On the history of $\pi\pi$ scattering

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EFT09, Valencia, February 2-6, 2009

- Introduction
- Annotated history of $\pi\pi$ scattering
- Paradise and real world
- Summary

INTRODUCTION

Notation

S-wave scattering lengths are amplitudes at threshold:

$$T^{\pi^+\pi^-\to\pi^0\pi^0} = 2N(a_2 - a_0)$$
$$T^{\pi^+\pi^-\to\pi^+\pi^-} = N(2a_0 + a_2)\ ; \quad N = 32\pi/6$$

scattering lengths: $a_{0,2\leftarrow \mathrm{isospin}}$

We will do quite a long journey $\Rightarrow$

Rochester Conference 1960

Yu. A. Batusov, S.A. Bunyatov, V.M. Sidorov, V.A. Yarba

Reaction	a_0	a_2	$a_2 - a_0$	References
$\pi^- p \to \pi^+ \pi^- n$			-(0.35 ± 0.30)	Batusov et al. 1960
$\pi N \to \pi N$	~ 1			Efremov et al. 1960
$\pi N \to \pi N$	~ 1			Ishida et al. 1960
$K^\pm \to 3\pi$	-0.8	-0.48	0.3	Sawyer and Wali 1960
$K^\pm \to 3\pi$	-1	-0.3	0.7	Khuri and Treiman 1960
$K^+ \to 3\pi$		~ 1		Thomas et al.,1959

40 - 50 years later

Theory

$$a_0 = 0.220(5), \ a_0 - a_2 = 0.265(4)$$

$$a_2 = -0.0444(10)$$

Roy+ChPT:Colangelo, J.G., Leutwyler 2000

$$a_2 = -0.04330(42)$$

NPLQCD 2007

Experiment

K_{e4} decays:

$$a_0 = 0.220 \pm 005_{\mathrm{stat}} \pm 0.002_{\mathrm{syst}} \pm 0.006_{\mathrm{theo}}$$

Cusp in $K^+ \to \pi^+\pi^0\pi^0$:

$$a_0 - a_2 = (0.266 - 0.268) \pm 0.003_{\mathrm{stat}} \pm 0.002_{\mathrm{syst}} \pm 0.001_{\mathrm{ext}} \pm 0.013_{\mathrm{theo}}$$

NA48/2 [B. Bloch-Devaux, Confinement 2008]

Pionium decay: $A_{\pi^+\pi^-} \to \pi^0\pi^0$

DIRAC 2005

$$a_0 - a_2 = 0.265^{+0.033}_{-0.022}$$

ANNOTATED HISTORY

Mission: Impossible — Tom Cruise and Jon Voight 1996

With $p = 1$, I have missed somebody's work, and I herewith wish to apologize for this. Please, let me know.

Prehistory: Prediction and discovery of pions

Prediction of intermediate particle of mass $\sim 200 m_e$, responsible for nuclear forces (*mesotron*)

Yukawa 1935

Discovery of π via the decay

$$\pi \longrightarrow \mu + \nu$$

Lattes, Muirhead, Occhialini, Powell 1947

"... by examining plates exposed in the Bolivien Andes at a height of 5500 m. . . In identifying the tracks, we employ the method of grain counting. "

"We represent the primary meson by the symbol π, and the secondary by μ."

Nobel Prizes: H. Yukawa (1949), C. Powell (1950)

Interacting pions 1960 - 1970

- Nambu: pions have small mass because strong interactions have approximate chiral symmetry, which is spontaneously broken

Y. Nambu 1960

Nobel Prize 2008

- Threshold anomalies

Wigner 1948

$\pi\pi$ interactions from threshold anomalies in K^+decays

P. Budini and L. Fonda, Phys. Rev. Lett. 1961

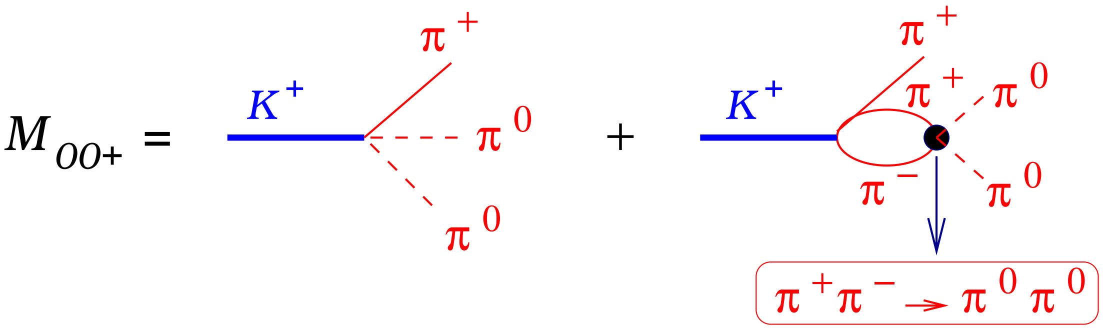
They say:

"The method is based on the observation of threshold anomalies of the kind of a cusp or rounded step in cross sections for reactions with three particles in the final state."

Budini and Fonda

$$K^+ \to \pi^+(p_3)\pi^0(p_1)\pi^0(p_2)$$

$$K^+ \to \pi^-(p_3)\pi^+(p_1)\pi^+(p_2)$$

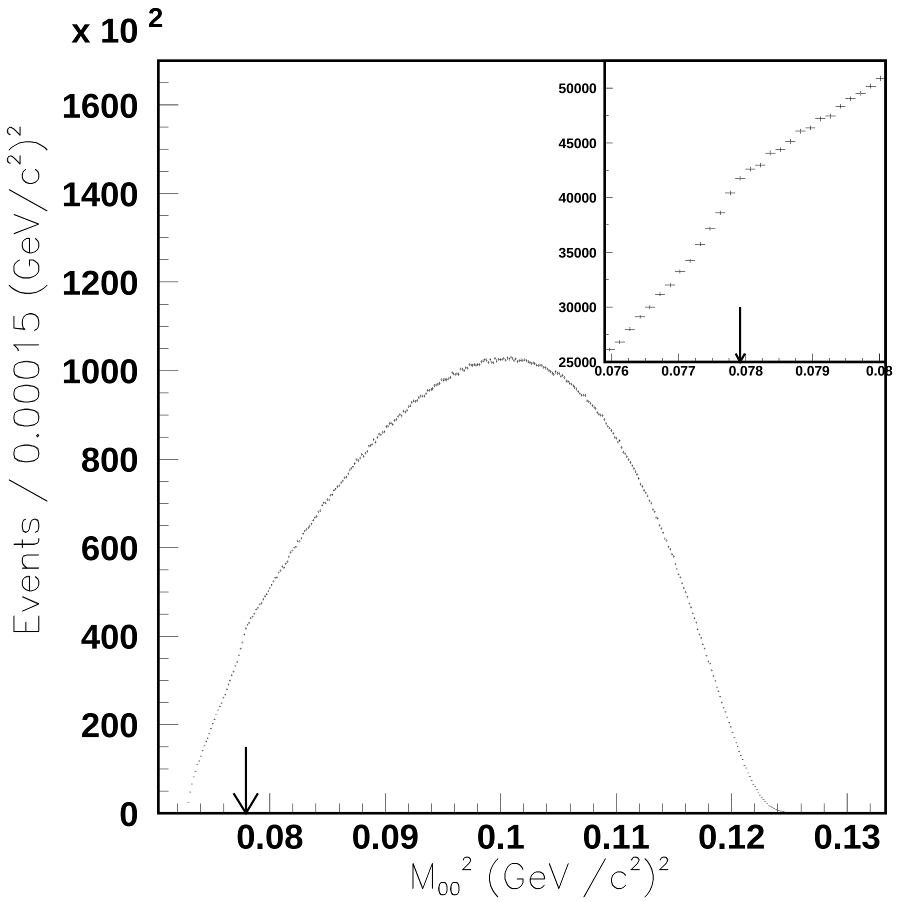


"We see, therefore, that if a detailed measurement of the π^+ distribution is performed around the $\pi^+\pi^-$ threshold, one can determine in principle, with the use of this equation, the pion-pion charge exchange cross section at zero energy."

14 years after discovery of the pion !!

Not enough data available in 1961. Their work was then forgotten.

The cusp in $K^+ \to \pi^+\pi^0\pi^0$ decays (NA48/2)



$K^+ \to \pi^+\pi^0\pi^0$

Partial sample of $\sim 2.3 \cdot 10^7$ decays

J. R. Batley *et al.* [NA48/2 Collaboration], PLB 633 (2006) 173

Still 1960 - 1970

- $\pi^+\pi^-$ bound states do exist (pionium). Decay into $2\pi^0$:

$$\Gamma_{\pi^0\pi^0} = C(a_0 - a_2)^2$$

Uretsky, Palfrey 1961

- Final state interactions in $K \to \pi\pi e\nu$ allow one to measure $\delta_0^0 - \delta_1^1$

Shabalin 1963; Cabibbo and Maksymovicz 1965

- Current algebra

$$a_0 = 7M_\pi^2/32\pi F_\pi^2 = 0.159$$

"...very much smaller than every one had thought." Weinberg 1966

"Greatest defeat of S-matrix theory" Weinberg 1997

1970 - 1990

- Roy equations
  Integral equations for partial waves. $a_{0,2}$ subtraction constants

  Roy 1971

- Solving Roy equations. Large uncertainties in a_0, a_2 from fit to data

  Early work reviewed by Pennington 1975

  Petersen CERN Yellow Report 1977

- $\pi\pi$ amplitude to one loop, in chiral limit

  Lehmann 1972

- $\pi\pi$ scattering: “Covariant chiral perturbation theory and chiral superpropagators”

  Ecker, Honerkamp 1972,1973

- Quantum Chromodynamics

  Fritzsch, Gell-Mann, Leutwyler 1973

- 30 000 K_{e4} decays at CERN PS

  $a_0 = 0.28 \pm 0.05$ Rosselet et al. 1977

  $a_0 = 0.26 \pm 0.05$ Froggatt and Peterson, using Roy eqns. 1977

  See, however, B. Bloch-Devaux, PANIC 2008

  “Somewhat above the original Weinberg prediction, but it appears that this prediction can be revised without any fundamental change in current algebra.” Rosselet et al. 1977

  Interest in Roy equations wanes

- Low energy effective theory for QCD (ChPT)

  Weinberg 1979; J.G., Leutwyler 1984

- $\pi\pi$ amplitude $A(s,t,u)$ to one loop. LECs $\bar{l}_{1,2,3,4}$

  $a_0 = 0.20 \pm 0.01$ J.G., Leutwyler 1984

- How can one calculate the $\pi\pi$ scattering lengths in lattice QCD?

  Lüscher 1986

1990 - 2005

- Does the quark condensate vanish? $\Leftrightarrow$ large a_0
  Generalized ChPT — Fuchs, Sazdjian, Stern 1991
- Proposal to measure $a_0 - a_2$ with pionium decay at CERN
  DIRAC collaboration 1994
- S-matrix calculation of $A(s,t,u)$ at order p^6
  Knecht, Moussallam, Stern, Fuchs 1995
- $A(s,t,u)$ to two loops — Bijnens et al. 1996
- Cusp in $\pi^0\pi^0 \to \pi^0\pi^0$ — Meißner, Müller, Steininger 1997
- Revival of Roy equations: given a_0, a_2, the scattering amplitude can be calculated rather precisely
  Ananthanarayan, Colangelo, J.G., Leutwyler 2001
  Descotes, Fuchs, Girlanda, Stern 2002

- More accurate method: Roy+ChPT

$$a_0 = 0.220 \pm 0.005\,,\; a_0 - a_2 = 0.265 \pm 0.004$$

and more threshold parameters a_ℓ^I, b_ℓ^I. Colangelo, J.G., Leutwyler 2001
Confirms the results of Amoros, Bijnens, Talavera [ChPT] 2000, with smaller uncertainties

- Other determinations

	[1]	[2]	[3]
a_0	0.228 ± 0.032	0.224 ± 0.013	0.230 ± 0.015
$-10 \cdot a_2$	0.382 ± 0.038	0.343 ± 0.036	0.480 ± 0.046

S. Descotes-Genon, N. H. Fuchs, L. Girlanda and J. Stern 2002 [1]

R. Kaminski, L. Lesniak and B. Loiseau 2003 [2]

J. R. Peláez and F. J. Yndur´ain 2005 [3]

See talk Colangelo at KAON07 for comments

- Experimental verification at Brookhaven: more than 400 000 K_{e4} decays

$$a_0 = 0.216 \pm 0.013 \pm 0.002 \pm 0.002$$

E865 collaboration 2001 - 2003

- Proposal to measure a_0, a_2 to percent precision from lifetime and energy levels of pionium

  Addendum to DIRAC proposal 2004; Nemenov, talk given at PS and SPS meeting in Villars, Sep. 25, 2004

- Cusp analysis by NA48/2 collaboration
  $2.3 \cdot 10^7 K^\pm \to \pi^\pm \pi^0 \pi^0$ decays analysed

$$\begin{aligned} a_0 - a_2 &= 0.268 \pm 0.010_{\rm stat} \pm 0.004_{\rm syst} \pm 0.013_{\rm ext} \\ a_2 &= -0.041 \pm 0.022_{\rm stat} \pm 0.014_{\rm syst} \end{aligned}$$

NA48/2 2005

Theory: Cabibbo, Isidori; Gamiz, Prades, Scimemi; Bissegger, Colangelo, Fuhrer, J.G., Kubis, Rusetsky;. . . >2004

See also talk by Zdrahal, this conference

- Theory of hadronic atoms in the framework of QFT

  Eiras, Gall, J.G., Ivanov, Jallouli, Kong, Labelle, Lipartia, Lyubovitskij, Raha, Ravndal, Rusetsky, Schweizer, Sazdjian, Soto, Zemp, . . . after 1996

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^*(a_0 - a_2)^2(1+\epsilon)\,;\,\epsilon = (5.8 \pm 1.2)\cdot 10^{-2}$$

- Theretical prediction for lifetime of $\pi^+\pi^-$ atom for decay into $\pi^0\pi^0$:

$$\tau_{2\pi^0} = (2.9 \pm 0.1)\cdot 10^{-15}\ \text{sec}$$

Gall, J.G., Lyubovitskij, Rusetsky 2001

Experiment:

$$\tau = (2.91^{+0.49}_{-0.62})\cdot 10^{-15}\ \text{sec}$$

DIRAC collaboration 2005

3 beautiful experimental confirmations of ChPT prediction!

More

- $a_{0,2}$ known $\Rightarrow$ partial waves precisely known below 800 MeV
- Precise determination of σ, ρ, f_0-poles from first principles
- Inverse amplitude method
- Unitarization of $\pi\pi$ amplitudes
- Estimates of uncertainties in $\pi\pi$ amplitude at order p^6
- Forward dispersion relations
- Once-subtracted Roy equations
- Resolving the “up-down” ambiguity in data analysis
- Quark mass and N_C dependence of poles on second Riemann sheet
- $\cdots$

Caprini, Colangelo, Dobado, Garcia Martin, J.G., Gomez-Nicola, Grynkiewicz, Hanhardt, Kaminski, Lesniak, Leutwyler, Loiseau, Moussallam, Peláez, Ruiz Arriola, Wang, Ynduráin, Zhou, Zheng,. . .

See also talks by J. Sanz Cillero, Rios Márquez, Garcia Martin, José Peláez

2006: the NA48/2 analysis

Large amount of $K \to \pi\pi l \nu_\ell$ data collected by NA48/2 collaboration in 2003 - 2004

2006: QCD conference in Montpellier

$$a_0 = \begin{cases} 0.253 \pm 0.036 & \text{Geneva-Saclay} \quad 30000 K_{e4} \\ 0.256 \pm 0.011 & \text{NA48/2} \quad 370000 K_{e4} \end{cases}$$

B. Bloch-Devaux for NA48/2, QCD2006

Several reports of NA48/2 at several conferences

Revival of small condensate scenario?

PARADISE and REAL WORLD

The paradise world

Predictions for $a_{0,2}$ are made in QCD (6 flavours), with

$$m_u = m_d = m, m_s, \Lambda_{\text{QCD}},$$

chosen such that

$$M_\pi = 139.6\,\text{MeV}\ ,\ M_K = 493.6\,\text{MeV}\ ,\ \ F_\pi = 92.4\,\text{MeV}$$

Precise values of heavy quark masses are irrelevant here
No photons: $F_{\mu\nu} = 0$

Lattice calculations of $a_{0,2}$ can also be done in this framework ($N_f = 2, 3$).

Experiments

Experiments are performed in the real world, described by the Standard Model

$$\alpha \neq 0,\ m_u \neq m_d$$

Then:

$K^+ \not\to \pi^+\pi^0\pi^0$
$K^+ \not\to \pi^+\pi^-e^+\nu_e$

ZERO probability that these processes occur in the laboratory

Bloch, Nordsieck 1937

$\Rightarrow$ Need a careful analysis of the situation

$\Rightarrow$ See also talk by H. Neufeld, this workshop

Phases in K_{e4}

The decay amplitude contains the axial current matrix element

$$\langle \pi^+ \pi^- | A_\mu | K^+ \rangle = \frac{1}{iM_K} [P_\mu F + Q_\mu G + L_\mu R]$$

with

$$F = f_s \exp i\delta_0^0 + f_p \exp i\delta_1^1 + D - \text{waves}$$

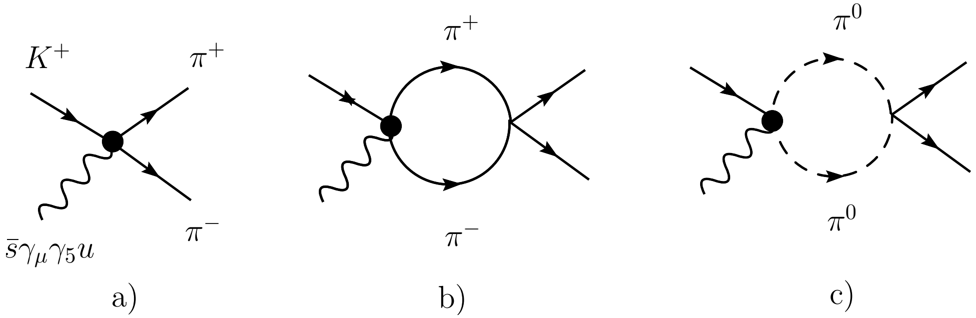
This is Watson's theorem, true in the isospin symmetry limit. Allows one to measure the phase difference $\delta_0^0 - \delta_1^1$ and thus to determine the scattering lengths:

$a_{0,2}$ known $\Rightarrow$ partial waves precisely known below 800 MeV via solutions of Roy equations.

Which phase is measured?

$\underline{m_u = m_d; e = 0}$

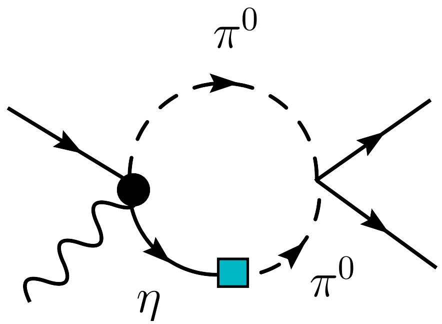
Lowest order:



$\rightarrow \delta_0^0$

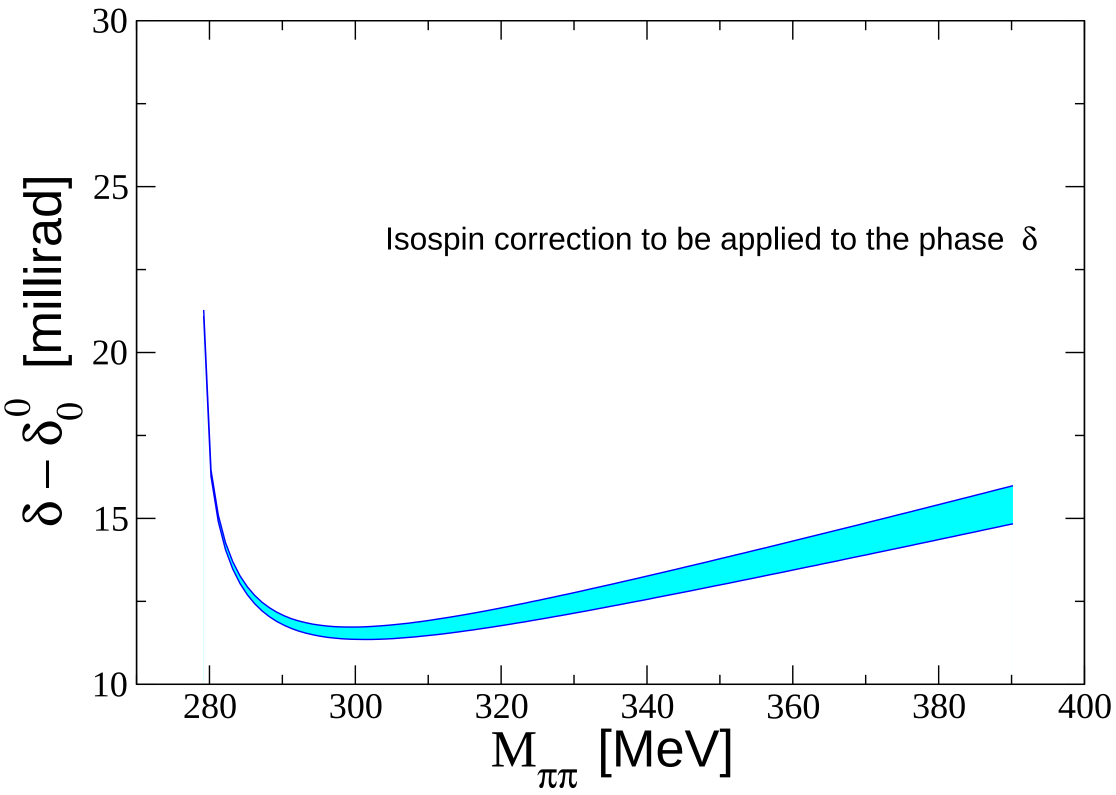
$\underline{m_u \neq m_d; e \neq 0}$

Analytic structure of diagrams change. Additional diagram:



Phase is changed: $\delta_0^0 \rightarrow \delta$. Experiment measures δ. Effect is large.

The missing piece



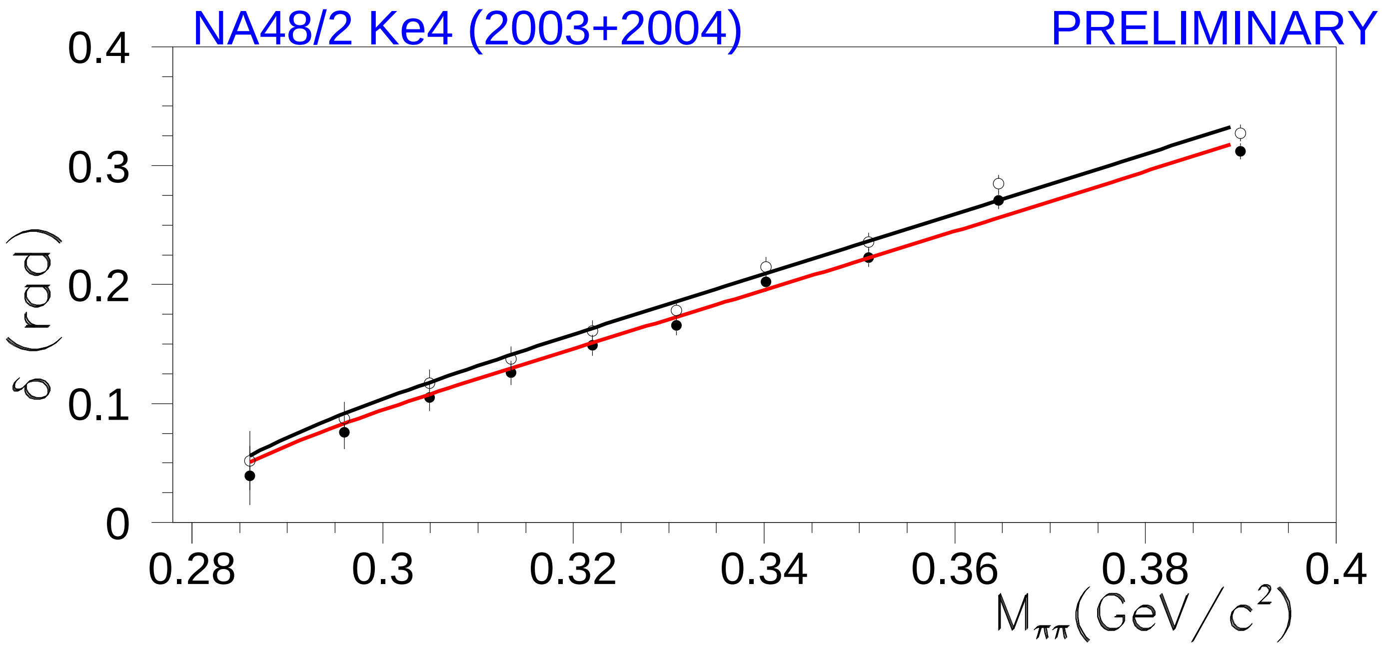
This part must be subtracted from the measured phase before a comparison with the prediction can be made

J.G. KAON07

Colangelo, J.G., Rusetsky 2008

see also Gevorkyan et al. 2007; Descotes, Knecht, FlaviaNet Meeting, Capri 2008

The effect I

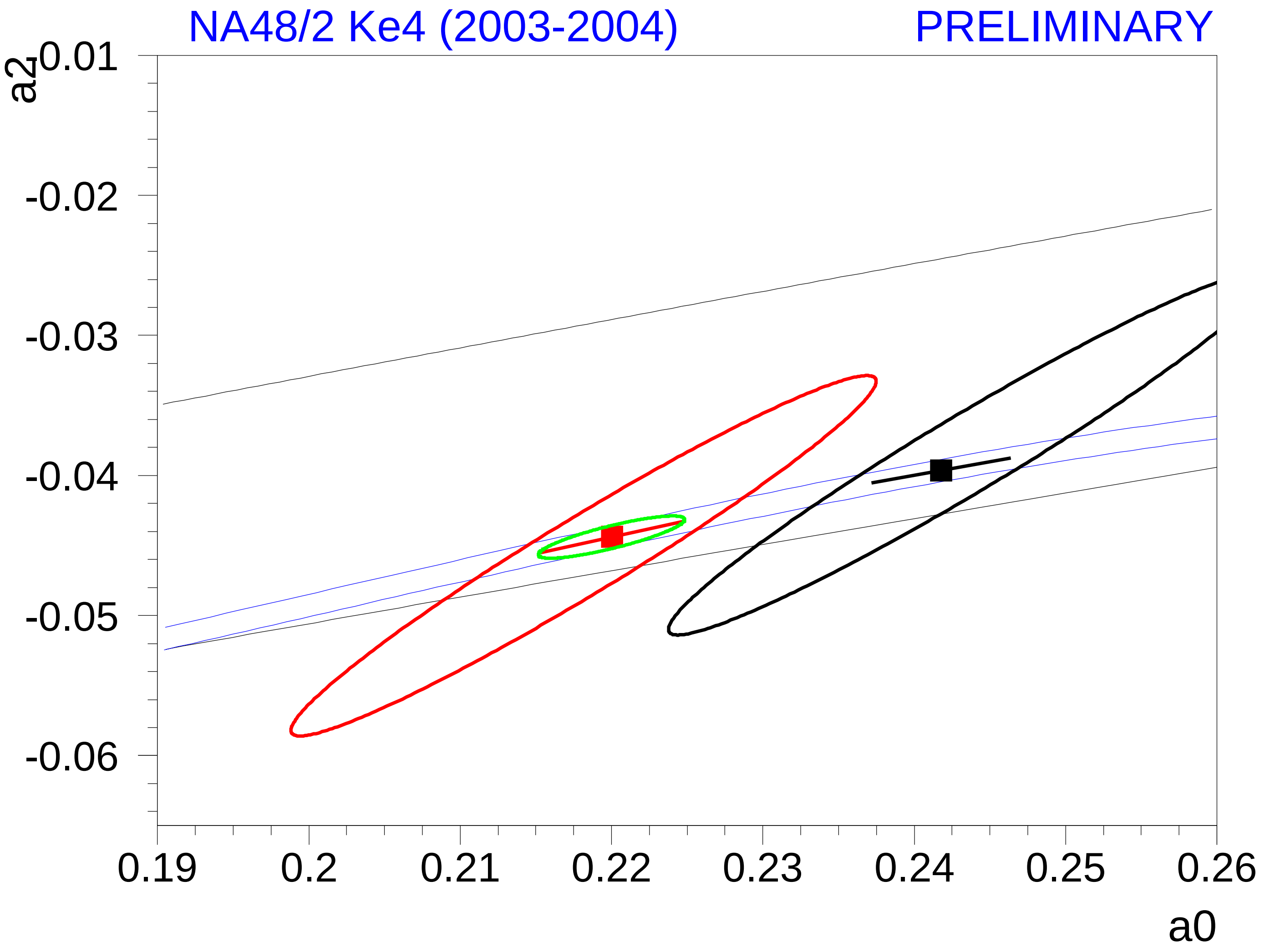


black: measured phase

red: phase in paradise world

Figure courtesy of Brigitte Bloch-Devaux

The effect II



red: paradise world black: $a_{0,2}$ from measured phase

Figure courtesy of Brigitte Bloch-Devaux

Summary and Comparison with other experimental measurements

Ke4 : apply isospin corrections to published phase points of all experiments and perform a_0 ChPT fit

Note : E865 number dominated by highest energy data point, otherwise compatible

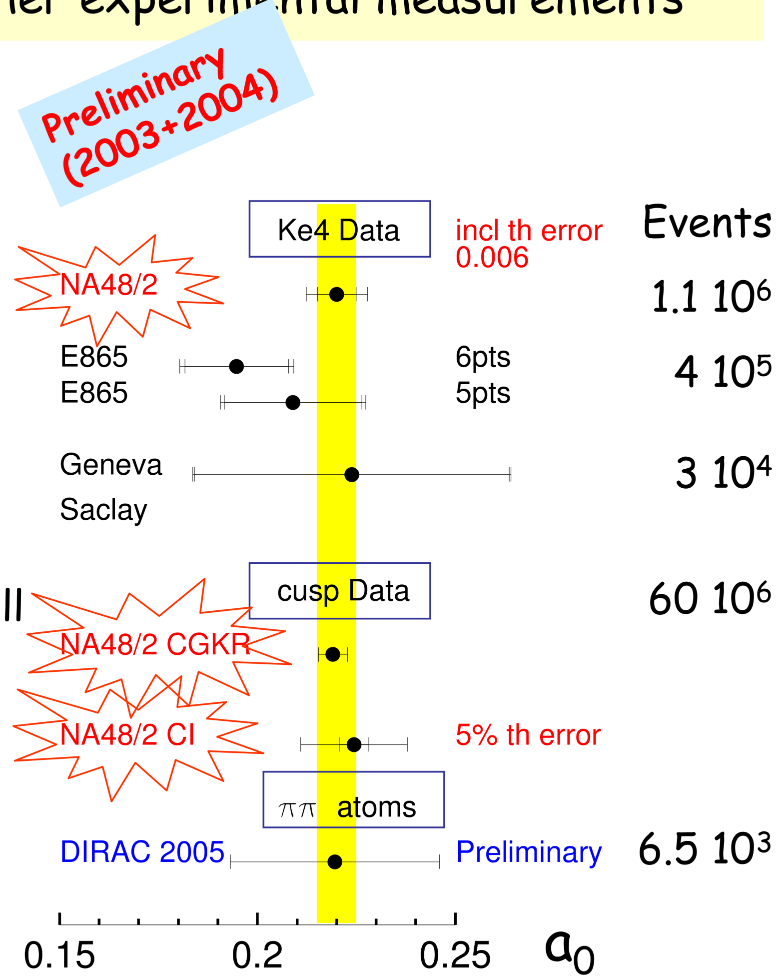
Cusp : (a_0-a_2) ChPT fit with 2 models

$\pi\pi$ atoms DIRAC: $|a_0-a_2|$ errors from PLB619 (2005), use ChPT constraint (still being revisited + more Data analyzed)

Yellow band is ChPT prediction

$a_0 = 0.220 \pm 0.005$

NA48/2 experimental precision now at the same level !



Final publications coming soon, fruitful collaboration with theory groups

Taken from B. Bloch-Devaux, Panic 2008

Including lattice calculations

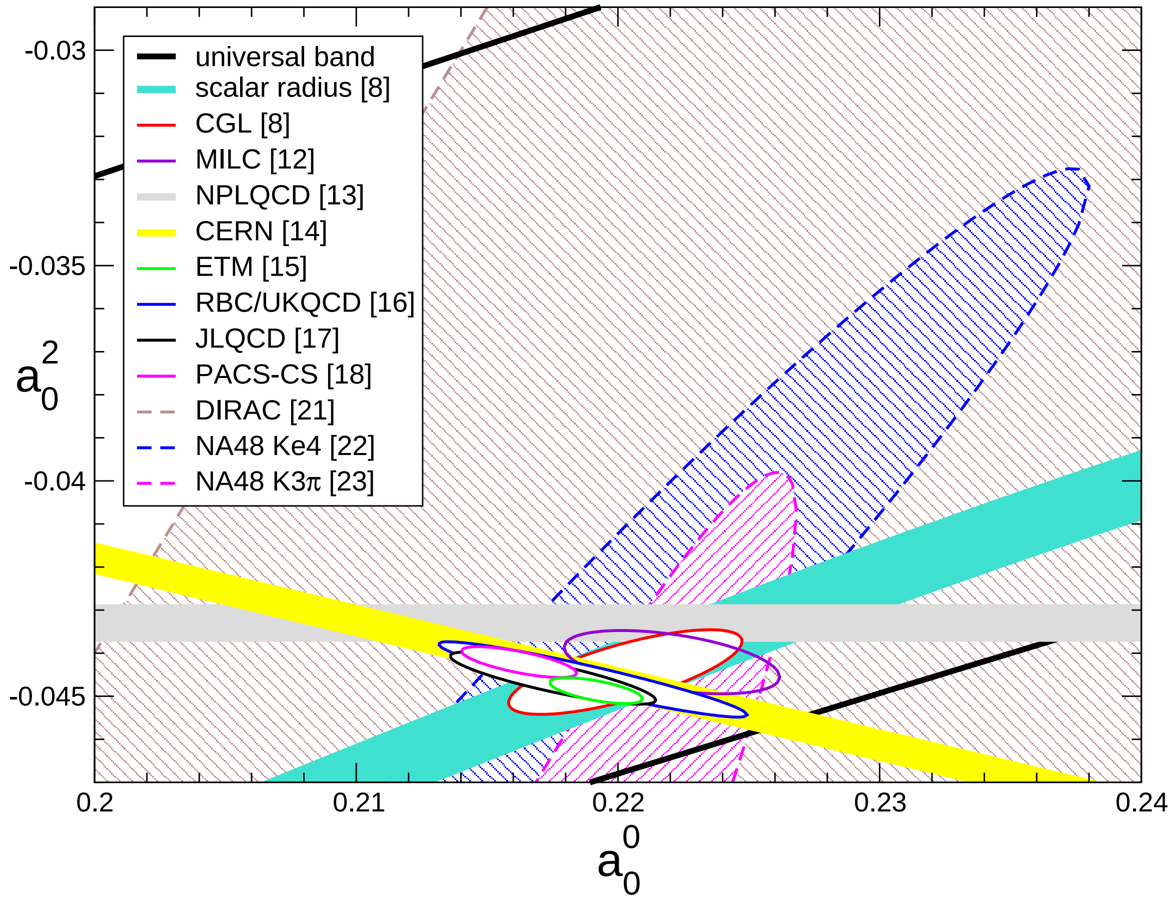


Figure taken from H. Leutwyler, arXiv:0812.4165 [hep-ph]

SUMMARY

- In the last years 10-20 years: tremendous progress in our understanding of the $\pi\pi$ system at low energies, both, theoretically and experimentally.
- Some care is needed when translating experimental data into the paradise world.
- Beautiful agreement between theory and experiment for the S-wave scattering lengths, in the standard picture.
- S, P – waves now known in the whole low-energy region, with high precision.
  $\Rightarrow$ Poles on second Riemann sheet from first principles.
- Lattice calculations have reached a precision that allows one to match part of the low–energy effective Lagrangian of QCD to the underlying theory.

EXTRAS


6th International Workshop on Chiral Dynamics
University of Bern, Switzerland, July 6-10, 2009
FNSNF
Swiss National Science Foundation
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b UNIVERSITÄT BERN
Topics
Goldstone Boson Dynamics
Meson-Baryon Interactions
Few-Body Physics
Lattice QCD and ChPT
Working group conveners
P.F. Bedaque (Maryland)
C. Bernard (St. Louis)
J.P. Chen (JLab)
S. Dürr (NIC)
E. Epelbaum (Jülich / Bonn)
S. Giovannella (Frascati)
B. Kubis (Bonn)
J. Portolés (IFIC)
H.R. Weller (Duke / TUNL)
Speakers
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S. Aoki (Tsukuba)
S.R. Beane (New Hampshire)
A.M. Bernstein (MIT)
J. Bijnens (Lund)
M.C. Birse (Manchester)
A. Denig (Mainz)
A. Deur (Virginia)
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QCD

Topics

- Goldstone Boson Dynamics
- Hadron Structure and Meson-Baryon Interactions
- Few-Body Physics
- Lattice QCD and Chiral Perturbation Theory

$\Rightarrow$ Invited plenary talks

$\Rightarrow$ Working Group contributions (open for every one)

Important dates (2009)

- Deadline for reserving hotel rooms at special rates: **March 31**
- Deadline for abstract submission to WG conveners: **March 31**
- Deadline for early registration: **April 30**
- Beginning of the meeting: **July 6**
- End of the meeting: **July 10**

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