Numerical techniques in Lattice QCD: Exercise II

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PhD course on Lattice field theory

Problem 1 *Simulate the harmonic oscillator using the HMC algorithm. The partition function is defined by*

$$\mathcal{Z}(\hat{m}) = \int \left(\prod_k \mathrm{d}x_k \right) e^{-S_{\mathrm{latt}}(x)} \,. \tag{1}$$

with

$$S_{\mathrm{latt}}(x) = \frac{1}{2} \sum_{k=0}^{T/a-1} \left\{ (x_{k+1} - x_k)^2 + \hat{m}^2 x_k^2 \right\} \tag{2}$$

(Use periodic boundary conditions $x_{T/a} = x_0$).

Fix $Tm = \frac{T}{a}\hat{m} = 8$. Start with $T/a = 4$, and take the continuum limit of the observable $\langle x^2 \rangle$. Show, using the viral theorem, that the energy of the ground state can be determined via

$$\frac{E_0}{m} = m\langle x^2 \rangle \tag{3}$$

Check that a continuum extrapolation of your data is consistent with this expectation.

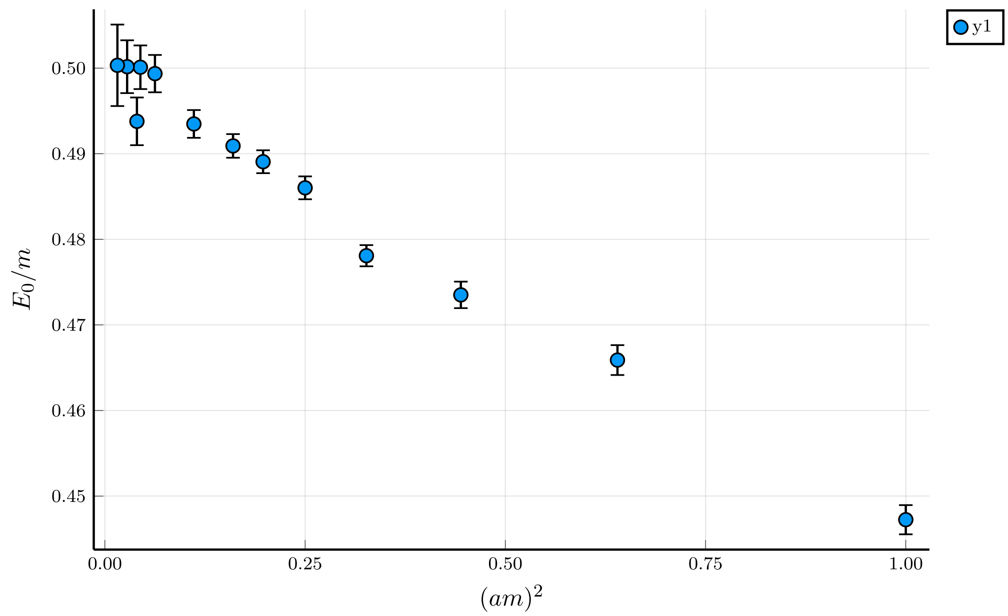
1 Algorithm

Code the HMC, and check that energy conservation is reasonable

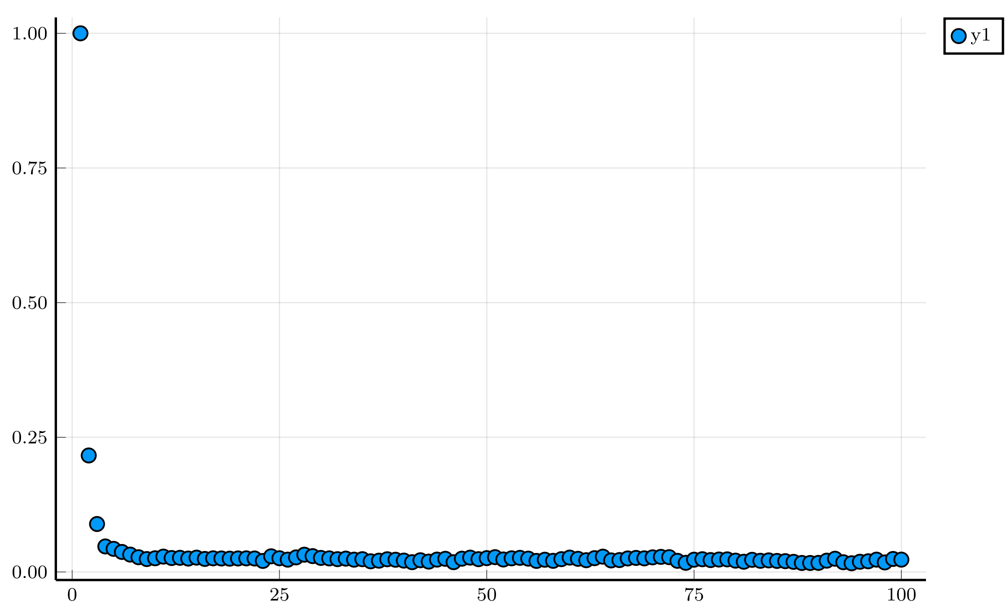
```
T/a:   128
mhat:  0.0625
m x T: 8.0
tau     eps      nmsm      pacc      E0/m
2.0     0.01     10000     100.0%     0.4662729873288891
2.0     0.02     10000     99.96%     0.507206688106228
2.0     0.05     10000     99.69%     0.4676117074113679
2.0     0.08     10000     98.92%     0.5016119482825717
2.0     0.1     10000     98.34%     0.4867416125582713
2.0     0.15     10000     96.85%     0.5403491411591572
2.0     0.2     10000     93.39%     0.4725722716089242
2.0     0.4     10000     69.3%     0.4799505736917153
2.0     0.5     10000     48.57%     0.4742203187841302
```

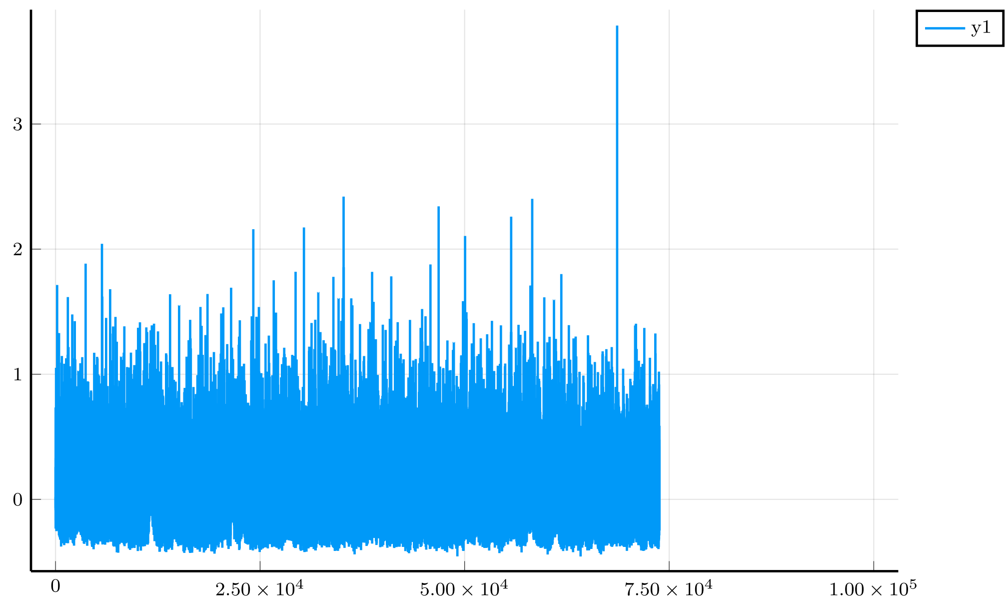
2 Experiment

We now fix $Tm = 4$, and explore the continuum limit



Look at the AC





y1
3
2
1
0
0
2.50×10^4
5.00×10^4
7.50×10^4
1.00×10^5