Gauge dependence in QCD correlation functions

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Motivations

- In QFT with gauge symmetry physical states are equivalent classes of field configurations connected by gauge transformations. In the continuum this requires to fixing the gauge, leaving the residual BRST symmetry.

- Correlation functions $\langle \phi(x_1)\ldots\phi(x_n)\rangle = \frac{1}{Z}\int D\phi\, \phi(x_1)\ldots\phi(x_n)\, e^{-S}$ are the building blocks of QFT from which gauge invariant physical observables are computed. They are gauge dependent quantities.

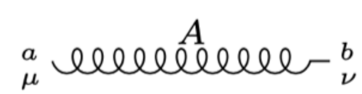
- PT: it would be useful to have an efficient way to transform at any order the expressions calculated with some gauge (i.e. Feynman gauge) to some other gauge.

- Non perturbative functional methods (DSE, FRG…) requiere truncation strategies and vertices modeling that can yield to spurious gauge dependencies in physical observables. Knowledge of gauge covariance constrains the possible *ansätze*.

linear covariant gauge (SU(N) symmetry) $\partial_\mu A_\mu = i\alpha b$

$$S = \int d^D x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}\left(\gamma_\mu D_\mu + m\right)\psi + i b^a \partial_\mu A^a_\mu + \frac{\alpha}{2} b^a b^a + \partial_\mu \bar{c}^a D^{ab}_\mu c^b\right]$$

Correlation functions acquire gauge dependence from the gauge field propagator



$$= \frac{1}{p^2}\left(\delta_{\mu\nu} + (\alpha - 1)\frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}$$

Can we factor out the dependence on the gauge parameter α? (Gauge covariance)

Ex: Fermion propagator in QED (U(1))

$$\langle \bar{\psi}_i(x)\psi_j(y)\rangle_\alpha = \langle \bar{\psi}_i(x)\psi_j(y)\rangle_{\alpha=0}\, e^{-g^2\left[\langle \xi(x)\xi(y)\rangle - \langle \xi(0)\xi(0)\rangle\right]}, \qquad \langle \xi(x)\xi(y)\rangle = \alpha \int \frac{d^D p}{(2\pi)^D}\frac{1}{p^4} e^{ip\cdot(x-y)}$$

L.D. Landau, I.M. Khalatnikov, Zh. Eksp. Teor. Fiz. 29 (1955).

LKF Transformations

Phys. Rev. D 97, 074017 (2018). arxiv:801.01703

Introducing composite gauge invariant operators $A_\mu^h, \quad \psi^h$
that contain an auxiliar scalar Stueckelberg field ξ

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$$

$$\psi^h = h^\dagger \psi$$

$$h = e^{ig\xi^a T^a}, \qquad h \to U^\dagger h \qquad\qquad U \in SU(N)$$

$$A_\mu \to U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U, \qquad \psi \to U^\dagger \psi$$

$$\Rightarrow A_\mu^h \to A_\mu^h, \quad \psi^h \to \psi^h$$

Note: transversal A_μ^h introduced to generalize Gribov-Zwanziger scenario to linear covariant gauges, preserving a local nilpotent BRST symmetry

$$A_\mu^h = \min_U \int d^D x \, \mathrm{Tr}\, A_\mu^U A_\mu^U$$
$$= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial_\sigma A_\sigma, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial_\sigma A_\sigma, \partial_\nu \frac{1}{\partial^2}\partial_\sigma A_\sigma\right] + \mathcal{O}(A^3)\right)$$

Classical action augmented with non-polynomial term:

$$S_h = \int d^D x \left[\tau^a \partial_\mu A_\mu^{h,a} + \bar{\eta}^a \partial_\mu D_\mu^{ab}[A^h] \eta^b \right]$$

- $\partial_\mu A_\mu^h = 0$
- $\langle \xi^a(p) \xi^b(-p) \rangle = \dfrac{\alpha}{p^4}$

renormalizability

$$\int D\xi \, \delta(\partial_\mu A_\mu^h) \det \left(-\partial_\mu D_\mu(A_\mu^h) \right) = 1$$

S_h Does not change physics!

Since A_μ^h is gauge invariant, BRST symmetry is not spoiled

$$s\tau = s\eta = s\bar{\eta} = 0,$$

$$sA_\mu^a = -D_\mu^{ab} c^b \Rightarrow \quad s\xi^a = -c^a + \frac{g}{2} f^{abc} c^b \xi^c - \frac{g^2}{12} f^{abc} f^{bde} c^d \xi^e \xi^c + \mathcal{O}(\xi^3)$$

Important: in Landau gauge ($\alpha = 0$) $A^h_\mu = A_\mu$ ($\psi^h = \psi$) and $\xi = 0$

$\langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_n}(x_n)\bar{\psi}^h(y_1)\psi^h(z_1)\ldots\bar{\psi}^h(y_m)\psi^h(z_m)\rangle$ do not depend on α

LKFT

$$\begin{aligned}
\Rightarrow \langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\bar{\psi}(y_1)\psi(z_1)\ldots\bar{\psi}(y_m)\psi(z_m)\rangle_{\alpha=0} &= \langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_n}(x_n)\bar{\psi}^h(y_1)\psi^h(z_1)\ldots\bar{\psi}^h(y_m)\psi^h(z_m)\rangle_{\alpha=0} \\
&= \langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_n}(x_n)\bar{\psi}^h(y_1)\psi^h(z_1)\ldots\bar{\psi}^h(y_m)\psi^h(z_m)\rangle_{\alpha} \\
&= \langle A_{\mu_1}(x_1)\ldots A_{\mu_n}(x_n)\bar{\psi}(y_1)\psi(z_1)\ldots\bar{\psi}(y_m)\psi(z_m)\rangle_{\alpha} + \mathscr{R}_\alpha
\end{aligned}$$

$\mathscr{R}_\alpha$ is given by expanding A^h_μ and ψ^h ($\bar{\psi}^h$) in terms of ξ and the elementary fields.

$\mathscr{R}_\alpha$ is a combination of $\langle E_i(x) O_j(y)\rangle$ and $\langle O_i(x) O_j(y)\rangle$

elementary → E_i; composite → O_j

$$\langle E_i(x)\, J_j(y)\rangle$$

where $J_i O_i$ is added to the action

Ex: LKFT for the quark propagator $\psi_i^h = \psi_i - ig\xi^a T^a_{ij}\psi_j - \frac{g^2}{2}\xi^a\xi^b T^a_{ik}T^b_{kj}\psi_j + O(\xi^3)$

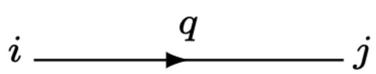
$$\langle\bar\psi_i(x)\psi_j(y)\rangle_{\alpha=0} = \langle\bar\psi_i^h(x)\psi_j^h(y)\rangle_\alpha$$
$$= \langle\bar\psi_i(x)\psi_j(y)\rangle_\alpha + 2\langle\bar\psi_i(x)O_{1j}(y)\rangle_\alpha + 2\langle\bar\psi_i(x)O_{2j}(y)\rangle_\alpha$$
$$+2\langle\bar O_{1i}(x)O_{2j}(y)\rangle_\alpha + \langle\bar O_{1i}(x)O_{1j}(y)\rangle_\alpha + \langle\bar O_{2i}(x)O_{2j}(y)\rangle_\alpha + \dots$$

Note: in QED the ξ field decouples and the expansion can be summed up to all orders:

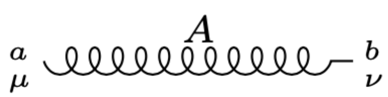
$$\langle\bar\psi_i(x)\psi_j(y)\rangle_{\alpha=0} = \langle\bar\psi_i(x)\psi_j(y)\rangle_\alpha\, e^{g^2\left[\langle\xi(x)\xi(y)\rangle - \langle\xi(0)\xi(0)\rangle\right]}$$

In QCD ξ interacts with the other fields and it is not clear yet whether such resummation is possible.

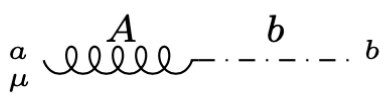
Feynman rules



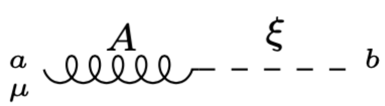
$$= \frac{-i\not{p}}{p^2}\delta^{ij},$$



$$= \frac{1}{p^2}\left(\delta_{\mu\nu}+(\alpha-1)\frac{p_\mu p_\nu}{p^2}\right)\delta^{ab},$$

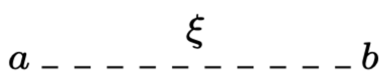


$$= \frac{p_\mu}{p^2}\,\delta^{ab},$$

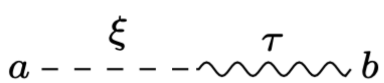


$$= -i\alpha\frac{p_\mu}{p^4}\,\delta^{ab},$$

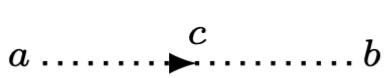
$$= \frac{i}{p^2}\,\delta^{ab},$$



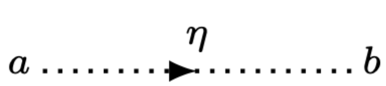
$$= \frac{\alpha}{p^4}\,\delta^{ab},$$



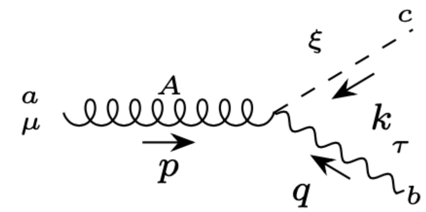
$$= \frac{1}{p^2}\,\delta^{ab}$$



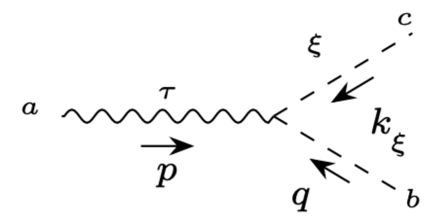
$$= \frac{1}{p^2}\,\delta^{ab}$$



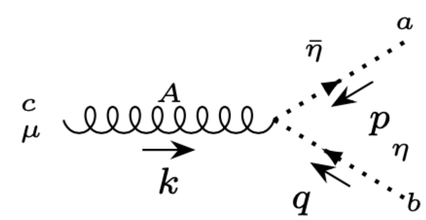
$$= \frac{1}{p^2}\,\delta^{ab},$$



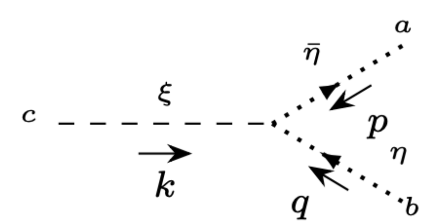
$$= -igf^{abc}q_\mu,$$



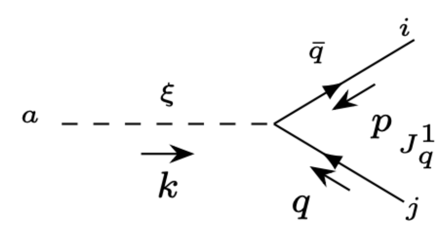
$$= \frac{g}{2}f^{abc}\,p\cdot(k-q),$$



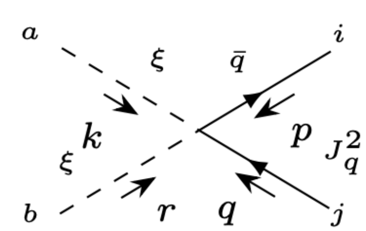
$$= igf^{abc}p_\mu,$$



$$= -gf^{abc}p\cdot k,$$



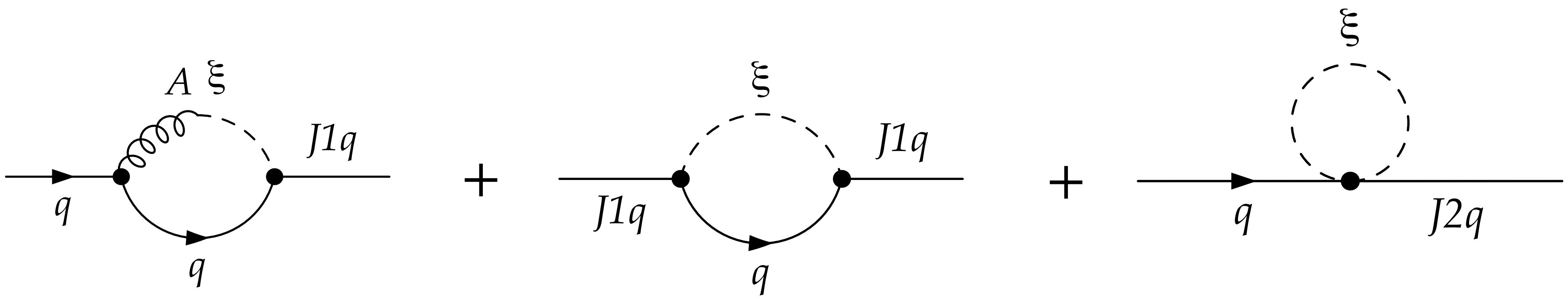
$$= -igT^a_{ij},$$



$$= \frac{g^2}{2}\left(T^aT^b+T^bT^a\right)_{ij}.$$

One loop (quark)

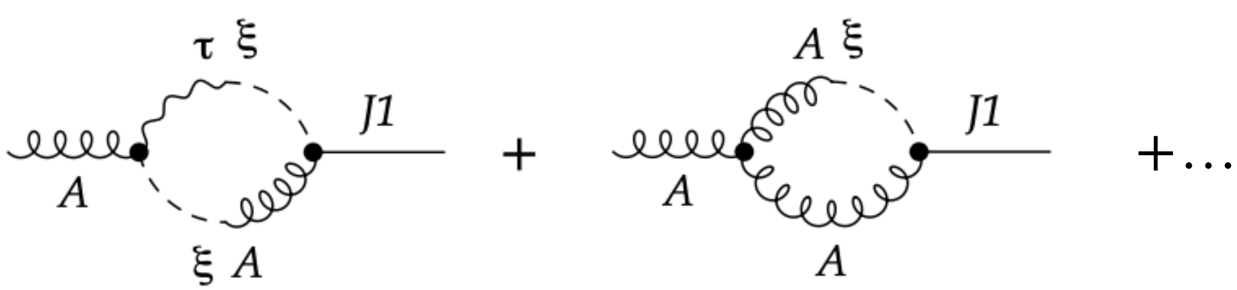
Nuclear Physics B 973, 115606 (2021). arXiv:2107.03910



$$\Rightarrow \quad \langle \bar{\psi}_i(p)\psi_j(-p)\rangle_\alpha = \langle \bar{\psi}_i(p)\psi_j(-p)\rangle_{\alpha=0} + \delta_{ij}\, i\gamma_\mu p_\mu (p^2)^{D/2-3}\frac{g^2 C_F}{(4\pi)^{D/2}}\frac{\alpha\,\Gamma(2-D/2)\Gamma(D/2)}{2\Gamma(D-1)}$$

One loop (gluon)

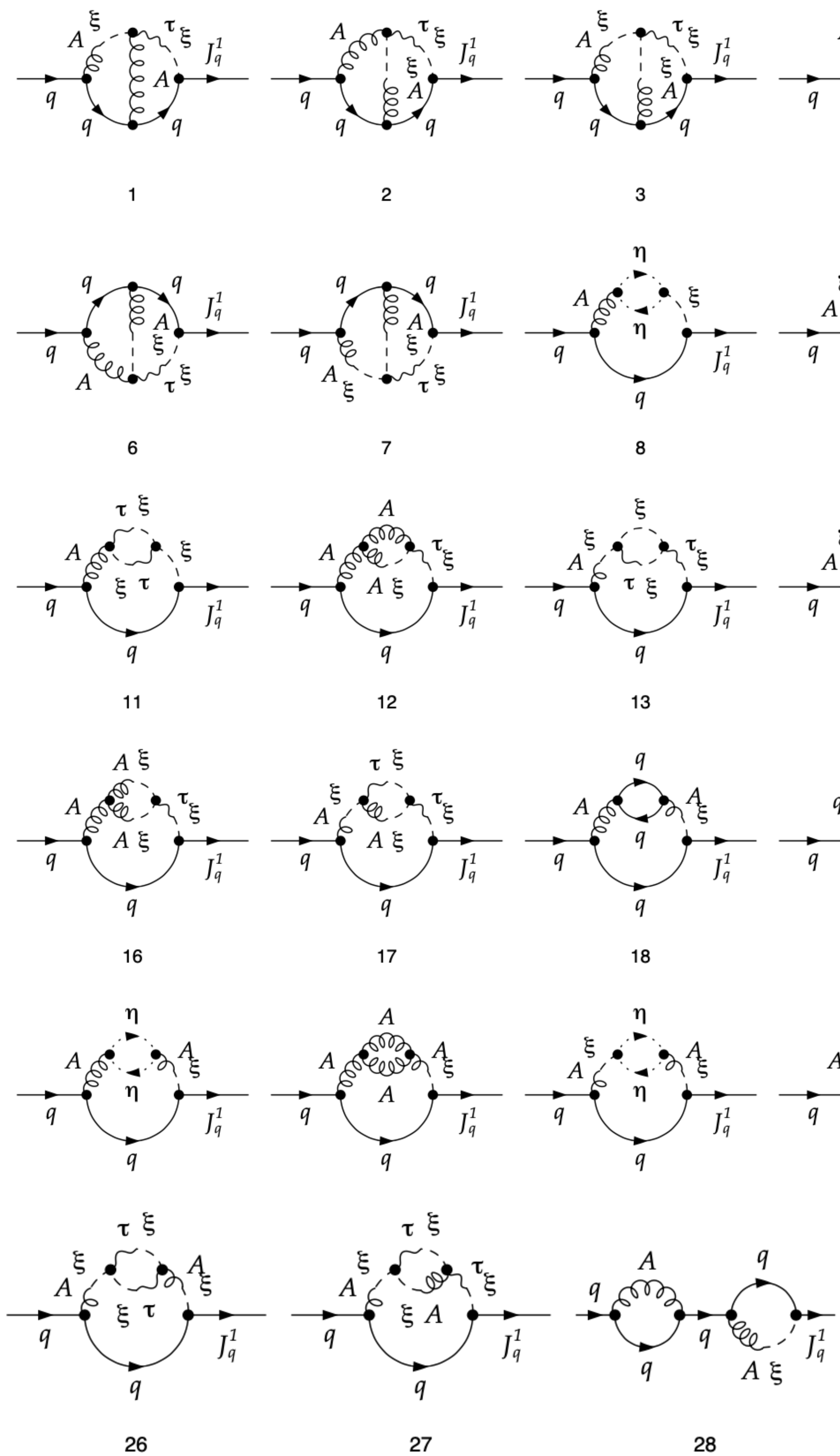
Phys. Rev. D 101, 085005 (2020). arXiv:1911.01907

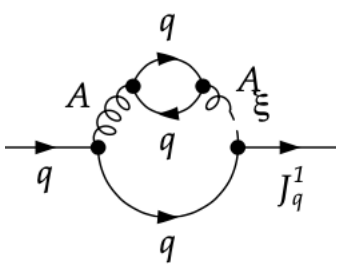


$$\Rightarrow \quad \langle A_\mu^a(p)A_\nu^b(-p)\rangle_\alpha = \langle A_\mu^a(p)A_\nu^b(-p)\rangle_{\alpha=0} + \alpha\frac{p_\mu p_\nu}{p^4}\delta^{ab}$$
$$-\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\delta^{ab}\frac{g^2 C_A}{(4\pi)^{D/2}}p^{D-6}\frac{\alpha(\alpha(D-4)+6D-20)\Gamma^2(D/2)\Gamma(2-D/2)}{2(D-2)\Gamma(D-1)}$$

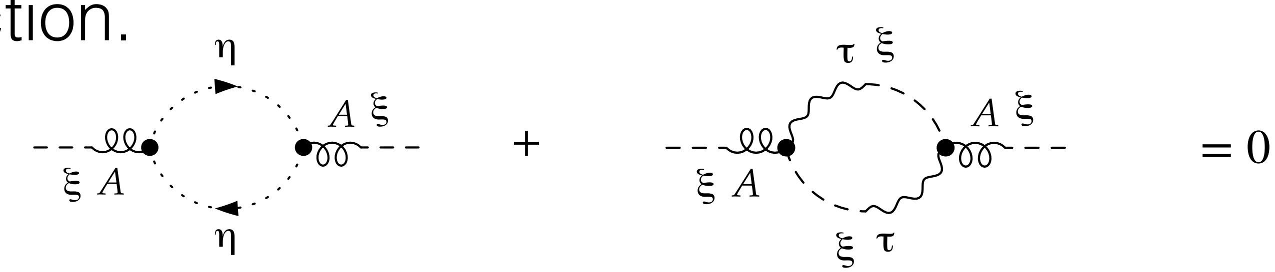
Two loops (quark)

Contribution to $\langle \bar{\psi}_i(x)\, O_{1_j}(y) \rangle$

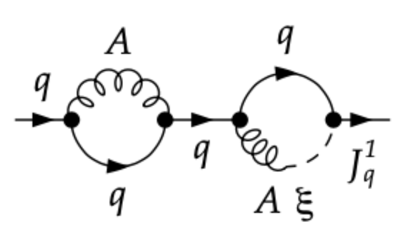


- At two loops there are 99 diagrams vs. 6 diagrams of usual PT. Mathematica takes half the time to calculate the 99 diagrams, that contain only the gauge dependent part, than the 6 that contain the full information. This is because the tensor structure is much simpler.
- 

automatically vanishes because $A_\mu - \xi$ is longitudinal.
- The diagrams containing loops of the new ghosts η cancel each others.
- In the one loop gluon LKFT η are essentials to guarantee the vanishing of longitudinal one loop correction.



- LKFT have been derived for connected correlation functions, not for 1PI. Reducible diagrams need to be included.



Final remarks

- We established a systematic framework to calculate only the gauge dependent part of correlation functions in QCD (order by order).
- This framework allows to derive both LKFT and Nielsen identities as Slavnov-Taylor identities (from the same BRST symmetry).
- We have to explore the massive case and study LKFT for the vertices
- The formalism allows to incorporate a gauge invariant massive term for the gluon field $\frac{1}{2}m^2 A_\mu^h A_\mu^h$. This should allow to generalize to linear covariant gauges the infrared safe renormalization group analysis done by Tissier and Wschebor in Landau gauge.