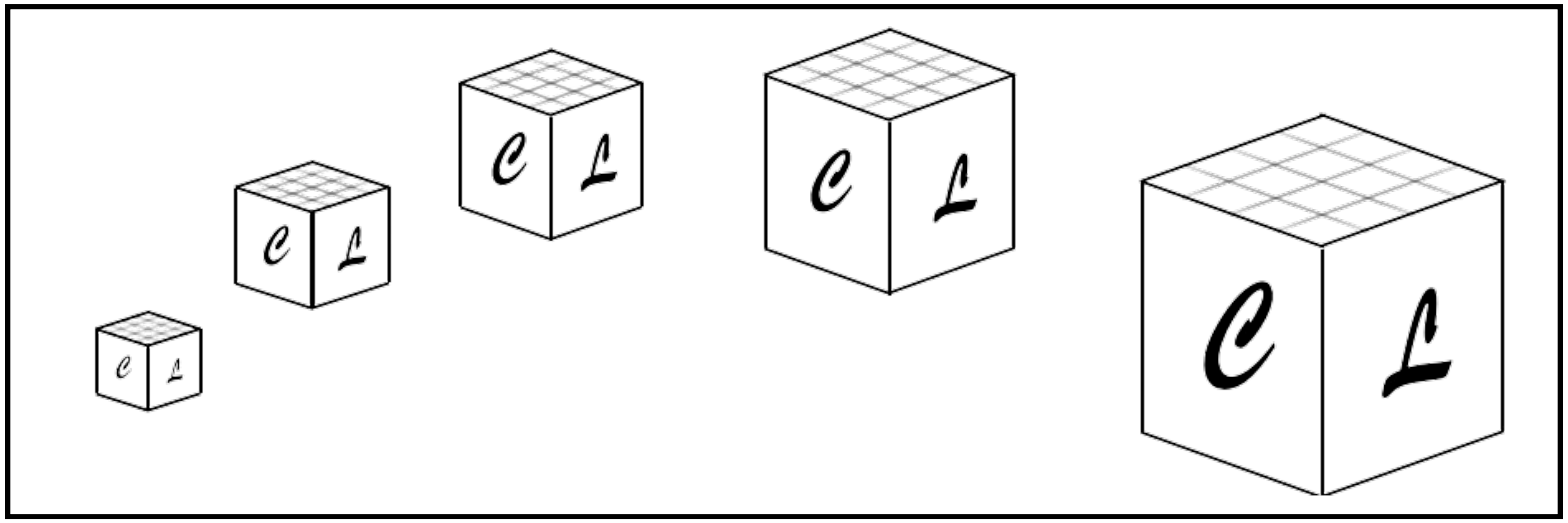

Cosmo$\mathcal{L}$attice:

simulating the early universe in a box

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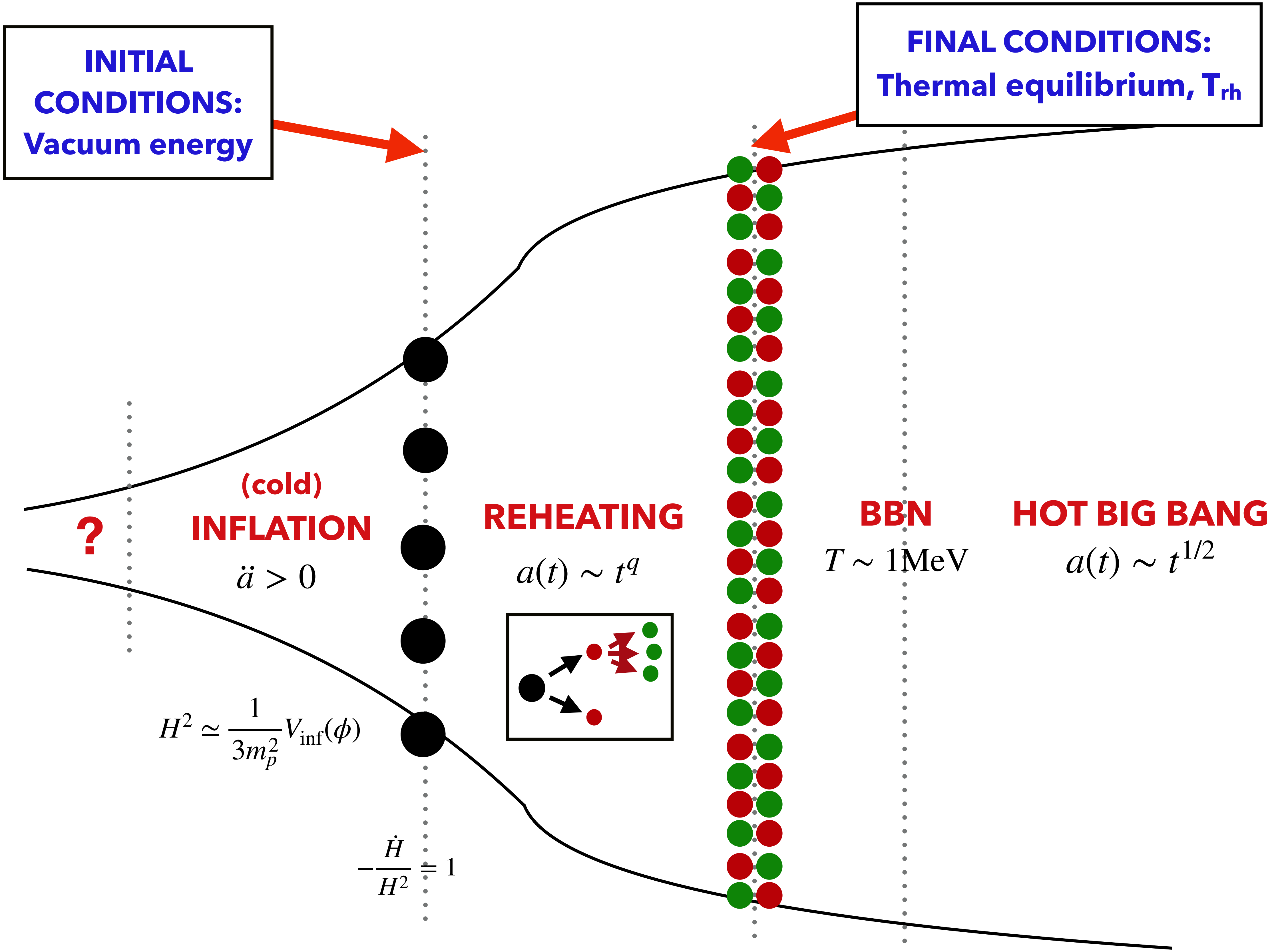
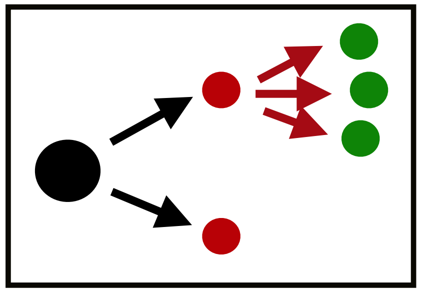
Jornadas Científicas IFIC, L5 - 28th April 2022

History of the early universe



Reheating



$\mathscr{L} = \mathscr{L}(\phi, \varphi_i, \psi_j, A_\mu, h_{\mu\nu}, \ldots)$**??**

- Details of reheating depend strongly on the high-energy physics model.
- Non-linear, non-perturbative, non-equilibrium physics.
- First stage is normally PREHEATING: field instabilities due to non-perturbative effects.

  - E.g: parametric resonance, self-resonance, tachyonic resonance…

- $n_k \sim |X_k|^2 \gg 1$ → **(classical) LATTICE SIMULATIONS**

Cosmo$\mathcal{L}$attice

- Several codes for classical lattice simulations have been publicly released in the past (e.g. `Latticeeasy`). Most can only simulate scalar fields (exception: `GFire`).

Cosmo$\mathcal{L}$attice:

JCAP (2020)
arXiv:2102.01031

- It can simulate **scalars**, as well as **U(1)** and **SU(2) gauge fields**.
- Written in **C++**, with a modular structure separating physics (`CosmoInterface` library) and technical details (`TempLat` library).
- Parallellized with **MPI** in multiple spatial dimensions.
- Includes numerical symplectic evolution algorithms, with accuracy ranging from $\delta\mathcal{O}(\delta t^2)$ - $\delta\mathcal{O}(\delta t^{10})$

Main developers: Daniel G. Figueroa (IFIC, Valencia U.), Adrien Florio (Stony Brook U.), Wessel Valkenburg (EPFL) and myself.

Field theory

➤ **Matter content:**

$$S = -\int d^4x\sqrt{-g}\left\{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + (D^A_\mu\varphi)^*(D^\mu_A\varphi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D^\mu\Phi) + \frac{1}{2}\mathrm{Tr}\{G_{\mu\nu}G^{\mu\nu}\} + V(\phi,|\varphi|,|\Phi|)\right\}$$

$$\phi \in \mathscr{R}e$$

Scalar sector

$$\varphi \equiv \frac{1}{\sqrt{2}}(\varphi_0 + i\varphi_1)$$
$$D^{\mathrm{A}}_\mu \equiv \partial_\mu - iQ^{(\varphi)}_A g_{_A} A_\mu$$
$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$$

U(1) gauge sector

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_0 + i\varphi_1\\ \varphi_2 + i\varphi_3\end{pmatrix}$$
$$D_\mu \equiv \mathscr{I}D^{\mathrm{A}}_\mu - ig_B Q_B B^a_\mu T_a$$
$$G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu - i[B_\mu, B_\nu]$$

SU(2) gauge sector

Scalar potential

➤ **Metric:**

$$ds^2 = -dt^2 + a^2(t)\,\delta_{ij}\,dx^i dx^j$$

- ➤ Self-consistent expansion (Friedmann equation)
- ➤ Fixed power-law background $a(t) \sim t^{\frac{2}{3(1+w)}}$

Field equations

- Equations are written as a set of **coupled first-order differential equations**, which can be solved with a Hamiltonian scheme:

- **Example:**

$$\phi'' - \frac{1}{a^2}\nabla^2\phi + \frac{3}{a}\frac{da}{dt}\frac{d\phi}{dt} = -\frac{\partial V}{\partial \phi} \xrightarrow{\pi_\phi \equiv \phi' a^{3-\alpha}}$$

KICK: $$(\pi_\phi)' = -a^{3+\alpha}\frac{\partial V}{\partial \phi} + a^{1+\alpha}\nabla^2\phi$$

DRIFT: $$\phi' \equiv \pi_\phi a^{\alpha-3}$$

We solve a discretized version of these equations in a lattice → **LATTICE SIMULATIONS**

- For gauge fields, equations are discretized with links and plaquettes to preserve gauge invariance.

Your model

- Equations are solved in a set of **dimensionless *program variables***:

Choose:

$$\{\alpha, f_*, \omega_*\} \rightarrow \begin{array}{c} d\tilde{\eta} \equiv a^{-\alpha}\omega_* dt \\ d\tilde{x}^i \equiv \omega_* dx^i \end{array}$$

Space and time

$$\tilde{\phi} = \frac{\phi}{f_*} \qquad \tilde{\varphi} = \frac{\varphi}{f_*} \qquad \tilde{\Phi} = \frac{\Phi}{f_*}$$

Scalar fields

$$\widetilde{A}_\mu = \frac{A_\mu}{\omega_*} \qquad \widetilde{B}^a_\mu = \frac{B^a_\mu}{\omega_*}$$

Gauge fields

- Write the (**program) scalar potential** and compute first and second derivatives:

$$\tilde{V}(\tilde{\phi}, |\tilde{\varphi}|, |\widetilde{\Phi}|) \equiv \frac{1}{f_*^2\omega_*^2} V(f_*\tilde{\phi}, f_*|\tilde{\varphi}|, f_*|\widetilde{\Phi}|) \rightarrow \frac{\partial \tilde{V}}{\partial \tilde{\phi}}, \frac{\partial \tilde{V}}{\partial |\tilde{\varphi}|}, \frac{\partial \tilde{V}}{\partial |\tilde{\Phi}|}, \frac{\partial^2 \tilde{V}}{\partial \tilde{\phi}^2}, \frac{\partial^2 \tilde{V}}{\partial |\tilde{\varphi}|^2}, \frac{\partial^2 \tilde{V}}{\partial |\tilde{\Phi}|^2},$$

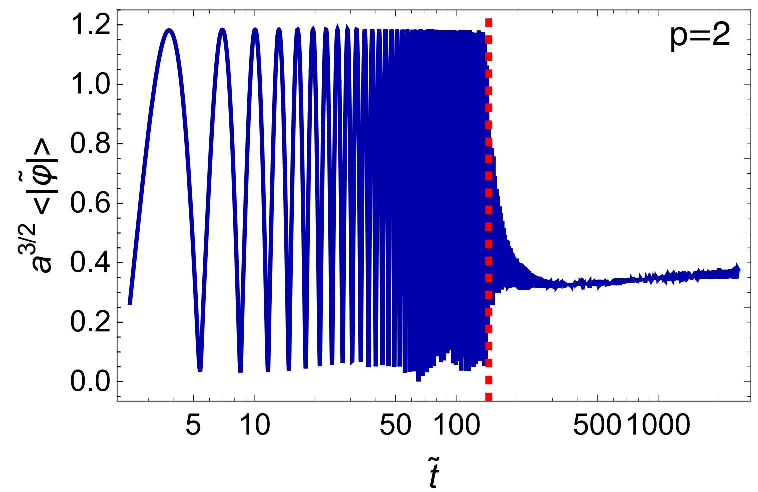
- These functions are introduced in a header (`model_name.h`). Parameters can be passed via a **text file**.

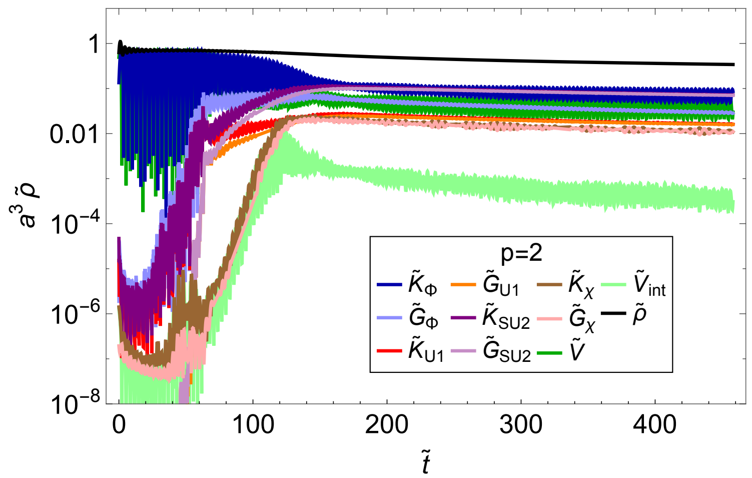
```
#Output
outputfile = ./

#Evolution
expansion = true
evolver = VV2

#Lattice
N = 32
dt = 0.01
kIR = 0.75
nBinsSpectra = 55

#Times
tOutputFreq = 0.1
tOutputInfreq = 1
tMax = 300

#IC
kCutOff = 1.75
initial_amplitudes = 7.42675e18 0 # homogeneous amplitudes in GeV
initial_momenta = -6.2969e30 0    # homogeneous amplitudes in GeV2

#Model Parameters
lambda = 9e-14
q = 100
```

Output

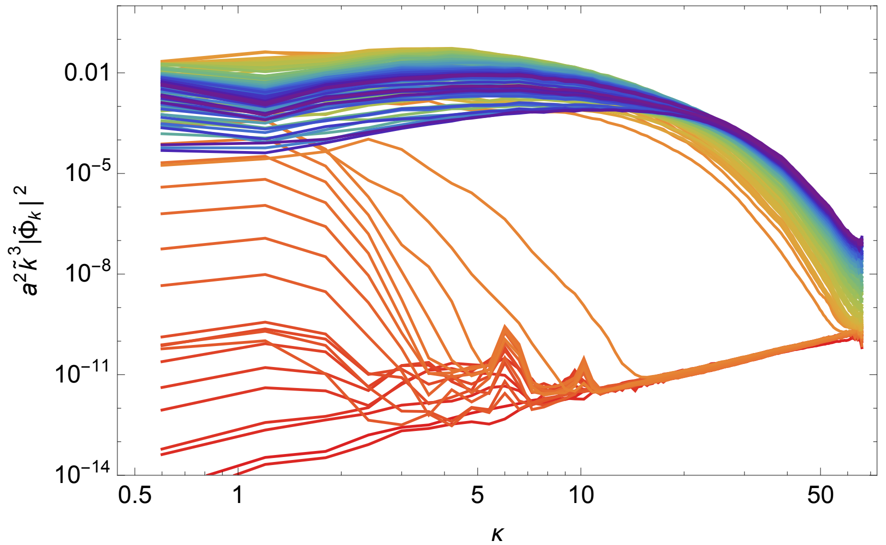
Three kinds of output

Volume averages: Spatial averages of certain quantities, such as field amplitudes or energies

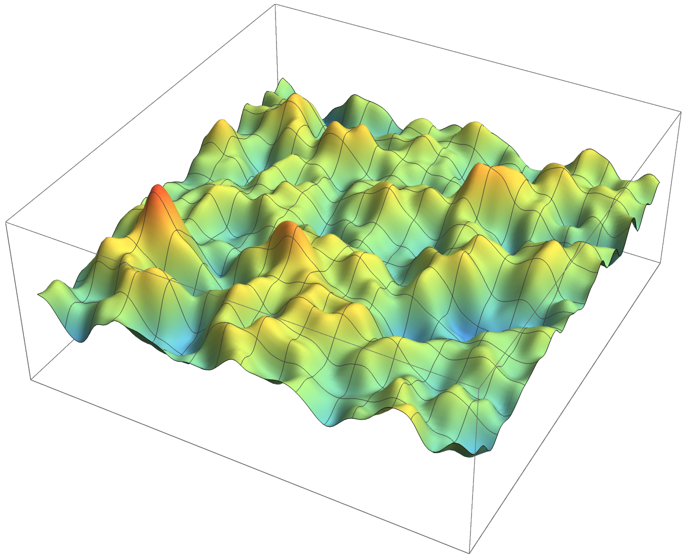


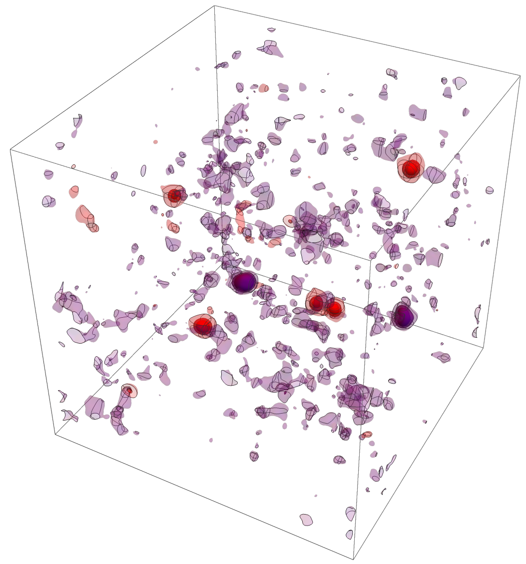


Spectra: Binned spectra in momentum space



Snapshots: Values of a certain quantity at all points of the lattice.





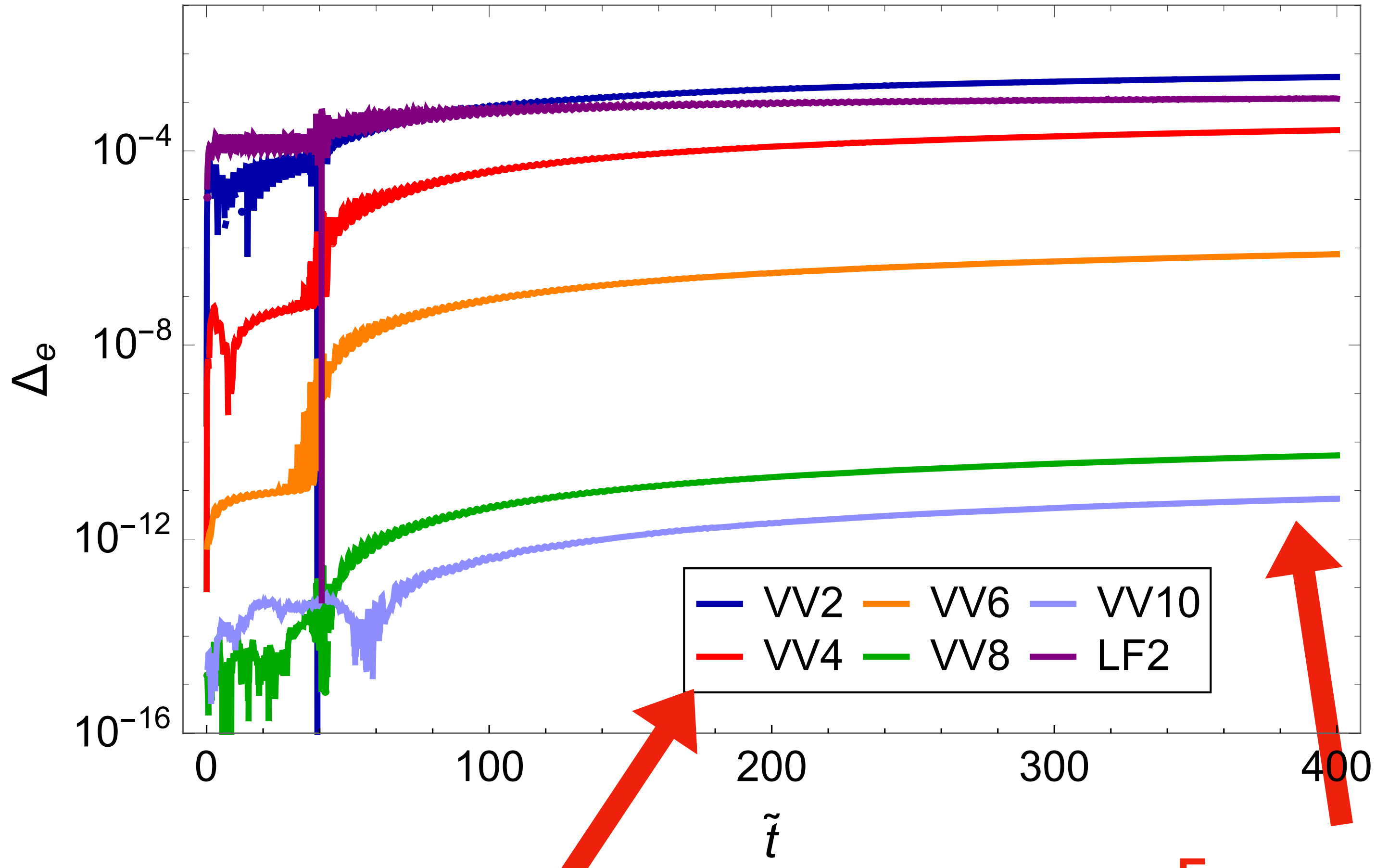
Energy conservation

- Algorithms use **second Friedmann equation** to **evolve the scale factor**.
- The **first Friedmann equation** is used to check the accuracy of the simulation.

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 \simeq \frac{\rho}{3m_p^2}}$$

$$\Delta_e \equiv \frac{\langle \mathrm{LHS} - \mathrm{RHS} \rangle}{\langle \mathrm{LHS} + \mathrm{RHS} \rangle}$$

(we must have $\Delta_e \ll 1$)



Energy conserved up to machine precision for VV10!

Evolution algorithms:

- **VVN:** Velocity-verlet of accuracy order $O(dt^n)$
- **LF2:** Staggered leapfrog, accuracy order $O(dt^2)$

Gauss constraint

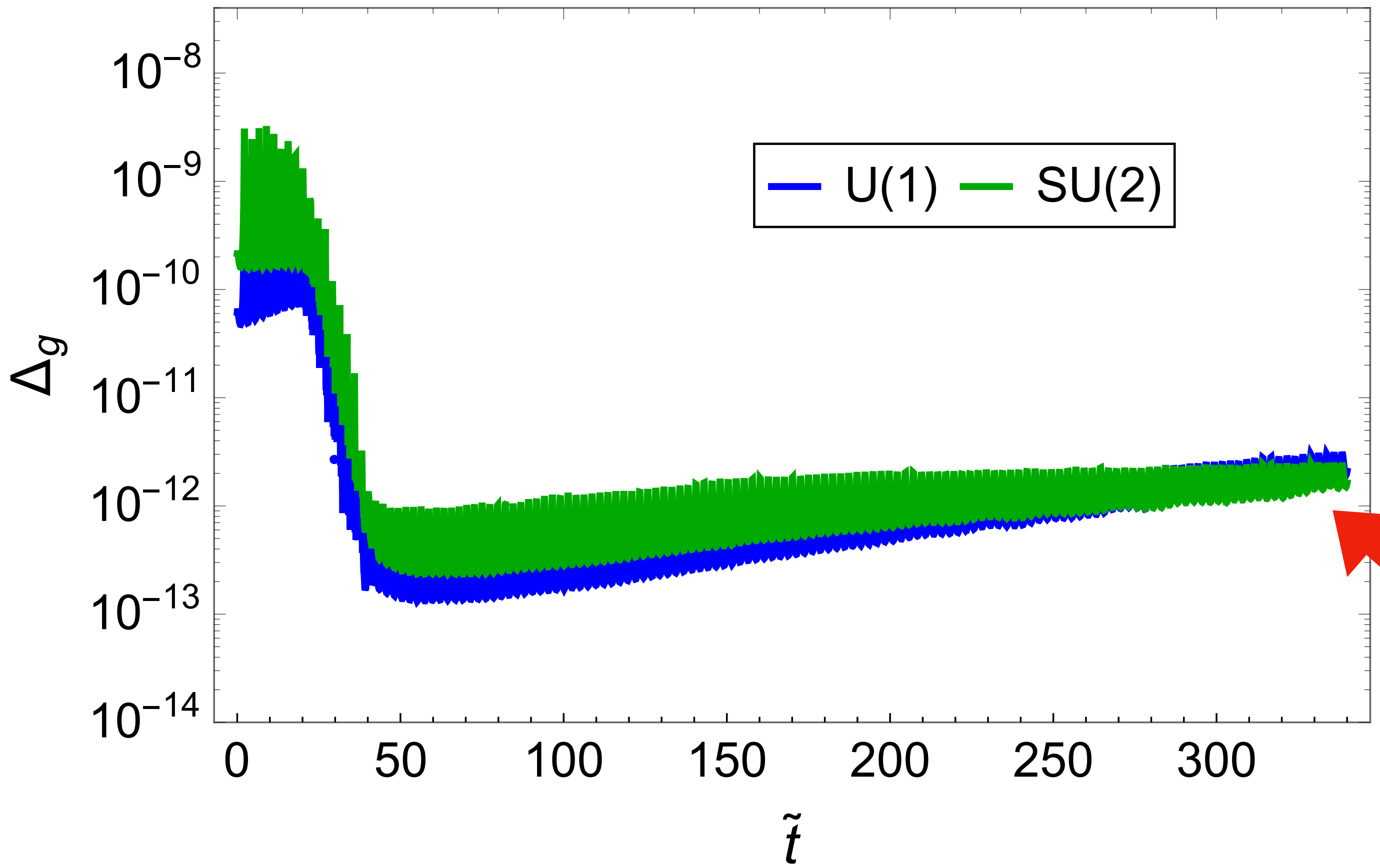
- Our gauge simulations preserve the U(1) and SU(2) **Gauss constraints**, independently of the accuracy of the integrator.

$$\boxed{\begin{aligned} \partial_i F_{0i} &= a^2 J_0^A \\ (\mathscr{D}_i)_{ab}(G_{0i})^b &= a^2 (J_0)_a \end{aligned}}$$

$$J_A^\mu \equiv 2g_A Q_A^{(\varphi)} \mathscr{I}m[\varphi^*(D_A^\mu \varphi)] + 2g_A Q_A^{(\Phi)} \mathscr{I}m[\Phi^\dagger (D^\mu \Phi)]$$

$$J_a^\mu \equiv 2g_B Q_B \mathscr{I}m[\Phi^\dagger T_a (D^\mu \Phi)]$$

$$\Delta_g \equiv \frac{\langle \sqrt{(\text{LHS} - \text{RHS})^2} \rangle}{\langle \sqrt{(\text{LHS} + \text{RHS})^2} \rangle}$$



Gauss constraint preserved up to machine precision

Future updates

- **COSMOLATTICE VERSION 2.0 (coming very soon!)**
  - **Gravitational waves** **[Baeza, Figueroa, Florio & Loayza]**
  - More accurate spectra **[Figueroa & Florio]**

- **Future potential updates:**
  - Lattice simulations in 2D
  - Initial power spectrum from input
  - Scalars non-minimally coupled to curvature
  - Axial couplings $\phi F\tilde{F}$
  - Other evolution algorithms (e.g. Runge-Kutta)
  - Non-minimal kinetic terms
  - …

CosmoLattice propaganda

Website:
http://www.cosmolattice.net

- **Documentation**:
  - *The art of simulating the early universe* - JCAP (2020) [arXiv:2006.15122]
  - CosmoLattice user manual [arXiv:2102.01031]
  - Technical notes (published in website)

COSMO$\mathcal{L}$ATTICE school

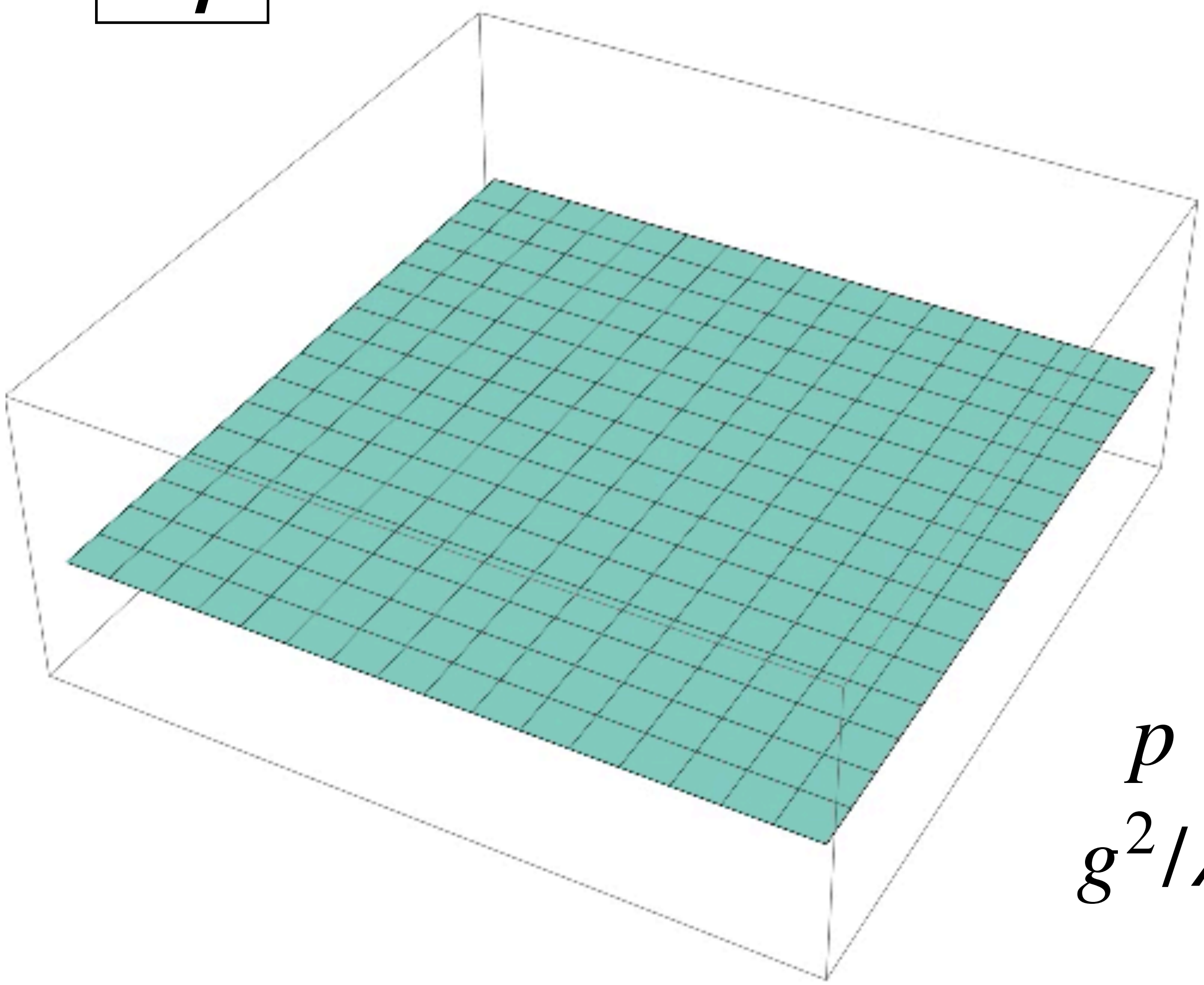
IFIC (Valencia)

5th-8th September 2022 (tentative dates)

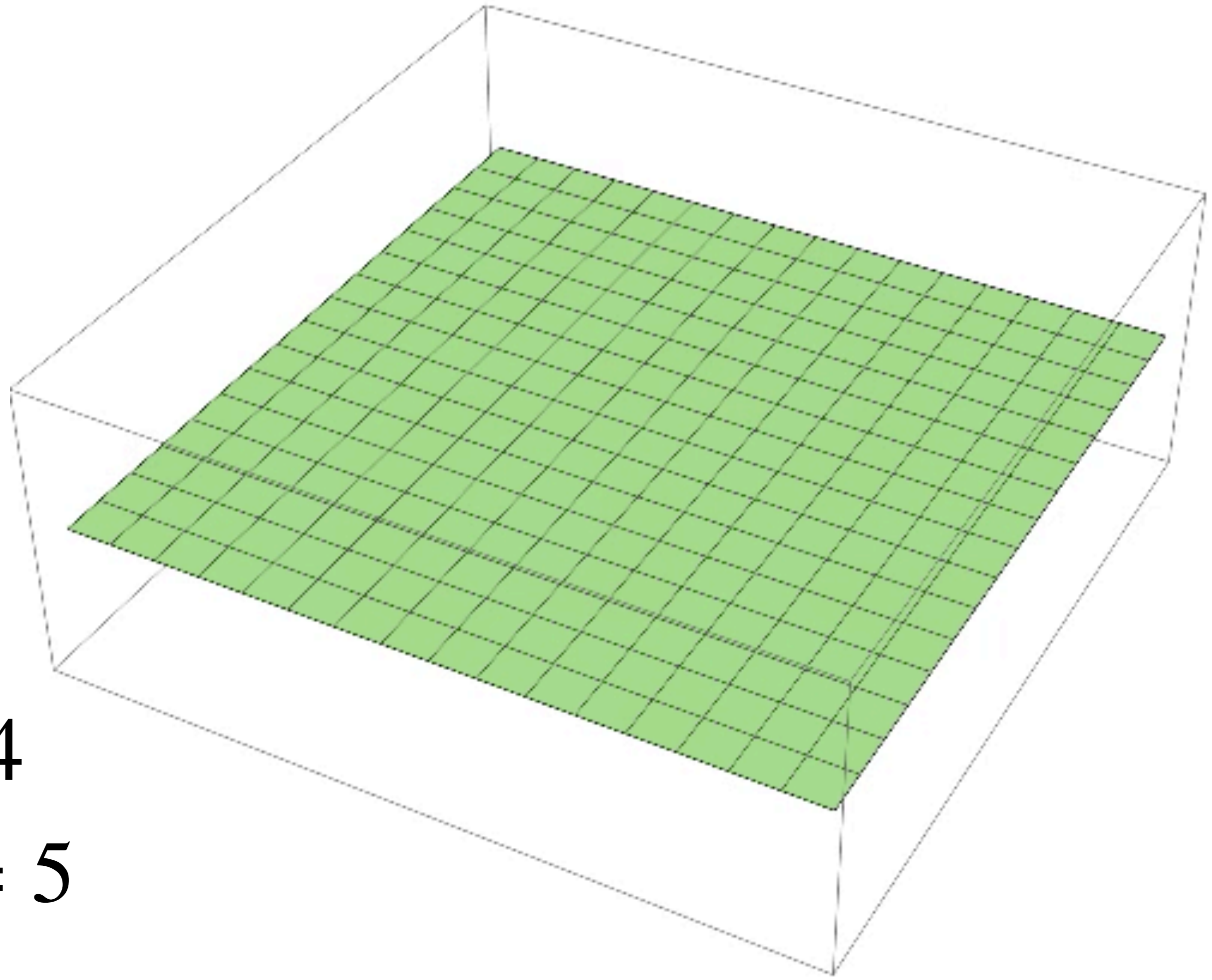
Application 1: Preheating after inflation

$$V(\phi) = \frac{1}{p}\lambda\mu^{4-p}|\phi|^p + \frac{1}{2}g^2\phi^2X^2$$

$a\phi$



aX



$p = 4$

$g^2/\lambda = 5$

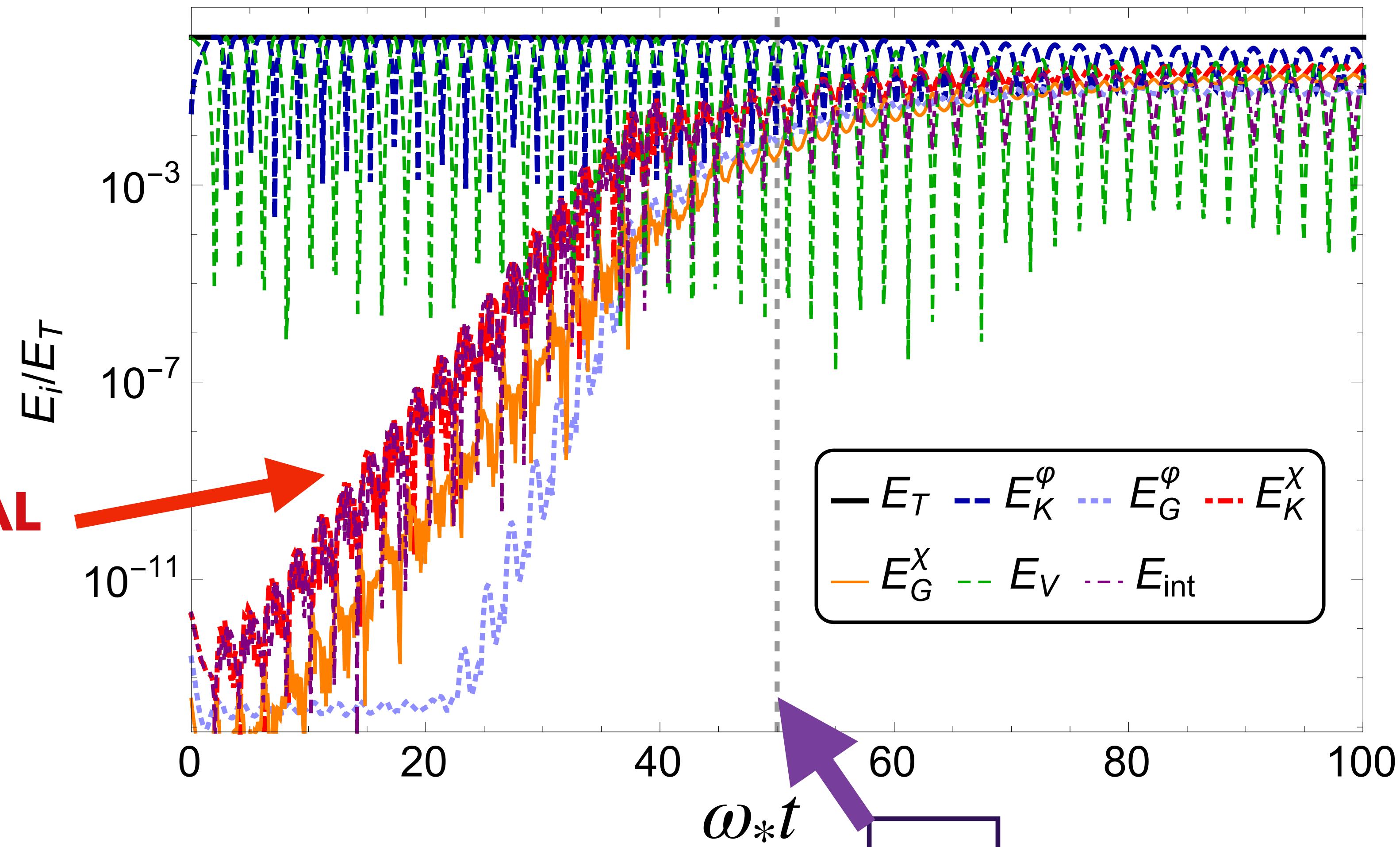
Inflaton oscillations trigger a broad parametric resonance of the "daughter field"

Figueroa & F.T. (JCAP 2016)

Application 1: Preheating after inflation

$$\rho_t(z) \equiv \frac{\lambda\phi_*^4}{a^4}E_t \equiv \frac{\lambda\phi_*^4}{a^4}\left(E_{K,\varphi} + E_{\rm V} + E_{G,\varphi} + E_{K,\chi} + E_{G,\chi} + E_{\rm int}\right)$$

Kinetic inflaton · Potential inflaton · Gradient inflaton · Kinetic daughter · Gradient daughter · Interaction energy



EXPONENTIAL GROWTH

$t_{\rm br}$ ***"BACKREACTION"*** **TIME**

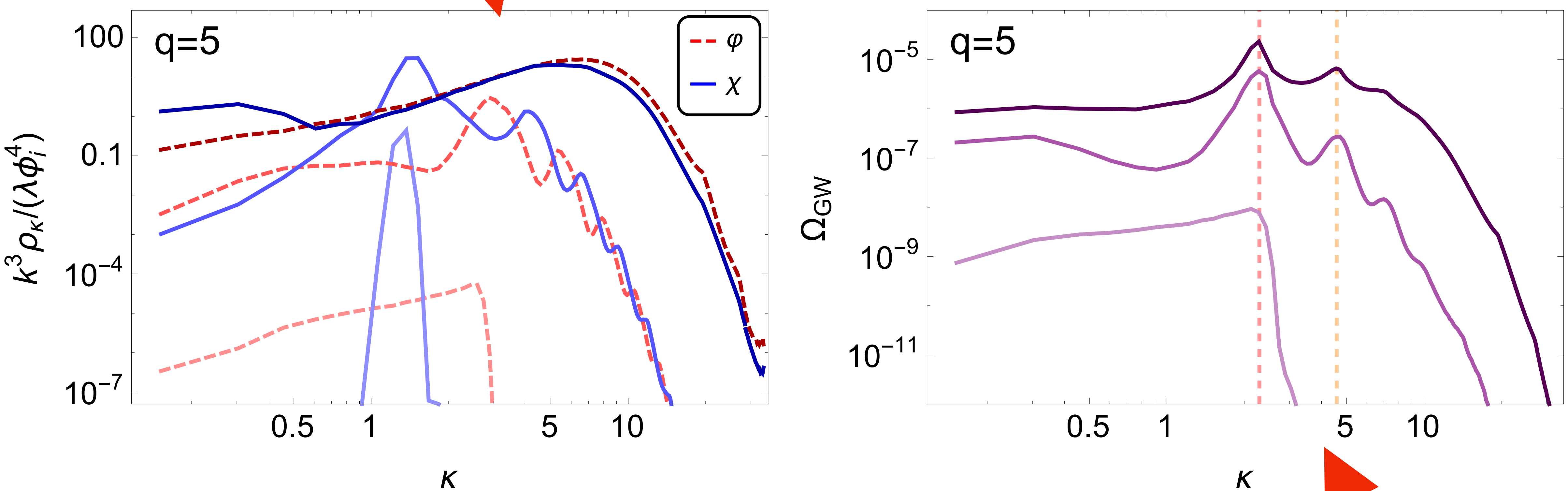
Application 2: Gravitational wave "spectroscopy"

- GWs from preheating (parametric resonance):

$$\ddot{h}_{ij} + 2\mathscr{H}\dot{h}_{ij} - \nabla^2 h_{ij} = \frac{2}{m_p^2}\left\{\partial_i X\,\partial_j X + \partial_i\phi\partial_j\phi\right\}^{\mathrm{TT}}$$

$$\boxed{V(\phi) = \frac{1}{4}\lambda\phi^4}$$

FIELD GRADIENTS...

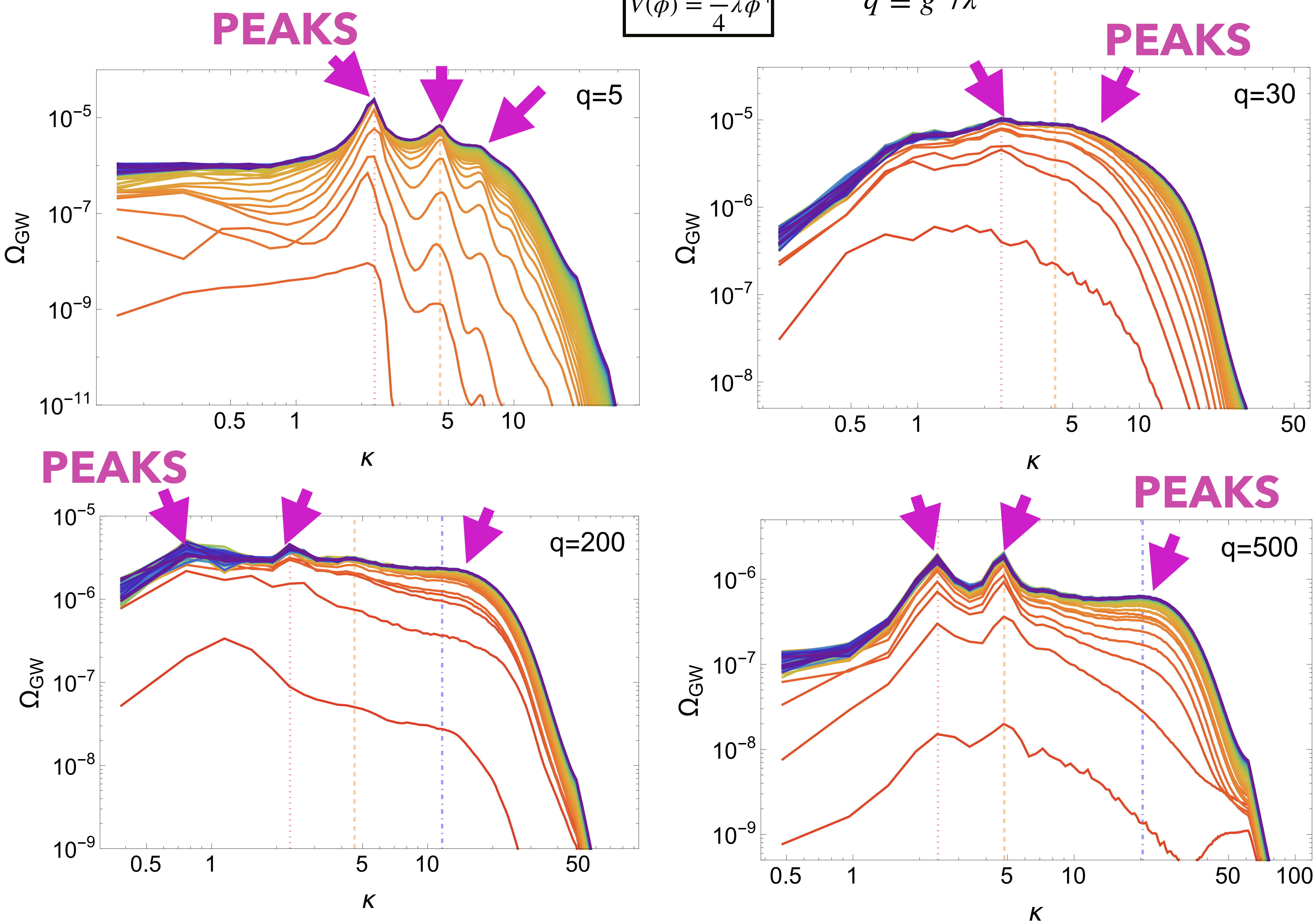


Figueroa & F.T. (JCAP 2017)

...SOURCE GWs!

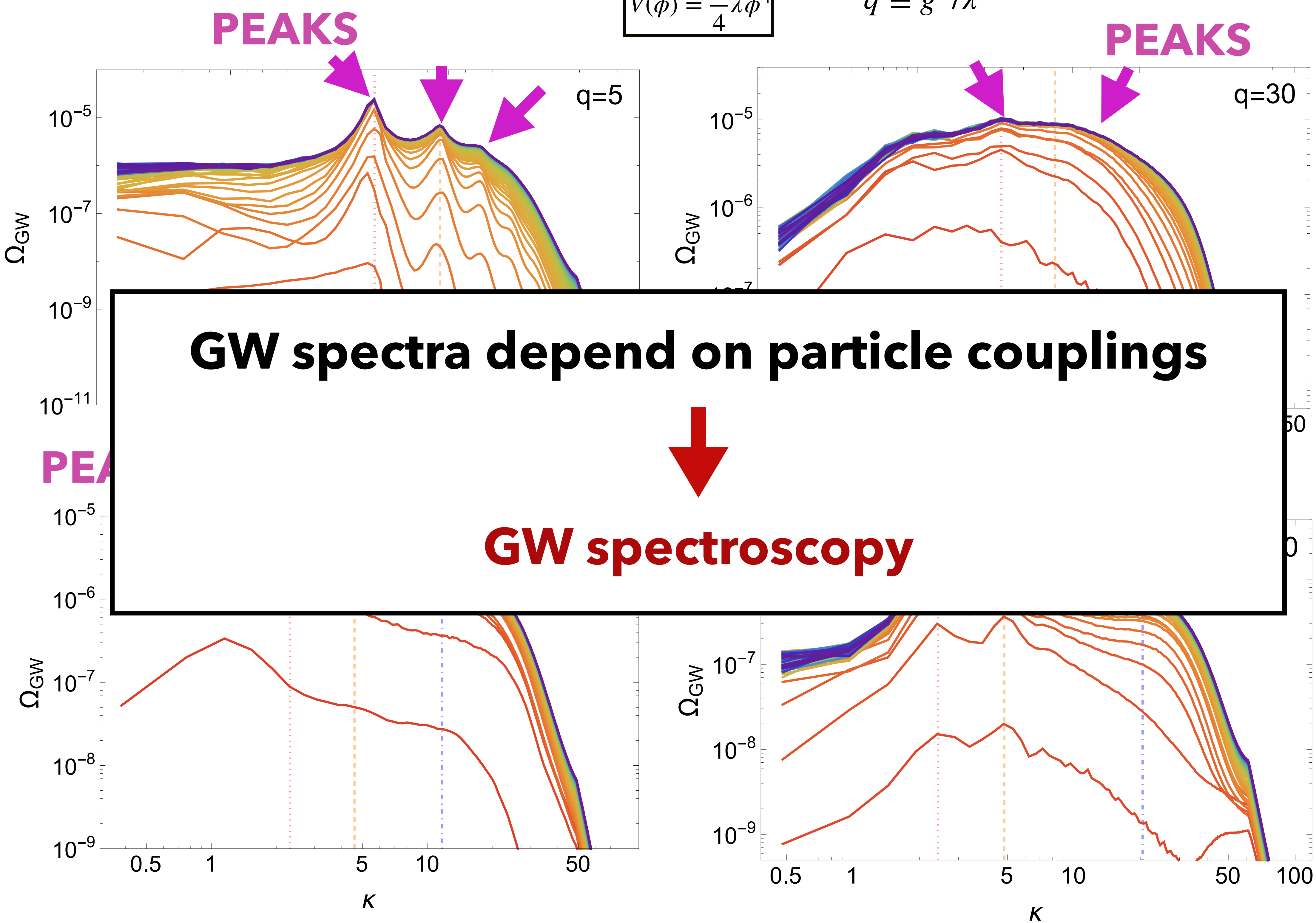
Application 2: Gravitational wave "spectroscopy"

$$\boxed{V(\phi) = \frac{1}{4}\lambda\phi^4} \qquad q \equiv g^2/\lambda$$



Application 2: Gravitational wave "spectroscopy"

$\boxed{V(\phi) = \frac{1}{4}\lambda\phi^4}$ $\qquad q \equiv g^2/\lambda$



GW spectra depend on particle couplings

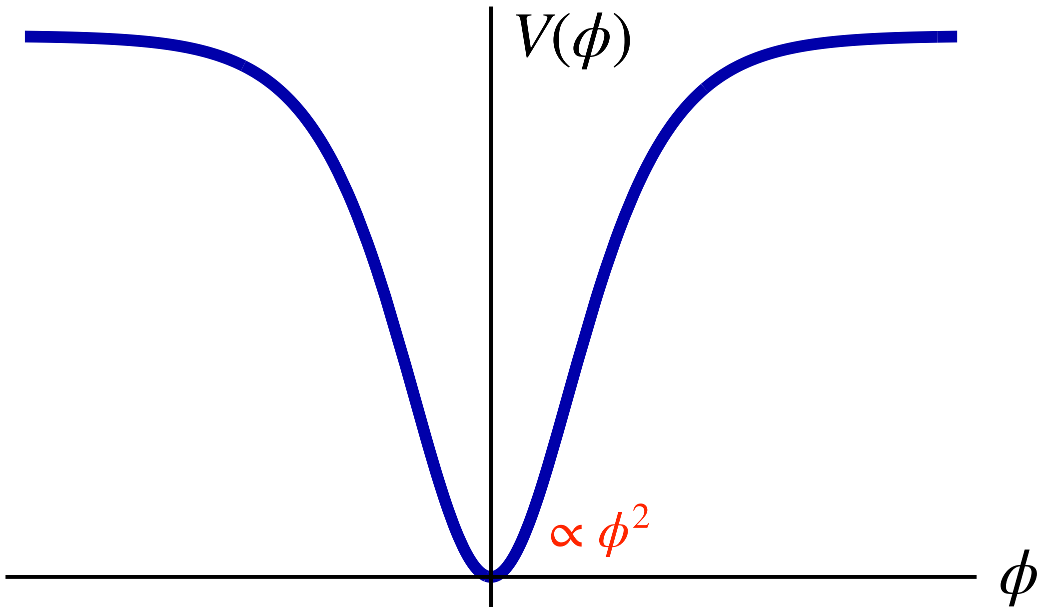
↓

GW spectroscopy

Application 3: Oscillons after inflation

What if the inflaton oscillates over shallower-than-quadratic regions?

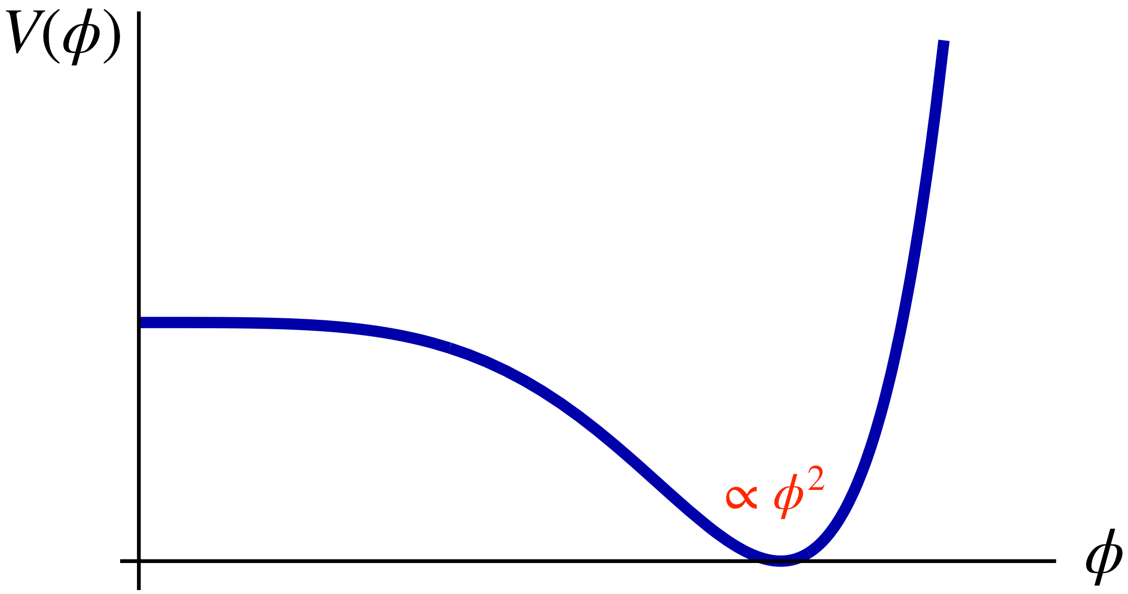
Symmetric:



$$V(\phi) = \frac{\Lambda^4}{2}\tanh^2\left(\frac{\phi}{M}\right)$$

$$(M \lesssim \mathcal{O}(0.1)m_p)$$

Asymmetric:

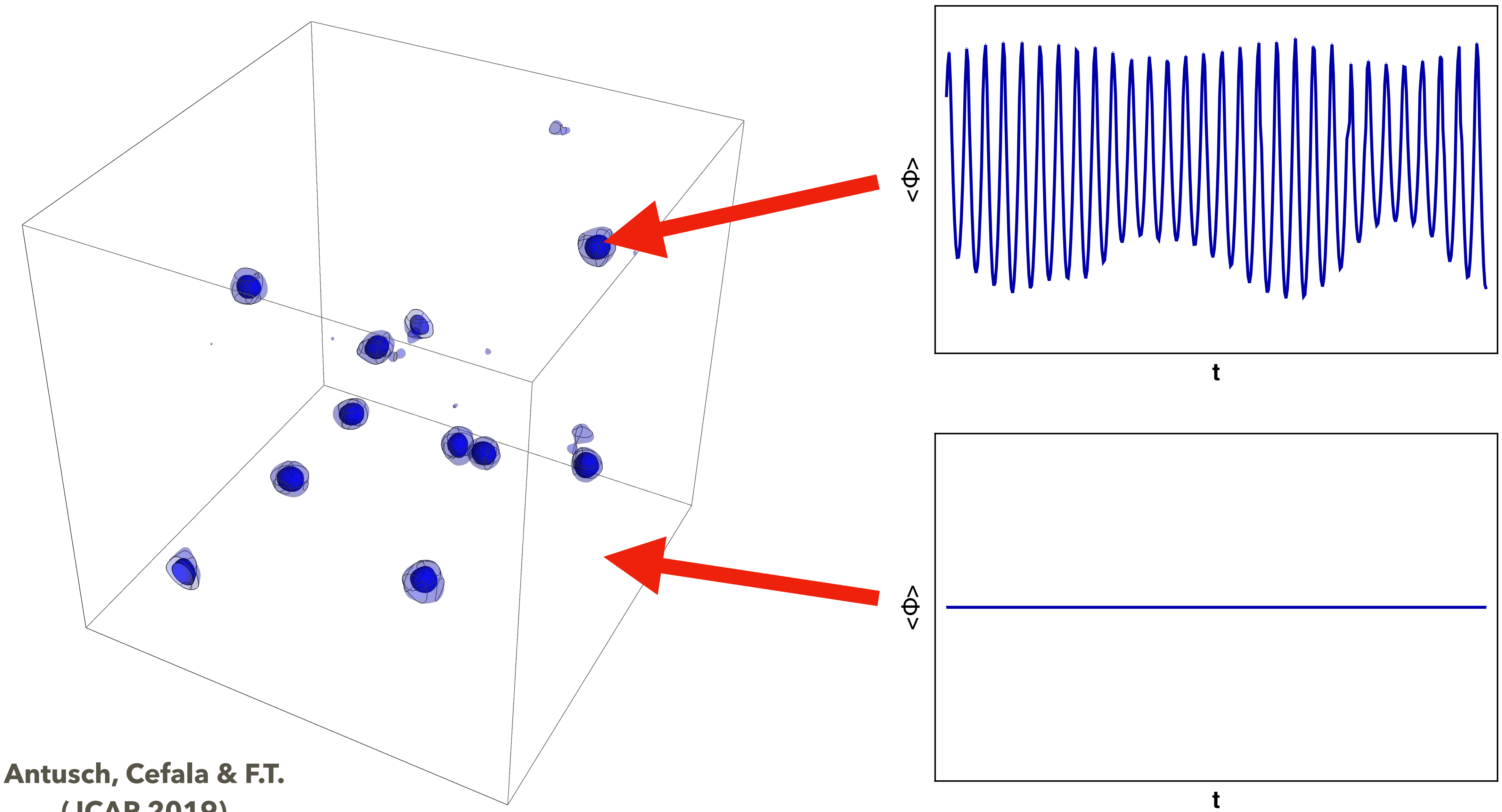


$$V(\phi) = V_0\left(1 - \frac{\phi^p}{v^p}\right)^2$$

$$p = 4,6,8\ldots$$

Application 3: Oscillons after inflation

OSCILLONS:
Spatially localised strong oscillations of a scalar field



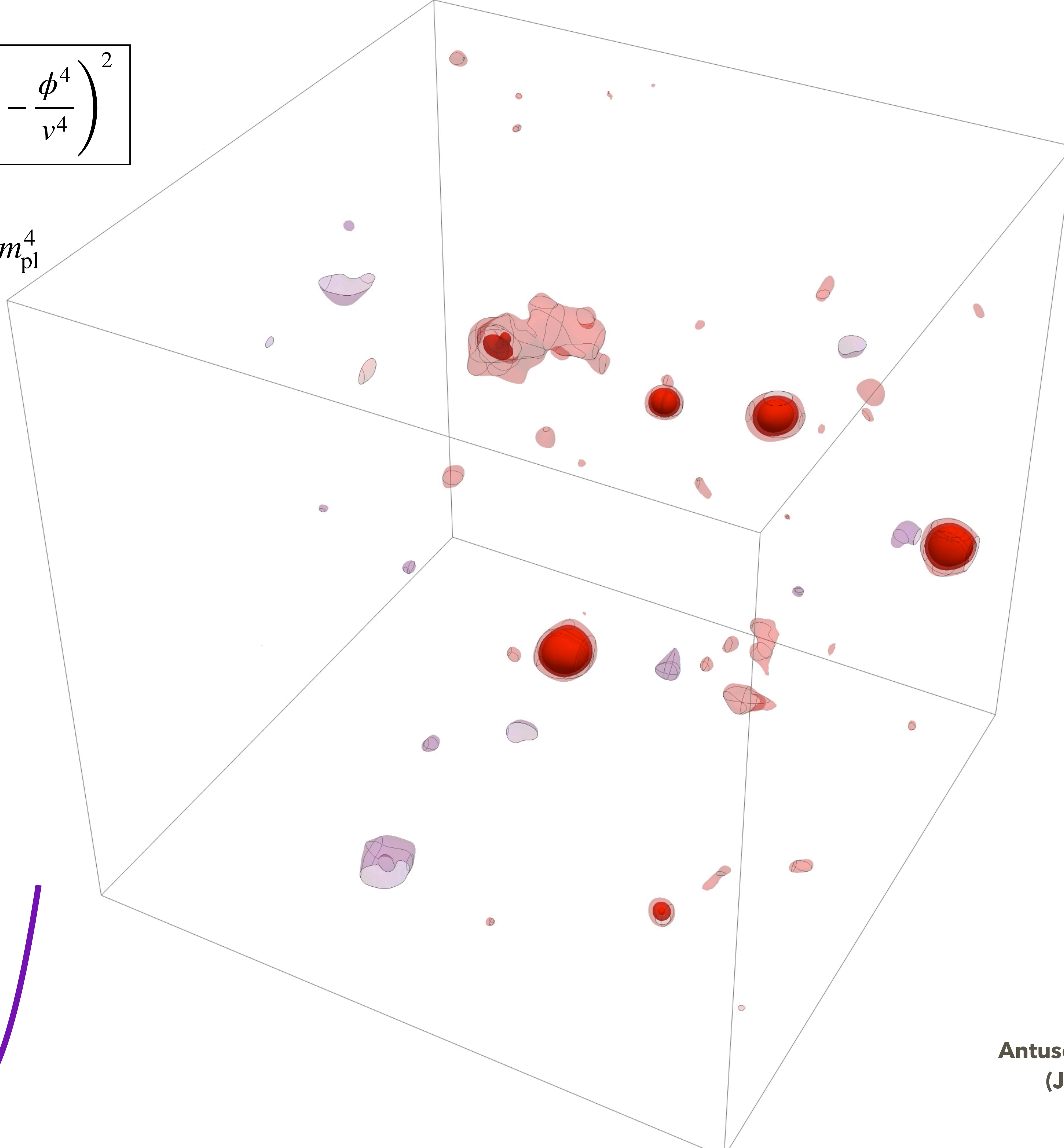
Antusch, Cefala & F.T.
(JCAP 2019)

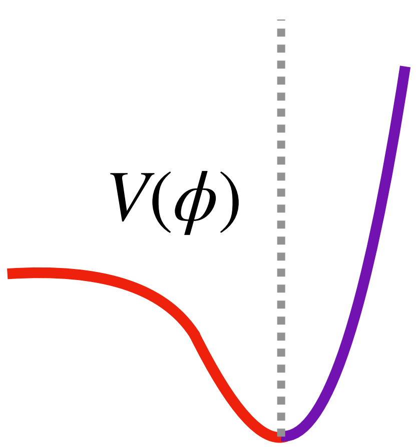
$$\boxed{V(\phi) = V_0\left(1 - \frac{\phi^4}{v^4}\right)^2}$$

$v = 0.01 m_{\rm pl}$

$V_0 = 2 \cdot 10^{-22}\, m_{\rm pl}^4$

$N^3 = (256)^3$

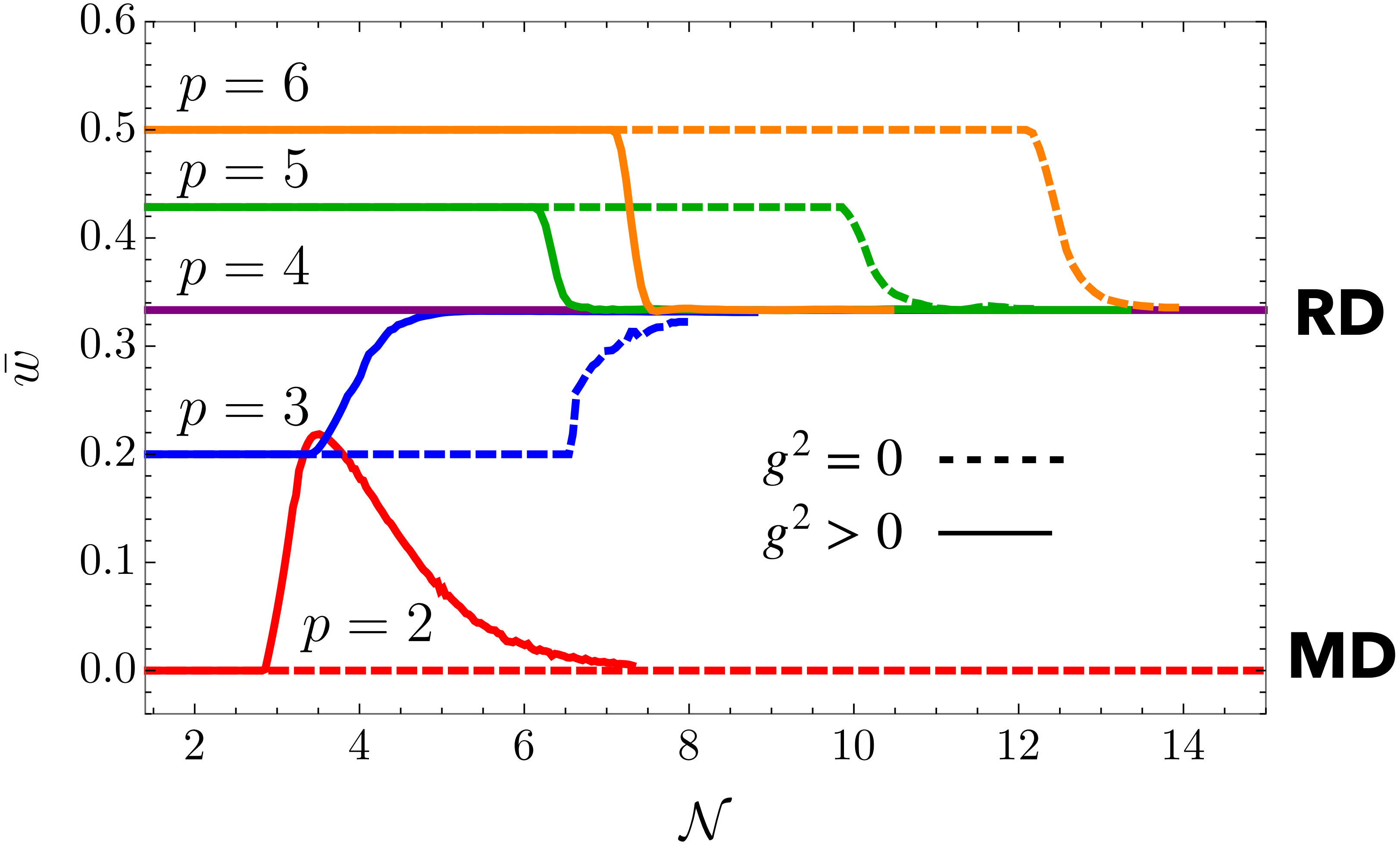




Antusch, Cefala & F.T.
(JCAP 2019)

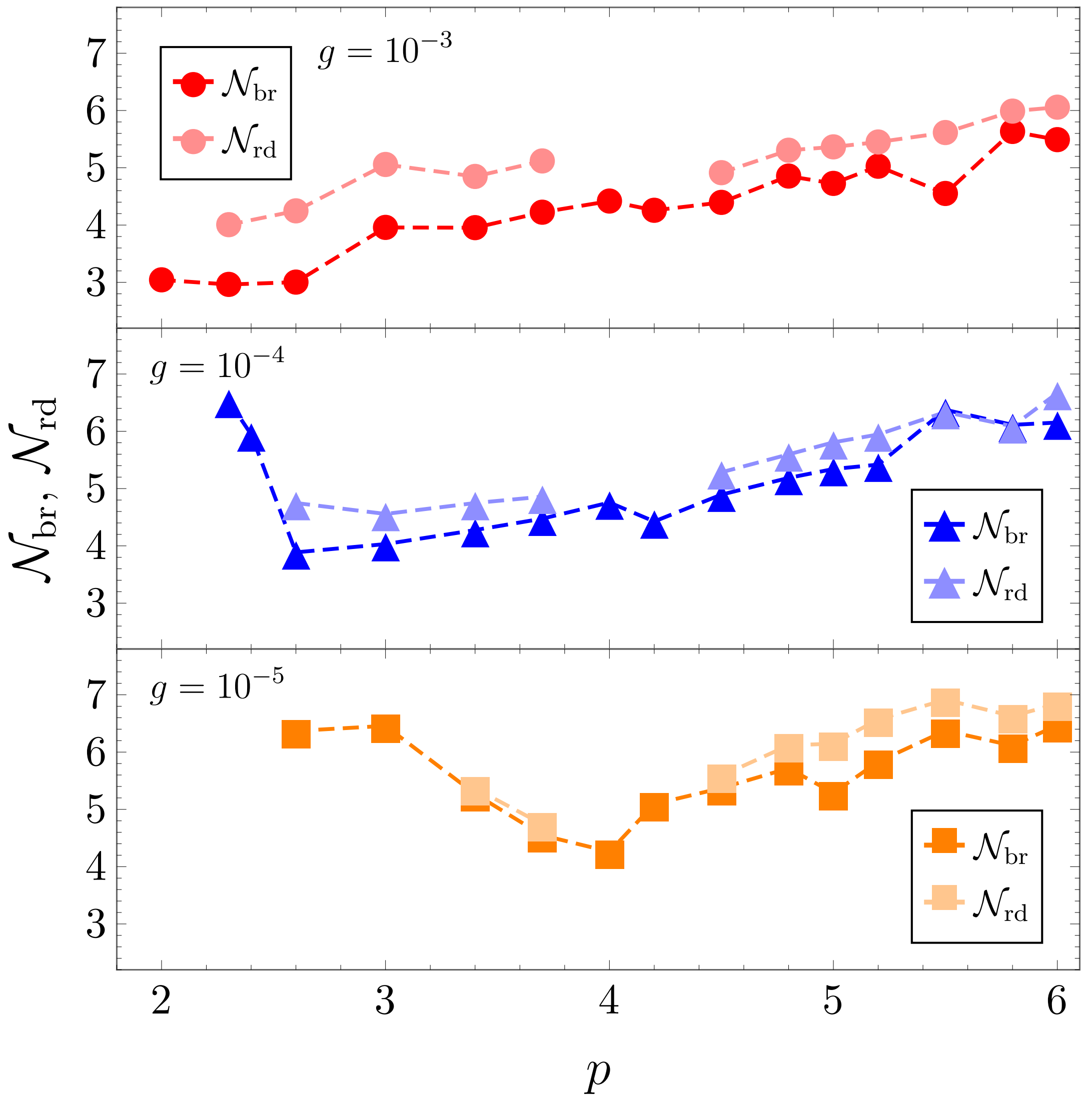
Application 4: Inflationary observables

$$\boxed{V(\phi) = \frac{\Lambda^4}{p}\tanh^p\left(\frac{|\phi|}{M}\right) + \frac{1}{2}g^2\phi^2X^2} \quad (M \gtrsim m_p)$$



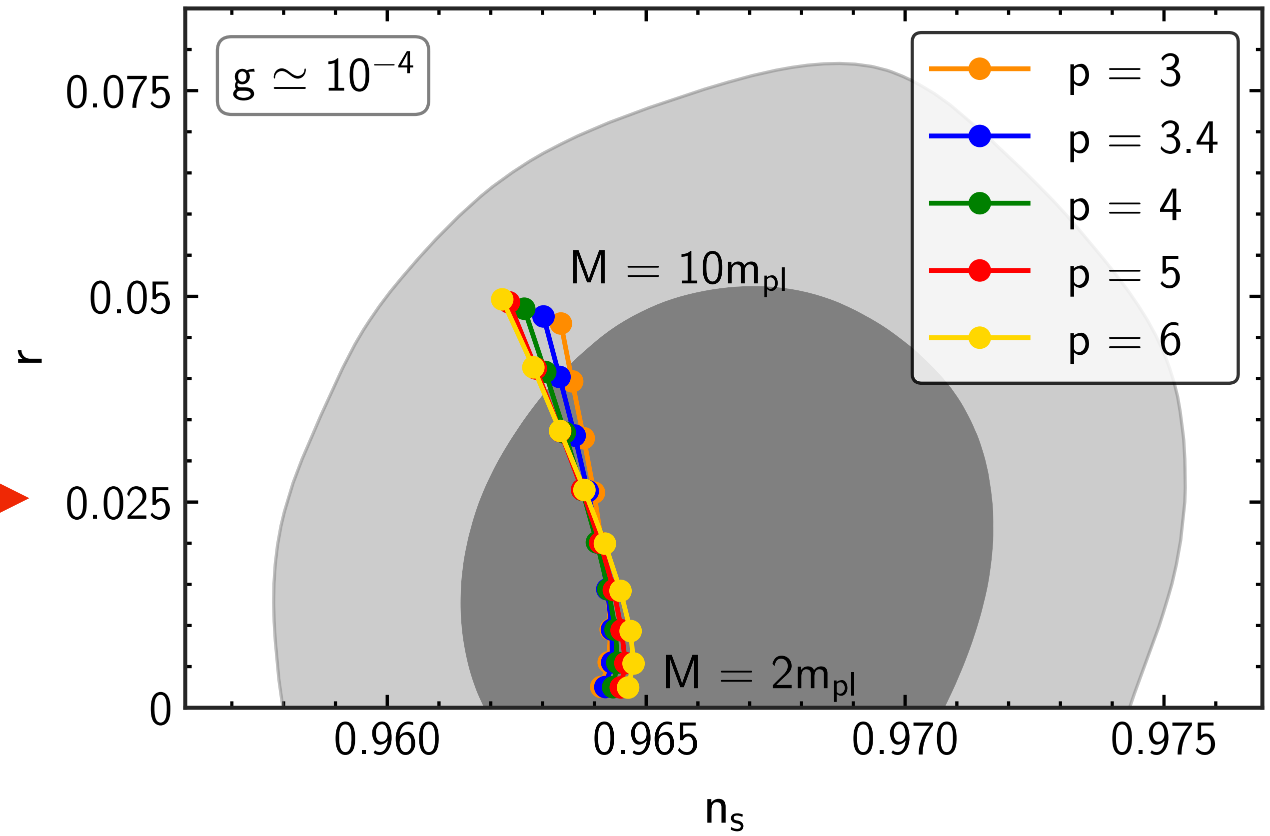
Antusch, Figueroa, Marschall & F.T. (PLB 2020, PRD 2021)

Application 4: Inflationary observables



Lattice simulations:
Number of e-folds after inflation when the Universe becomes RD

Exact predictions for n_s and r!



Thank you!