Chiral Effective Field Theory on the Light Front

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Motivations

- Calculation of baryon properties
  - need a relativistic non-perturbative formulation for bound states
- Chiral effective Lagrangian expanded in momentum p
  - but any calculation of baryon properties relies on an expansion in pion fields
- Appropriate formalism based on Light Front Dynamics

  and a decomposition of the baryon state vector in Fock sectors, with Fock space truncation
- Problems to solve
  - Control of rotational invariance
  - Adequate renormalization scheme consistent with Fock space truncation
  - Appropriate regularization procedure which preserves symetries

Covariant formulation of Light Front Dynamics

➢ State vector $\phi(p)$ of any bound state defined on the light front with aribtrary position ω

$$\omega\cdot x = 0 \quad \text{with} \quad \omega^2 = 0$$

$$\omega = (1, 0, 0, -1) \quad \text{for the standard formulation of LFD}$$

➢ Explicitly covariant, and control of any violation of rotational invariance through the ω dependence of observables (in any approximate calculation)

➢ Bound state eigenvalue equation

$$\hat{P}^2\,\phi(p) = M^2\,\phi(p) \qquad \text{with} \qquad \hat{P}_\mu = \hat{P}^0_\mu + \hat{P}^{int}_\mu$$

$$\hat{P}^{int}_\mu = \omega_\mu \int H^{int}(x)\delta(\omega\cdot x)\; d^4x = \omega_\mu \int_{-\infty}^{+\infty} \tilde{H}^{int}(\omega\tau)\frac{d\tau}{2\pi}$$

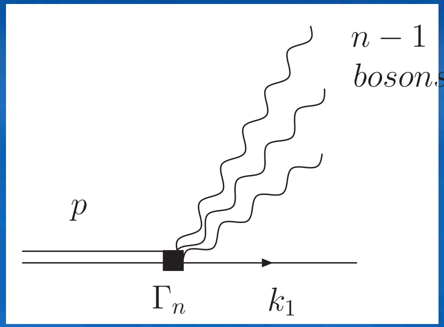
➢ **Fock state decomposition**

$$|p\rangle = |1\rangle + |2\rangle + |3\rangle + \ldots$$

➢ **Non-perturbative many-body vertices** Γ_n

for the two-body component in the Yukawa model

$$\bar{u}(p)\Gamma_2 u(k_1) = \bar{u}(p)\left[b_1 + b_2 \frac{m\not{\omega}}{\omega.p}\right] u(k_1)$$



➢ **For any truncation of the Fock space of order N, one can write down the relevant effective Lagrangian**

$$\mathcal{L}_{lfd}^{N} \equiv \mathcal{L}_{eff}^{p=2(N-1)}$$

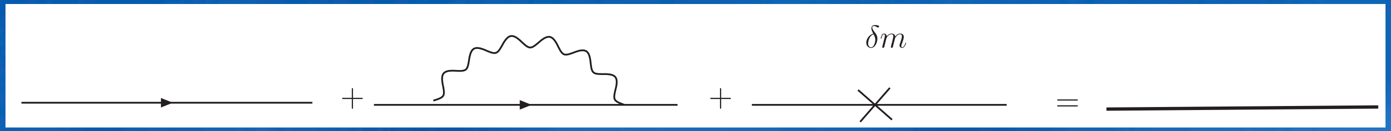
V. Karmanov, Sov. Phys. JETP 44 (210) 1976

J. Carbonell, V. Karmanov, JFM, B. Desplanques, Phys. Rep. 300 (215) 1998

Non-perturbative renormalization scheme on Light Front Dynamics

Ex. Yukawa model (or QED)

➢ In first order perturbation theory



at $p^2 = m^2$

➢ The renormalization condition couples contributions from two different Fock components

➢ One should keep track of the physical content of the counterterm as a function of the number of particles it corresponds to : $\delta m^{(2)}$ and more generally $\delta m^{(n)}$

➢ The same is true for the bare coupling constant $g_0 \longrightarrow g_0^{(n)}$

➢ This is the only way to prevent uncancelled divergences when the Fock space is truncated

- A calculation of order N involves $\delta m^{(1)} \ldots \delta m^{(N)}$

  $g_0^{(1)} \ldots g_0^{(N)}$

- $\delta m^{(n)}$ and $g_0^{(n)}$ are calculated by successive solutions of the N=1, N=2 , N=3 ...N systems

- The explicit covariance of our formalism enables to write down immediately the structure of counterterms, which may depend on ω

V. Karmanov, JFM, S. Smirnov, Phys. Rev. D77 (2008) 085028

Test function regularization method

➢ Field operators are treated as distributions which are defined on specific test functions

$$T_x\Phi(\rho) = \langle \varphi, \rho \rangle = \int d^D \varphi(y)\rho(x-y)$$

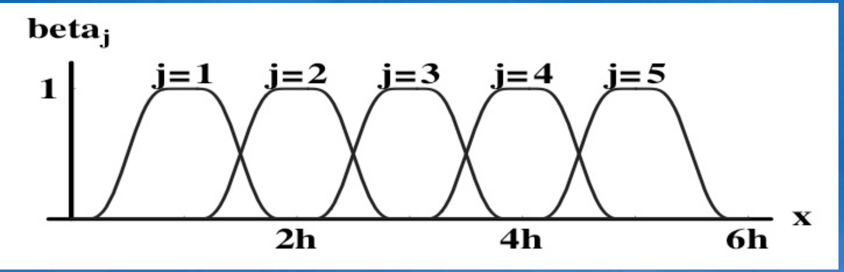
➢ Decomposition in momentum space

$$\phi(x) = \int \frac{d^{(D-1)}}{(2\pi)^{(D-1)}} \frac{f(\omega_p^2, \vec{p}^2)}{2\omega_p} \left[a_p^+ e^{ipx} + a_p e^{-ipx}\right]$$

➢ Adequate choice of test fuctions

- partition of unity: observables should be independent of the choice of test functions

$$f(x) = \sum_{j=0}^{N-1} u(x - jh)$$



- Super regular test functions with all their derivatives equal to zero at the boundaries to treat all types of singularities at once

- Scaling properties provided by the boundary condition

In the UV domain

$f(X) = 0$ at $X = 1 + h$ with $h(X) = \mu^2 X^\alpha - 1$

the limit $f \to 1$ corresponds to $\alpha \to 1^-$

Scale

- Using the Lagrange formula

$$f(X) = -\frac{X}{k!} \int_1^\infty \frac{dt}{t} (1-t)^k \partial_X^{(k+1)} \left[X^k f(Xt) \right]$$

one can define the extension of any distribution

$\langle T, f \rangle \equiv \langle \tilde{T}, f \rangle \to \langle \tilde{T}, 1 \rangle$ by partial integration. $\tilde{T}$ is finite

- Direct relation to BPHZ scheme

H. Epstein , V. Glaser, Ann. Inst. Henri Poincaré, XIX A (1973) 211

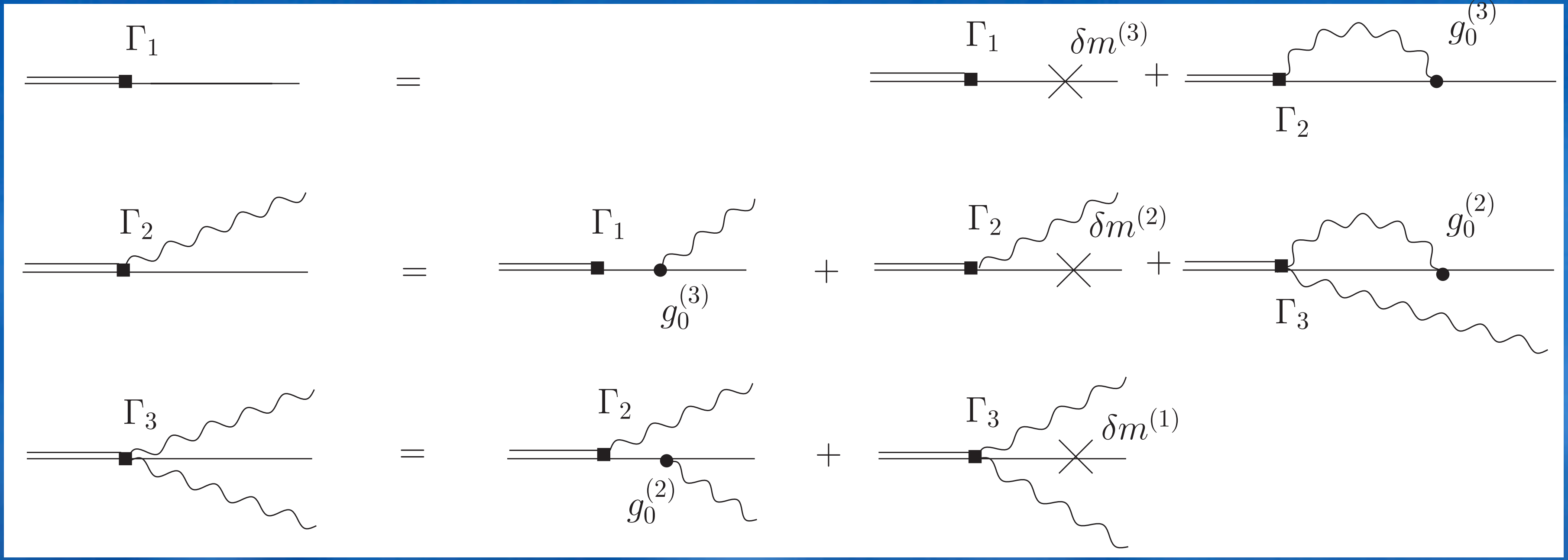
P. Grangé, E. Werner, Nucl. Phys. B, Proc. Supp. 161 (2006) 75
arXiv: math-ph/0612011

First application to the Yukawa model

Scalar bosons coupled to one fermion

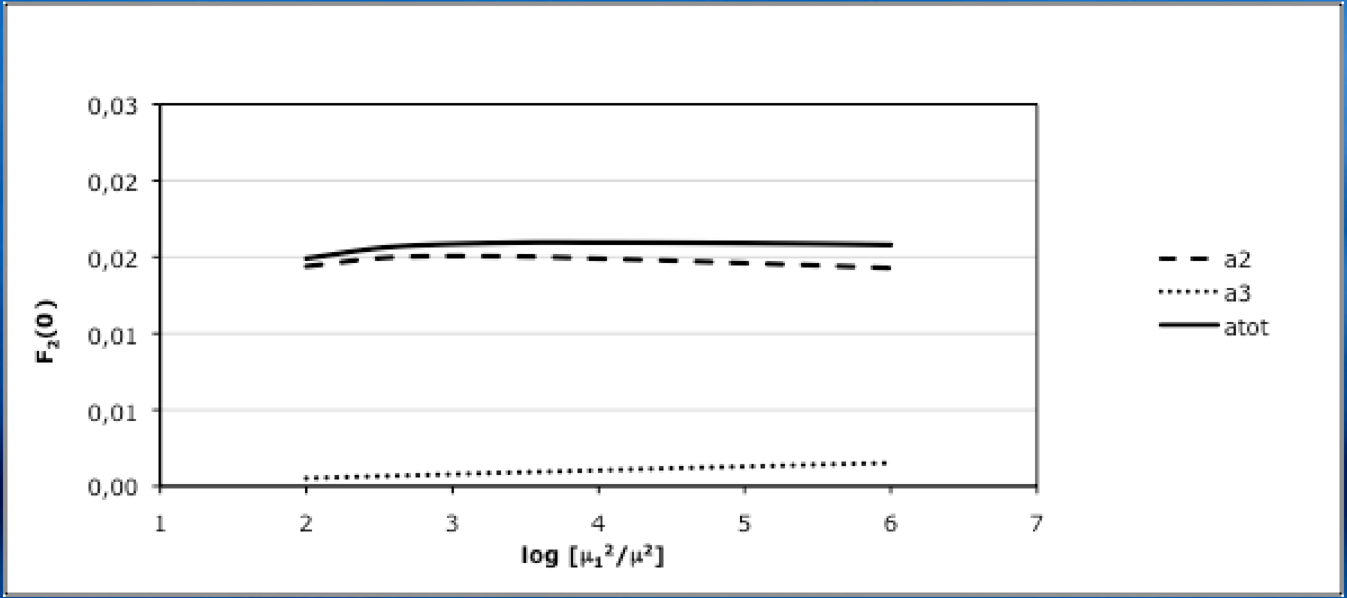
➢ In the non trivial Fock space truncation N=3 (up to two bosons "in flight")

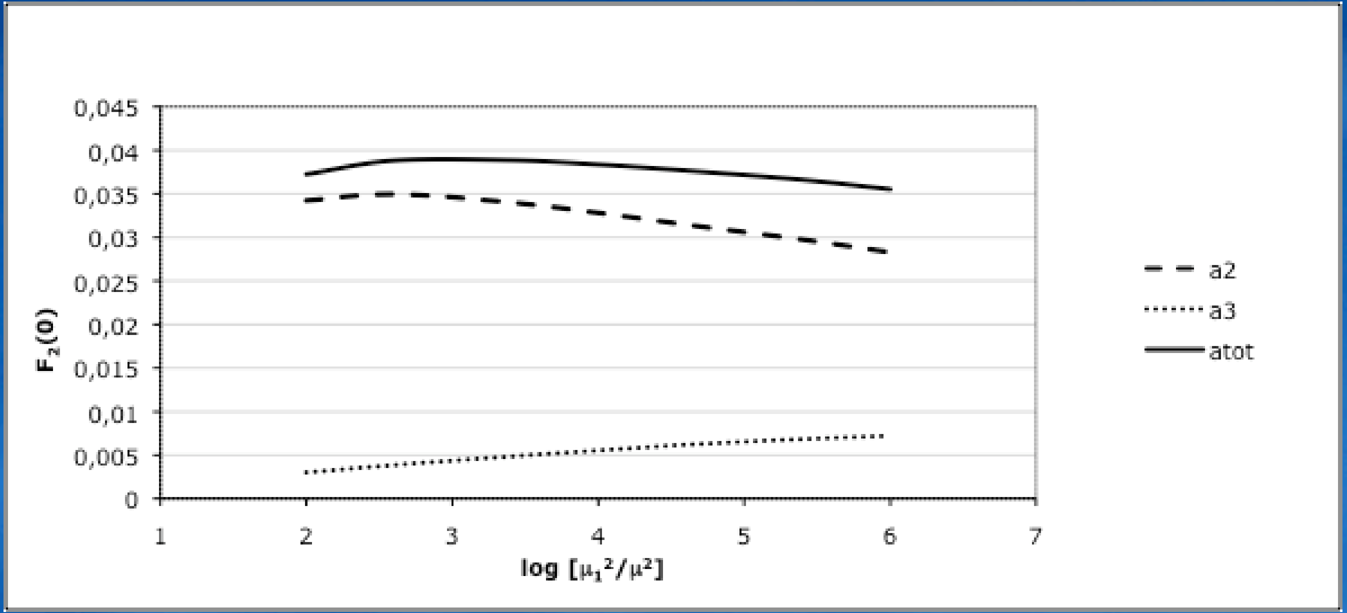
➢ Eigenvalue equation



➢ Regularization scheme used here : Pauli-Villars

Anomalous magnetic moment





$m = 1$ GeV

$\mu = 1$ GeV

$\alpha = 0.2$

$\alpha = 0.5$

➢ **Still residual dependence on the mass of the Pauli-Villars boson** μ_1 **to be cured by appropriate counterterms**

Perspectives

- **Coherent scheme**

  in order to describe both the effective Lagrangian and the state vector of any bound state

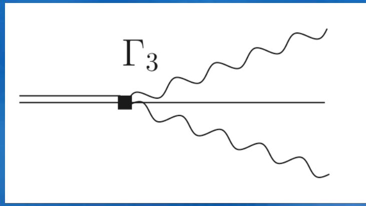
- **Systematic relativistic and non-perturbative framework**

- **Expansion in Fock components, with truncation**

  Should be valid at low energies: Fock components with high number of particles correspond to short time fluctuations

- **All πN states considered in a consistent way**

  Resonances like the Δ (1232 MeV) or the Roper (1440 MeV) resonances are already taken into account at the N=3 Fock state truncation



Δ

Δ , R degrees of freedom