Numerical techniques in Lattice QCD: Exercise I

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PhD course on Lattice field theory

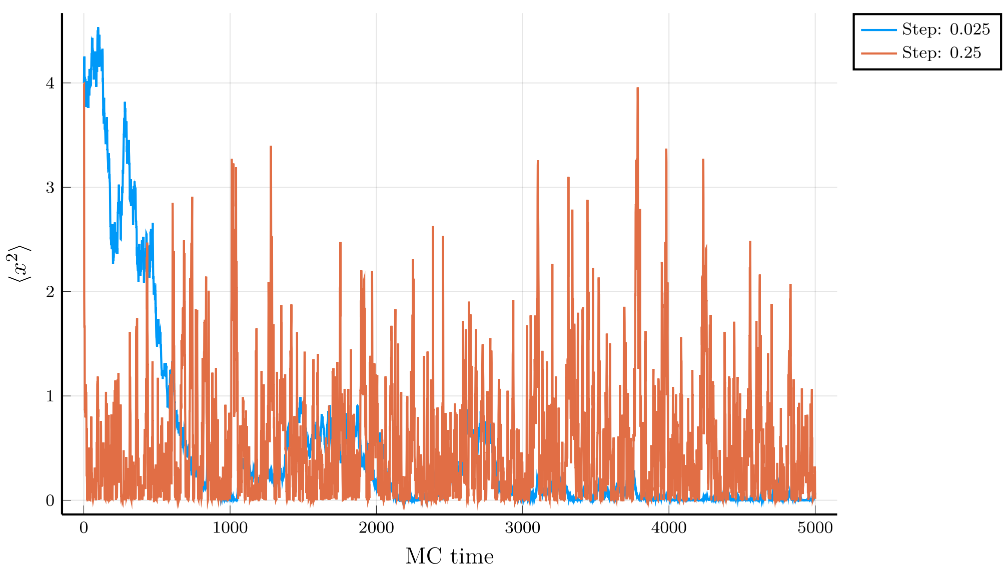
Problem 1 *We will use the zero-dimensional model defined by the partition function*

$$\mathcal{Z}(\lambda) = \int \mathrm{d}x \, e^{-\frac{1}{2}x^2 - \lambda x^4} \, . \tag{1}$$

Code an algorithm to simulate the model (simple accept reject or HMC if you want). Investigate a few observables near $\lambda = 0$ and compare with the perturbative predictions.

1 Accept reject

Code an accept-reject algorithm based on a random walk with step ϵ and a symmetric proposal $\mathcal{N}(0, \epsilon)$. Start with $x = -2.0$ and look on how thermalization depends on ϵ. (If you are ambitious, you can try to code an HMC for this model).



2 Check

Check your results with the trivial case $\lambda = 0$:

$$\langle x^2 \rangle = 1.0; \langle x^4 \rangle = 3.0 \, . \tag{2}$$

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Number of measurements: 50000
<x^2>: 0.9779271927055091 +/- 0.031754172484145825
<x^4>: 2.881574549168074 +/- 0.2031009685683552
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3 Comparison with perturbation theory

Determine the observables

$$O_n(\lambda) = \langle x^{2n} \rangle \tag{3}$$

for a few values of n. Make a plot of its values as a function of λ and compare with the perturbative prediction for $\lambda \in [0, 0.05]$.

We first need the perturbative coefficients. The general formula is

$$O_n(\lambda) = \frac{\sum_k b_k \lambda^k}{\sum_k a_k \lambda^k} = \sum_k c_k \lambda^k \,. \tag{4}$$

with

$$b_k = \frac{(-)^k}{k!}(2(2k+n)-1)!!\,, \tag{5}$$

$$a_k = \frac{(-)^k}{k!}(4k-1)!!\,, \tag{6}$$

$$c_k = \frac{1}{a_0}\left(b_k - \sum_{i=1} a_i c_{n-i}\right)\,. \tag{7}$$

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Perturbative coeff <x^2>: [1.0, -12.0, 384.0, -19008.0, 1.253376e6]
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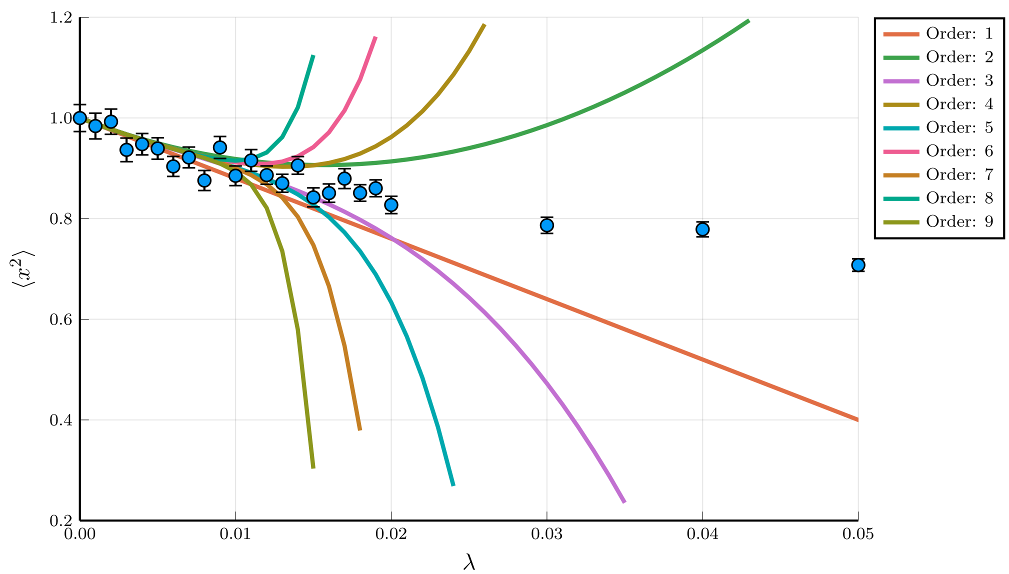
Now we compare the "non-perturbative" simulations with the perturbative predictions.



Note that the bad perturbative behavior is related with the **asymptotic** nature of the series: it is not convergent ($\mathcal{Z}(\lambda)$ does not exist for $\lambda < 0$, so the expansion **around** $\lambda = 0$ must have zero convergence radius). This feature is shared by basically all perturbative expansions in QFT.