A Numerical Study of Oscillations of Highly Magnetized Non-rotating Axisymmetric Neutron Stars

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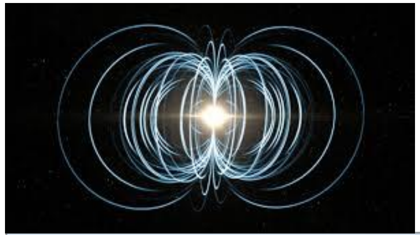
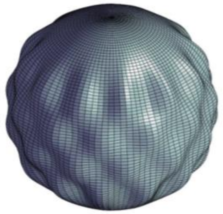
Introduction

Neutron stars (NSs)





Magnetars:

→ Extreme magnetic fields (10^{15-16} G)

→ Soft gamma repeaters, X-ray pulsars…

Oscillations under perturbation:

→ Stellar composition, internal structure, equation of state (EoS)

→ Possible sources of gravitational waves

Oscillations of highly magnetized NSs

*Figures from Google

Introduction

Literature:

✓Newtonian approach [1-8]

✓General relativistic approach [9-16]:

- ✓Cowling approximation: Evolving stellar matter but not spacetime metric
- ✗ Expected field strengths of magnetars in non-Cowling regime

Our numerical study:

→ Highly magnetized NSs with fields $\sim 10^{15-17}$ G

→ In ideal GRMHD without Cowling approximation

Compute equilibrium stellar models → Perform dynamical oscillation simulations → Apply Fourier analysis to study magnetization effects on eigenfrequencies of oscillation modes

Equilibrium models by XNS code

XNS code [17]:

→ Relativistic equilibrium models of highly magnetized non-rotating axisymmetric NSs (purely toroidal fields)

→ Assumptions:

1. Axisymmetric and stationary spacetime, conformal flatness condition (CFC) approximation:
   $\mathrm{d}s^2 = -\alpha^2 \mathrm{d}t^2 + \psi^4(\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \mathrm{d}\phi^2)$,
   where $\alpha(r,\theta)$=lapse function, $\psi(r,\theta)$=conformal factor
2. Polytropic EoS for stellar fluid:
   $p = K\rho^\gamma$,
   where K=polytropic constant, γ=polytropic index
3. Polytropic expression for toroidal field:
   $B_\phi = \alpha^{-1} K_\mathrm{m} (\rho h \varpi^2)^m$,
   where K_m=toroidal magnetization constant, h=specific enthalpy,
   $\varpi^2 = \alpha^2 \psi^4 r^2 \sin^2\theta$, m=toroidal magnetization index

Equilibrium models by XNS code

Total 12 models:

→ All with $K = 1.6 \times 10^5\ \mathrm{cm^5 g^{-1} s^{-2}}$, $\gamma = 2$, $M_0 = 1.68\ M_\odot$

Non-magnetized reference model

	ρ_c (10^{14} g cm^{-3})	M ($M_\odot$)	R_{circ} (km)	r_e (km)	r_p/r_e	$\mathscr{H}/\mathscr{W}$	B_{max} (10^{17} G)
REF	8.56	1.55	14.25	11.85	1.00	0.00	0.00
T1K1	8.56	1.55	14.25	11.85	1.00	3.97×10^{-6}	3.45×10^{-2}
T1K2	8.56	1.55	14.25	11.85	1.00	1.58×10^{-5}	6.89×10^{-2}
T1K3	8.57	1.55	14.25	11.85	1.00	3.95×10^{-4}	3.44×10^{-1}
T1K4	8.63	1.55	14.32	11.92	1.01	6.21×10^{-3}	1.36
T1K5	8.81	1.56	14.54	12.15	1.02	2.35×10^{-2}	2.63
T1K6	9.10	1.58	16.79	14.43	1.09	1.23×10^{-1}	5.52
T1K7	8.81	1.59	18.55	16.21	1.12	1.69×10^{-1}	6.01
T1K8	8.27	1.60	20.97	18.64	1.15	2.14×10^{-1}	6.14
T1K9	7.53	1.61	24.28	21.97	1.17	2.58×10^{-1}	5.96
T1K10	6.64	1.62	28.92	26.62	1.21	3.02×10^{-1}	5.53
T1K11	5.69	1.63	35.48	33.19	1.24	3.44×10^{-1}	4.93

11 magnetized models with $m = 1$ but different K_m

(ρ_c=central density, M=gravitational mass, R_{circ}=circumferential radius, r_e=equatorial radius, r_p=polar radius, $\mathscr{H}$=magnetic energy, $\mathscr{W}$=binding energy, B_{max}=maximum field inside the star)

Oscillation simulations by `Gmunu` code

`Gmunu` code [18, 19]:

→ Initially perturb and dynamically evolve our equilibrium models without Cowling approximation

→ 3 different initial fluid perturbation functions to excite oscillations [20]:

1. The $l = 0$ perturbation: $\delta v^r = a \cdot \sin\left[\pi \frac{r}{r_s(\theta)}\right]$ — $Y_{0,0}$
2. The $l = 2$ perturbation: $\delta v^\theta = a \cdot \sin\left[\pi \frac{r}{r_s(\theta)}\right] \sin\theta\cos\theta$ — $Y_{2,-1}$
3. The $l = 4$ perturbation: $\delta v^\theta = a \cdot \sin\left[\pi \frac{r}{r_s(\theta)}\right] \sin\theta\cos\theta(3 - 7\cos^2\theta)$ — $Y_{4,1}$

where a=perturbation amplitude (0.001c for $l = 0$; 0.01c for $l = 2, 4$), $r_s(\theta)$=surface of NS

→ Each equilibrium model: perform `Gmunu` simulation once for each initial perturbation

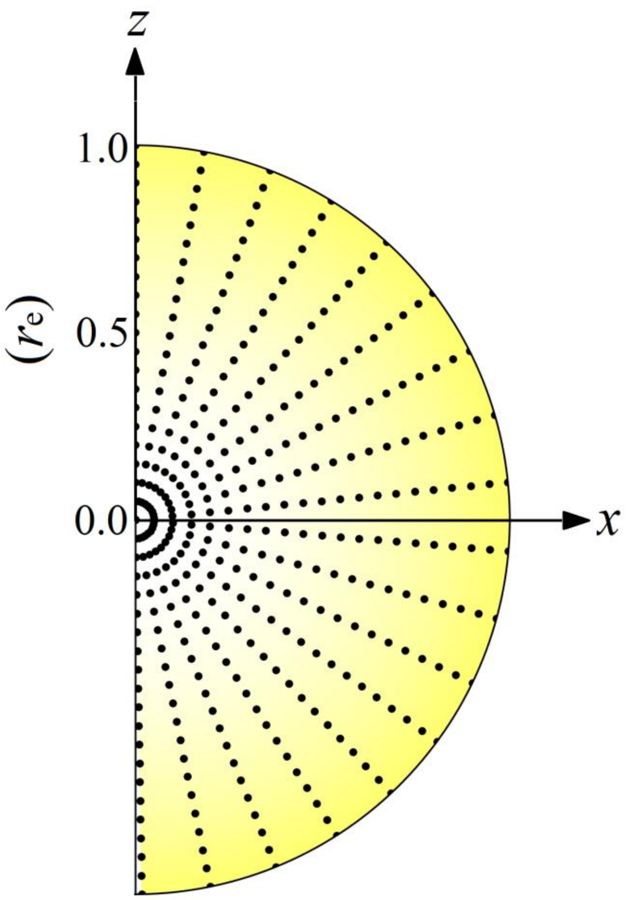
Data analysis for each Gmunu run

<u>Step 1:</u>

→ At each (r, θ)-point,

1. Extract time evolution of the initially perturbed component of three-velocity field
2. Compute Fast Fourier Transform (FFT) of temporal data
3. Obtain an FFT spectrum, plot of FFT magnitude in frequency domain

→ Perturbative regime: Overall evolution of the NS model ~ superposition of a few global oscillation modes [20]



361 spatial (r, θ)-points in the NS model

Data analysis for each Gmunu run

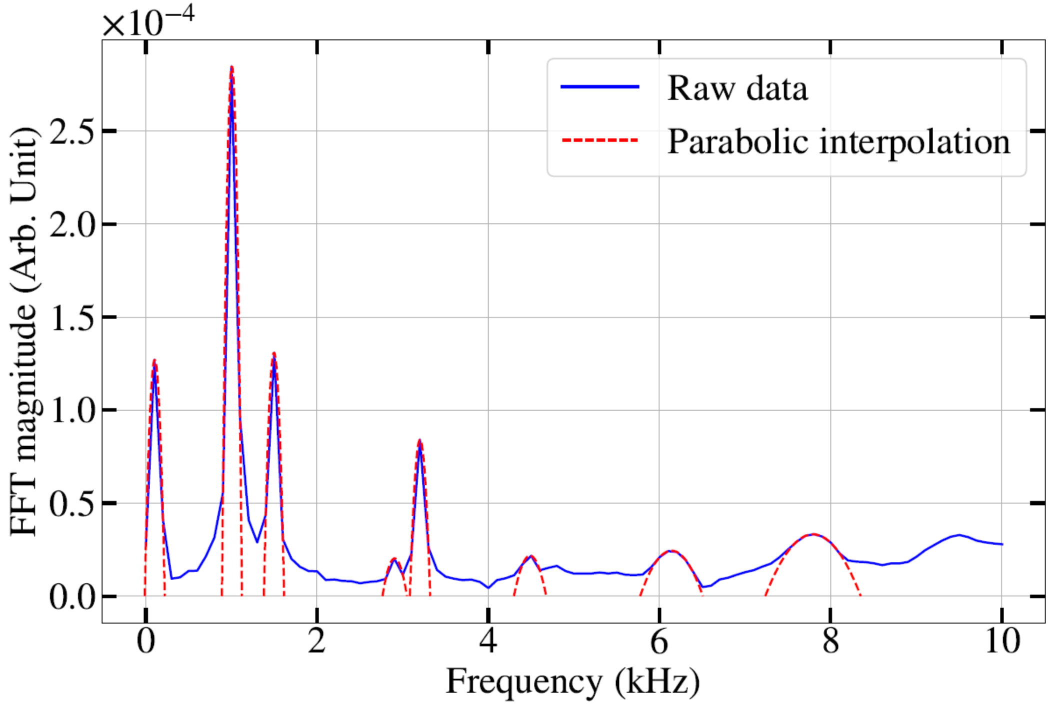
Step 2: Extract eigenfrequencies of excited oscillation modes

→ Use FFT spectrum at a single point[1] or integrated FFT spectrum along a radial line[2]

→ Apply parabolic interpolation to the FFT peaks

→ Interpolated peak position = measured eigenfrequency f_{eig}

→ Full-width-at-half-maximum (FWHM) of parabolic interpolation = Uncertainty in f_{eig}



[1] $(r, \theta) \sim \left(\frac{r_e}{2}, \frac{\pi}{2}\right), \left(\frac{r_e}{2}, \frac{\pi}{4}\right), \left(\frac{r_e}{2}, \frac{2\pi}{15}\right)$

[2] $\theta \sim \frac{\pi}{2}, \frac{\pi}{4}, \frac{2\pi}{15}$

for $l = 0$, $l = 2$, $l = 4$ perturbation respectively

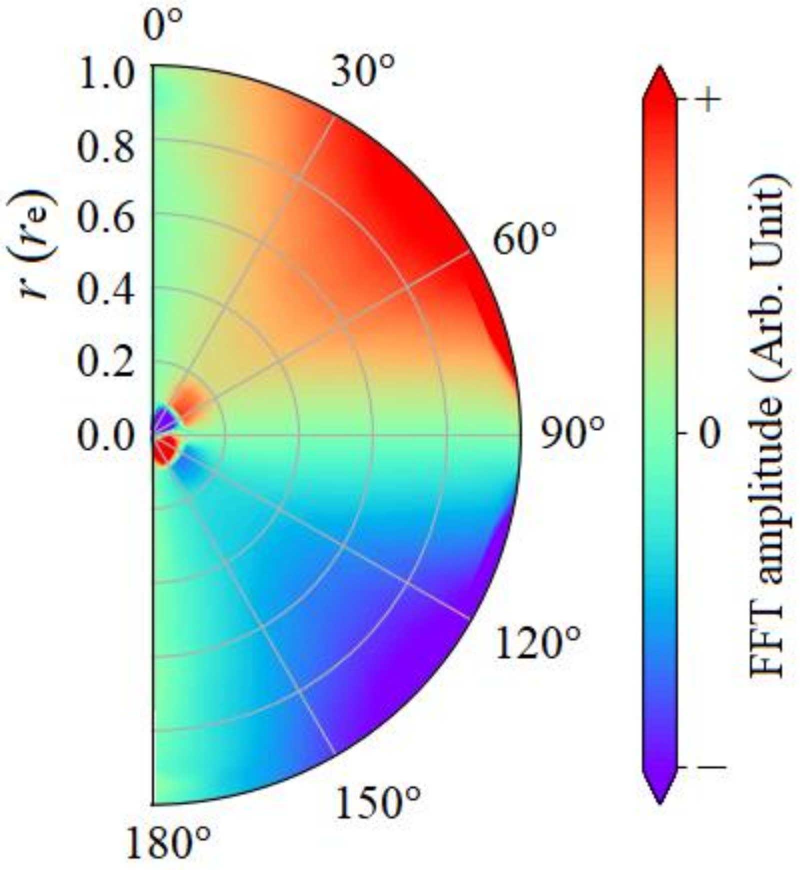
Data analysis for each `Gmunu` run

<u>Step 3: Visualize eigenfunctions of excited oscillation modes</u>

→ Eigenfunction ~ spatial map of FFT amplitude at eigenfrequency of the mode [20, 21]

→ Spatially map FFT amplitude at the frequency to which f_{eig} is the closest in the discretized frequency domain of our Fourier analysis

→ Trademark to identify the same mode excited in different `Gmunu` simulations in order to find $f_{eig}(\mathscr{H}/\mathscr{W})$



Data analysis: Very last step

After obtaining eigenfrequency trends $f_{eig}(\mathscr{H}/\mathscr{W})$ of different oscillation modes:

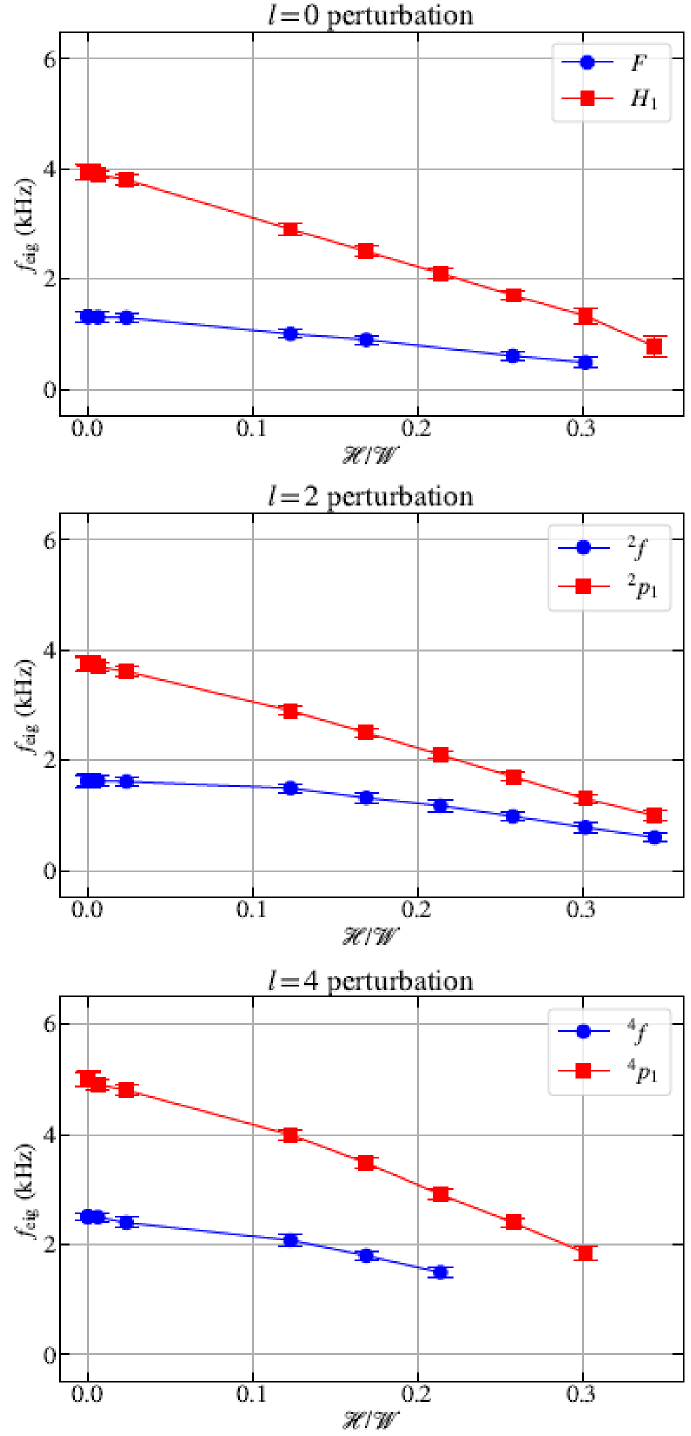
→ Compare $f_{eig}(\mathscr{H}/\mathscr{W} = 0)$ of the modes found here with mode frequencies previously reported for a non-magnetized non-rotating NS model with a similar mass [20]

→ Determine correspondence between modes found here and modes reported in literature

Model	F	H_1	${}^2f$	${}^2p_1$	i_{-2}	i_1	i_2	${}^4f$	${}^4p_1$
A0	1.458	3.971	1.586	3.726	0.000	0.000	0.000	2.440	4.896

*Adapted from Tables 2. and 6. of Ref.[20]. All frequencies are in kHz.

Analysis results

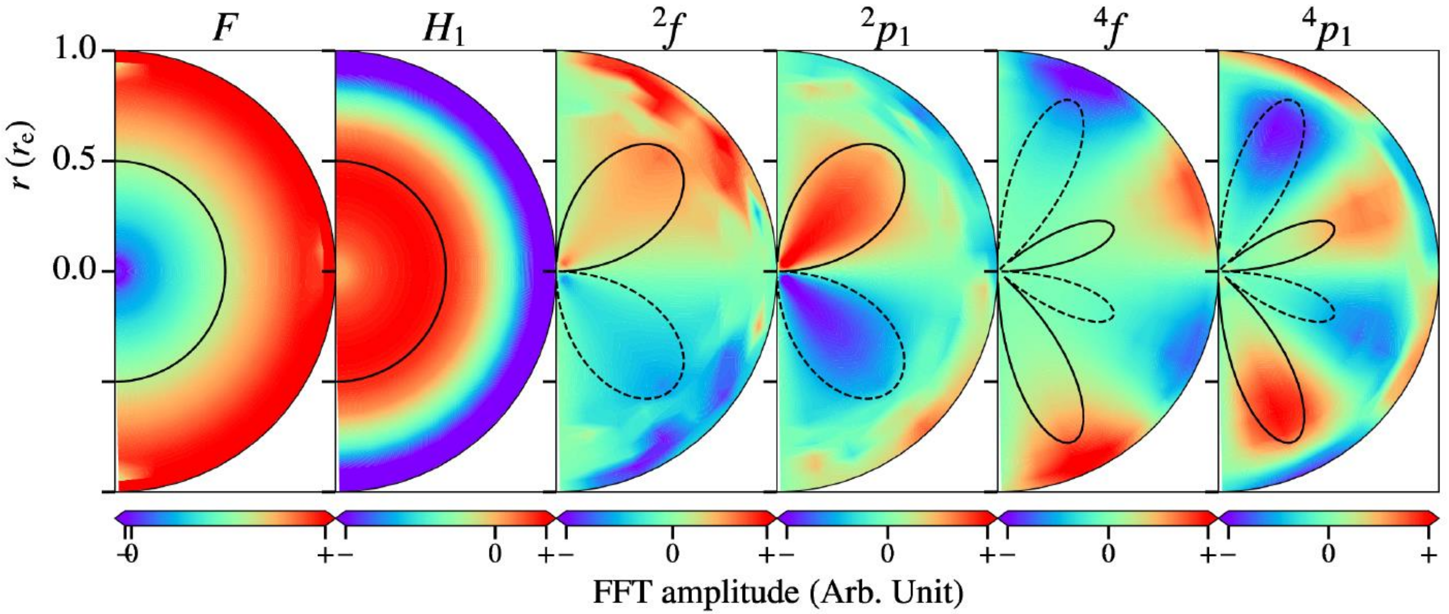


Oscillation modes found:

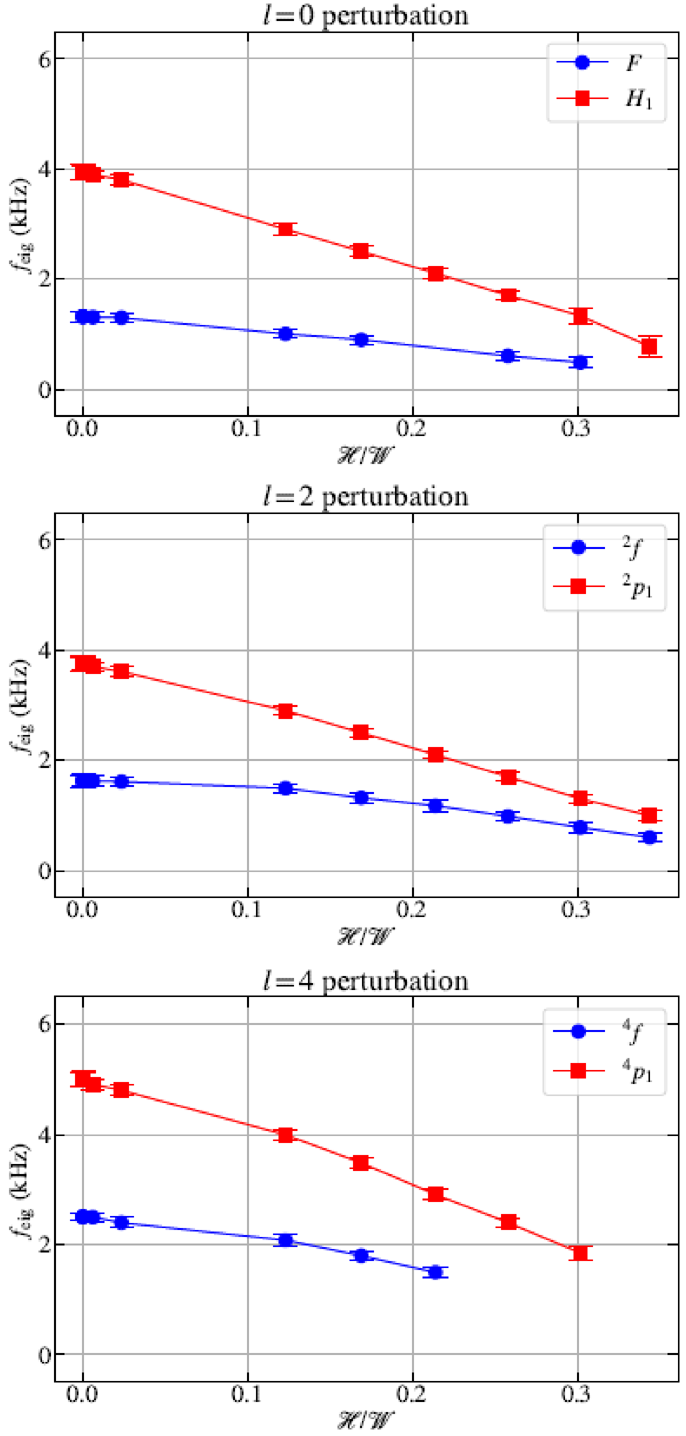
	Fundamental	Overtone
Quasi-radial ($l=0$) modes	F	H_1
Quadrupole ($l=2$) modes	^2f	$^2p_1$
Hexadecapole ($l=4$) modes	^4f	$^4p_1$

More nodes in θ-direction

More nodes in r-direction



Analysis results

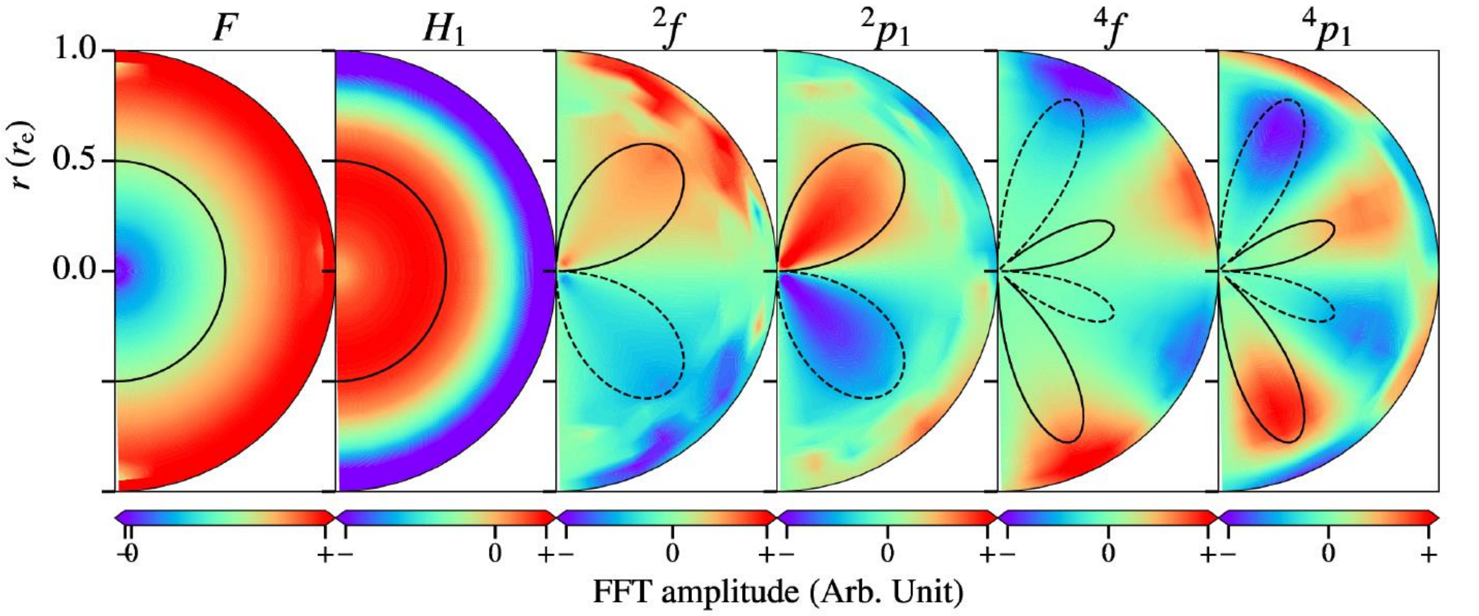


Some remarks:

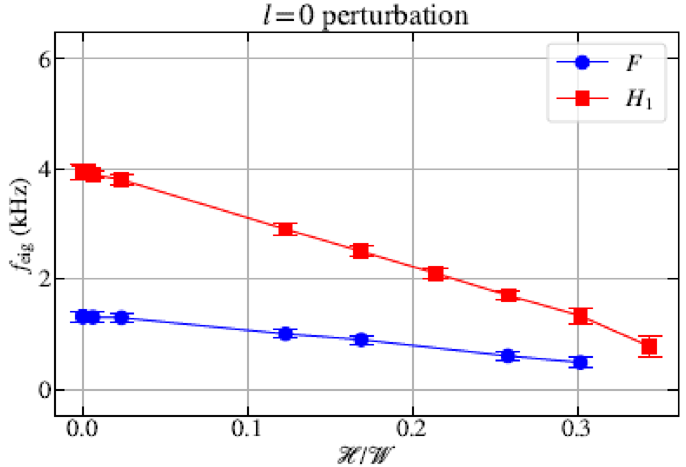
→ Each initial perturbation predominantly excites modes with the corresponding l index

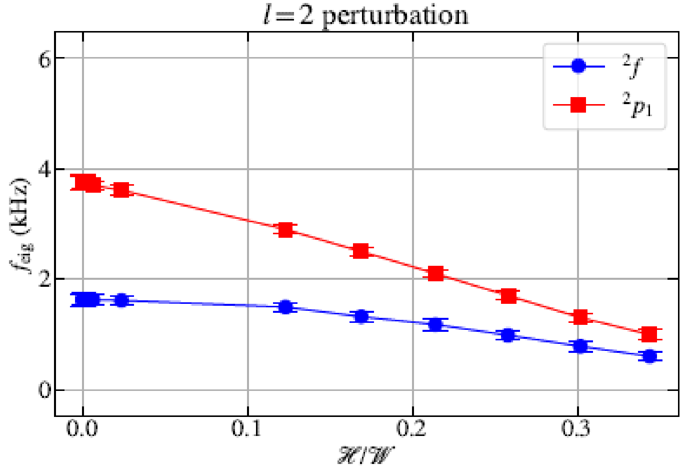
→ Missing data points:

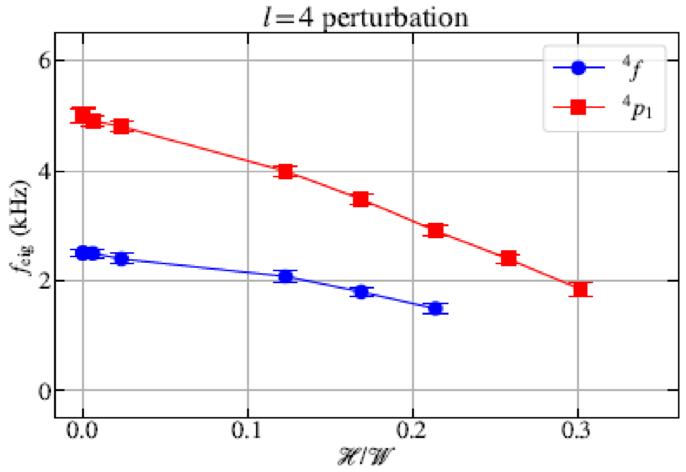
1. Curve of H_1: Unsatisfactory data quality
2. Curves of ${}^4f$ and ${}^4p_1$: Hexadecapole ($l = 4$) modes are masked by quadrupole ($l = 2$) modes



Analysis results





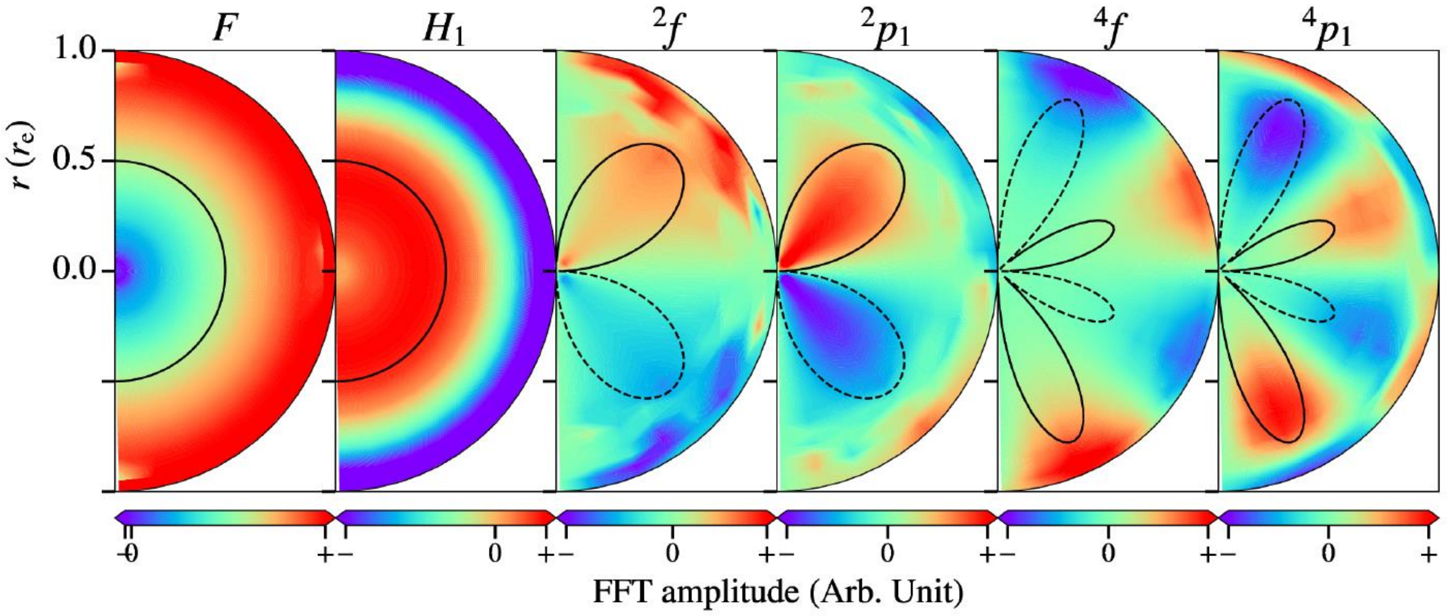


Main discoveries:

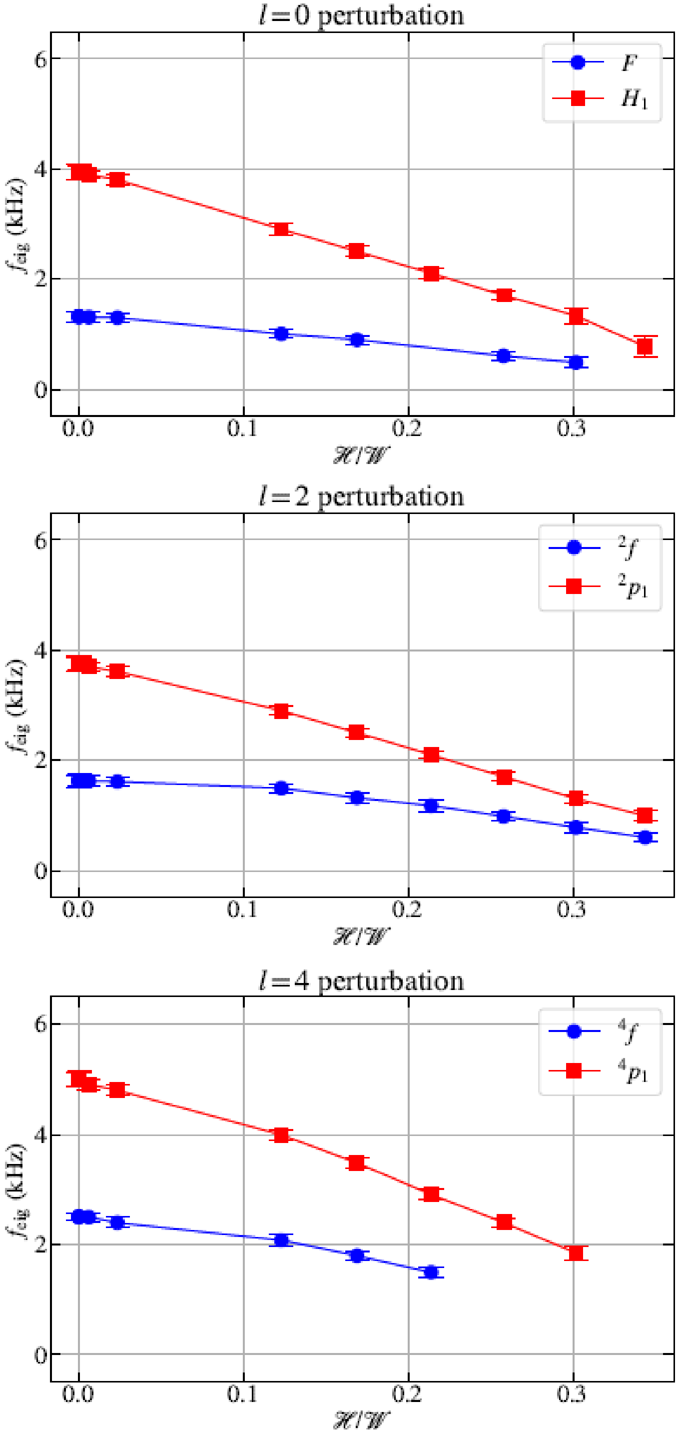
→ For $\mathscr{H}/\mathscr{W} \leq 10^{-4}$ and $B_{max} \leq 10^{16}$ G, oscillation modes are insensitive to magnetization effects

→ For $\mathscr{H}/\mathscr{W} > 10^{-4}$ and $B_{max} > 10^{16}$ G, oscillation modes are suppressed in a more magnetized NS:

1. f_{eig} decreases with $\mathscr{H}/\mathscr{W}$
2. $l = 4$ perturbation cannot predominantly excite hexadecapole ($l = 4$) modes anymore in the more magnetized NS models



Analysis results

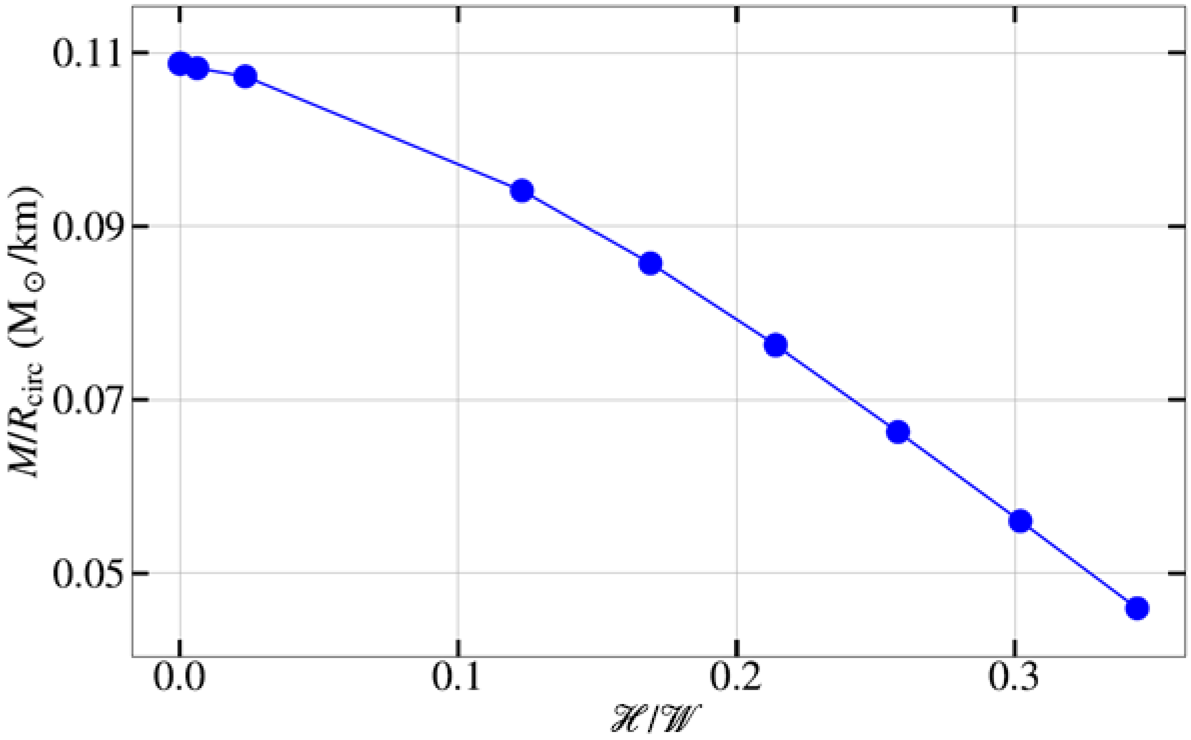


Possible reason behind eigenfrequency trends:

→ For non-magnetized NSs, $f_{\rm eig}$ is largely affected by stellar compactness $M/R_{\rm circ}$ [20]

→ For highly magnetized NSs here, $f_{\rm eig}$ and compactness show similar dependence on $\mathscr{H}/\mathscr{W}$

→ Intimate relation between $f_{\rm eig}$ and compactness is applicable to magnetized cases too



Conclusions and Outlooks

Conclusions:

→ Suppression of oscillations in a more magnetized NS: f_{eig} of $F, H_1,\ {}^2f,\ {}^2p_1\ ,\ {}^4f,\ {}^4p_1$ decrease with $\mathscr{H}/\mathscr{W}$

→ Correlation between oscillations and stellar compactness is also valid in cases of highly magnetized NSs

Outlooks:

→ Future updates on `Gmunu`:

1. A resistive GRMHD solver
2. Even better handling of sensitive stellar boundaries
3. Further enhanced numerical stability and efficiency

→ More realistic models of highly magnetized NSs:

1. More complicated field configurations
2. Uniform or differential rotations
3. Non-axisymmetries
4. Different fluid EoS

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THE END

Thank you!