A probabilistic approach to the hierarchy problem

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1. Introduction

- **Motivation**: no new particles have been observed yet at current accelerators energy scale $\Rightarrow$ to introduce new scalar particles, we need to have a **mass hierarchy**.

  Is it possible to obtain hierarchy in a *natural* way? (i.e. general model/models & wide range of parameters).

- **Proposal**: toy model to study large separation of scales from quantum origin.

  We start from a massless lagrangian and make use of the Coleman & Weinberg mechanism to obtain spontaneous symmetry; effective potential formalism (1-loop).

  We define a probability proportional to the size of the hyper-volume of the parameter space of the model associated with an interval of mass ratios between two sectors.

2. The model

$SU(2)_L \times SU(2)_H \times U(1)_X$ + 2 scalars

$$\mathcal{L}_0 = |D_\mu \Phi|^2 + |D_\mu \Theta|^2 - V_0(\Phi, \Theta)$$

$$\mathrm{D}^\mu_{L,H} = \partial^\mu - \frac{i}{2} g_{L,H} \sigma_a W^{a\mu}_{L,H} - \frac{i}{2} g_X Q_{L,H} X^\mu$$

$$V_0(\varphi, \eta) = \frac{1}{4!}\lambda_L \varphi^4 + \frac{1}{4!}\lambda_H \eta^4 + \frac{1}{4!}\lambda_{LH}\varphi^2\eta^2$$

Classical fields: $\phi \to (0, \varphi)$, $\Theta \to (0, \eta)$

Particle content of the model

2 scalars (φ, η) and 7 gauge bosons whose masses m_j depend on the scalar backgrounds and gauge couplings:

$L - H$ decoupled limit $(g_X = 0)$:

- $m_{W_{L,j}} = g_L \varphi/2$
- $m_{W_{H,j}} = g_H \eta/2$
- $m_X = 0$

$(j = 1, 2, 3)$

General case $(g_X \neq 0)$:

- $W^\mu_{L,1}$, $W^\mu_{L,2}$, $W^\mu_{H,1}$ and $W^\mu_{H,2}$: the same m as with $g_X = 0$
- mixing between $W^\mu_{L,3}$, $W^\mu_{H,3}$ and $X^\mu \to Z^\mu_L$, Z^μ_H and $\hat{\gamma}^\mu$
- $\hat{\gamma}^\mu$ is always massless
- Z^μ_L and Z^μ_H masses: combination of the three gauge couplings $g_{L,H,X}$

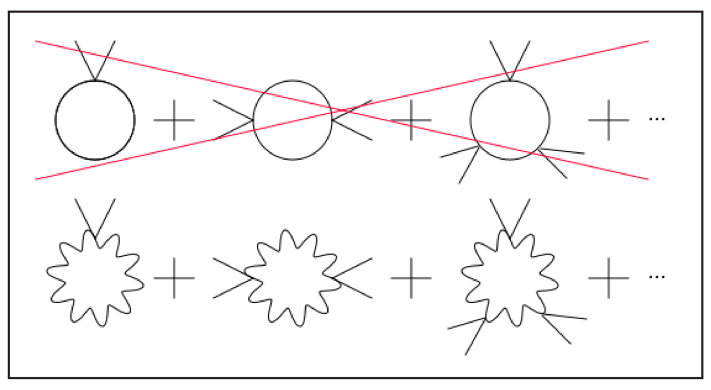
Effective potential (1-loop + Coleman & Weinberg hypothesis)

$$V(\varphi, \eta) = V_0(\varphi, \eta) + \frac{3}{64\pi^2} \sum_{j=1}^{7} m_j^4 \left[\ln\left(\frac{m_j^2}{\mu^2}\right) - \frac{5}{6} \right]$$

Restrictions $\Rightarrow$ *CW-triangle*

CW hypothesis
$\to |\lambda_j| < \epsilon_{CW} \cdot g_j^2$

Perturbativity
$\to g_j^2 < \epsilon_{g^2} \cdot 4\pi \equiv g^2_{max}$



3. Phenomenolgy

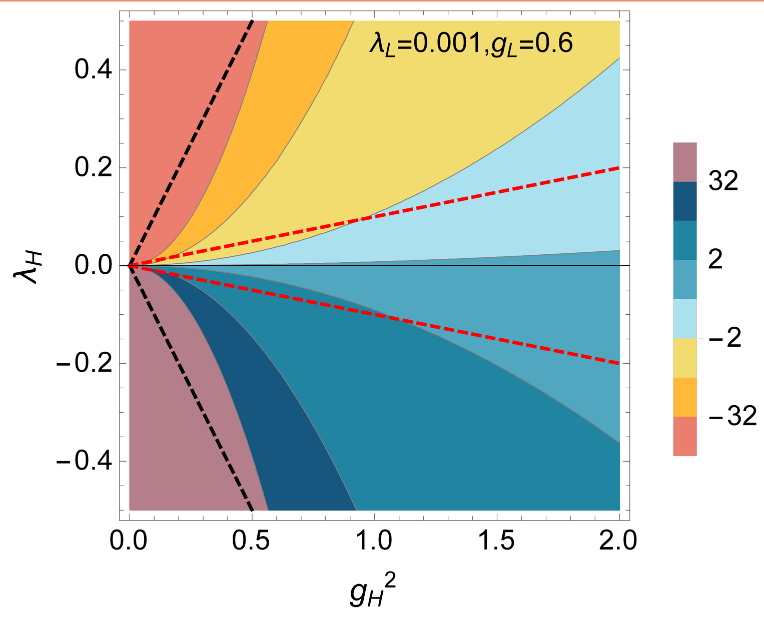
Hierarchy between the L and H sectors

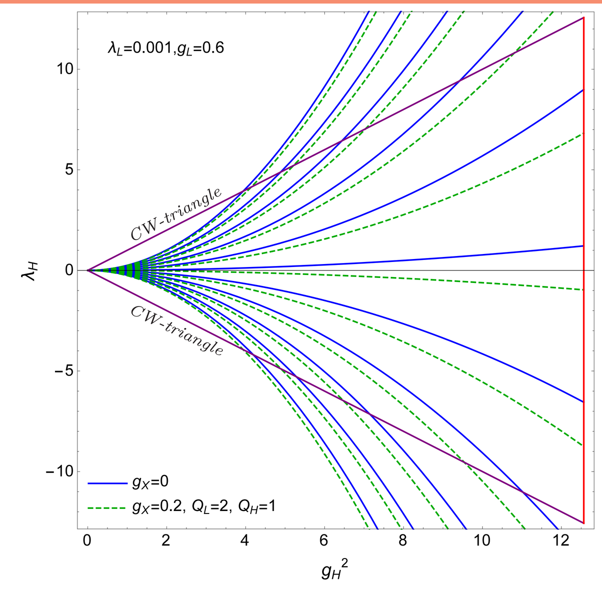
$$\mathfrak{R} = \frac{m^2_{W_H}}{m^2_{W_L}} = \frac{g_H^2 \langle\eta\rangle^2}{g_L^2 \langle\varphi\rangle^2} \quad \Rightarrow \quad \lambda_H = \mathcal{P}(\mathfrak{R}) g_H^4, \ |\lambda_H| = \varepsilon g_H^2$$

Constant hierarchy $\mathfrak{R}$ lines in the (g_H^2, λ_H) plane $(\lambda_{LH} = 0)$

$$\mathfrak{R} = Exp\left[\frac{128\pi^2}{27}\left(\frac{\lambda_L}{g_L^4} - \frac{\lambda_H}{g_H^4}\right)\right] \qquad (g_X = 0)$$

$$\mathfrak{R} = Exp\left[\frac{128\pi^2}{27}\left(\frac{(\lambda_L + \frac{9}{128\pi^2} g_L^2 Q_L^2 g_X^2)}{(g_L^4 + \frac{2}{3} g_L^2 Q_L^2 g_X^2)} - (L \leftrightarrow H)\right)\right]$$

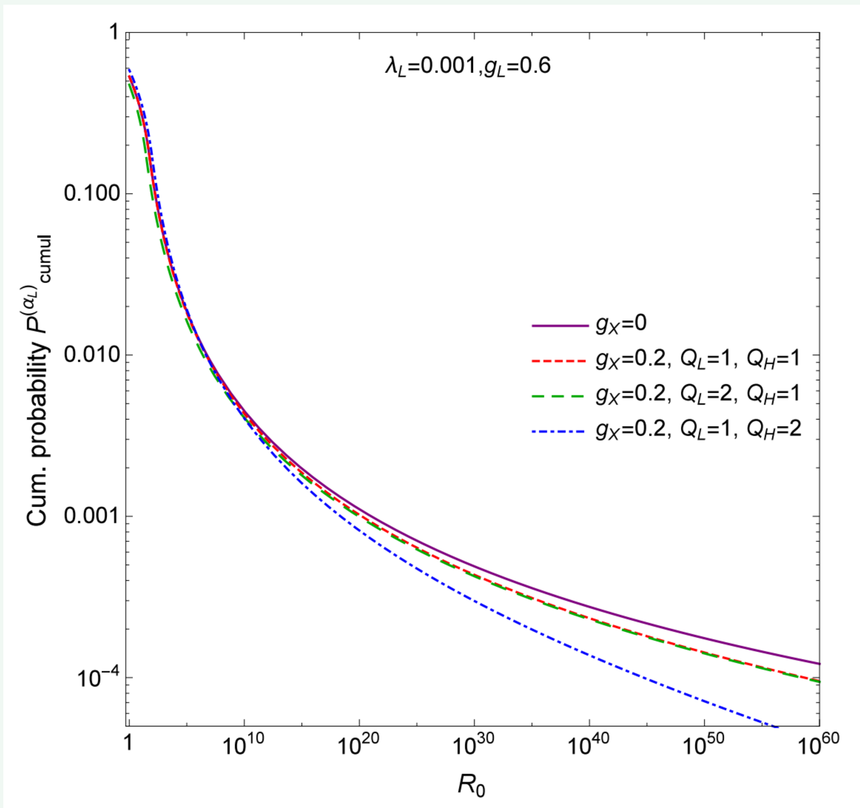




Fixed L sector $(\alpha_L = \{g_L^2, \lambda_L\})$

Probability $\mathfrak{P}^{(\alpha_L)}_{\text{cumul}}$: ratio of the area with $\mathfrak{R} \in [\mathfrak{R}_0, \infty]$ and the total allowed area in the (g_H^2, λ_H) plane (*CW-triangle*)
$\to$ cumulative probability for $\mathfrak{R}$ from $\mathfrak{R}_0$ to ∞

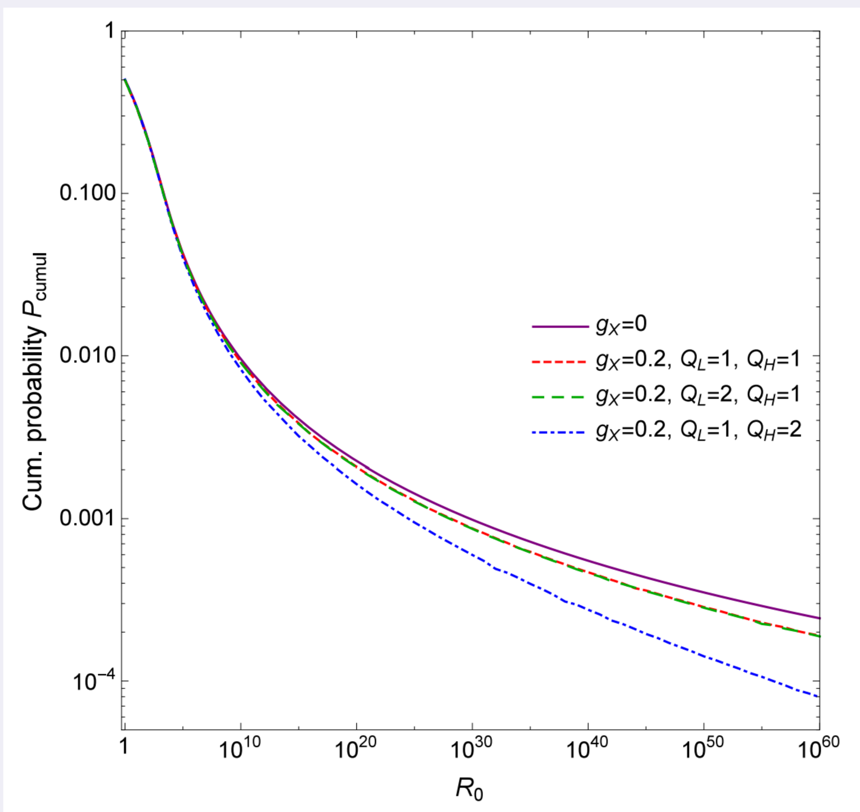
$$\mathfrak{P}^{(\alpha_L)}_{\text{cumul}} = \frac{1}{6}\left(\frac{27 \ln \mathfrak{R}_0}{32\pi} - \frac{4\pi\lambda_L}{g_L^4}\right)^{-2} \overset{\mathfrak{R}_0 \gg 1}{\simeq} \frac{0.44}{(\log_{10}\mathfrak{R}_0)^2} \qquad (g_X = 0)$$



Both L and H sectors integrated out

Probability $\mathfrak{P}_{\text{cumul}}$ from the integration to the whole $(g_L^2, \lambda_L, g_H^2, \lambda_H)$ allowed parameter space

$$\mathfrak{P}_{\text{cumul}} \overset{\mathfrak{R}_0 \gg 1}{\simeq} \frac{1}{3}\left(\frac{32\pi}{27 \ln \mathfrak{R}_0}\right)^2 \simeq \frac{0.87}{(\log_{10}\mathfrak{R}_0)^2} \qquad (g_X = 0)$$



4. Conclusions and future work

- model with $SU(2)_L \times SU(2)_H \times U(1)_X$ symmetry $\to$ 2 sectors of particles with **possible hierarchy** between them
- hierarchy: depends on the **couplings** $g_L, g_H, \lambda_L, \lambda_H, g_X(Q_L, Q_H), \lambda_{LH}$
- **promising results**: wide region of parameter space giving place to hierarchy
- ⋆ probability of obtaining very large hierarchies is suppressed, but only logarithmically $\to$ not excluded
- ⋆ $\mathfrak{R} \sim (M_P/m_{EW})^2 \sim 10^{32}$ only suppressed by $\mathfrak{P} \sim 10^{-3}$–$10^{-4}$
- ⋆ same results if more symmetry groups are included as $\mathcal{G} = \prod_\chi SU(2)_\chi$

References

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