Probing dark Universe with gravitational Landau damping

Part I: Theoretical setting

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Outline

- Landau damping in electromagnetic plasmas
- Gravitational waves in modified theories of gravity: dealing with additional polarizations
- Introducing the notion of gravitational plasma
- The kinetic approach to gravitational waves in matter
- Gravitational Landau damping

Based on:

***"Gravitational Landau Damping for massive scalar modes"*, F. Moretti, F. Bombacigno, G. Montani**

***"The Role of Longitudinal Polarizations in Horndeski and Macroscopic Gravity: Introducing Gravitational Plasmas"*, F. Moretti, F. Bombacigno, G. Montani**

Introducing Landau damping

- Enhancing/damping of wave perturbations travelling in a medium in the absence of collisions (i.e. viscosity)

It’s just a definition: let’s see it in more detail!

Surfing the wave

- If you are a little bit slower than the wave, you can surf the crest of the wave (**the wave is pushing you**)
- If you are a little bit faster than the wave, you are trapped by the crest in front of you (**the wave is pushed by you**)

It’s not so easy…

To keep in mind: in order to interact (in “Landau” sense) with the wave, your velocity must be very similar, otherwise…

If you are at rest, you just move up and down

If you are too fast, you barely see the crest of the waves

Let’s introduce some physics

- Electromagnetic plasma: gas of ions and electrons interacting via Maxwell equations
- The temporal evolution of the probability distribution function is described by the Boltzmann equation

$$f(\vec{x}, \vec{v}, t) \longrightarrow \frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_v f = \hat{C}[f]$$

- The external force drives out of the equilibrium the system
- We look at processes which take place on temporal scales much smaller than the typical time of collisions

In order to deal with the problem in a self-consistent way we need to relate the dynamics of Maxwell equations to the distribution function

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = 4\pi\vec{j} + \frac{\partial \vec{E}}{\partial t}$$

$\longleftarrow$

$\longrightarrow$

$$\rho = \sum_S q_S \int d^3v\, f_S$$

$$\vec{j} = \sum_S q_S \int d^3v\, \vec{v}\, f_S$$

We perturb the equilibrium configuration with an electrostatic field $\left(\vec{B}=0,\ \vec{E}=(E,0,0)\right)$:

$$f(x,\vec{v},t) = f_0(\vec{v}) + \delta f(x,\vec{v},t)$$

$$\frac{\partial \delta f}{\partial t} + v_x \frac{\partial \delta f}{\partial x} + E\frac{e}{m_e}\frac{\partial f_0}{\partial v_x} = 0$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E}{\partial x} = 4\pi e \int d^3v\,\delta f$$

Since we look for damped solutions we perform the analysis in the Fourier-Laplace space:

$$\omega = \omega_r + i\omega_i \quad \longrightarrow \quad e^{-i(\omega t - kx)} = e^{\omega_i t} e^{-i(\omega_r t - kx)}$$

Damping / Enhancement

The solution is given by

$$\tilde{E}(k,\omega) = \frac{4\pi e}{k^2 \epsilon(k,\omega)} \int dv_x \, \frac{\delta \tilde{f}(\vec{v},0)}{v_x - \frac{\omega}{k}}$$

Where we defined the complex dielectric function:

$$\epsilon(k,\omega) = 1 + \frac{\omega_P^2}{k^2} \int dv_x \frac{g(v_x)}{\left(\frac{\omega}{k} - v_x\right)^2}$$

Hypothesis of phase velocity of the wave greater than the electron thermal velocity and weak damping scenario

$$v_p = \frac{\omega}{k} >> v_e \qquad |\omega_i| << \omega_r$$

SOME COMPLEX PLANE INTEGRATION MAGIC

$$\epsilon_r(k, \omega_r) = 0$$

Langmuir modes:
Longitudinal oscillations of frequency ω_r

$$\omega_i = \left.\frac{\epsilon_i(k,\omega)}{\partial \epsilon_r / \partial \omega}\right|_{\omega=\omega_r}$$

Landau damping

- We can have self-sustained oscillations of frequency ω_r in the electron distribution: ions are much heavier than electrons and just contribute to neutralize the background field

- The Langmuir modes are longitudinal, i.e. parallel to the electrostatic field: effective massive behaviour of electromagnetic field in the plasma

- In the limit of weak damping and phase velocity of the wave perturbation much greater than the thermal velocity of electrons, we can derive analytically the damping coefficient

⬇

How does this apply to GRAVITY?

- Neutralizing background: in a local inertial frame we can neglect at the first order the curvature (analysis on a “effective” Minkowski background metric)

→ In the Einstein-Vlasov equation we only deal with the gravitational wave perturbation

- Longitudinal stresses: additional polarizations in the gravitational wave spectrum, such as vector and scalar modes, can carry longitudinal excitations

→ We look at modified theories of gravity endowed with additional degrees of freedom: f(R), Horndeski, …

- In a fully relativistic treatment the “physical” velocity of particles and signals must be subluminal (phase velocity vs. group velocity)

→ We have to pay attention to the limits and to the path of integration in the expression for the gravitational dielectric function

We look at small perturbations around Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \; \varphi = \varphi_0 + \phi$$

We obtain the linearized equations of motion:

$$\Box \bar{h}_{ij} = -2\kappa' \delta T_{ij} \qquad \left(\Box - M^2\right) \phi = \kappa'' \delta T$$

Effective coupling and mass depending on the form of the theory

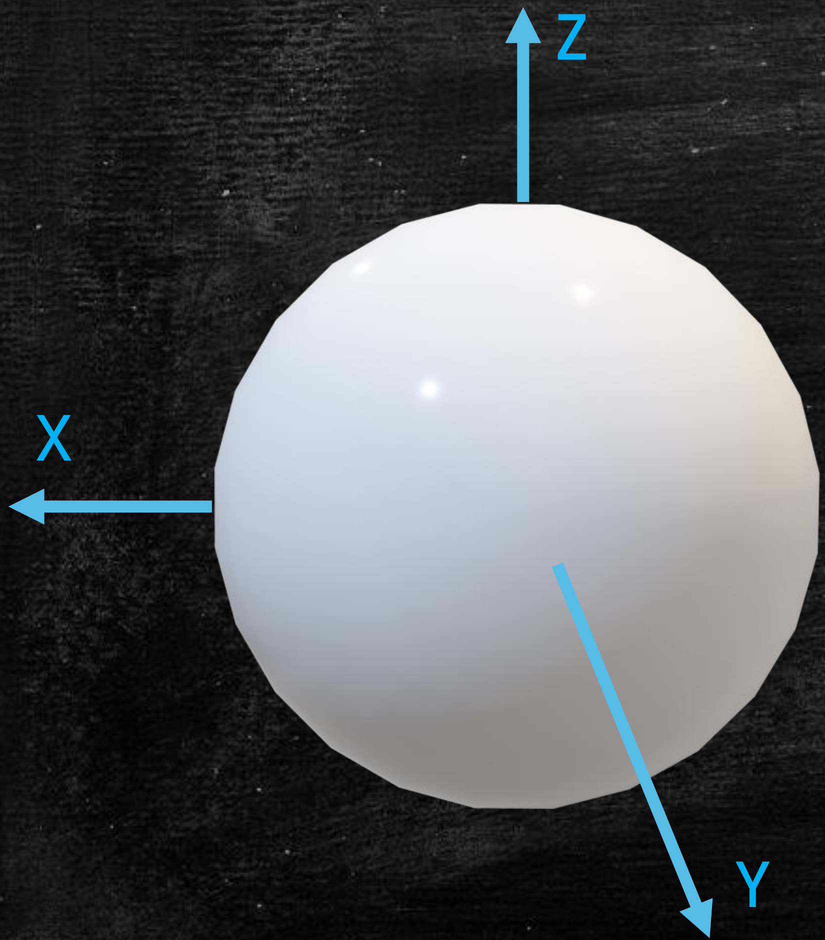
Perturbations on the matter distribution induced by the passing of the wave itself
(the background curvature is absorbed in the L.I.F.)

We restrict our attention to tensor and scalar modes which propagate in vacuum

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\left(h + 2\alpha\phi\right), \; \bar{h} = 0, \; \partial_\mu \bar{h}^{\mu\nu} = 0$$

How does the scalar mode affect the geodesic deviation equation?

We consider a sphere of test masses and we look at the deformations induced by a monochromatic planar wave travelling along the z direction

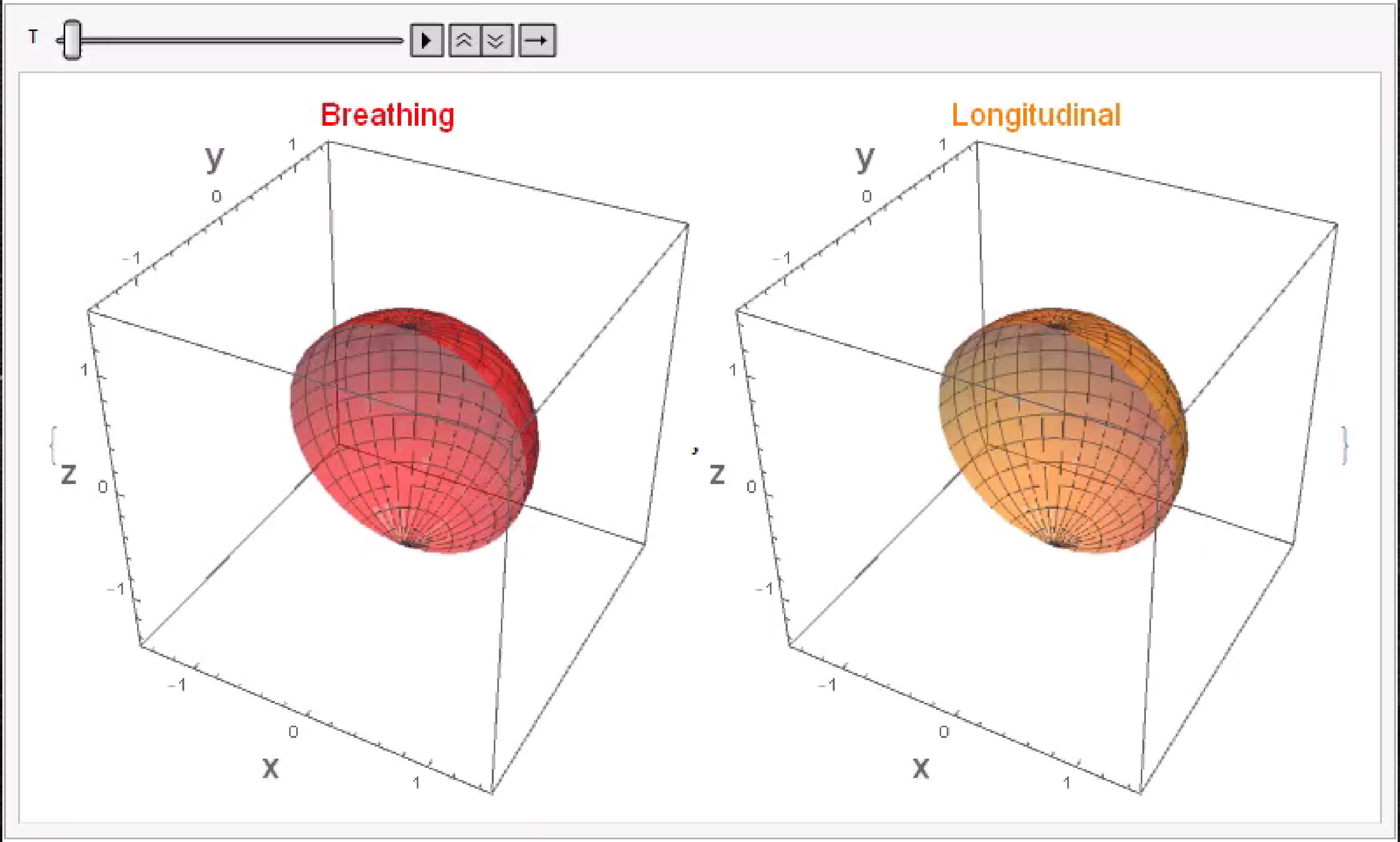


$$\frac{d^2\delta X}{dt^2} \simeq \frac{\alpha\omega^2}{2}\phi\, X_0$$
$$\frac{d^2\delta Y}{dt^2} \simeq \frac{\alpha\omega^2}{2}\phi\, Y_0$$

Breathing mode (transverse plane)

$$\frac{d^2\delta Z}{dt^2} \simeq \frac{\alpha M^2}{2}\phi\, Z_0$$

Longitudinal mode (for M → 0 cancels out)


T
Breathing
Longitudinal
y
z
x
1
0
-1


Direction of propagation

Vlasov equation in the presence of gravity

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_v f = 0 \longrightarrow \frac{\partial f}{\partial t} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i} + \frac{dp_i}{dt}\frac{\partial f}{\partial p_i} = 0$$

The external force can be evaluated from the geodesic equation:

$$\frac{dp^\mu}{dt} + \Gamma^\mu_{\alpha\beta}\frac{p^\alpha p^\beta}{p^0} = 0 \longrightarrow \boxed{\frac{dp_i}{dt} = \frac{1}{2p^0}\left(p_k p_l \frac{\partial \bar{h}_{kl}}{\partial x^i} + \alpha m^2 \frac{\partial \phi}{\partial x^i}\right)}$$

It depends on the gravitational wave perturbation

A complete dynamical description of the system:

1. The linearized Vlasov equation for $\delta f(\vec{x},\vec{p},t)$ in terms of the metric perturbation

$$\frac{\partial \delta f}{\partial t} + \frac{p^m}{p^0}\frac{\partial \delta f}{\partial x^m} - \frac{f_0'(p)}{2p}\left(p_i p_j \frac{\partial \bar{h}_{ij}}{\partial t} - \alpha p^2 \frac{\partial \phi}{\partial t} - \alpha p^0 p^m \frac{\partial \phi}{\partial x^m}\right) = 0$$

2. The equations for the gravitational wave with the sources in terms of $\delta f(\vec{x},\vec{p},t)$

$$\Box \bar{h}_{ij} = -2\kappa' \delta T_{ij} = -2\kappa' \int d^3p \, \frac{p_i p_j}{p^0} \delta f(\vec{x},\vec{p},t)$$

$$\left(\Box - M^2\right)\phi = \kappa'' \delta T = -m^2 \kappa'' \int d^3p \, \frac{\delta f(\vec{x},\vec{p},t)}{p^0}$$

We observe that in principle $\bar{h}_{ij}$ and ϕ can be coupled via the perturbation

In Fourier-Laplace space it is an algebraic problem which actually decouples:

$$\phi^{(k,\omega)} = \frac{\left(-i\omega + i\alpha\pi m^2\kappa' \int d\rho dp_3\, \rho\, \frac{f_0'(p)p}{p^0\omega - kp_3}\right)\phi^{(k)}(0)}{(k^2 + M^2 - \omega^2)\epsilon^{(\phi)}(k,\omega)}$$

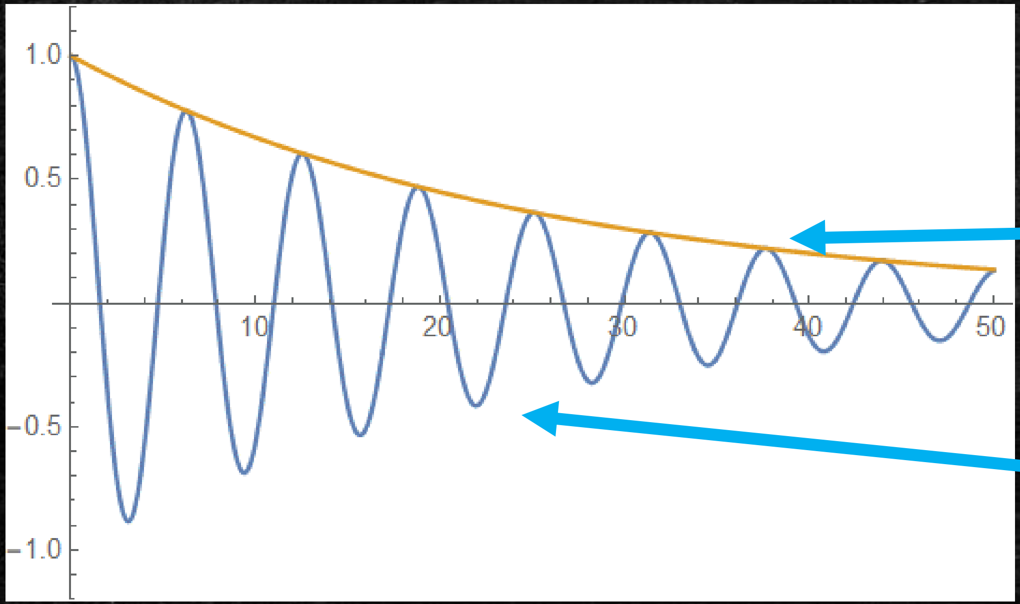
$$\bar{h}_{ij}^{(k,\omega)} = \frac{\left(-i\omega - \frac{i\pi\kappa''}{2} \int d\rho dp_3\, \rho^5\, \frac{f_0'(p)}{p(p^0\omega - kp_3)}\right)\bar{h}_{ij}^{(k)}(0)}{(k^2 - \omega^2)\epsilon^{(h)}(k,\omega)},$$

The inverse Laplace transform is dominated, for long times, by the contribution stemming from the poles in the complex plane, i.e. **the zeros of the dielectric functions**

Gravitational dielectric functions:

$$\epsilon^{(\phi)}(k,\omega) = 1 + \frac{\alpha\pi m^2\kappa'}{k^2+M^2-\omega^2}\int d\rho dp_3\,\rho\frac{f_0'(p)}{p}\frac{p^2\omega - kp^0p_3}{p^0\omega - kp_3}$$

$$\epsilon^{(h)}(k,\omega) = 1 - \frac{\pi\kappa''}{2(k^2-\omega^2)}\int d\rho dp_3\,\rho^5\frac{f_0'(p)}{p}\frac{\omega}{p^0\omega - kp_3}$$



Weak damping: $|\omega_i| << |\omega_r|$

$$\omega_i = \left.\frac{\epsilon_i(k,\omega)}{\partial\epsilon_r/\partial\omega}\right|_{\omega=\omega_r}$$

$$\epsilon_r(k,\omega_r) = 0$$

The damping is related to the imaginary part of the dielectric function

$$\epsilon^{(\phi)}(k,\omega) = 1 + \frac{\alpha\pi m^2\kappa'}{k^2+M^2-\omega^2}\int d\rho dp_3\, \rho \frac{f_0'(p)}{p}\frac{p^2\omega - kp^0p_3}{p^0\omega - kp_3}$$

If the Landau pole is included in the path of integration the imaginary part of $\epsilon^{(\phi,h)}$ is not vanishing

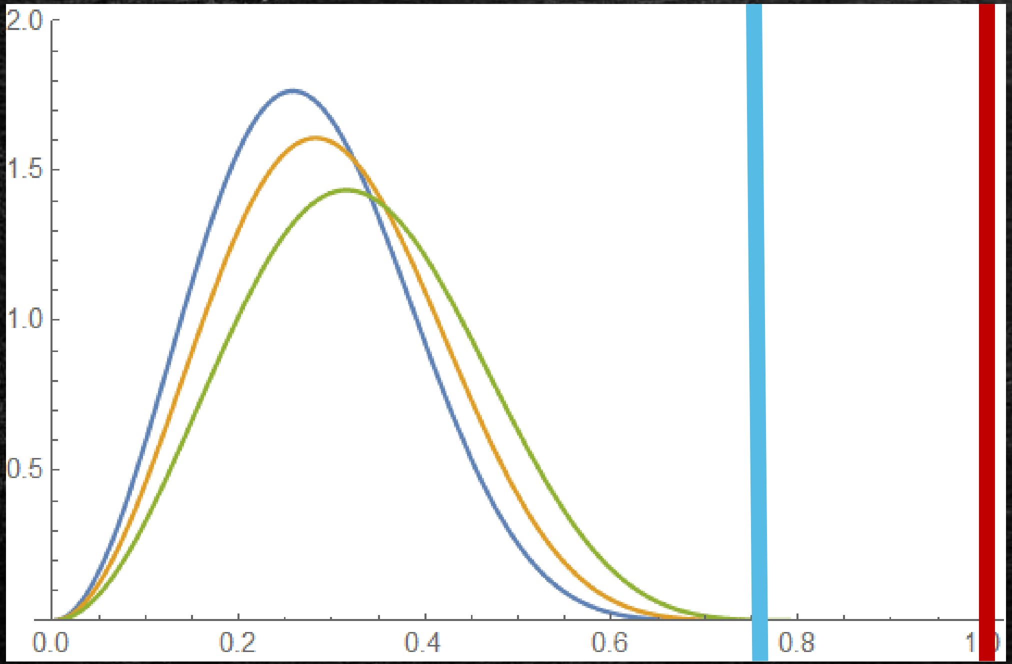
It only occurs if the phase velocity of the wave perturbation is subluminal!

$$v_p = \frac{\omega}{k} < 1$$

It has to be checked *a posteriori!*

We describe at the equilibrium the medium with the Jüttner distribution (relativistic generalization of Maxwell)

$$f_0(p) = \frac{n}{4\pi m^2 \Theta K_2\left(\frac{m}{\Theta}\right)} e^{-\frac{\sqrt{m^2+p^2}}{\Theta}}$$



$$v_T << v_p < 1$$

When the phase velocity is much greater than the thermal velocity the dielectric function can be expanded in power series around the pole and the real / imaginary part evaluated analytically

Langmuir modes for TENSOR polarizations

By imposing $\epsilon_r^{(\bar{h})}(k, \omega_r) = 0$ we obtain:

$$\omega_r^2(k) = \frac{k^2 + 12\omega_h^2 \frac{x-\gamma(x)}{x^2} + \sqrt{\left(k^2 + 12\omega_h^2 \frac{x-\gamma(x)}{x^2}\right)^2 + 48k^2\omega_h^2 \frac{\gamma(x)}{x^2}}}{2}$$

It turns out that the Langmuir frequency is always greater than 1:
There are no poles in the integration path and the imaginary part of the dielectric function vanishes

LANDAU DAMPING FOR TENSOR MODES CANNOT OCCUR

Langmuir modes for SCALAR polarizations

By imposing $\epsilon_r^{(\phi)}(k,\omega_r) = 0$ we obtain:

$$\omega_r^2(k) = \frac{k^2 + M^2 - 9\gamma\omega_0^2 + \sqrt{\left(k^2 + M^2 - 9\gamma\omega_0^2\right)^2 + 12\gamma\omega_0^2 k^2}}{2}$$

It turns out that the Langmuir frequency is smaller than 1 only if:

$$M^2 < 6\gamma\omega_0^2 = \kappa'\gamma(x)\alpha\, n\, m$$

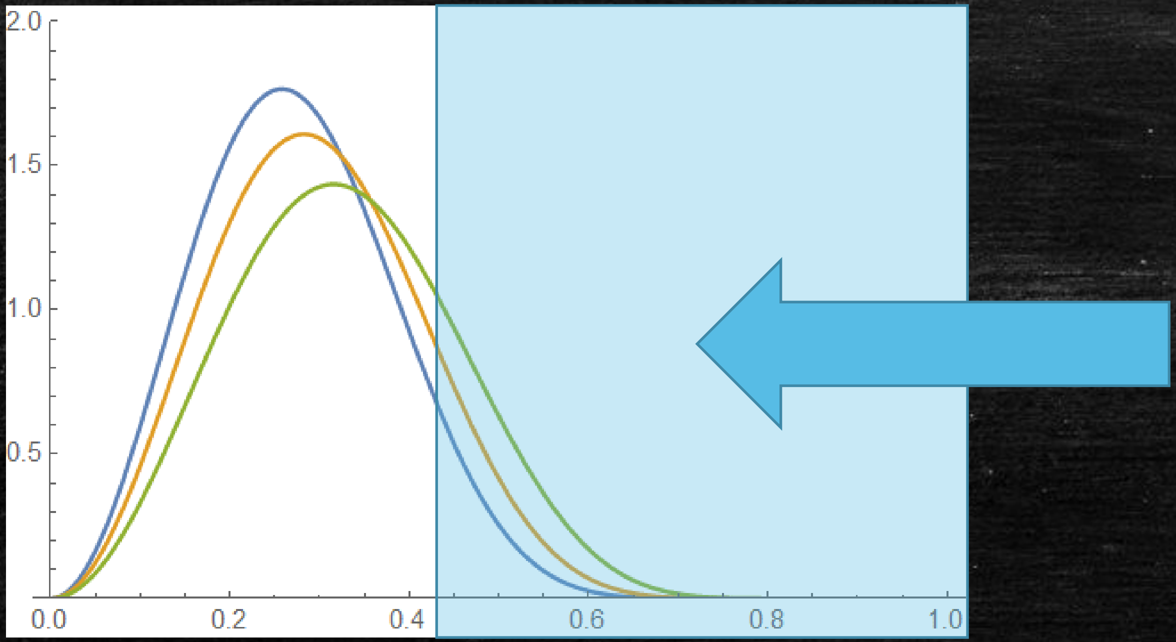
LANDAU DAMPING FOR SCALAR MODES CAN OCCUR!

$$M^2 < 6\gamma\omega_0^2$$

- It relates the model (M) with the thermodynamic properties of the medium (in ω_0)
- Given a medium with particular temperature, density and particle mass, it can interact only with specific scalar modes (and the contrary)
- The upper bound on the mass of the graviton given by GW170817-GRB170817A does not represent a limitation: smaller the mass M, wider the media where Landau can occur
- In the limit of vanishing mass, the scalar polarization becomes purely transverse and Landau damping always take place *(as opposed to the electromagnetic case!)*

$$\omega_i(k) = -\frac{\pi x}{4kK_1(x)} \frac{\omega_r^4(k^2+M^2-\omega_r^2)e^{-\frac{x}{\sqrt{1-\frac{\omega_r^2}{k^2}}}}}{3\omega_r^4 - 2\omega_r^2 + (k^2+M^2)} < 0$$

It is always negative: the sign coincides with the sign of the derivative of the background distribution function considered at a velocity equal to the phase velocity of the signal.



fast-tail region of the distribution

- Gravitational scalar waves can be absorbed by noncollisional media (Landau damping), even in the absence of viscosity
- Damping can occur if the effective mass of the scalar polarization and the thermodynamic properties of trasversed medium satisfy a peculiar inequality (which has no electromagnetic counterpart)
- Relativistic and dense medium are more likely to experience Landau damping: interaction with cold dark matter
- Also vector modes of the gravitational polarizations are endowed with longitudinal stresses: Do they suffer kinetic damping?

Thank you for your attention

Introducing longitudinal stresses: Horndeski theories

The most general scalar-tensor theory of gravity with second order equations of motion

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\sum_{i=2}^{5} L_i$$

where

$$L_2 = K(\varphi, X)$$
$$L_3 = -G_3(\varphi, X)\Box\varphi$$
$$L_4 = G_4(\varphi, X)R + G_{4,X}\left((\Box\varphi)^2 - \varphi_{\mu\nu}\varphi^{\mu\nu}\right)$$
$$L_5 = G_5(\varphi, X)G_{\mu\nu}\varphi^{\mu\nu} + \frac{1}{6}G_{5,X}\left((\Box\varphi)^3 - 3\Box\varphi\,\varphi_{\mu\nu}\varphi^{\mu\nu} + 2\varphi^{\mu}{}_{\nu}\varphi^{\nu}{}_{\rho}\varphi^{\rho}{}_{\mu}\right)$$

They are still compatible with GW170817-GRB170817A

with

$$X \equiv -\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi, \quad \varphi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\varphi$$