

Dependence of microdosimetric mean values on the cell nucleus size and eccentricity for radiopharmaceutical alpha emitters

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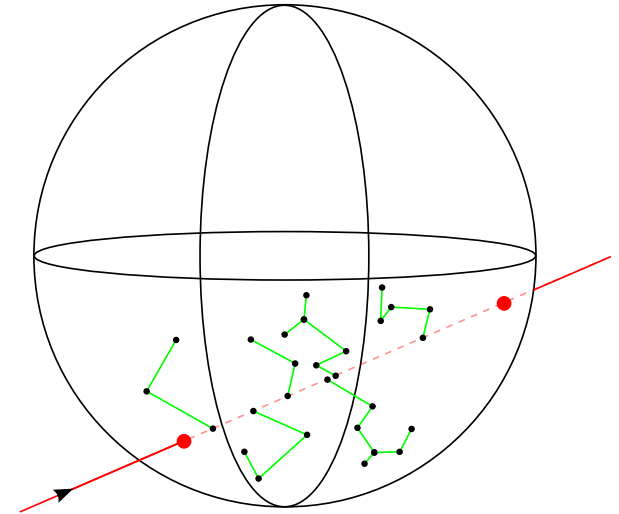
Introduction

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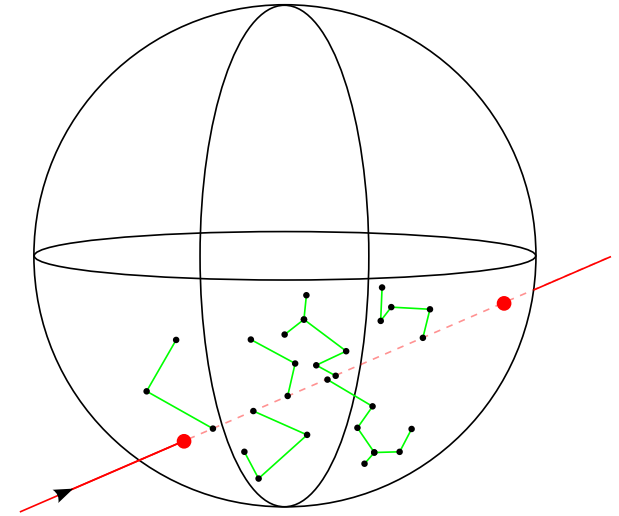
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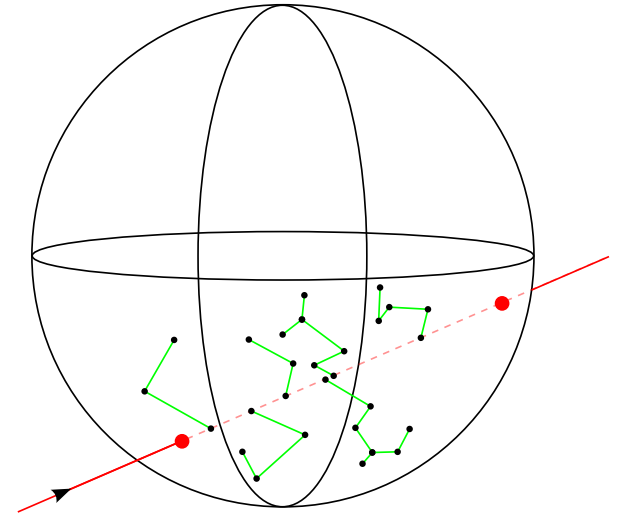
- Specific energy $z = \frac{\varepsilon}{m}$
 - Energy absorbed
 - Mass of the specified region
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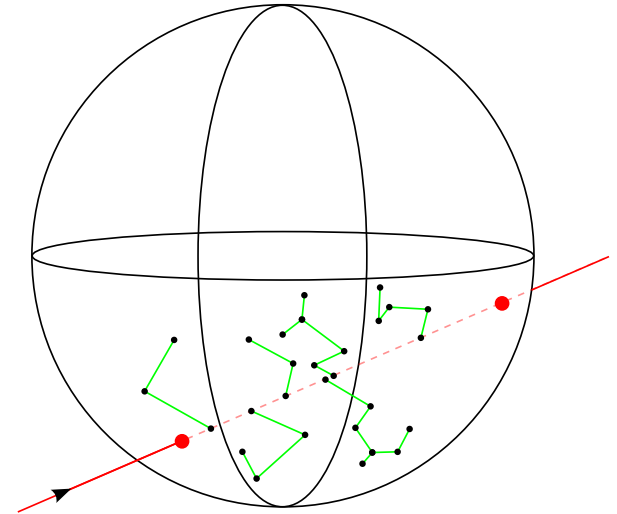
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- Dose-weighted average → defined in terms of the absorbed dose

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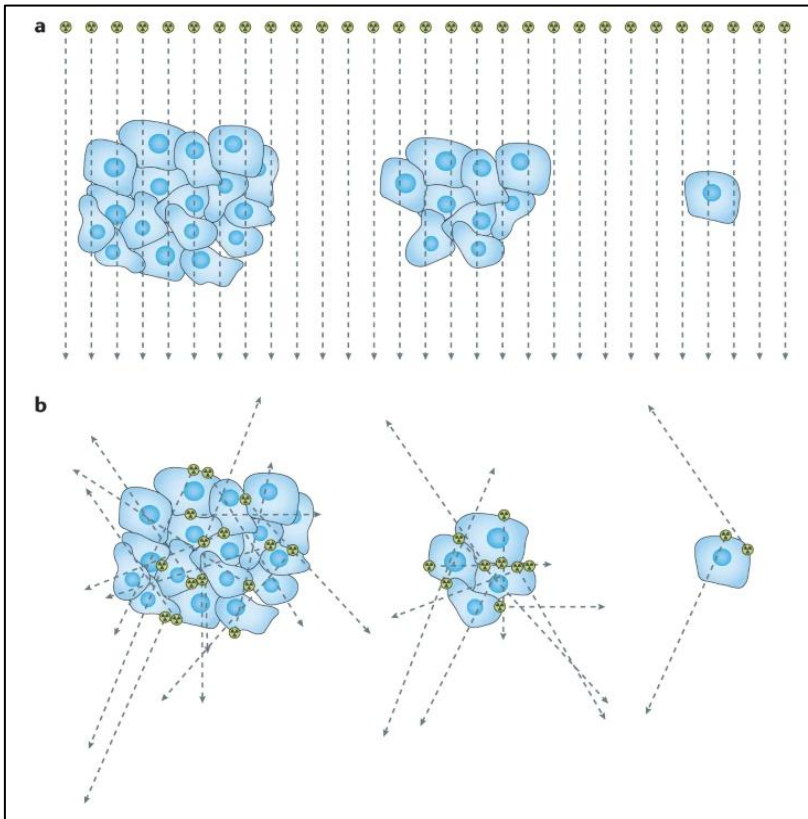


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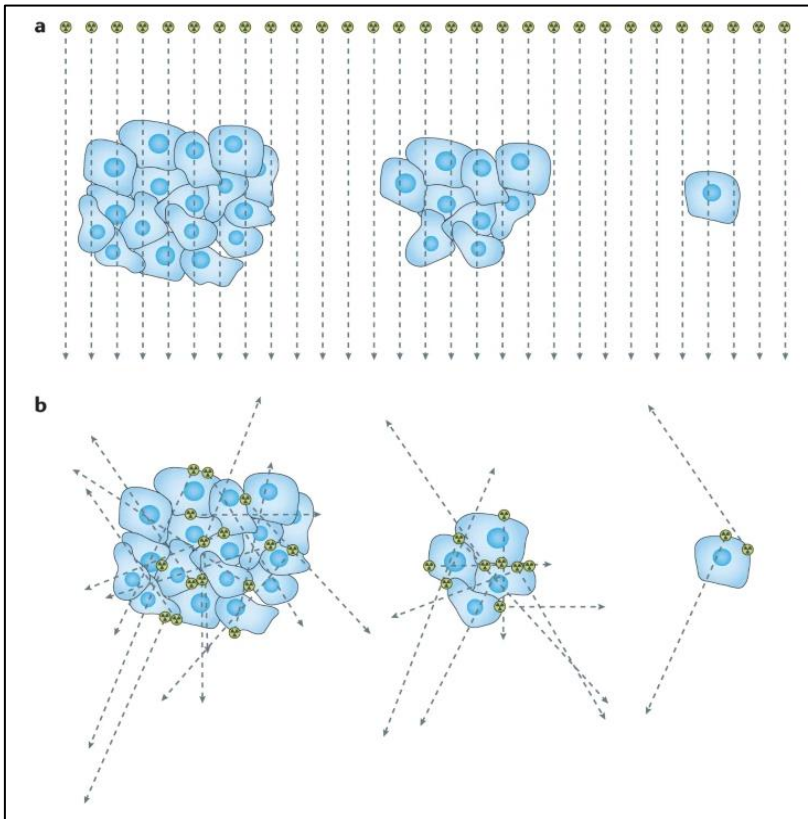
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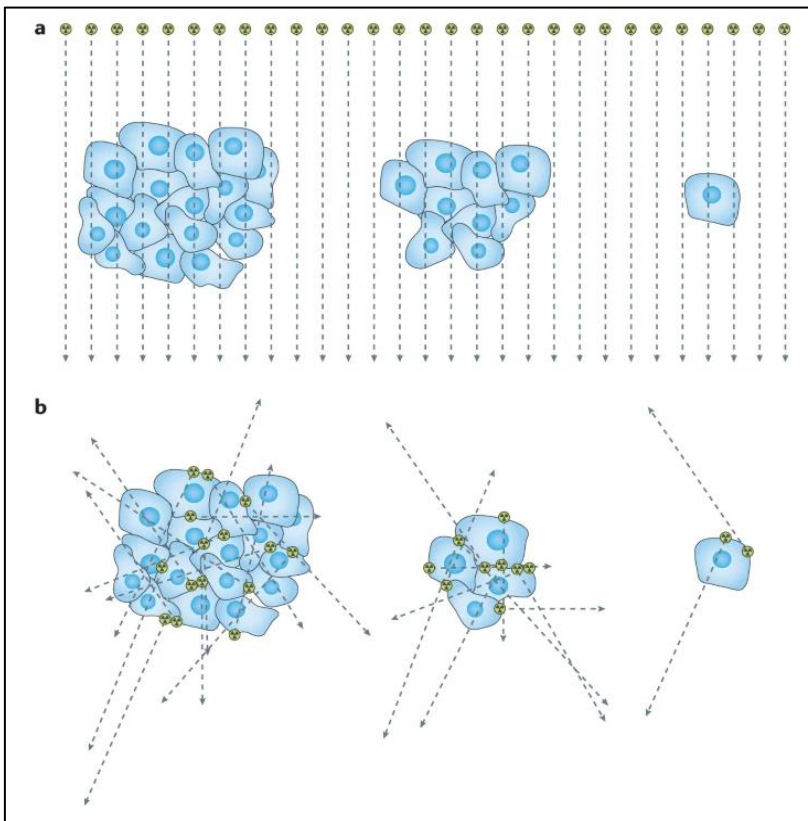


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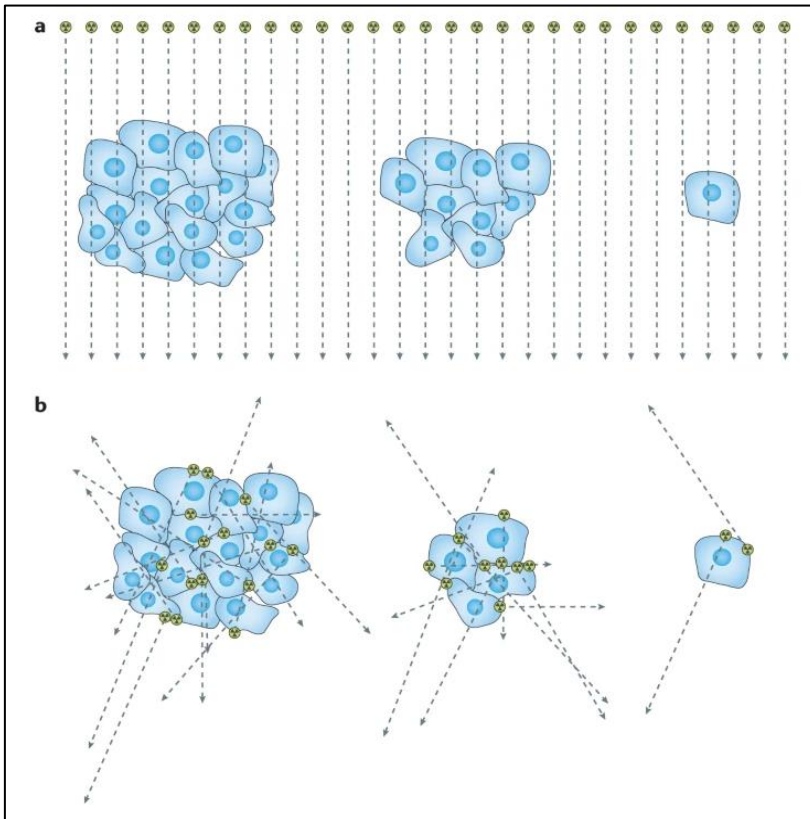


- Beta emissions → the range for typical these beta emissions is of the order of millimeters
- Alpha emissions → Range from 50 μm to 100 μm

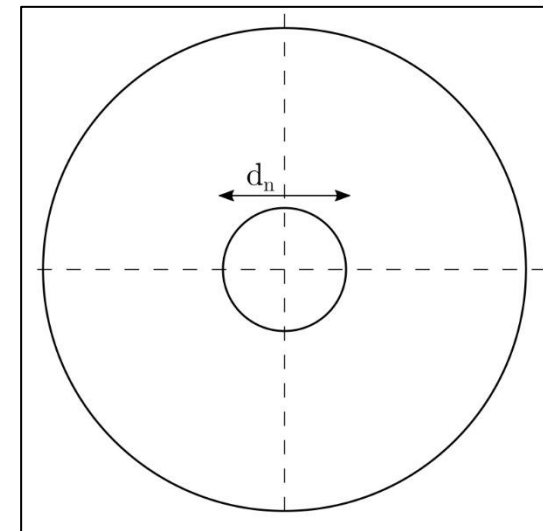
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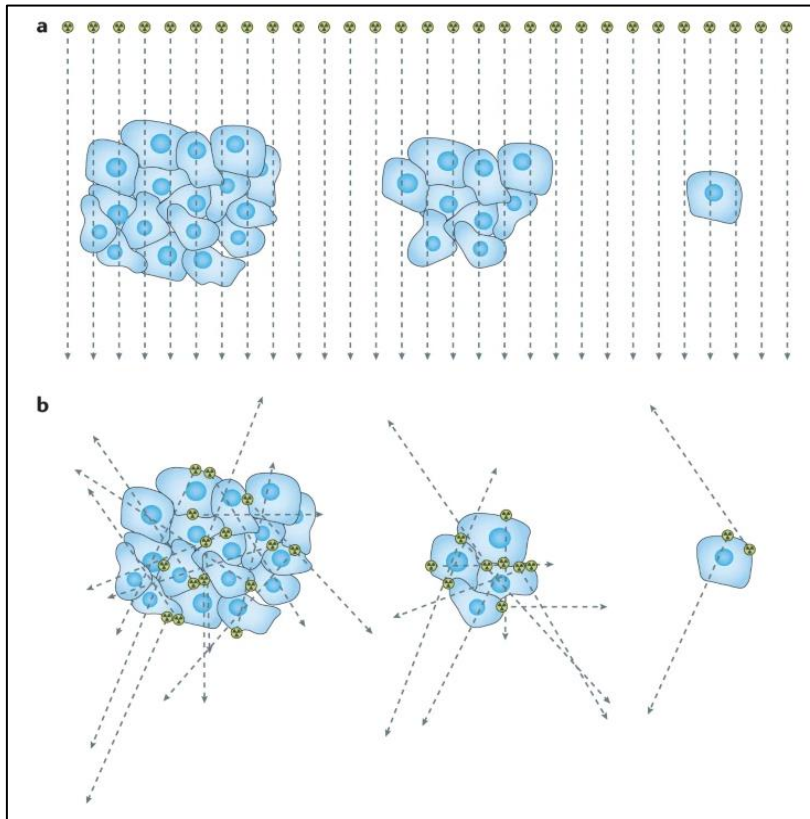
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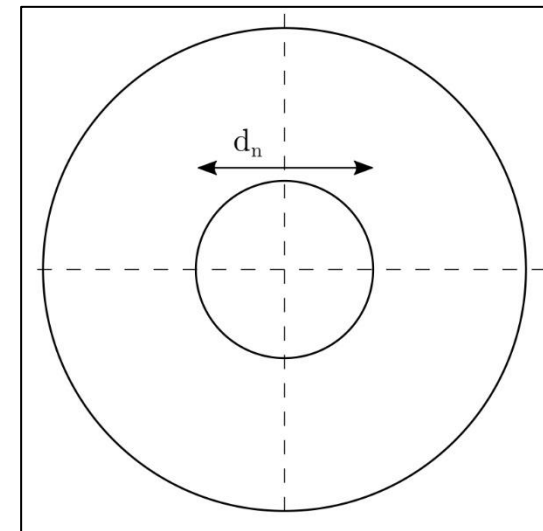
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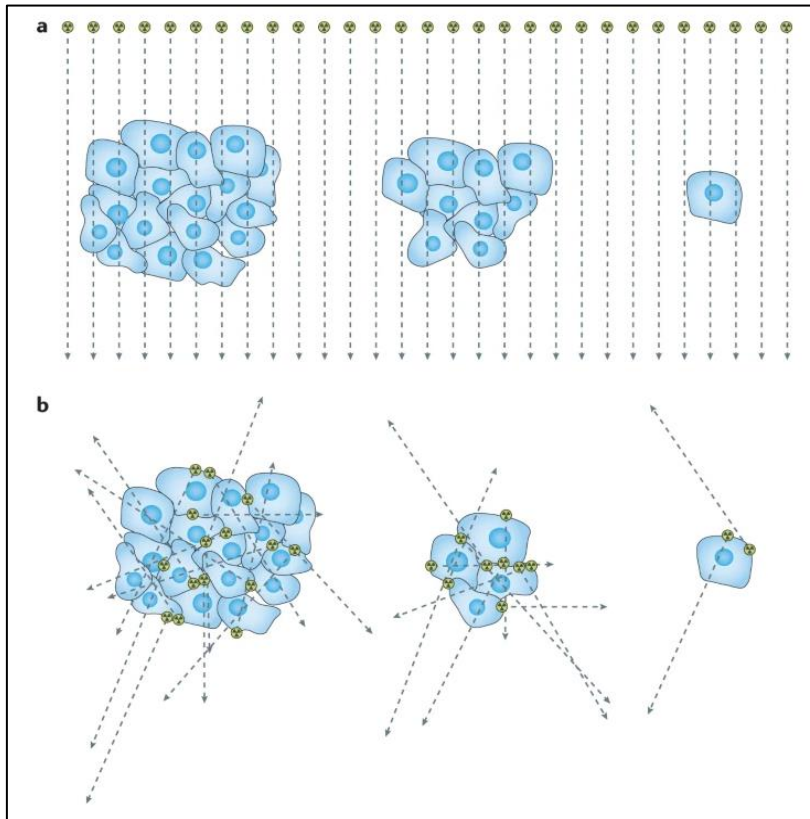
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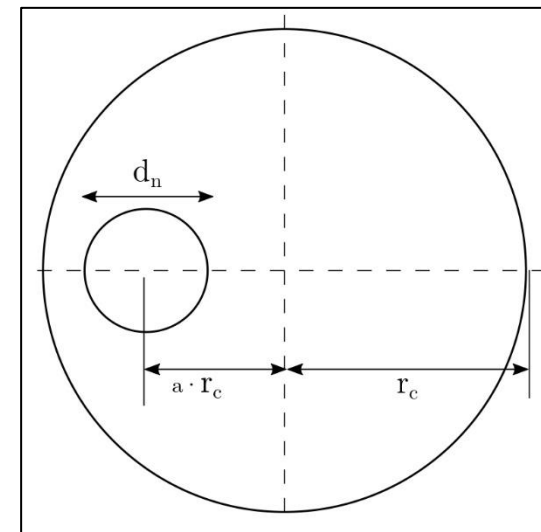
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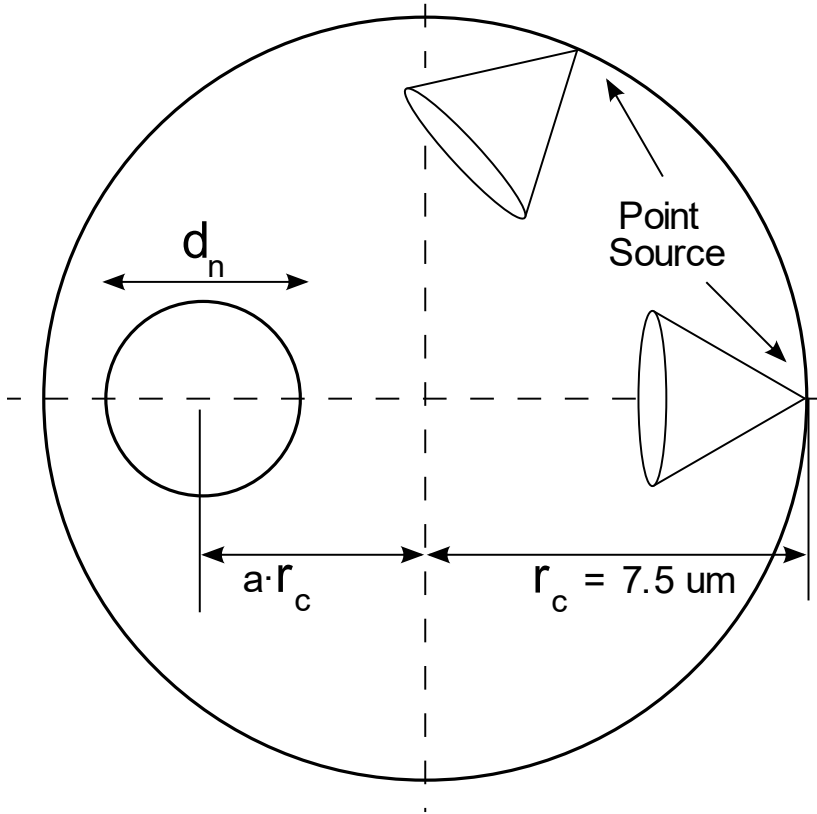
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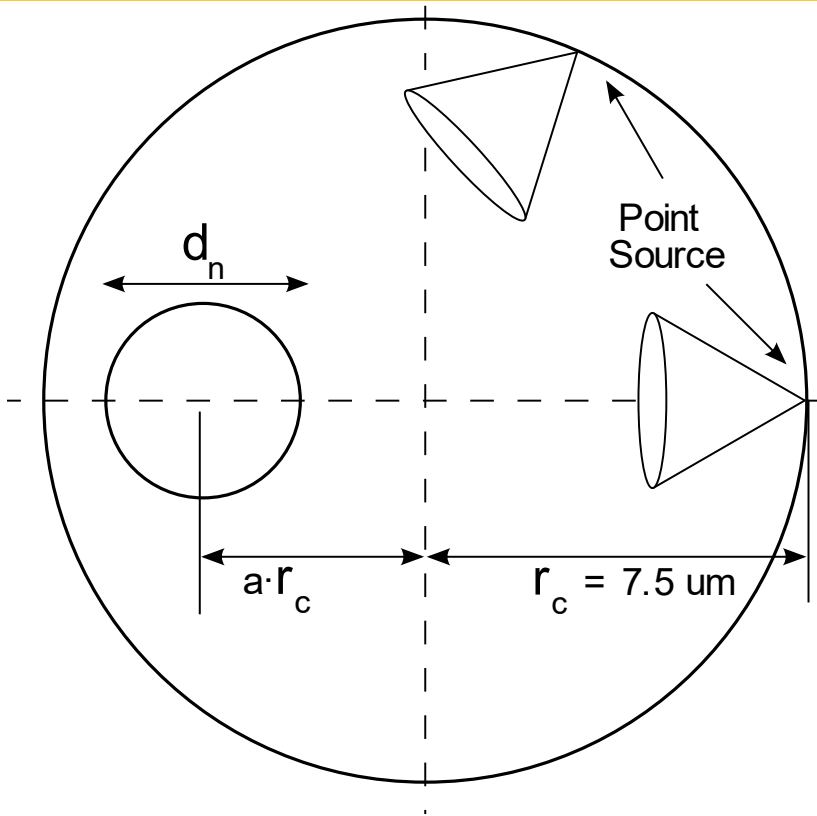
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Geant4 Simulation



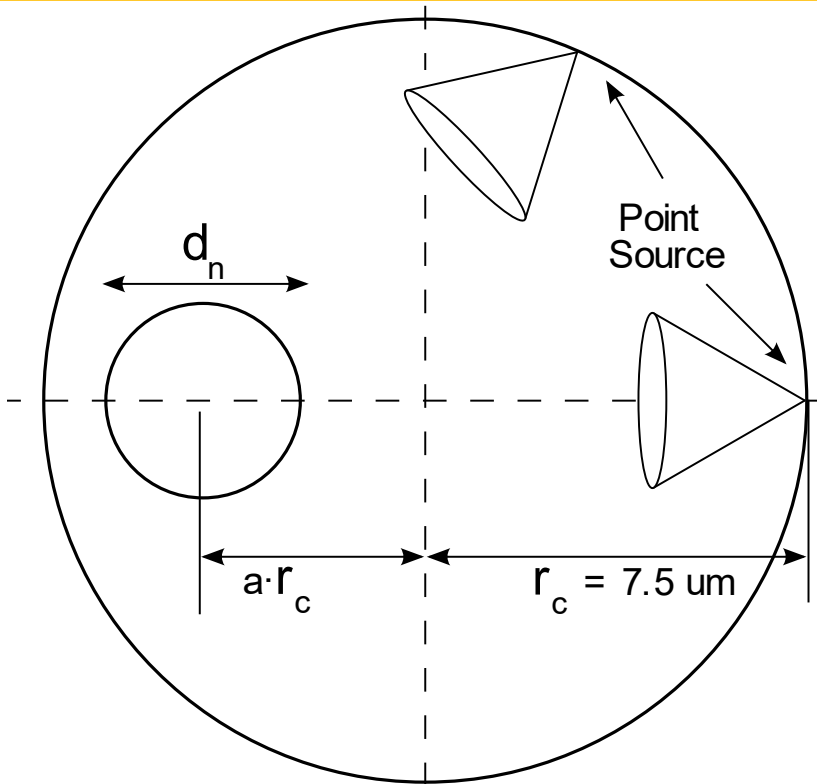
Geant4 Simulation



Source ^{211}At

- 5.87 MeV --- 42%
- 7.45 MeV --- 58%

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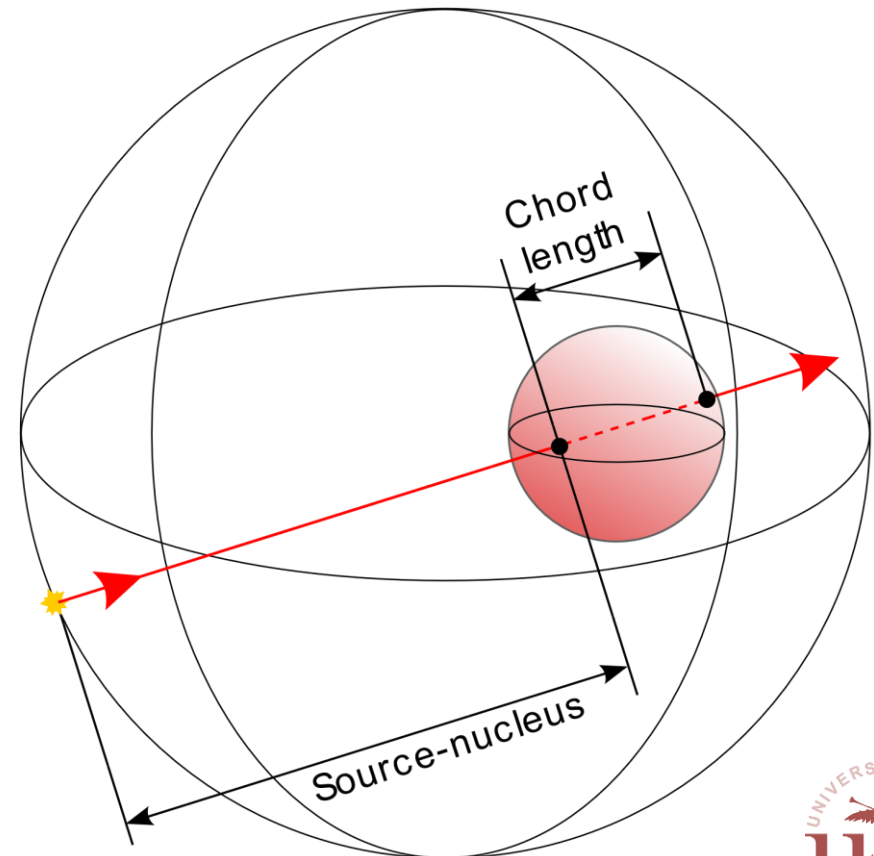


Source ^{211}At

- 5.87 MeV --- 42%
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Output

- Distribution of deposited energy
- Distribution of distance source-nucleus
- Distribution of mean chord length



Analytical method - Bertolet et al. Radiat. Res. (2020)

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- $\bar{\varepsilon}_S(E)$ → Mean energy absorbed as a function of the incident energy

Analytical method - Bertolet et al. Radiat. Res. (2020)

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- $\bar{\varepsilon}_S(E) \rightarrow$ Mean energy absorbed as a function of the incident energy

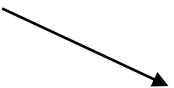
We can get the mean energy absorbed of a polyenergetic beam if the incident spectrum is known

- $\phi_E(E) \rightarrow$ Spectrum incident energy

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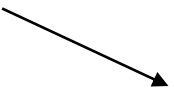
- $\phi_E(E) \rightarrow$ Spectrum incident energy


$$\bar{\varepsilon}_S = \frac{\int \phi_E(E) \bar{\varepsilon}_S(E) dE}{\int \phi_E(E) dE}$$

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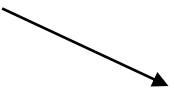

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Similar process with σ_ε^2

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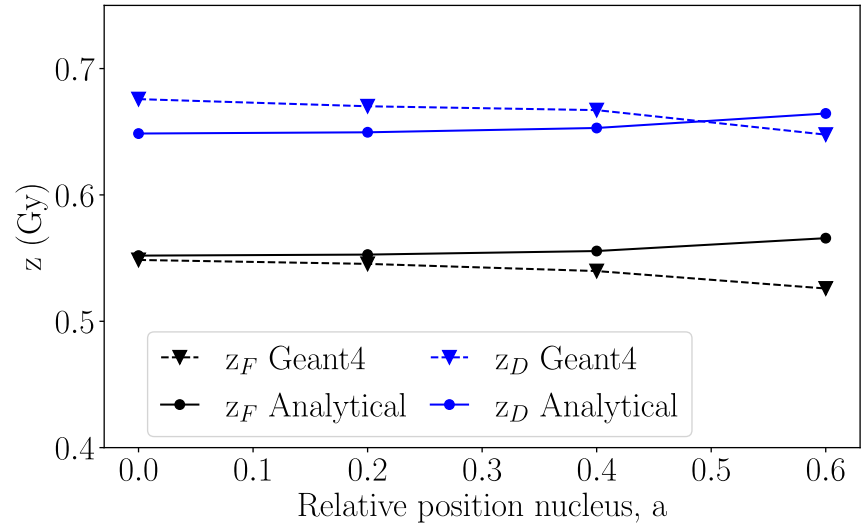
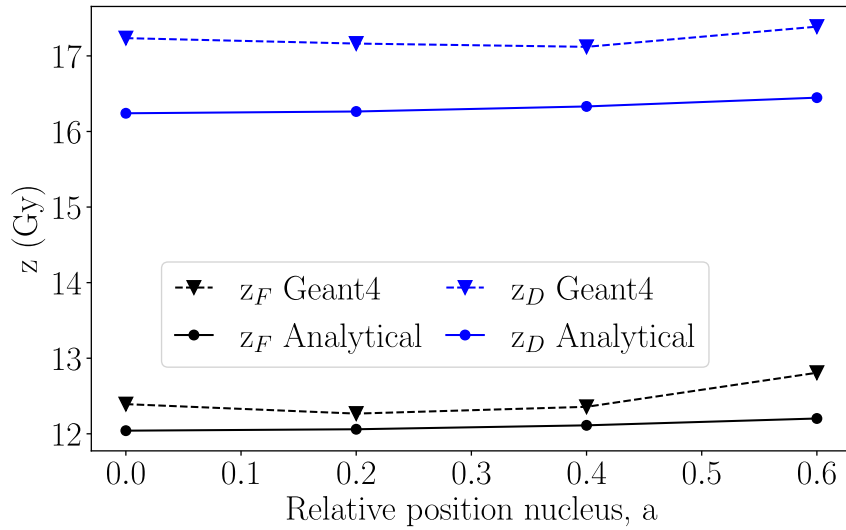

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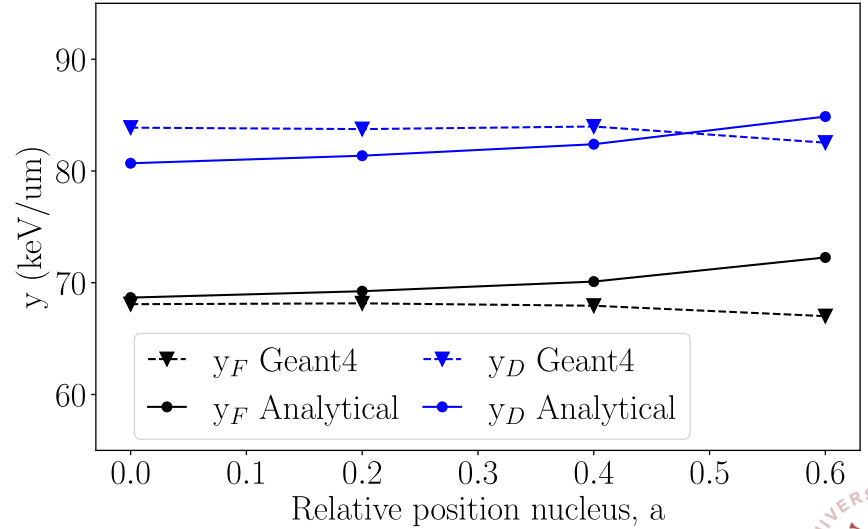
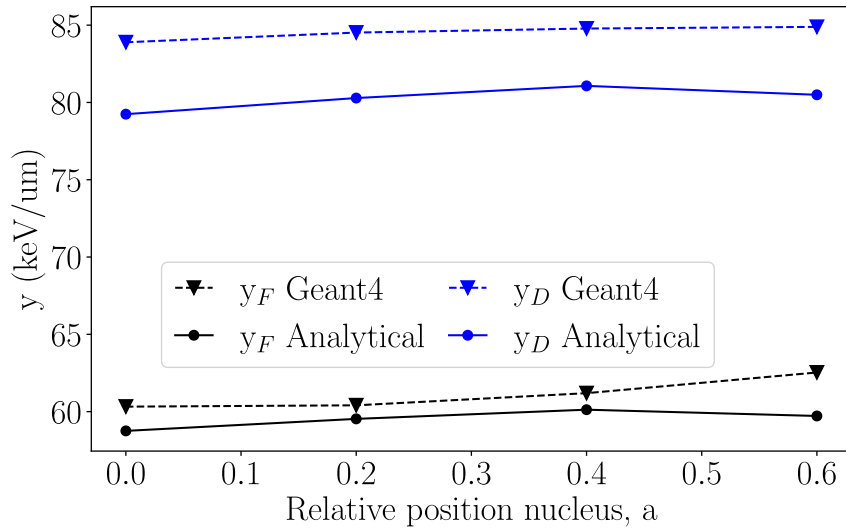
$$\bar{z}_F \quad \bar{z}_D \quad \bar{y}_F \quad \bar{y}_D$$

Results

Nucleus radius 0.5 μm



Nucleus radius 2.5 μm



Conclusions

- Our preliminary results show that the analytical method agrees reasonably well with the Geant4 simulations, even when the cell nucleus is displaced.
- Microdosimetric quantities are not affected significantly by the cell nucleus position.
- The dependence of these microdosimetric quantities on the nucleus radius is clearly more significant.
- In simulations with similar conditions, the displacement of the nucleus within the cell can be disregarded. This phenomenon can help to simplify future studies.

Future...

- To explore the cases where the cell size is comparable with the range of the alpha emitted

Acknowledgments

This work was funded in part by Junta de Andalucía under grant no. P18-RT-1900, cofunded by the Operational Program ERDF Andalusia 2014-2020 "Growth smart: an economy based on knowledge and innovation", and by Grant RTI2018-098117-B-C21 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe"



Analytical method - Bertolet et al. Radiat. Res. (2020)

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Energy absorbed as a function of the energy of the incident particle

$$\bar{\varepsilon}_s(E) = C_s \cdot \operatorname{erf}(k_s E^{q_s}) \cdot \log(a \cdot E + b_s) \cdot \frac{\alpha \cdot E^{p-1} + e_s}{\alpha \cdot E^p + \frac{4r_s}{3}}$$

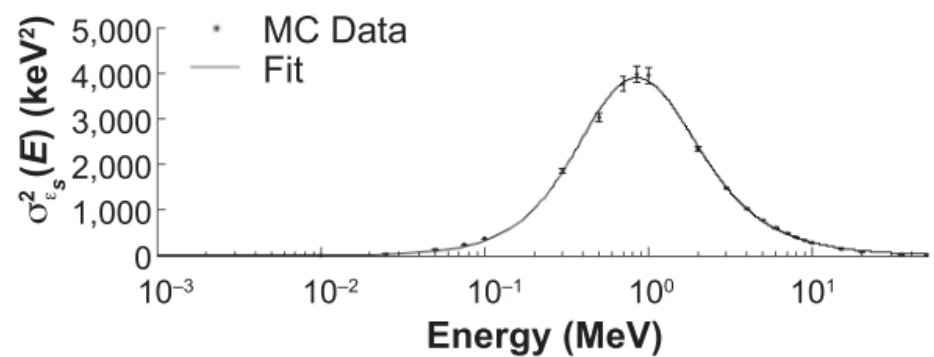
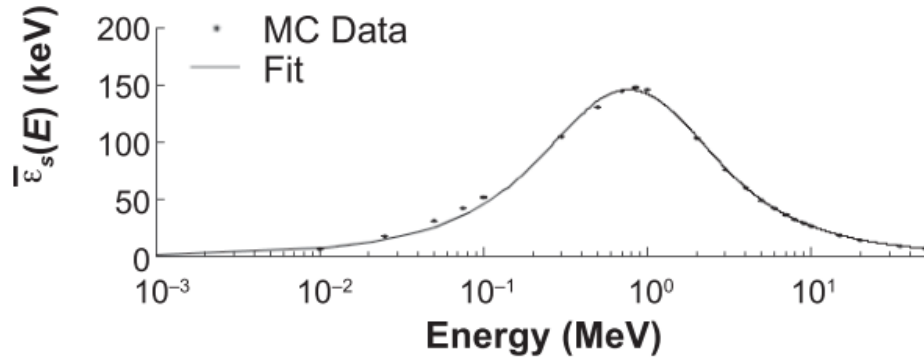
$$\sigma_{\varepsilon_s}(E) = C'_s \cdot \operatorname{erf}(k'_s E^{q_{s'}}) \cdot \log(a \cdot E + b'_s) \cdot \frac{\alpha \cdot E^{p-f_{s'}} + e'_s}{\alpha \cdot E^p + \frac{4r_s}{3}}$$

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spherical sites with diameters of 1 μm

Analytical method - Bertolet et al. Radiat. Res. (2020)

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Distribution of distance source-nucleus $\rightarrow \phi_E(E)$ Distribution of incident energy

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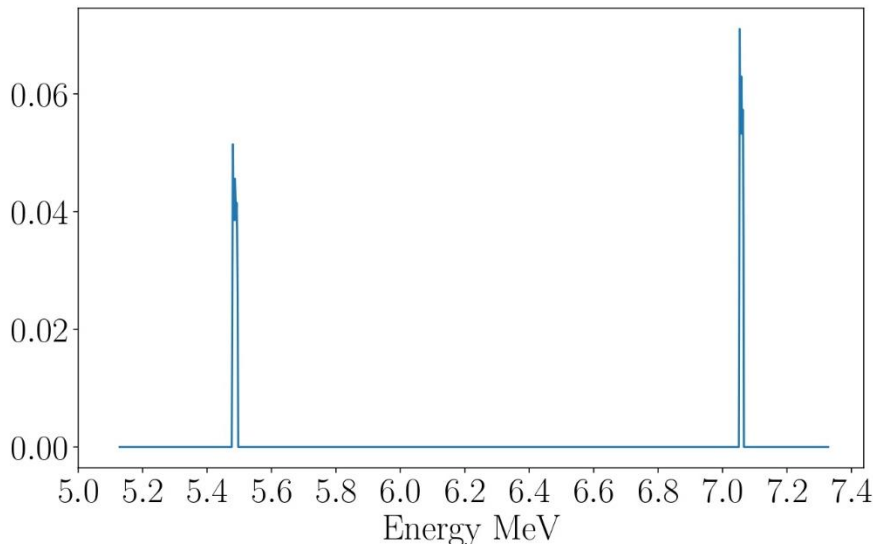
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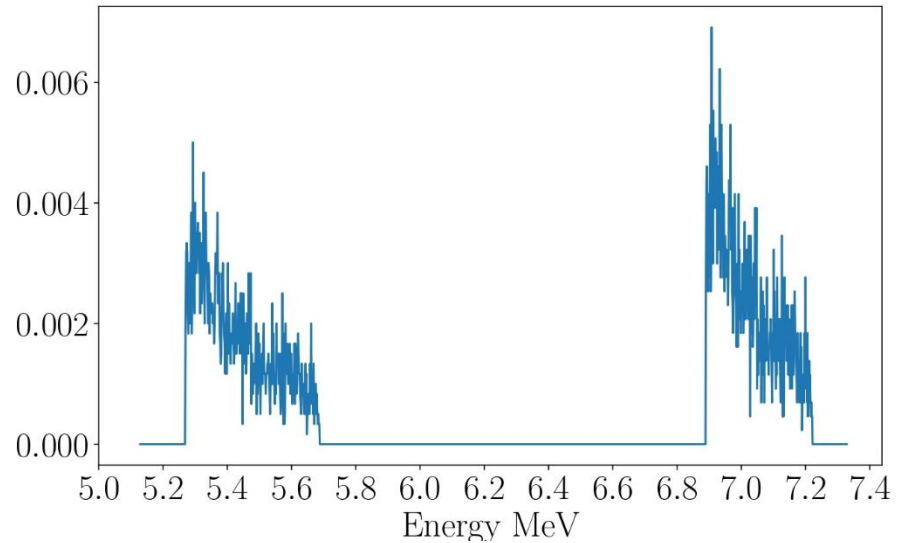
Distribution of distance source-nucleus

→ $\phi_E(E)$

Distribution of incident energy



$$r_n = 2.5 \text{ um} \quad a = 0.00$$



$$r_n = 2.5 \text{ um} \quad a = 0.60$$



Analytical method - Bertolet et al. Radiat. Res. (2020)

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Mean energy absorbed and variance for polyenergetic beams

$$\sigma_{\varepsilon_s}^2 = \frac{\int \phi_E(E) \sigma_{\varepsilon_s}^2(E) dE}{\int \phi_E(E) dE} + \frac{\int \phi_E(E) (\bar{\varepsilon}_s(E) - \bar{\varepsilon}_s)^2 dE}{\int \phi_E(E) dE}$$

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$$\bar{Z}_F = \frac{\bar{\varepsilon}_s}{m}$$

$$\bar{y}_F = \frac{\bar{\varepsilon}_s}{l}$$

$$\bar{Z}_D = \frac{\bar{\varepsilon}_s}{m} \left(1 + \frac{\sigma_{\varepsilon_s}^2}{\bar{\varepsilon}_s^2} \right)$$

$$\bar{y}_D = \frac{\bar{\varepsilon}_s}{l} \left(1 + \frac{\sigma_{\varepsilon_s}^2}{\bar{\varepsilon}_s^2} \right)$$

