Effective QG, Cosmological Constant and the Standard Model of Particle Physics

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Partially supported by CNPq and FAPEMIG

XIII CPAN days meeting, Huelva - March 22, 2022

Based on: *Breno Giacchini, Tibério P. Netto, I.Sh, 2112.06314.*

Previous works: *2006.04217, 2009.04122*

Starting from the paper *R. Utiyama & B. DeWitt, J.M.Phys. (1962)*, **we know that renormalizable theory of matter fields on classical curved background requires classical action of vacuum**

$$S_{vac} = S_{EH} + S_{HD}\,, \qquad S_{EH} \,=\, -\,\frac{1}{16\pi G}\,\int d^4x\,\sqrt{-g}\,\,(R + 2\Lambda)\,.$$

Without independent vacuum parameter $\Lambda = \Lambda_{vac}$ **the theory is inconsistent because loops of massive fields give divergences of this type. If** $\Lambda_{vac} \equiv 0$**, these divergences cannot be removed by renormalization, and we have a kind of theoretical disaster.**

What is typical magnitude of $\rho_\Lambda = \rho_\Lambda^{vac}$**? RG running in the UV:**

$$(4\pi)^2\,\mu\frac{d\,\rho_\Lambda^{vac}}{d\,\mu} \,=\, \frac{N_s m_s^4}{2} - 2N_f m_f^4\,, \qquad \rho_\Lambda^{vac} = \frac{\Lambda_{vac}}{8\pi G_{vac}}.$$

ρ_Λ^{vac} **cannot be much smaller then typical** $mass^4$ **of the theory.**

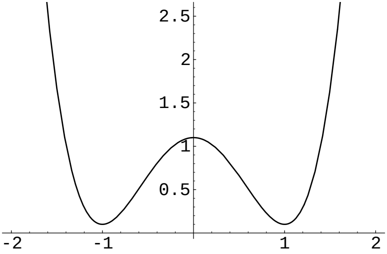
Consequence: the natural value from the MSM perspective is

$$\rho_\Lambda^{vac} \,\sim\, M_F^4 \qquad \textbf{or} \qquad \rho_\Lambda^{vac} \,\sim\, 10^8\,GeV^4.$$

Induced CC from SSB in the Standard Model.

In the stable point of the Higgs potential $V = -m^2\phi^2 + f\phi^4$, we meet $\rho_\Lambda^{ind} = \langle V \rangle \approx 10^8\, GeV^4$ – of the same magnitude as ρ_Λ^{vac} !

This is induced CC, similar to the one found by Zeldovich (1967).



The observed CC is a sum

$$\rho_\Lambda^{obs} = \rho_\Lambda^{vac} + \rho_\Lambda^{ind} \approx 10^{-56} \rho_\Lambda^{ind}.$$

This is the Cosmological Constant (CC) Problem. In what follows, we accept that the CC fine tuning takes place and explore the physical consequences on this fact.

Renormalization group running for ρ_Λ^{vac} or ρ_Λ^{ind}

may break down the fine tuning and produce significant effect, even if the running is very weak compared to basic values.

In semiclassical theory, such a running may take place only because of the effects of massive particles, and it is supposed to be weakened by AC-like decoupling (*Gorbar & Sh. 2003 ...*).

Another possibility is the running in QG, i.e., in the quantum theory of metric, which is the main subject of this talk.

There are a few serious challenging problems on this way:
1) How to apply renormalization group in the nonrenormalizable theory such as quantum gravity?;
2) Construct unambiguous beta functions.

Let us discuss these issues in the effective QG framework.

Running in the effective QG

In effective QG, Feynman technique, e.g., propagators and vertices, should be constructed on the basis of GR.

J.F. Donoghue, Phys. Rev. D 50 (1994) 3874.

Power counting tells us log. divergences have number of derivatives $d(G) = 2 + 2L - 2K_\Lambda$. **With** $\Lambda = 0$, **at the one-loop level there are divergences**

$$\mathcal{O}(R^2_{\dots}) \,=\, R^2_{\mu\nu\alpha\beta}\,,\;\; R^2_{\mu\nu}\,,\;\; R^2$$

and no sign of the renormalization of the R**- term or** Λ.

Without the cosmological term, we have only the renormalization in higher and higher derivative sectors – too boring to explore.

What about QG with the nonzero cosmological constant?

In this case, yes - we have all kinds of divergences and certainly can construct the renormalization group equations which make sense and are potentially interesting.

Gauge invariant renormalizability in QG/GR

The Faddeev-Popov approach, with $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$

$$S_{tot} \,=\, S(h) \,+\, \frac{1}{2}\,\chi^{\mu} Y_{\mu\nu} \chi^{\nu} \,+\, \bar{C}^{\mu} M^{\nu}_{\ \mu} C_{\nu}, \qquad M^{\nu}_{\ \mu} \,=\, \frac{\delta \chi^{\nu}}{\delta h_{\lambda\sigma}}\, R_{\lambda\sigma,\mu}\,.$$

The useful choices of gauge fixing conditions and the weight function depend on the theory, e.g., the background gauge

$$\chi_{\mu} = \nabla^{\nu} h_{\mu\nu} - \beta \nabla_{\mu} h\,, \qquad Y_{\mu\nu} = \alpha g_{\mu\nu},$$

Independent on the parametrization and gauge fixing, one can prove that the divergent part of effective action, $\Gamma_{div} = \Gamma_{div}(g_{\mu\nu})$, **in all loop orders, is local and satisfies the gauge identity**

$$\frac{\delta \Gamma_{div}}{\delta g_{\mu\nu}}\, R_{\mu\nu,\alpha} \,=\, 0\,.$$

Multiplicative renormalizability depends on a power counting.

Gauge-fixing dependence

The gauge and parametrization ambiguities in the running of Newton and cosmological constants are typical for quantum GR.

R.E. Kallosh, O.V. Tarasov, I.V. Tyutin, Nucl. Phys. B137 (1978).

One can analyse gauge and parametrization dependencies using general QFT theorems, see, e.g.,

I.Y. Aref'eva, A.A. Slavnov, L.D. Faddeev, Theor. Math. Phys. (1974).

B.L. Voronov, P.M. Lavrov, I.V. Tyutin, Sov. J. Nucl. Phys. (1982).

This formalism was applied to quantum gravity in

E.S. Fradkin, A.A. Tseytlin, Nucl. Phys. B201 (1982) 469.

I.Sh, A. Jacksenaev, PLB 324 (1994) 284.

Also, it was confirmed by explicit calculations, e.g., in

M. Kalmykov, Class. Quant. Grav. 12 (1995) 1401.

J. Gonçalves, T. de Paula Netto, I.Sh., PRD 97 (2018), 1712.03338.

How can we get an invariant definition of RG in QG?

The best solution is based on the Vilkovisky–DeWitt (VdW) scheme for constructing effective action in quantum gravity.

*G.A. Vilkovisky, Nucl. Phys. B***234** *(1984) 125.*

A.O. Barvinsky and G.A. Vilkovisky, Phys. Repts. 119 (1985) 1.

B.S. DeWitt, The effective action, (Essays in honor of the sixtieth birthday of E.S. Fradkin, 1987).

We need that (at least) one-loop divergences do not depend on the gauge-fixing and parametrization of the quantum metric.

Then, we can achieve the universal running of the coefficients of $\rho_\Lambda, \; R, \; R^2$ **and, in fact, of all other terms of the action.**

And the VdW approach makes it possible, and even gives more

T. Taylor and G. Veneziano, Nucl. Phys. B ***345*** *(1990).*

B. Giacchini, T. de Paula Netto, I.Sh., JHEP (2020), 2009.04122.

For the effective QG based on Einstein's GR with the CC, this prescription gives

$$\bar{\Gamma}_{\rm div}^{(1)} = -\frac{1}{\epsilon}\int d^4x\sqrt{-g}\,\left\{\frac{121}{60}C^2 - \frac{151}{180}E + \frac{31}{36}R^2 + 8\Lambda R + 12\Lambda^2\right\}.$$

This, completely invariant, result enables us to construct the renormalization group equations

$$\mu\frac{d}{d\mu}\left(\frac{1}{16\pi G}\right) = \frac{8\Lambda}{(4\pi)^2}\,, \qquad \mu\frac{d\Lambda}{d\mu} = -\frac{2(16\pi G)\Lambda^2}{(4\pi)^2}\,. \qquad (*)$$

The solutions can be easily found in the form ($\gamma_0 = 16\pi G_0\Lambda_0^2$).

$$G(\mu) = \frac{G_0}{\left[1+\frac{10}{(4\pi)^2}\,\gamma_0\ln\frac{\mu}{\mu_0}\right]^{4/5}}\,, \qquad \Lambda(\mu) = \frac{\Lambda_0}{\left[1+\frac{10}{(4\pi)^2}\,\gamma_0\ln\frac{\mu}{\mu_0}\right]^{1/5}}.$$

We get a well-defined renormalization group running of the Newton and cosmological constants between the Planck and Hubble scales!

This result is because of effective QG, as we assume that below the Planck scale all extra degrees of freedom get inactive.

The renormalization group (RG) equations in effective QG:

$$\mu \frac{d}{d\mu}\left(\frac{1}{16\pi G}\right) = \frac{8\Lambda}{(4\pi)^2}\,, \qquad \mu \frac{d\Lambda}{d\mu} = -\frac{2(16\pi G)\Lambda^2}{(4\pi)^2}\,. \qquad (*)$$

The two most remarkable properties of these equations and their solutions are as follows:

i) Universality. Eqs. (*) do not depend on the gauge fixing, parametrization of quantum fields or any kind of hypothesis and assumptions, except using the VdW effective action.

ii) Can be regarded exact, i.e., not restricted to one-loop order. All higher-loop corrections are suppressed by the powers of

$$\frac{\Lambda}{M_P^2} = \frac{\rho_\Lambda}{M_P^4}.$$

In the present-day Universe this quantity is of the order of 10^{-120}, **but even in the inflationary epoch it is at least** 10^{-12}.

Thus, RG equations (*) describe an effectively exact running.

For the cosmological constant

(vacuum energy) density, is there a chance that the running of vacuum CC breaks down the fine tuning between $\rho_\Lambda^{vac} = \rho_\Lambda$ and ρ_Λ^{ind}, making an extra trouble related to the CC problem?

Consider the strongest option - the SUSY GUT case. Then $M_X \sim 10^{16}$ GeV **and hence**

$$\big|\rho_\Lambda\big| = \big|\rho_\Lambda^{vac}\big| \approx \rho_\Lambda^{ind} \propto M_X^4 \qquad \Longrightarrow \qquad \frac{\gamma_0}{(4\pi)^2} \sim 8\Big(\frac{M_X}{M_P}\Big)^4 \approx 10^{-11}.$$

Accordingly, we get, as a very good approximation,

$$G(\mu) = G_0\Big[1 - \frac{8\,\gamma_0}{(4\pi)^2}\ln\frac{\mu}{\mu_0}\Big] \quad \textbf{and} \quad \rho_\Lambda(\mu) = \rho_\Lambda^0\Big[1 + \frac{6\,\gamma_0}{(4\pi)^2}\ln\frac{\mu}{\mu_0}\Big].$$

The main point is that the effective quantum gravity running depends on the vacuum part ρ_Λ only.

But what happens with the observed sum $\rho_\Lambda^{obs} = \rho_\Lambda + \rho_\Lambda^{ind}$??

The answer depends of the magnitude of the running, i.e., of

$$\delta\rho_\Lambda^{obs} \;=\; \rho_\Lambda^{obs}(UV) - \rho_\Lambda^{obs}(IR) \;=\; \frac{6\gamma_0}{(4\pi)^2}\,\rho_\Lambda^0\,\ln\left(\frac{\mu_{\rm UV}}{\mu_{\rm IR}}\right).$$

In cosmology, standard identification is $\mu \sim H$ (Hubble const).

This running produces a discrepancy with the cosmological observations much more than 10^{50} for a change of just about an order of magnitude in the parameter H.

As we already know, for a typical SUSY GUT model

$$\frac{\gamma_0}{(4\pi)^2} \propto 10^{-11}, \qquad \text{also} \qquad \ln\left(\frac{\mu_{\rm UV}}{\mu_{\rm IR}}\right) = \ln\left(\frac{H_{\rm inflation}}{H_0}\right) \approx 130.$$

Consequently, if we “believe” in the VDW effective action in QG, then SUSY GUT’s are ruled out.

What about lower energy physics?

What if we have to "rule out" the Minimal Standard Model of particle physics (MSM)??

Assuming that the MSM is valid up to the Planck scale, we get

$$\frac{6}{(4\pi)^2}\,\gamma_0 \,\sim\, 48\left(\frac{M_F}{M_P}\right)^4 \,\approx\, 10^{-65}.$$

Multiplying by $\rho_\Lambda^0 \sim M_F^4$, the variation is

$$\delta\rho_\Lambda^{obs} \approx 10^{-55}\ln\left(\frac{H}{H_0}\right)\mathrm{GeV}^4 \approx 10^{-53}\,\mathrm{GeV}^4 \approx 10^{-6}\,\rho_\Lambda^{obs}.$$

Thus, the model of effective QG running is lucky enough to pass the test related to MSM. The opposite output would mean the disproval of the Vilkovisky and DeWitt approach in QG.

However, since the result is $\mathcal{O}(M_F^8)$, the existence of any kind of "new physics" based on the symmetry breaking beyond the scale $10M_F$, contradicts the CC running in effective QG.

Conclusions

- **The great CC problem looks impossible to solve, as it comes from summing up completely independent contributions: induced and vacuum, the last has to be fine-tuned.**

- **In the IR region, i.e., below the mass spectrum of the fundamental higher derivative gravity, we meet an effective QG, which is remarkable in several respects.**

- **Assuming the Vilkovisky-DeWitt unique effective action, we arrive at the well-defined RG equations, which turn out free from the higher-loop corrections.**

- **It is remarkable that the effective CC running, derived in this framework, provides a relation between the cosmological constant problem and the particle physics.**