A numerical approach to stochastic inflation and primordial black holes

TAUP2021

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Based on 2012.06551, in collaboration with D. Figueroa, S. Raatikainen, S. Räsänen

Concepts

Cosmic inflation

Concepts

Cosmic inflation

Cosmological perturbations

Concepts

Cosmic inflation

Cosmological perturbations

Primordial black holes

Concepts

Stochastic inflation

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Stochastic inflation

- Includes non-linear effects

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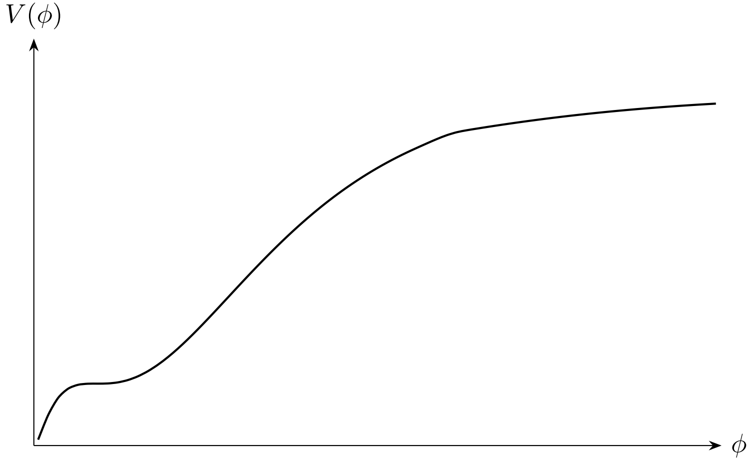
Stochastic inflation

- Includes non-linear effects
- Numerical method: even more non-linearities

Model of inflation fits CMB



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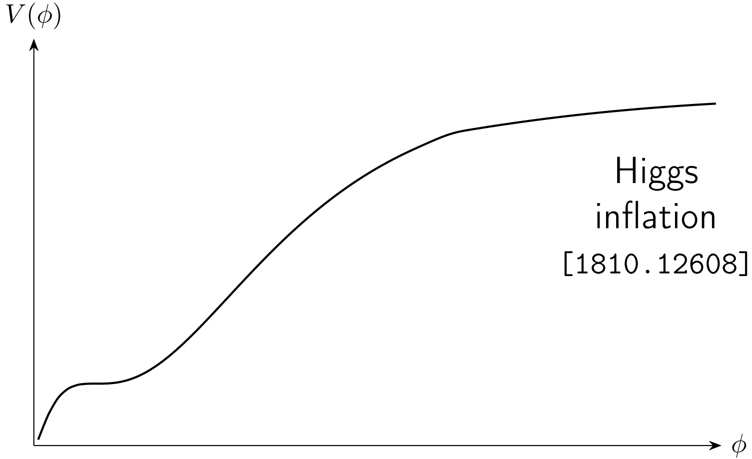
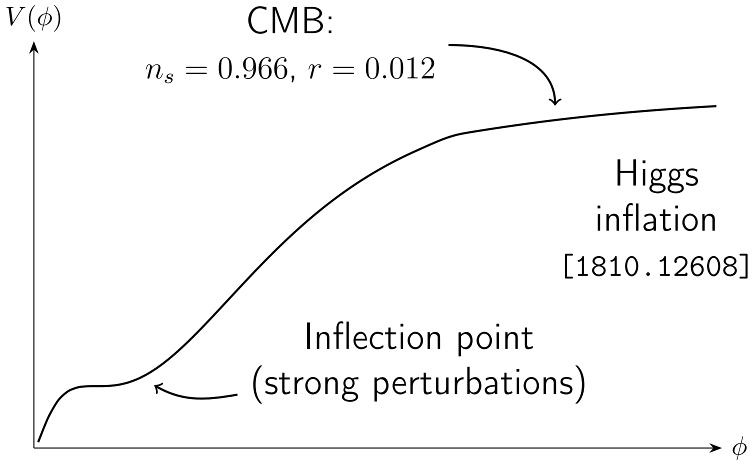


Higgs
inflation
[1810.12608]

Model of inflation fits CMB



Perturbations depend on scale

Origin of perturbations: fluctuations of quantum vacuum

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Space expands and perturbations get stretched

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Strong perturbations from ultra-slow-roll inflation

Must go beyond linear perturbations

Coarse-grain perturbations over super-Hubble scales

Gradient expansion: to leading order, coarse-grained perturbations follow locally (non-linear) FLRW equations [Class.Q.Grav.9,1943(1992)]

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ΔN formalism: from FLRW variables to perturbation variables [astro-ph/9507001]

- Change in e-folds of expansion $\Delta N =$ curvature perturbation $\mathcal{R}$

Stretching perturbations give stochastic kicks

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Stochastic evolution of local coarse-grained field
`[Lect.Notes Phys.246,107(1986)]`

PBHs form from strong perturbations

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[1309.4201, 1405.7023, 2011.03014]

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BH mass = all the mass inside one Hubble radius when the scale re-enters

ALGORITHM

Track numerically evolution of coarse-grained field $\bar{\phi}$ and linear perturbations $\delta\phi$

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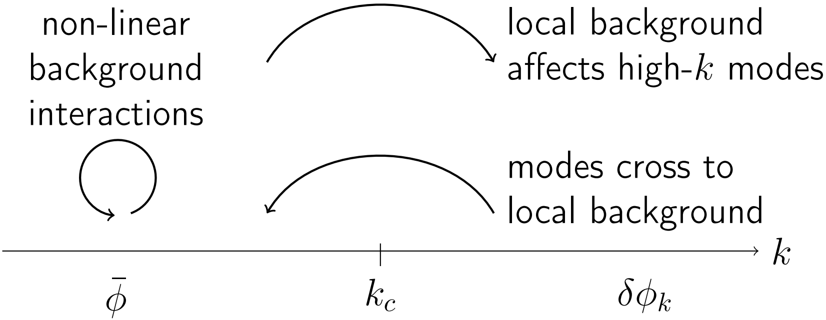
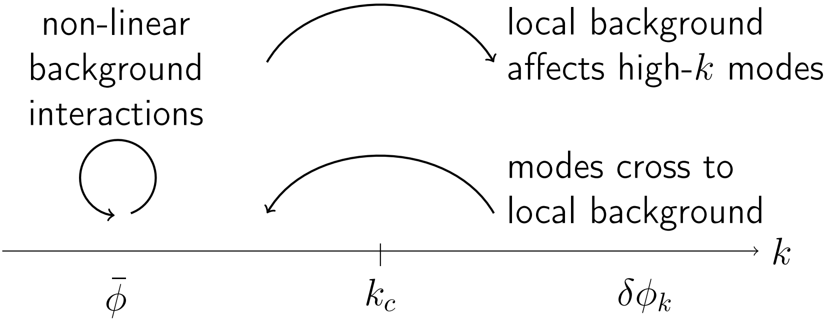
Stochastic evolution with backreaction

Non-linear interactions included



Non-linear interactions included



Compare to simpler approach with noise $\sim \frac{H^2}{2\pi^2}$

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Repeat 10^{11} times, collect statistics

Want tiny initial PBH fraction

Scale $M_{\rm PBH} = 10^{-14} M_\odot$, $k_{\rm PBH} = 10^{13}$ Mpc^{-1} chosen so that USR ends when $k_{\rm PBH}$ gets coarse-grained

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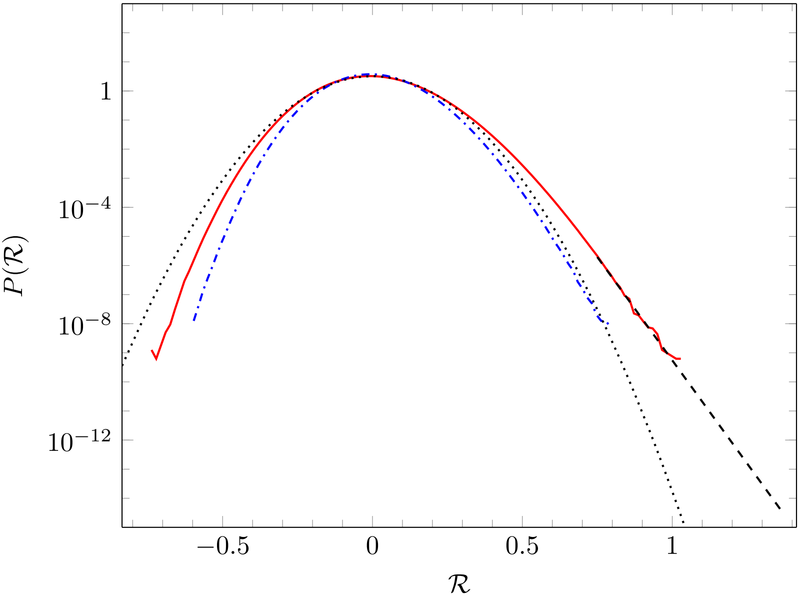
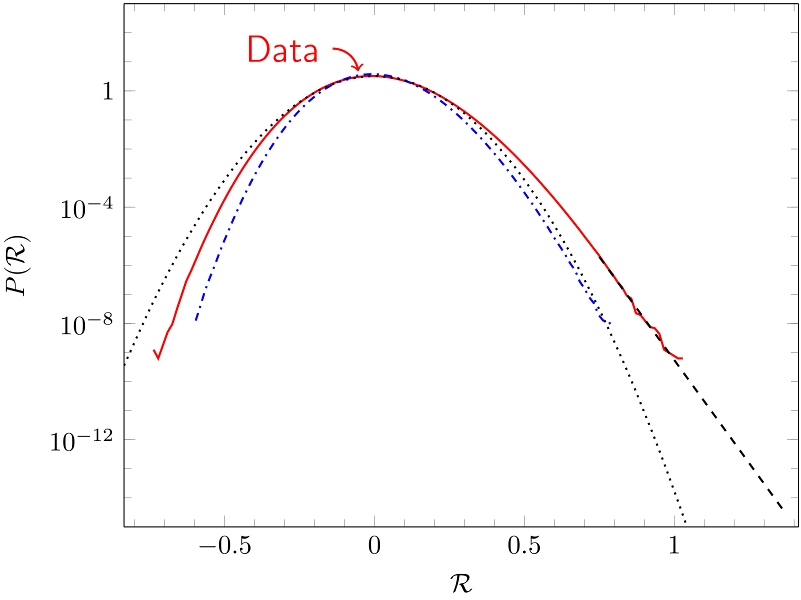
Gaussian statistics:

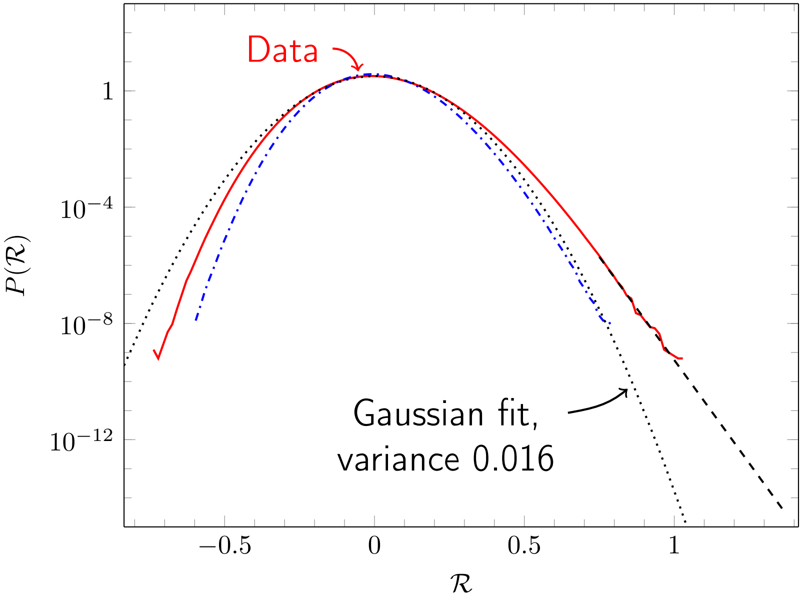
$$\sigma_{\mathcal{R}}^2 = \int^{k_{\rm PBH}} \mathrm{d}(\ln k) \mathcal{P}_{\mathcal{R}}(k)$$

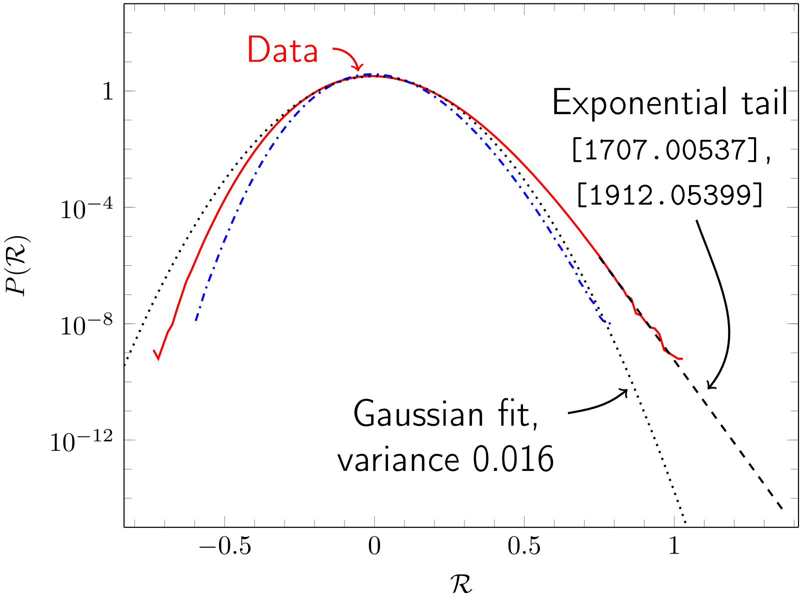
$$\beta = 2 \int_{\mathcal{R}_c}^{\infty} \mathrm{d}\mathcal{R} \frac{1}{\sqrt{2\pi}\sigma_{\mathcal{R}}} e^{-\frac{\mathcal{R}^2}{2\sigma_{\mathcal{R}}^2}} \approx \frac{\sqrt{2}\sigma_{\mathcal{R}}}{\sqrt{\pi}\mathcal{R}_c} e^{-\frac{\mathcal{R}_c^2}{2\sigma_{\mathcal{R}}^2}}$$

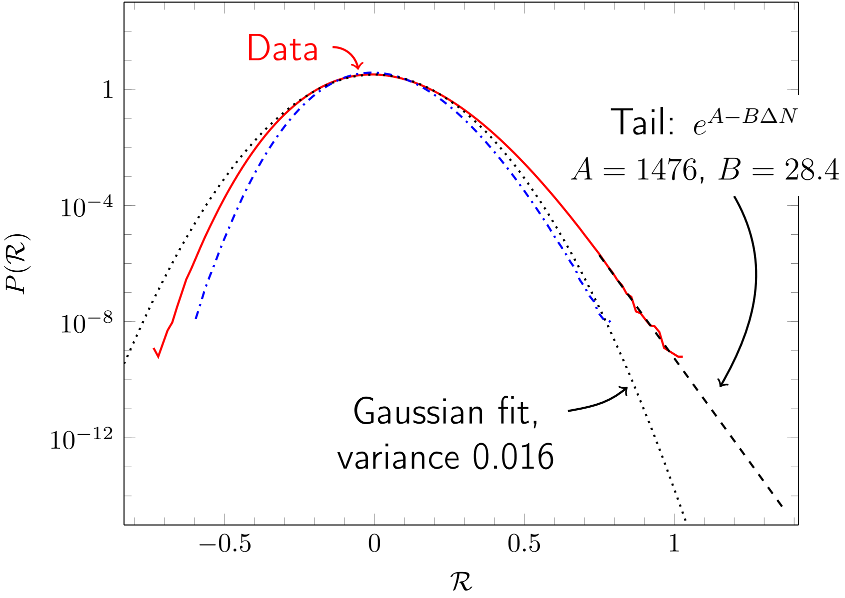


1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$

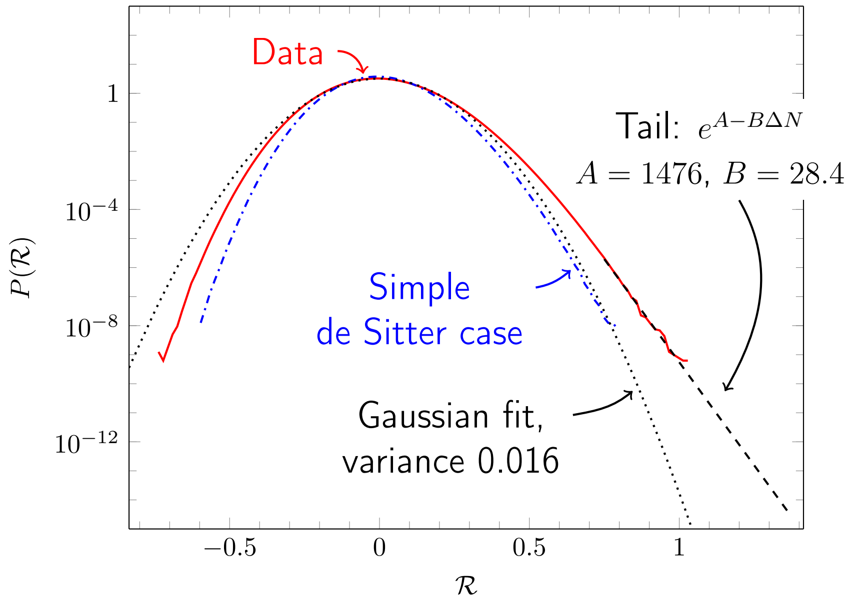

Data
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
-0.5
0
0.5
1
$\mathcal{R}$

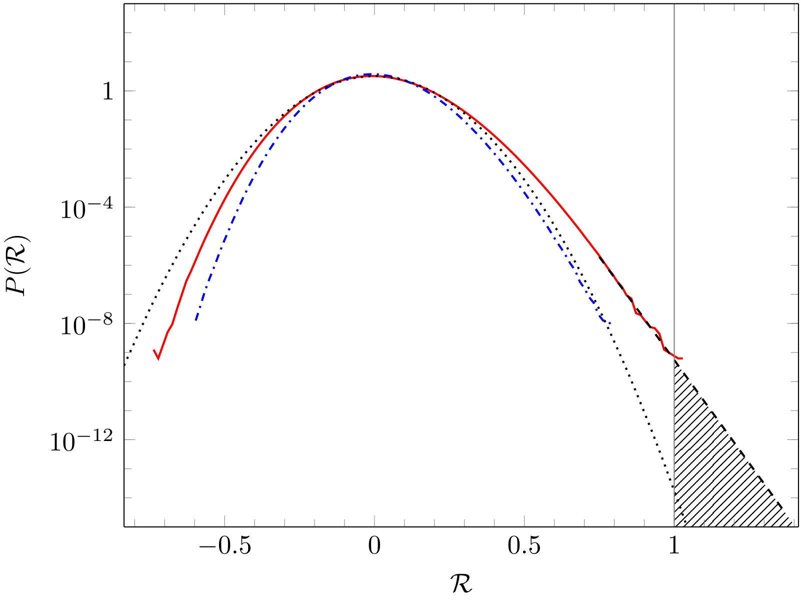

Data
Gaussian fit,
variance 0.016
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$

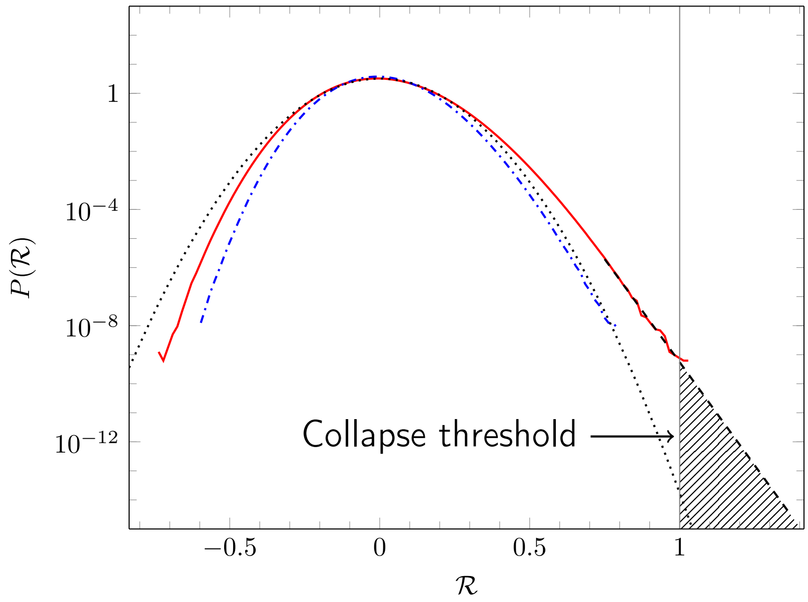

Data
Exponential tail
[1707.00537],
[1912.05399]
Gaussian fit,
variance 0.016
1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$



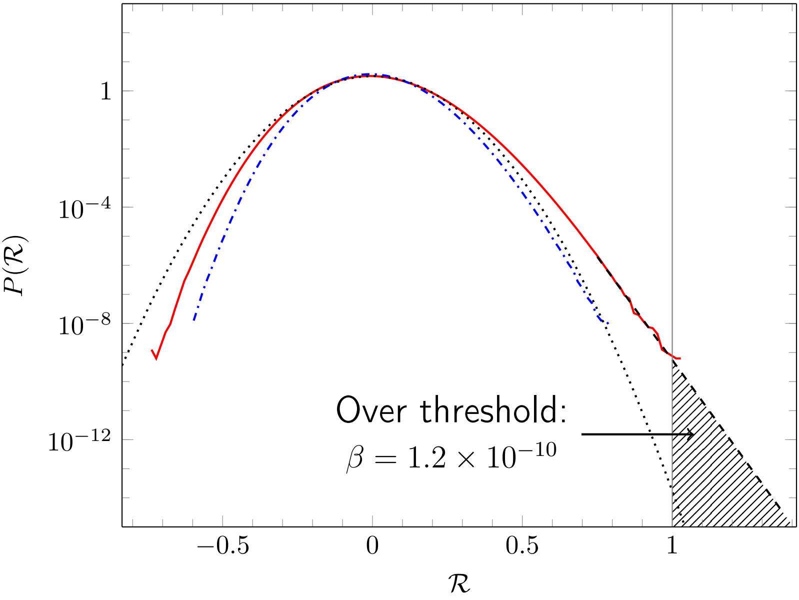
Data
Tail: $e^{A-B\Delta N}$
$A = 1476$, $B = 28.4$
Gaussian fit,
variance 0.016
$P(\mathcal{R})$
1
10^{-4}
10^{-8}
10^{-12}
-0.5
0
0.5
1
$\mathcal{R}$


Data
Simple
de Sitter case
Gaussian fit,
variance 0.016
Tail: $e^{A-B\Delta N}$
$A = 1476$, $B = 28.4$
$P(\mathcal{R})$
$\mathcal{R}$
1
10^{-4}
10^{-8}
10^{-12}
-0.5
0
0.5
1

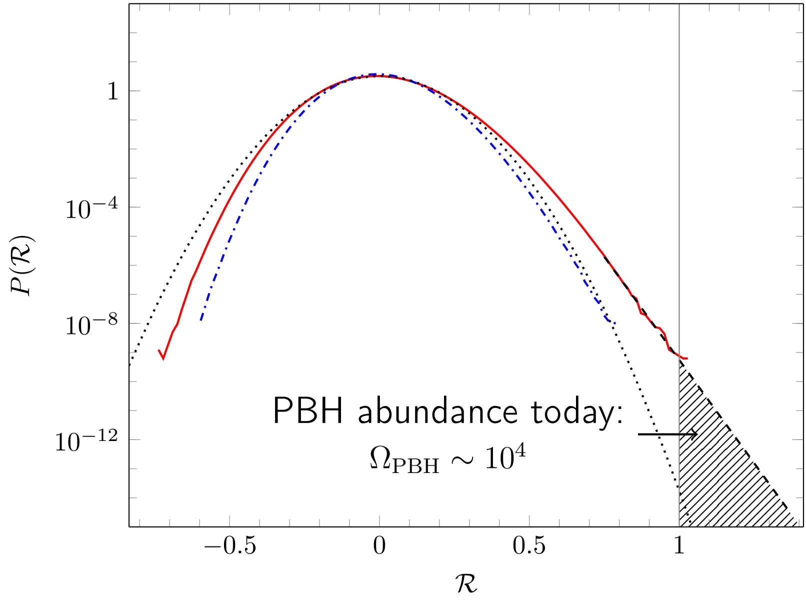

1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
-0.5
0
0.5
1
$\mathcal{R}$



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$
Collapse threshold



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
-0.5
0
0.5
1
$\mathcal{R}$
Over threshold:
$\beta = 1.2 \times 10^{-10}$



1
10^{-4}
10^{-8}
10^{-12}
$P(\mathcal{R})$
−0.5
0
0.5
1
$\mathcal{R}$
PBH abundance today:
$\Omega_{\mathrm{PBH}} \sim 10^4$

True abundance much higher than Gaussian estimate

Numerics: exponential tail, with

$$\beta = 1.2 \times 10^{-10}\,, \quad \Omega_{\rm PBH} = 5.4 \times 10^{4}$$

Larger than Gaussian result by factor 10^5!

Future directions

More statistics

More models

Full mass spectrum

Correlations between different scales

Conclusions

Stochastic inflation captures non-linearities of cosmological perturbations

Crucial for PBH formation

Introduced a numerical recipe to calculate these in a general single-field model

Thank you!

[2012.06551]