Axial-vector meson contributions to the HFS of μH

Pablo Sanchez-Puertas
psanchez@ifae.es

Instituto de Fisica d'Altes Enrgies (IFAE)
Barcelona Institute of Science and Technology (BIST)
Barcelona, Spain

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Institut de Física d'Altes Energies

EXCELENCIA SEVERO OCHOA

BIST Barcelona Institute of Science and Technology

Outline

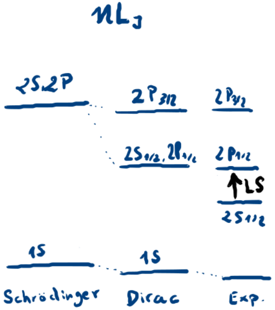
- Atomic spectroscopy and the proton structure
- Warming up: $A \to \ell^+\ell^-$ decays
- Axial-vector meson contributions to the HFS of μH
- Conclusions

Section 1

Atomic spectroscopy and the proton structure

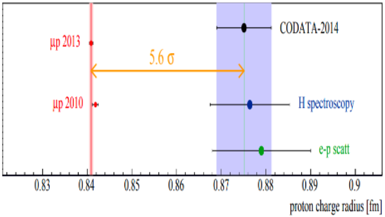
Atomic spectroscopy and the proton structure

- Schrödinger Eq. degenerate (L, s) levels
- Dirac Eq. degenerate J levels
- But Lamb shift (LS) came!
  $2P_{1/2}, 2S_{1/2}$ splitting!
- Lamb shift $\propto \delta_{l0}|\psi_{nls}(0)|^2$ captures short-distance (spin-independent) effects.
  $\Rightarrow$ QED: loop diagrams
  $\Rightarrow$ Proton structure (finite size) ...
- QED extremely precise allows for a precise charge radius $< r_E^2 >$ determination
- Compares to scattering determination
  $G_E(Q^2) = 1 - \frac{<r_E^2>}{6} Q^2 + \ldots$



Atomic spectroscopy and the proton structure

- CODATA(2010) (eH + scattering)
  $< r_E^2 >= (0.8775(51)\ \mathrm{fm})^2$
- But μH came (CREMA Coll. 2010)
  Bohr radius $\frac{(a_0)_{\mu\mathrm{H}}}{(a_0)_{e\mathrm{H}}} \sim \frac{m_e}{m_\mu}$
  More sensitive to short distances
- Proton radius puzzle
  $< r_E^2 >= (0.8418(7)\ \mathrm{fm})^2$ (5σ!)
- Recent results support μH
  While controversy with results



Take-home message: Atomic spectroscopy allows deciphering the proton structure, but requires careful evaluation of all contributions (QED ones well-known; hadronic ones are the bottelneck)
This talk: HFS and Zemach radius

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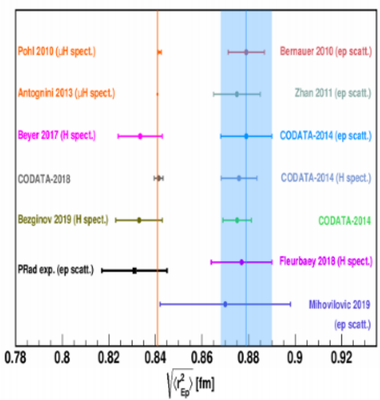

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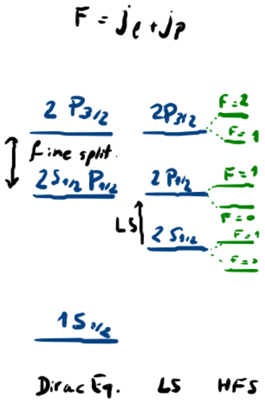

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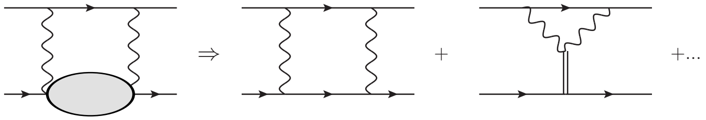
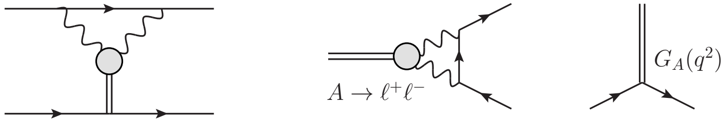
Atomic spectroscopy and the proton structure

- Want to extract info about the proton structure from to the Lamb shift/HFS
- Other corrections perturbing Coulomb potential need be removed, among them 2γ effects



- In this talk, focus on the axial exchange contribution



Section 2

Warming up: $A \to \ell^+\ell^-$ decays

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- Computing $\langle \ell^+\ell^- | \, S \, | A \rangle$ demands the nonperturbative matrix element

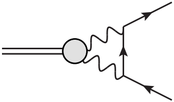
$$i \int d^4x e^{iq_1 \cdot x} \langle 0| \, T\{j^\mu(x) j^\nu(0)\} \, |A(q_1+q_2)\rangle \equiv \mathcal{M}^{\mu\nu\rho}_{A\gamma^*\gamma^*} \varepsilon_\rho$$

- Based on general principles, following structures [refs]

$$\mathcal{M}^{\mu\nu\rho}_{A\gamma^*\gamma^*} = i\Big\{ \epsilon_{\mu\alpha\rho q_1} \left[q_2^\alpha q_{2\nu} - g_\nu^\alpha \, q_2^2 \right] B_2(q_1^2, q_2^2) + \\ \epsilon_{\nu\alpha\rho q_2} \left[q_1^\alpha q_{1\mu} - g_\mu^\alpha \, q_1^2 \right] B_2(q_2^2, q_1^2) + \epsilon_{\mu\nu q_1 q_2} \left[\bar{q}_{12\rho} \, C_A(q_1^2, q_2^2) \right] \Big\}$$

It is useful to define $B_2(x,y) = B_{2S}(x,y) + B_{2A}(x,y)$

- The final result from the loop diagram can be expressed as



$$\mathcal{M}_{A\to\ell^+\ell^-} = \bar{u}[A_1(q^2)\gamma^\rho + A_2(q^2) q^\rho]\gamma^5 v \; \varepsilon_\rho$$

$$\Gamma_{A\to\ell^+\ell^-} = \frac{m_A}{12\pi} \beta_\ell^3 |A_1(m_A^2)|$$

Warming up: $A \to \ell^+\ell^-$ decays

- Need to specify form factors. Focus on B_{2S} (leading for the HFS)
- Information from $e^+e^- \to e^+e^- A$; L3 fitted $B_{2S}(q^2, 0) = \frac{B_{2S}(0,0)}{(1-q^2/\Lambda^2)^2}$
- We use different parametrizations (pQCD predicts Q^{-4}), among them

$$B_{2S}^{\mathrm{OPE}}(q_1^2, q_2^2) = \frac{B_{2S}(0,0)\Lambda^4}{(q_1^2+q_2^2-\Lambda^2)^2}$$

$$B_{2S}^{\mathrm{heVMD}}(q_1^2, q_2^2) = \frac{B_{2S}(0,0)m_V^4 M^4(1+q_1^2 q_2^2 \Lambda_{\mathrm{OPE}}^{-2})}{(q_1^2-m_V^2)(q_1^2-M^2)(q_1^2-m_V^2)(q_1^2-M^2)}$$

- Compare with previous model

$$B_{2S}^{\mathrm{Dorokhov}}(q_1^2, q_2^2) = \frac{B_{2S}(0,0)\Lambda^8}{(q_1^2-\Lambda^2)^2(q_2^2-\Lambda^2)^2}$$

- Partial predictions for $A \to e^+e^-$ (see PRD105 '22)
- Cross-check against previous models for $A \to e^+e^-$

Section 3

Axial-vector meson contributions to the HFS of μH

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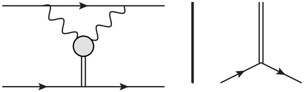
- We got our first subamplitude, then coupling axials to protons



$$\mathcal{L}_{a_1NN} = -g_{a_1NN}(\bar{N}\gamma_\mu\gamma^5\vec{\sigma}N)\vec{a}_1^\mu$$
$$\mathcal{L}_{f_1NN} = -g_{f_1NN}(\bar{N}\gamma_\mu\gamma^5 N)f_1^\mu$$

- Joining them together, then expanding NR ($\boldsymbol{q} \sim \alpha$)

$$i\mathcal{M}_{\ell p} = -ig_{ANN}[\bar{u}\gamma^\nu\gamma^5 u]_N i\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_A^2}}{q^2 - m_A^2} i[\bar{u}(A_1\gamma^\mu + A_2 q^\mu)\gamma^5 u]_\ell$$

- Then perturbation theory $\Delta E = \langle V_{\rm NR}(r)\rangle$

$$\Delta E_1^{\rm HFS} = \frac{4g_{ANN}A_1(0)}{\pi}\frac{(\mu\alpha)^3}{m_A^2} \qquad \Delta E_2^{\rm HFS} = \frac{g_{ANN}A_1(0)}{2\pi}\frac{(\mu\alpha)^3}{m_A^2}$$

- Important findings at this stage: $-1/2$ factor wrt previous results[1]
- Next major difference: linking g_{ANN} to $G_A(q^2)$ (model) $\Rightarrow$ Let's go!

[1] Dorokhov et al., Phys. Lett. B 776, 105 (2018)

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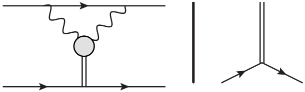

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- Joining them together, then expanding NR ($\boldsymbol{q} \sim \alpha$)

$$\tilde{V}_{NR}(\boldsymbol{q}^2) = g_{ANN}\left[\frac{A_1(-\boldsymbol{q}^2)}{m_A^2+\boldsymbol{q}^2}\left\{(\hat{\boldsymbol{\sigma}}_\ell\cdot\hat{\boldsymbol{\sigma}}_N) + \frac{(\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}_\ell)(\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}_N)}{m_A^2}\right\} - \frac{\tilde{A}_2(-\boldsymbol{q}^2)}{m_A^2}(\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}_\ell)(\boldsymbol{q}\cdot\hat{\boldsymbol{\sigma}}_N)\right]$$

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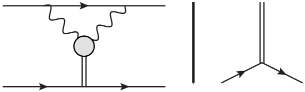

$$\mathcal{L}_{a_1 NN} = -g_{a_1 NN}(\bar{N}\gamma_\mu\gamma^5\vec{\sigma}N)\vec{a}_1^\mu$$
$$\mathcal{L}_{f_1 NN} = -g_{f_1 NN}(\bar{N}\gamma_\mu\gamma^5 N)f_1^\mu$$

- Joining them together, then expanding NR ($\boldsymbol{q} \sim \alpha$)

$$V_{\mathrm{NR}}(r) = \frac{g_{ANN}A_1(0)}{4\pi r}e^{-m_A r}(\hat{\boldsymbol{\sigma}}_\ell \cdot \hat{\boldsymbol{\sigma}}_N) \Rightarrow \text{Only } B_{2S} \text{ matters for } A_1(0)$$

- Then perturbation theory $\Delta E = \langle V_{\mathrm{NR}}(r)\rangle$

$$\Delta E_1^{\mathrm{HFS}} = \frac{4g_{ANN}A_1(0)}{\pi}\frac{(\mu\alpha)^3}{m_A^2} \qquad \Delta E_2^{\mathrm{HFS}} = \frac{g_{ANN}A_1(0)}{2\pi}\frac{(\mu\alpha)^3}{m_A^2}$$

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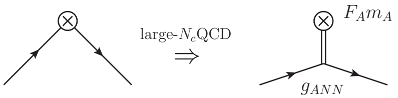
- Let's model the whole thing based on $G_A^a(q^2) \sim \langle N|\, \bar{q}\gamma^\mu\gamma^5 t^a q\, |N\rangle$:



$$G_A^a = \sum_i \frac{2F_{A_i} m_{A_i} g_{A_i NN}}{m_A^2 - q^2}$$

- $G_A^a(0)$ from $n \to pe\bar{\nu}$ and lattice; F_A from τ decays;
- Simplest option: 1 resonance, ok low Q^2

A	$A_1(0)/\alpha^2 B_{2S}(0,0)$			$\Delta E_A^{\mathrm{HFS}}(1S)$	$\Delta E_A^{\mathrm{HFS}}(2S)$
	Dorokhov	heVMD	OPE	(meV)	(meV)
$a_1(1260)$	0.9(2)	1.2(5)	1.4(3)	0.029(12)	0.0036(15)
$f_1(1285)$	1.0(2)	1.3(5)	1.5(2)	0.011(2)	0.0014(3)
$f_1(1420)$	0.8(1)	1.0(3)	1.2(2)	−0.001(3)	−0.0001(3)

- Systematics: 2 res. with Q^{-4} pQCD (+model for excited $A \to \gamma^*\gamma^*$[2])

$$\Delta E_A^{\mathrm{HFS}}(1S) = 0.039(^{+12}_{-13})(^{+3}_{-0})(^{+22}_{-0})\ \mathrm{meV}$$
$$\Delta E_A^{\mathrm{HFS}}(2S) = 0.0049(^{+14}_{-16})(^{+3}_{-0})(^{+29}_{-0})\ \mathrm{meV}$$

[2] P. Masjuan, P. Roig, P. SP, *J.Phys.G 49 (2022)*

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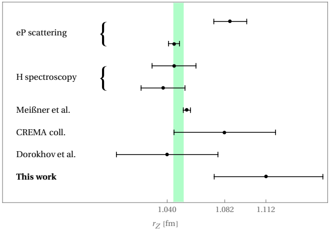
- Our final result

$$\Delta E_A^{\rm HFS}(1S) = 0.039(^{+12}_{-13})(^{+3}_{-0})(^{+22}_{-0})\ \text{meV}$$
$$\Delta E_A^{\rm HFS}(2S) = 0.0049(^{+14}_{-16})(^{+3}_{-0})(^{+29}_{-0})\ \text{meV}$$

Compare to previous: $-0.05(2)$ meV $(1S)$, $-0.007(2)$ meV $(2S)$

- Important impact in the Zemach radius r_Z [3]

$$\Delta^{\rm th}_{\rm HFS}(2S) = 22.984(3) - 0.162(1) r_Z + \Delta^A_{HFS} \Rightarrow r_Z = 1.112(31)_{\rm exp}(19)_{\rm th}(^{+20}_{-10})_{\rm axials}$$



[3] A. Antognini et. al, Annals. Phys. 331 (2013)

Section 4

Conclusions

Conclusions

- μH spectroscopy allows extraction of proton structure
- Focused on the Zemach radius r_Z, related to the HFS
- Found a factor of $-1/2$ wrt previous results
- Proposed a model: fixes signs unambiguously and relates parameters to other hadronic quantities
- Better knowledge of $A \to \gamma^* \gamma^*$ as well as understanding of $G_A(q^2)$ hepful
- Our shift for r_Z in slight tension, yet still compatible with other extractions

Thanks for your attention!

Section 5

Backup

- 1 Resonance

$$g_{a_1 NN} = 5.6(1.1), \quad g_{f_1 NN} = 2.01(0.17), \quad g_{f_1' NN} = -0.33(0.08), \qquad (\phi = 0),$$
$$g_{a_1 NN} = 5.6(1.1), \quad g_{f_1 NN} = 1.93(0.16), \quad g_{f_1' NN} = 0.71(21), \qquad (\phi_{\mathrm{L3}} = 27(5)^\circ)$$

- 2 Resonances

A	$\frac{A_1(0)}{\alpha^2 B^A_{2S}}$	g_{ANN}	$B^A_{2S}(0,0)$ [GeV^{-2}]	$\Delta E^{HFS}_A(1S)$ [meV]	$\Delta E^{HFS}_A(2S)$ [meV]
$f_1(1285)$	1.53	4.78	0.269	0.0269	0.0034
$f_1(1^{\mathrm{st}}$ excitation$)$	3.05	−3.64	0.093	−0.0082	−0.0010
Subtotal				0.0187	0.0024
$a_1(1260)$	1.41	11.8	0.245	0.0605	0.0076
$a_1(1^{\mathrm{st}}$ excitation$)$	2.93	−8.6	0.082	−0.0162	−0.0020
Subtotal				0.0443	0.0056
$f_1(1420)$	1.20	−0.90	0.197	−0.0024	−0.0003
$f_1'(1^{\mathrm{st}}$ excitation$)$	2.72	0.71	0.051	0.0007	0.0001
Subtotal				−0.0017	−0.0002
Total				0.0613	0.0078