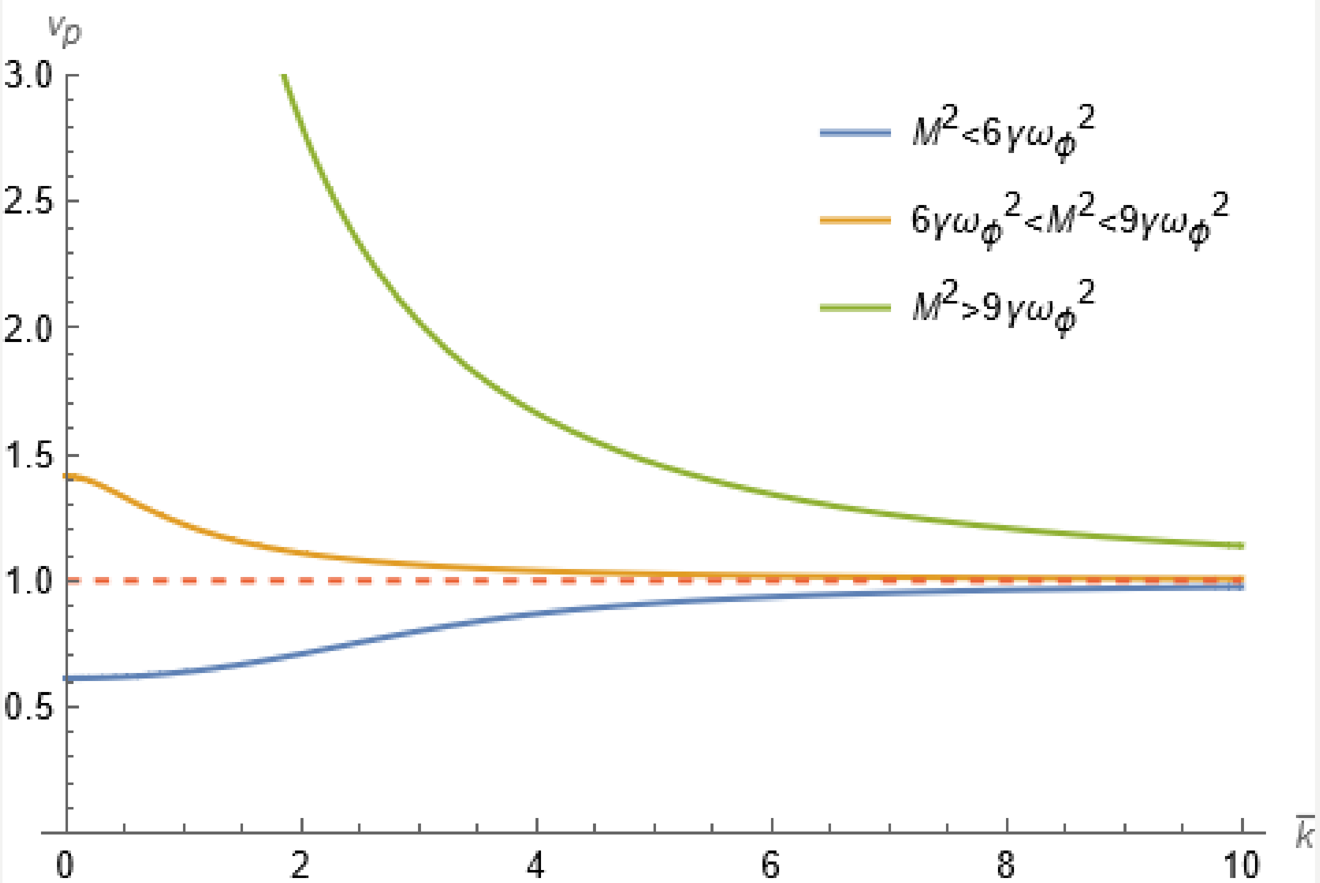
Probing dark universe with gravitational Landau damping

Part II: Numerics and extension to the beam-plasma system

Fabio Moretti - Universitat de València & IFIC

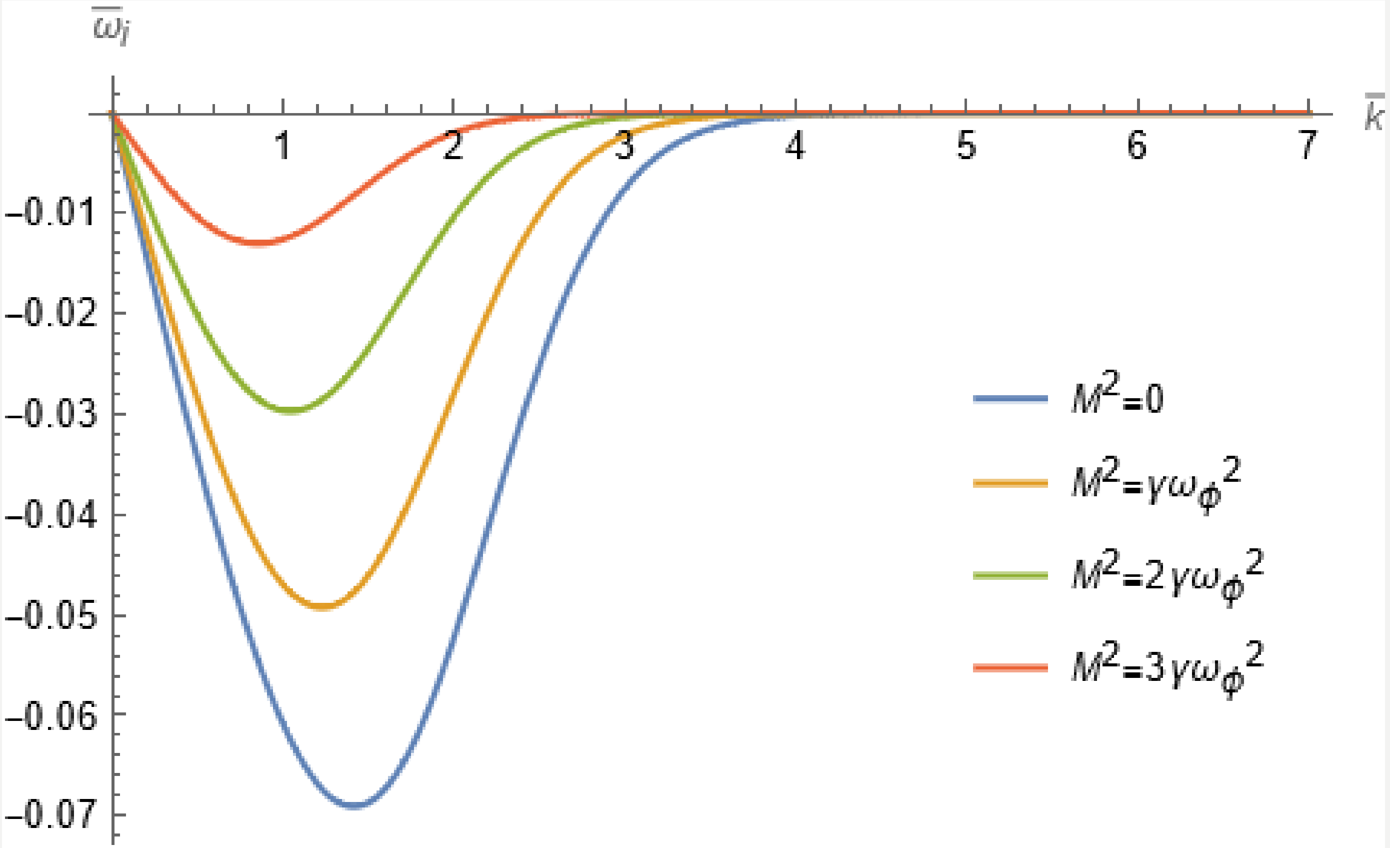
Jornadas Científicas IFIC, 28th December 2021

- Phase velocity

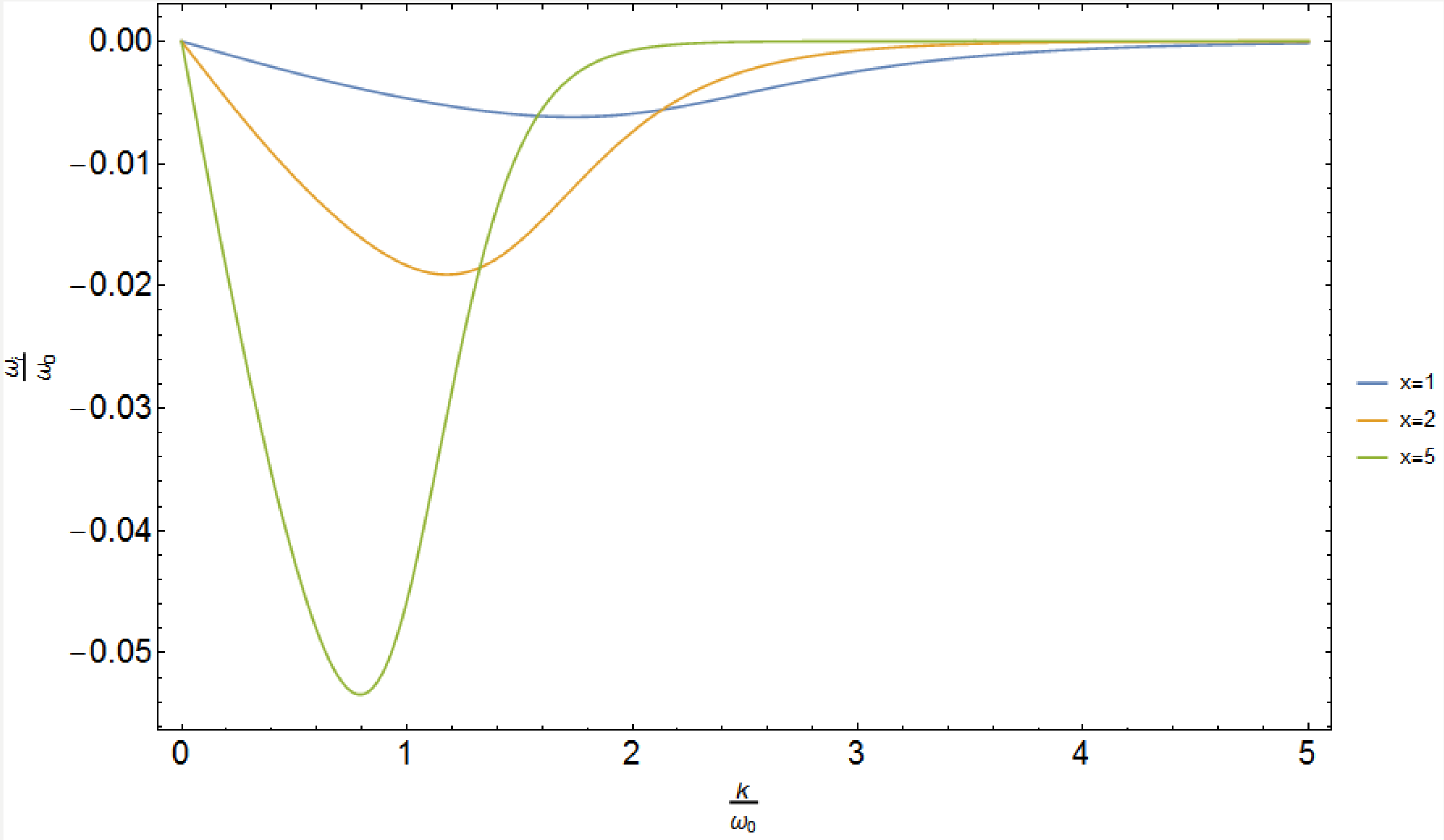


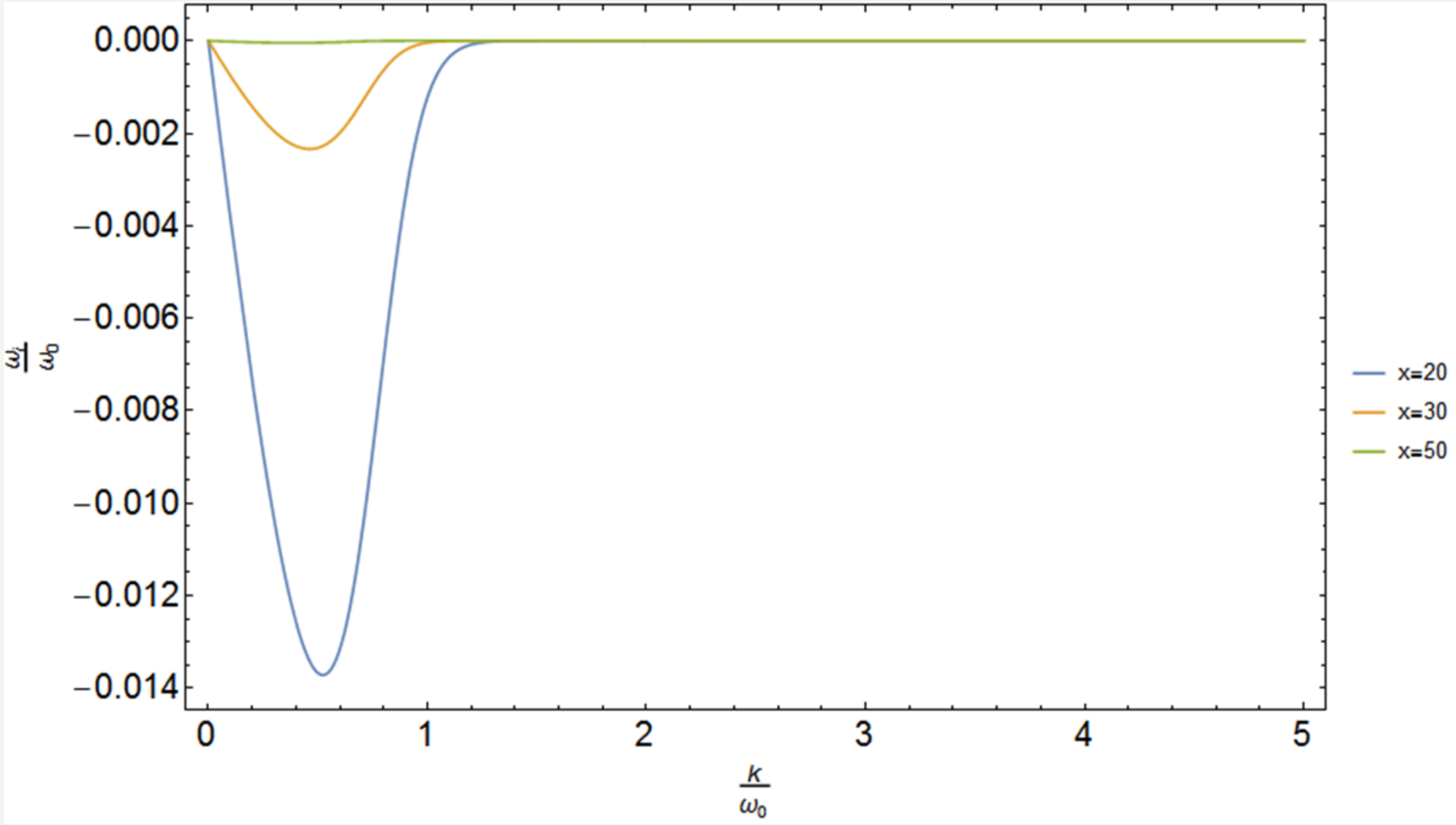
$$\lim_{k\to 0} v_p(k) \equiv v_{\min} = \frac{1}{\sqrt{3 - \frac{\bar{M}^2}{3\gamma}}} \qquad \bar{M} = \frac{M}{\omega_\phi}$$

- Damping rate: dependence from M



- Damping rate: dependence from x



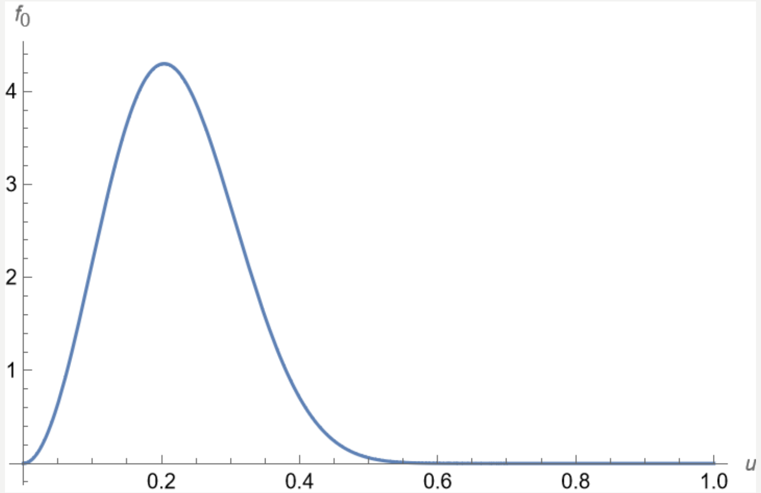

0.000
−0.002
−0.004
−0.006
−0.008
−0.010
−0.012
−0.014
$\frac{\omega_i}{\omega_0}$
0
1
2
3
4
5
$\frac{k}{\omega_0}$
x=20
x=30
x=50

- Maximum damping: $\bar{k} \approx 1$

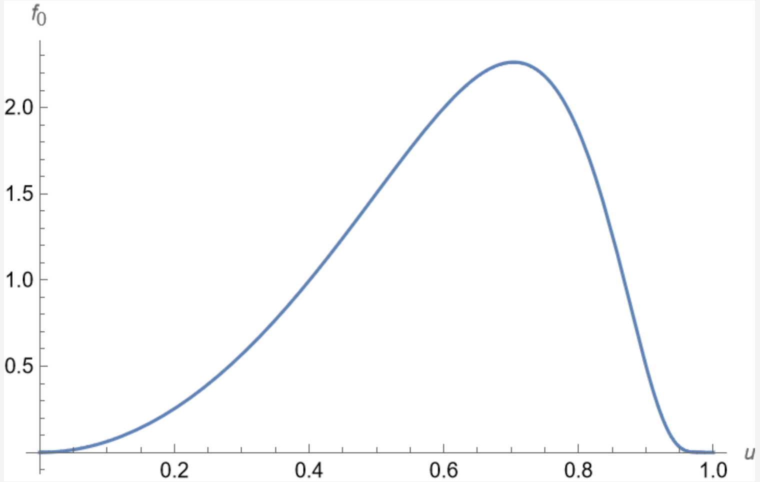
- The proper frequency is $\omega_\phi^2 = \dfrac{\kappa\rho}{6}$

- The parameter x must be of order unity (maximum damping for $x \approx 5$)

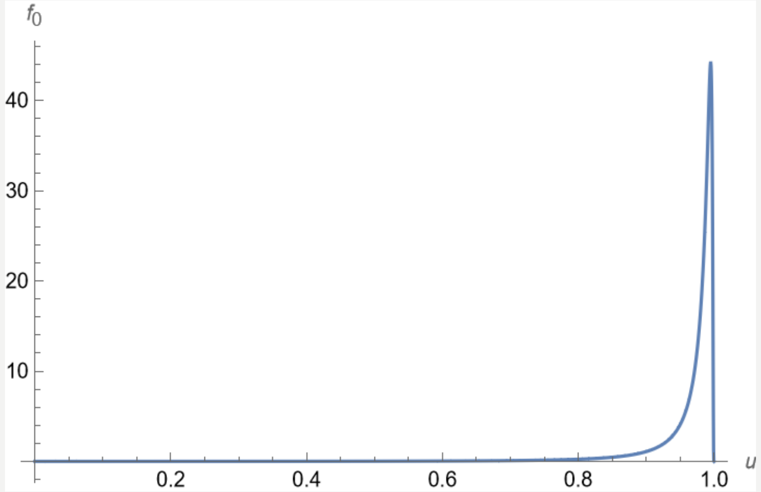
Background distribution



$$x = 50$$



$$x = 5$$



$$x = 0.5$$

- Velocities domain $u_i = \frac{p_i}{p^0}$

$$\epsilon(k,\omega) = 1 - \frac{3\omega_\phi^2 x^2}{4\pi\left(k^2 + M^2 - \omega^2\right) K_2(x)} \int_{-1}^{1} du_3 \, \frac{F(u_3)}{u_3 - v_p}$$

$$F(u_3) = \frac{2\pi}{x^3}\left(2\left(u_3 - v_p\right) + \frac{2x\left(u_3 - v_p\right)}{\sqrt{1-u_3^2}} + \frac{x^2 u_3\left(1 - v_p u_3\right)}{1-u_3^2}\right) e^{-\frac{x}{\sqrt{1-u_3^2}}}$$

- Integration with rectangles: step width 10^{-2}

$$\epsilon(\bar{k},\bar{\omega}) = 1 - \frac{3\cdot 10^{-2}}{2xK_2(x)\left(\bar{k}^2 + \bar{M}^2 - \bar{\omega}^2\right)} \times$$

$$\times \left[e^{-7.09x}\left(2 + 14.18x + 50.25x^2 + \frac{v_p x^2}{-0.99 - v_p}\right) + \right.$$

$$\left. + e^{-5.03x}\left(2 + 10.05x + 25.25x^2 + \frac{v_p x^2}{-0.98 - v_p}\right) + \cdots \right]$$

- Polynomial of degree 200: expansion of ϵ up to third order in $\delta = 1 - v_p > 0$
- Selection of δ_0 solution of the cubic satisfying the ansatz
- The dispersion relation is obtained as $\bar{\omega}_r\left(\bar{k}; x, \bar{M}\right) = \bar{k}\left(1 - \delta_0\right)$
- We retain the frequency imaginary part exactly calculated through residue theorem

- Dependence from redshift

$$k = k^{(0)}(1+z)$$
$$\omega_\phi = \omega_\phi^{(0)}(1+z)^{3/2}$$
$$x = x^{(0)}(1+z)^{-1}$$

- Parameters selection

$$k^{(0)} = 5.6 \cdot 10^{-3}\, Mpc^{-1}$$
$$M = 2.5 \cdot 10^{-28}\, eV$$
$$m = 1.75\, eV$$

$\longrightarrow$

- Modes detectable on the CMB
- Constraints on graviton mass
- $\bar{k} \approx 1$ and $x \approx 5$ at a chosen redshift ($z \approx 2000$)

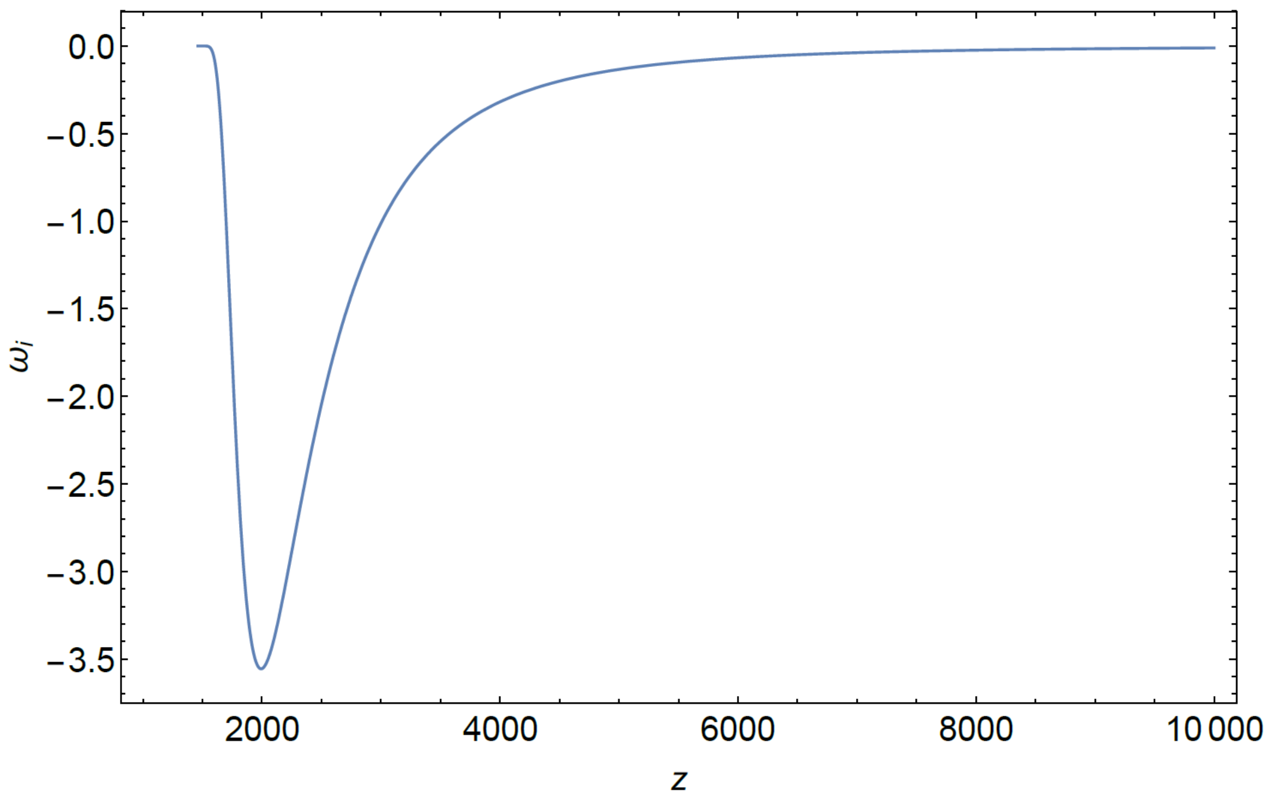


Figure 8.3. Damping coefficient ω_i (in 10^{-16} Hz) vs redshift z.

$$\bar{M}^2 < 6\gamma$$

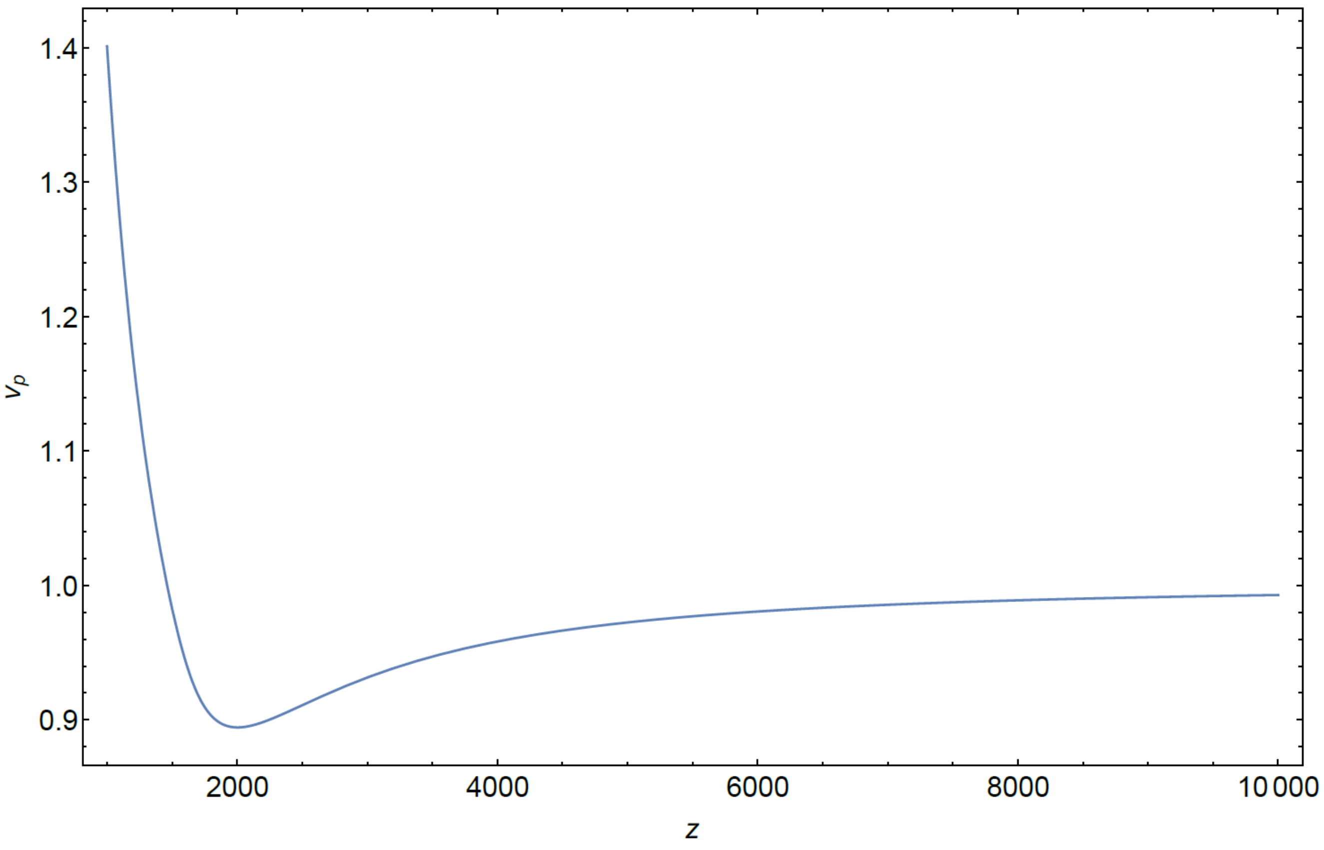


Figure 8.4. Phase velocity v_p vs redshift z.

- Time-integrated effect

$$\mathcal{A}(t) = e^{\int_{t_0}^{t} \omega_i(t')\, dt'} \longrightarrow \mathcal{A} = e^{\int_{t(z_0)}^{t(z)} \omega_i(z(t'))\, dt'(z)}$$

- Radiation dominated era $z > 3400$

$$z(t) \simeq \left(2H_0\sqrt{\Omega_{0,r}}t\right)^{-\frac{1}{2}}$$

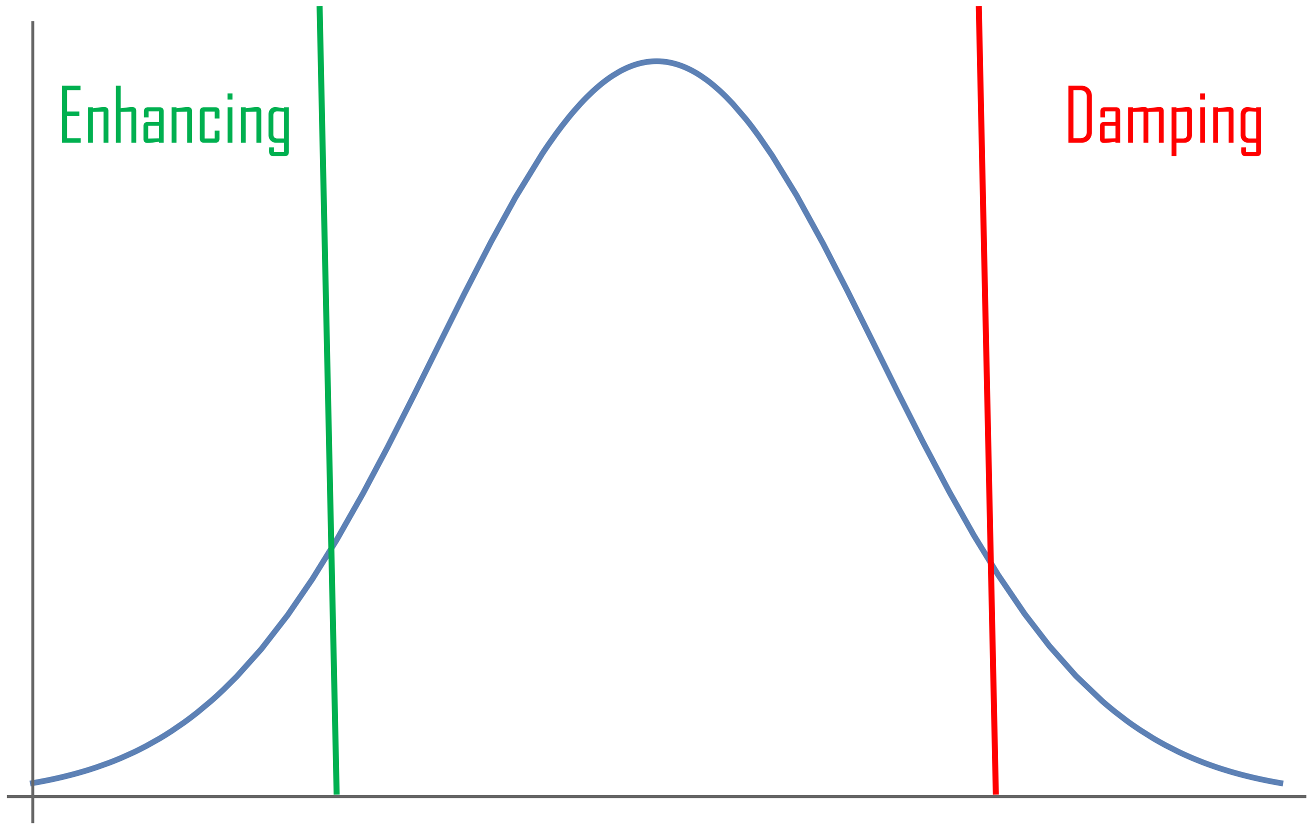
- Radiation dominated era $z < 3400$

$$z(t) \simeq \left(\frac{3}{2}H_0\sqrt{\Omega_{0,m}}t\right)^{-\frac{2}{3}}$$

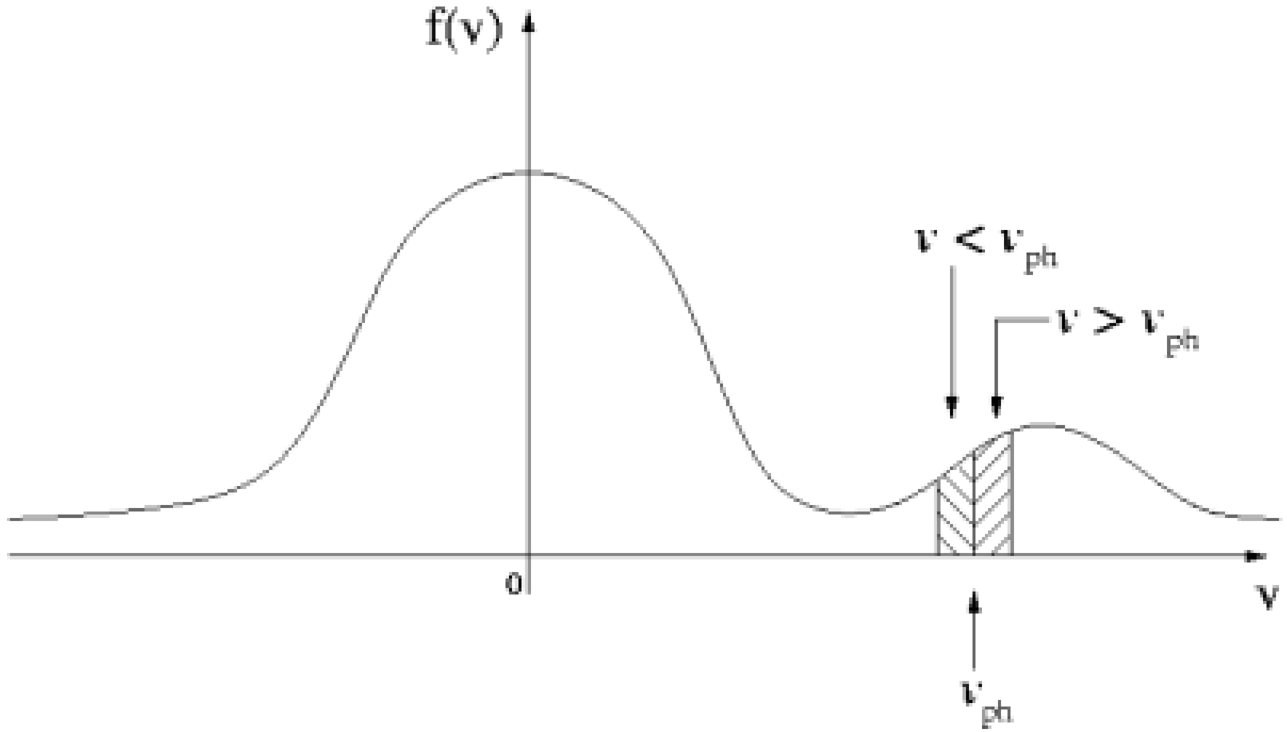
Absorption on the specific $k^{(0)}$

$$\boxed{1 - \mathcal{A} = 8.2 \cdot 10^{-4}}$$

$$z \in (1450, 10^4)$$


Enhancing
Damping

Bump-on-tail distribution



BEAM-PLASMA SYSTEM

- Medium in the absence of gravitational waves

$$f_0^{TOT}(p_1,p_2,p_3) = f_0^P(p) + f_0^B(p_1,p_2,p_3)$$

- Vlasov equation: only scalar perturbations

$$\frac{\partial \delta f}{\partial t} + \frac{p^i}{p^0}\frac{\partial \delta f}{\partial x^i} + \frac{df_0^P}{dp}\frac{\alpha}{2p}\left(p^2\frac{\partial \phi}{\partial t} + p^i p^0 \frac{\partial \phi}{\partial x^i}\right) + \frac{\alpha m^2}{2p^0}\frac{\partial f_0^B}{\partial p_i}\frac{\partial \phi}{\partial x^i} = 0$$

- Solution in the Fourier-Laplace space

$$\phi_{k,s} = \frac{\left(s + \frac{\alpha m^2 \kappa'}{2}\int d^3p \frac{df_0^P}{dp}\frac{p}{p^0 s + ikp_3}\right)\phi_k(0)}{\left(s^2+k^2+M^2\right)\left(\epsilon^P(k,s)+\epsilon^B(k,s)\right)}$$

- Dielectric function

$$\epsilon^P(k,s) = 1 + \frac{\alpha m^2 \kappa'}{2\left(s^2+k^2+M^2\right)}\int d^3p \frac{p^2 s + ikp_3 p^0}{p\left(p^0 s + ikp_3\right)}\frac{df_0^P}{dp}$$

$$\epsilon^B(k,s) = \frac{ik\alpha m^4 \kappa'}{2\left(s^2+k^2+M^2\right)}\int d^3p \frac{1}{p^0\left(p^0 s + ikp_3\right)}\frac{\partial f_0^B}{\partial p_3}$$

MONOCHROMATIC BEAM

- Beam distribution

$$f_0^B(p_1, p_2, p_3) = n_B \delta(p_1)\delta(p_2)\delta(p_3 - p_B)$$

- Dielectric functions

$$\epsilon^P(k, \omega) = 1 + 3\gamma\omega_\phi^2 \frac{k^2 - 3\omega^2}{\omega^2(k^2 + M^2 - \omega^2)}$$

$$\epsilon^B(k, \omega) = \frac{3\eta\omega_\phi^2}{k^2 + M^2 - \omega^2} \frac{\left(1 - v_B^2\right)^{\frac{3}{2}} \left(1 + v_B^2 - 2\frac{\omega}{k} v_B\right)}{\left(v_B - \frac{\omega}{k}\right)^2}$$

$$v_B = \frac{p_B}{\sqrt{p_B^2 + \mu^2}} \qquad \eta = \frac{n_B}{n_P}$$

- The condition of vanishing total dielectric function implies

$$\frac{\epsilon^P(k,\omega)\left(\omega^2-k^2-M^2\right)\left(v_B-\frac{\omega}{k}\right)^2}{\left(1+v_B^2-2\frac{\omega}{k}v_B\right)\left(1-v_B^2\right)^{\frac{3}{2}}}=3\omega_\phi^2\eta \qquad (\star)$$

- Tenuous beam: $\eta \ll 1$

- Plasma solution: $\epsilon^P = 0$

- Beam solution: $\omega = kv_B$

- Degenerate points: $\epsilon^P(k_0, v_B k_0) = 0$

$$k_0=\sqrt{\frac{3\gamma\omega_\phi^2\left(3v_B^2-1\right)-M^2v_B^2}{v_B^2\left(1-v_B^2\right)}} \xrightarrow{\text{Reality}} v_B \geq v_{\min}=\frac{1}{\sqrt{3-\frac{\bar{M}^2}{3\gamma}}}$$

NEAR THE DEGENERATE POINT

- Expansion of $(\star)$ up to third order in the proximity of the degenerate point

$$\left(\frac{\delta\omega}{\omega_0}-\frac{\delta k}{k_0}\right)^2\left(P\,\frac{\delta\omega}{\omega_0}+Q\,\frac{\delta k}{k_0}\right)=C$$

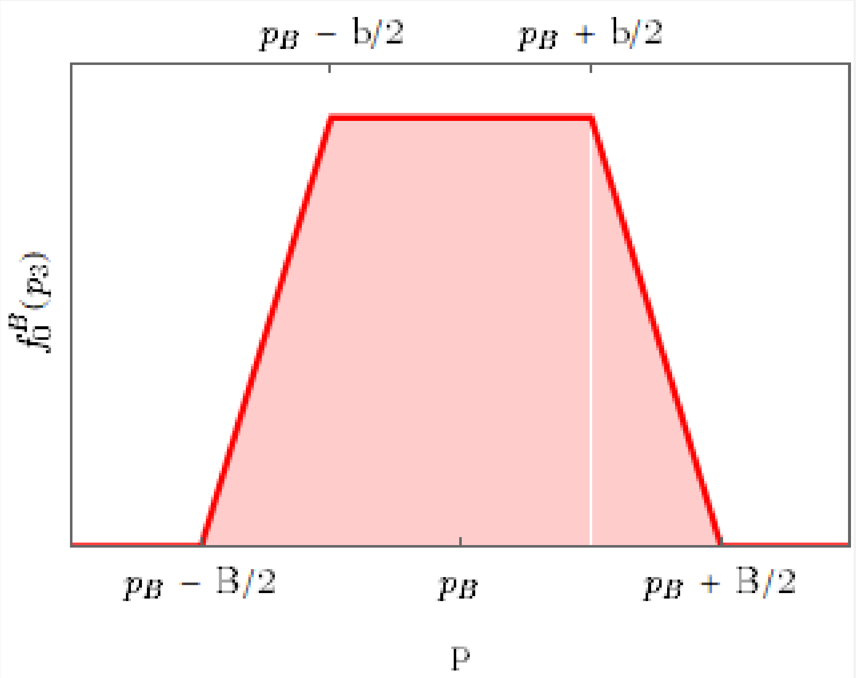
- A single real solution + a couple of complex conjugates solutions

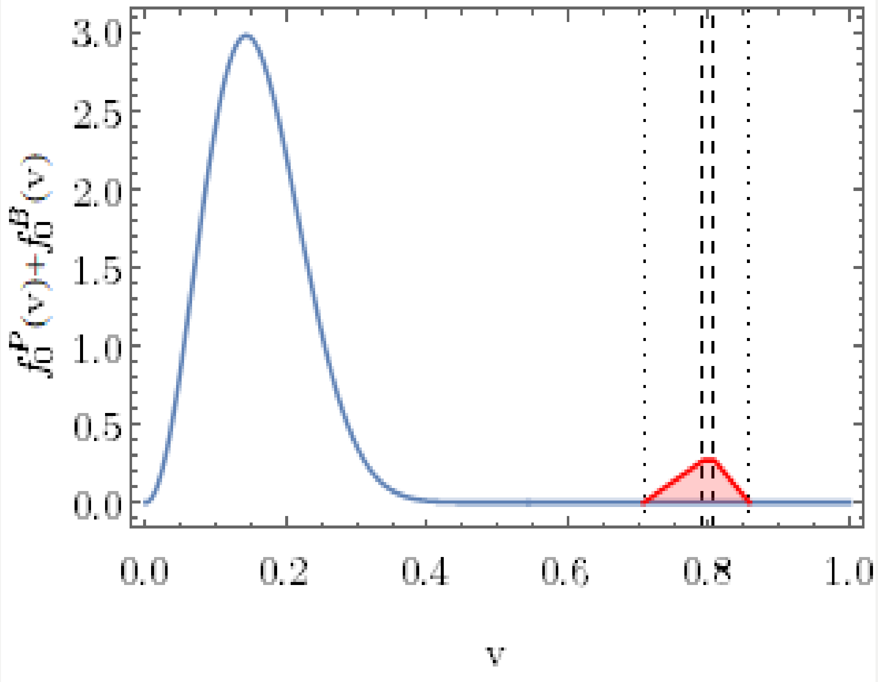
- We select the complex solution with positive imaginary part: $\delta\omega^*\left(\delta k\right)$

$$\frac{\Im\left(\delta\omega^{\star}(0)\right)}{\omega_0}=\left(\frac{\sqrt{27}\eta\left(1-v_B^2\right)^{\frac{7}{2}}}{16\left(1-2v_B^2+\left(\frac{v_B^2}{v_{\min}}\right)^2\right)}\right)^{\frac{1}{3}}$$

NUMERICAL ANALYSIS

- Trapezoidal beam





- Beam contribution

$$\epsilon^B(k,\omega) = \frac{3\omega_\phi^2}{k^2+M^2-\omega^2}\frac{4\eta m^2}{B^2-b^2}\left[\Gamma(u_1,u_2)-\Gamma(u_3,u_4)\right]$$

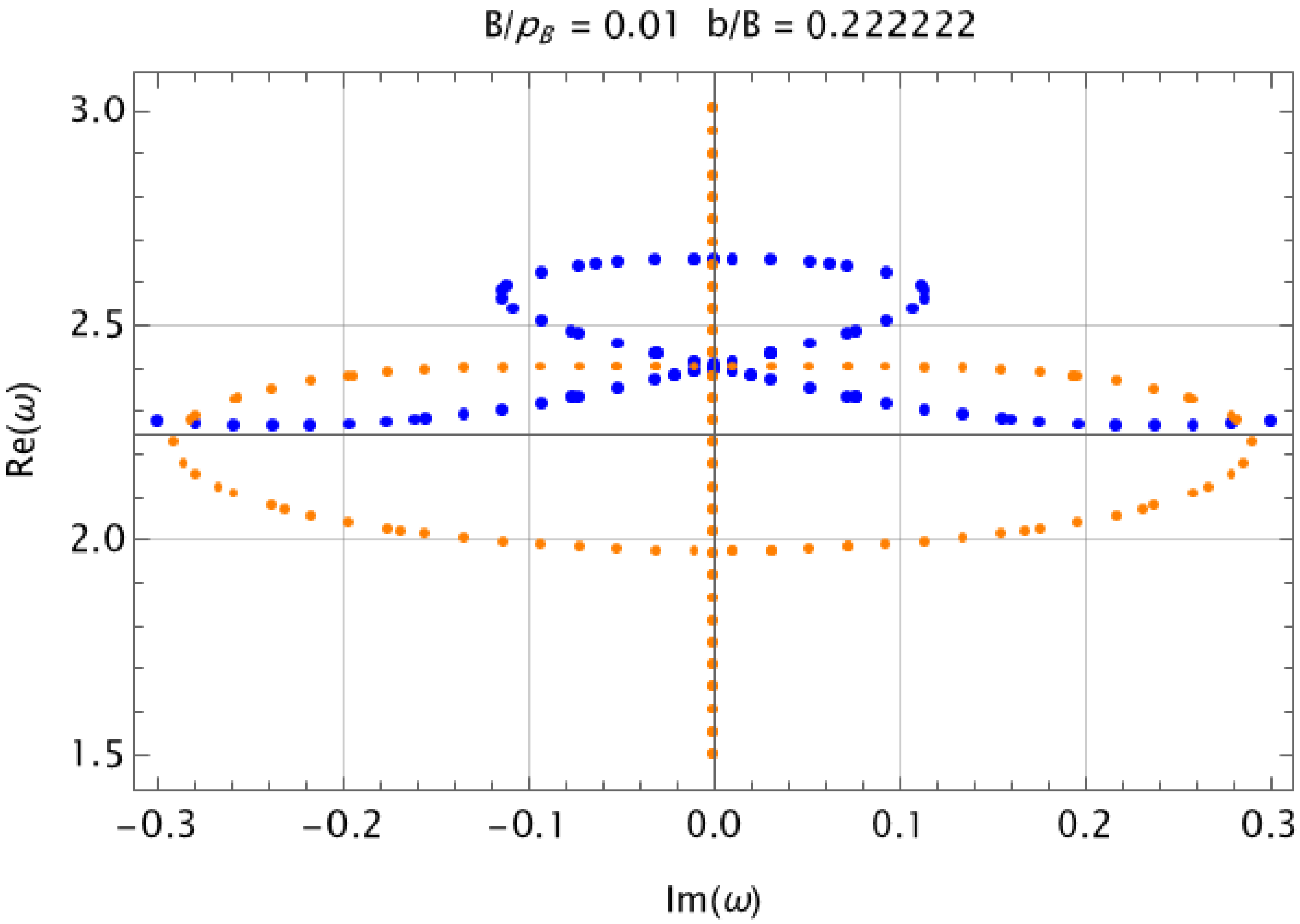
$$\Gamma(u_i,u_j) = \int_{u_i}^{u_j}\frac{du}{\sqrt{1-u^2}}\frac{1}{u-\frac{\omega}{k}}$$

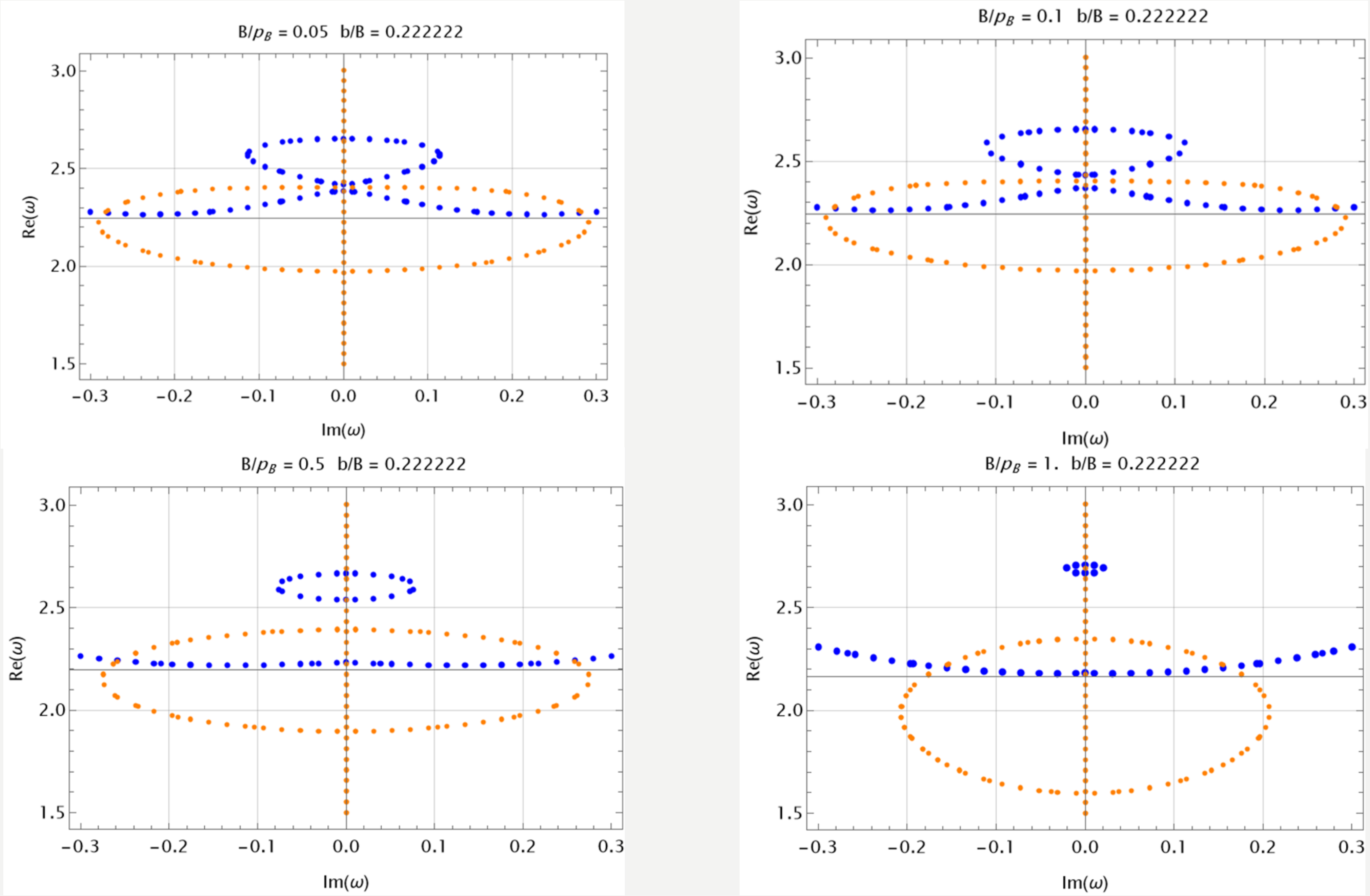
$$u_1 = \frac{p_B-\frac{B}{2}}{\sqrt{m^2+\left(p_B-\frac{B}{2}\right)^2}} \qquad u_2 = \frac{p_B-\frac{b}{2}}{\sqrt{m^2+\left(p_B-\frac{b}{2}\right)^2}}$$

$$u_3 = \frac{p_B+\frac{b}{2}}{\sqrt{m^2+\left(p_B+\frac{b}{2}\right)^2}} \qquad u_4 = \frac{p_B+\frac{B}{2}}{\sqrt{m^2+\left(p_B+\frac{B}{2}\right)^2}}$$

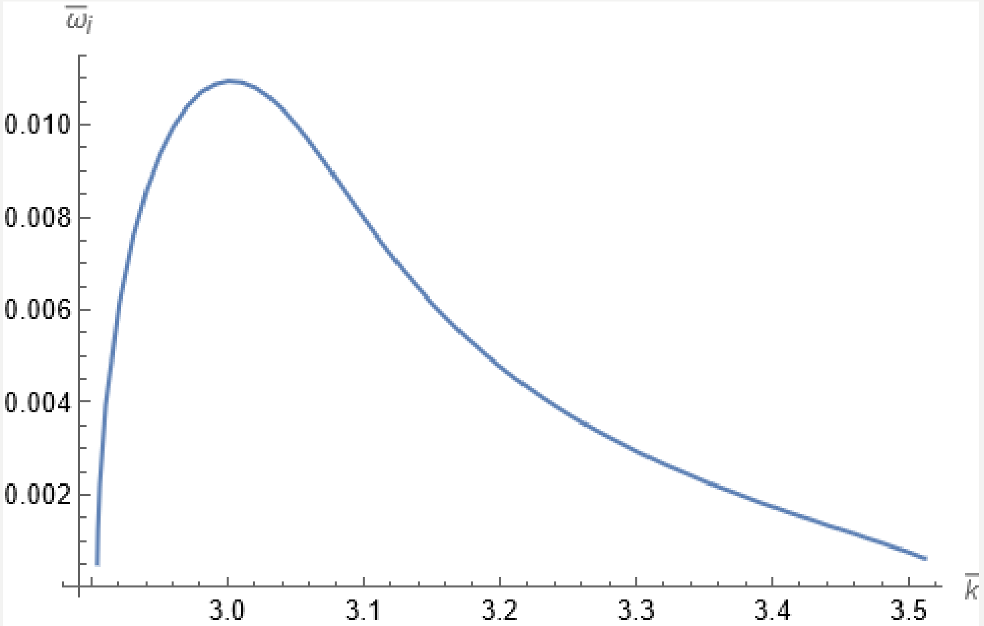
- The integration is exact

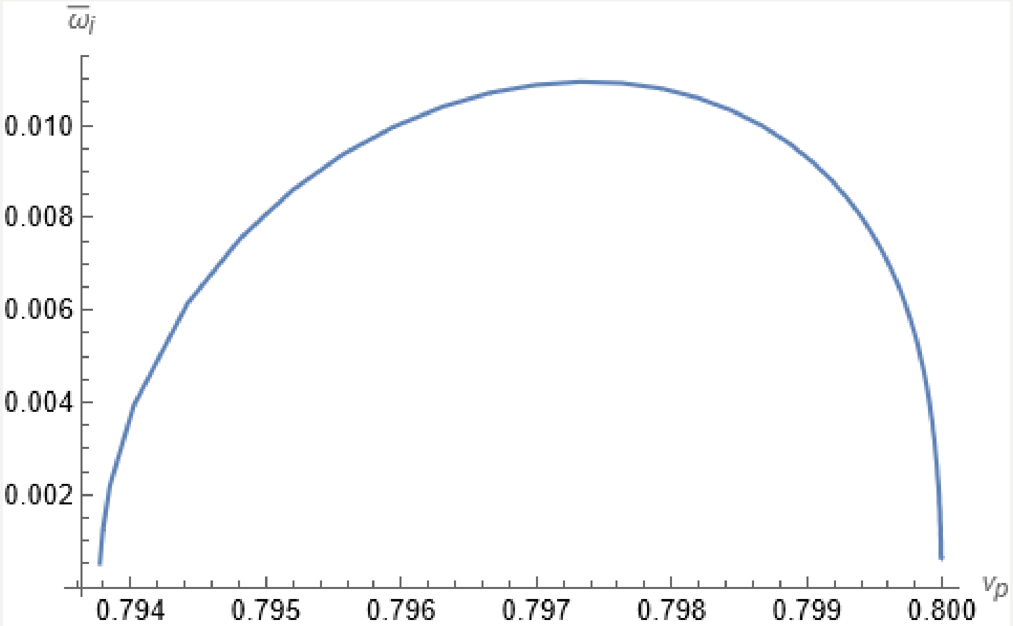
$$\int\frac{du}{\sqrt{1-u^2}}\frac{1}{u-v} = \frac{\log\left(u-v\right)-\log\left(1-uv+\sqrt{1-u^2}\sqrt{1-v^2}\right)}{\sqrt{1-v}}$$


B/ρ_B = 0.01 b/B = 0.222222
Re(ω)
Im(ω)
3.0
2.5
2.0
1.5
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3

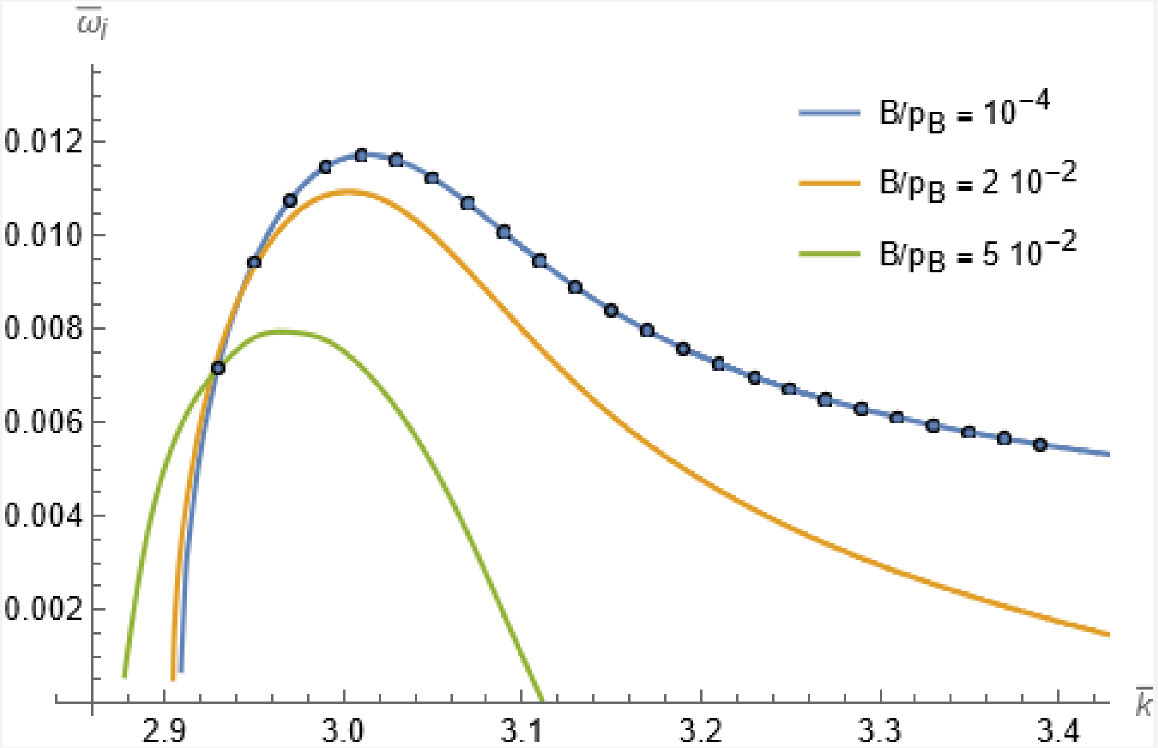

$B/p_B = 0.05$ b/B = 0.222222
$B/p_B = 0.1$ b/B = 0.222222
$B/p_B = 0.5$ b/B = 0.222222
$B/p_B = 1.$ b/B = 0.222222
Re(ω)
Im(ω)

- Growth rate as a function of the wavenumber (left) or the phase velocity (right)

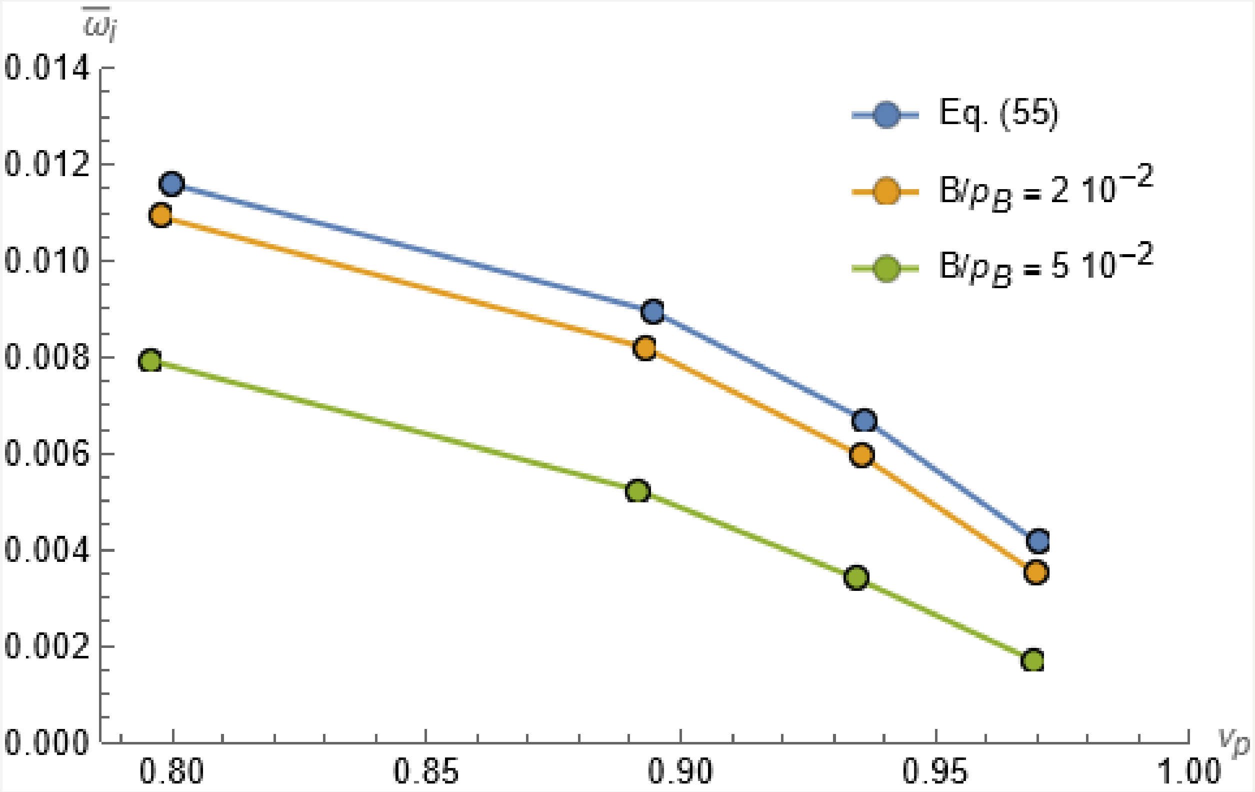




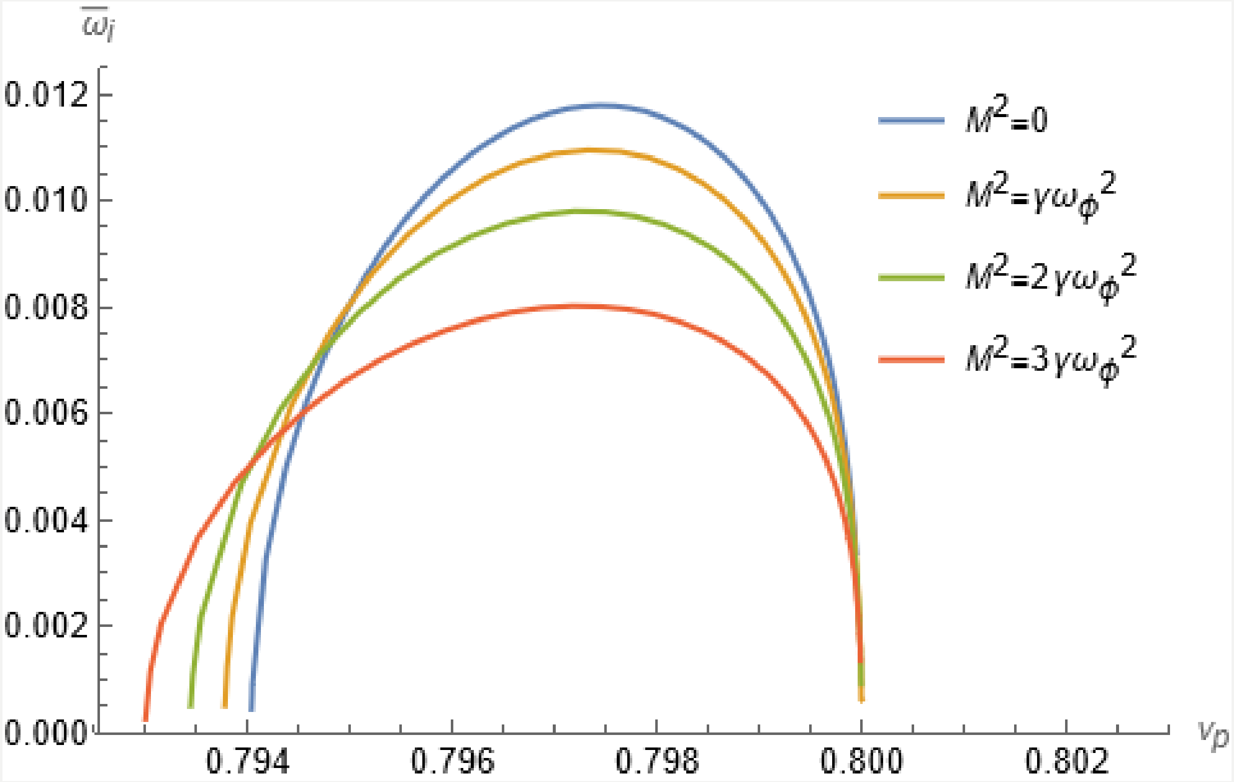
- Dependence of the growth rate w.r.t. the beam temperature



- Maximum growth rate vs. beam velocity



- Dependence of the growth rate from the scalar mode mass



THANKS FOR YOUR ATTENTION