Turnaround physics beyond spherical symmetry

Valerio Faraoni[1]

[1]Bishop's University, Canada

A. Giusti & VF, *Phys. Rev. D* 103 (2021) 044049;
Phys. Dark Universe 26 (2019) 100353

TAUP 2021

OVERVIEW

1. Turnaround radius
2. Turnaround physics without spherical symmetry
3. Outlooks

Turnaround radius in spherical symmetry

In the present accelerated era of the universe, consider the largest bound objects in the sky. The turnaround radius was suggested as a possible way to test dark energy

(Roupas + 2014, PRD 89, 083002; Pavlidou & Tomaras 2014, JCAP 09, 020; Pavlidou + 2014, JCAP 05, 017)

but the concept of TR is older

(Souriau 1981; Stuchlik 1983; Stuchlik + 1989-2005; Mizony & Lachiéze-Rey 2005; Blau & Rollier 2008, ...)

In 2015 it was reported that the upper bound set by GR on the turnaround radius is significantly exceeded in the galaxy group NGC 5353/4 (Lee '015, '16).

Claim taken back in Lee & Yepes '16 because the error introduced by the non-sphericity of the system was underestimated.

Moreover, one should expect a *distribution* of values of the turnaround radius for different astronomical systems: an excess in this quantity would be significant from the statistical point of view rather than for individual systems (Lee '17).

Observational search focusses on galaxy groups with web-like structures in their neighbouring zones, and six more groups exceeding the general-relativistic prediction for the turnaround radius have been reported (Lee '17).

Consider an accelerated FLRW universe with a spherical inhomogeneity; massive test particles with zero radial initial velocity cannot collapse if they are outside a critical radius R_c (*turnaround radius*), but only expand.

For $R < R_c$, dust shells reach zero radial acceleration and collapse under self-gravity. If you cross outside R_c in geodesic motion, you will never fall back.

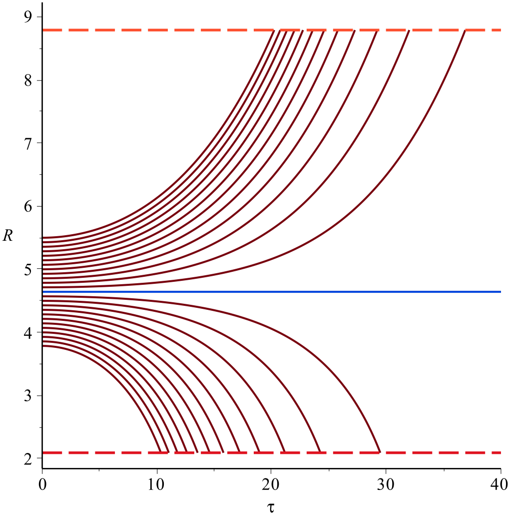
R_c is the upper limit to the radius of spherical bound structures in an accelerated universe.

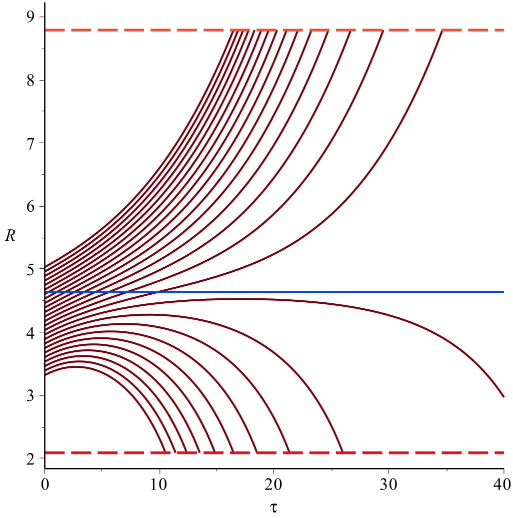
Schwarschild-de Sitter (heuristic)**:**

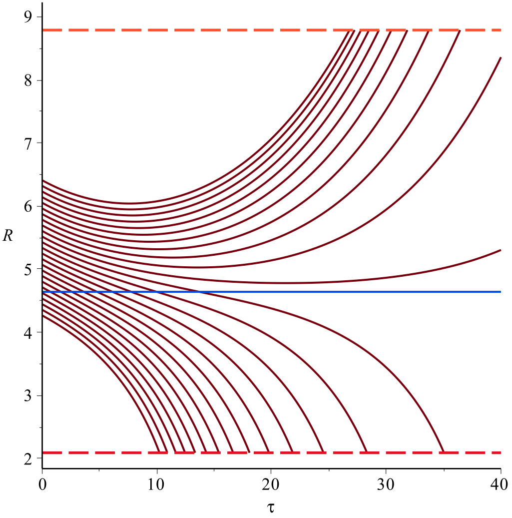
$$ds^2 = -\left(1 - \frac{2M}{R} - H^2 R^2\right) dt^2 + \frac{dR^2}{1 - \frac{2M}{R} - H^2 R^2} + R^2 d\Omega^2_{(2)}$$

$$H = \sqrt{\Lambda/3} \, ,$$

$$R_c = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$


9
8
7
6
5
4
3
2
R
0
10
20
30
40
τ


R
τ
9
8
7
6
5
4
3
2
0
10
20
30
40


9
8
7
6
5
4
3
2
R
0
10
20
30
40
τ

More realistic: post-FLRW space (1st order)

$$ds^2 = a^2(\eta)\left[-\left(1+2\phi\right)d\eta^2 + \left(1-2\phi\right)\left(dr^2 + r^2 d\Omega_{(2)}^2\right)\right]$$

timelike radial geodesics obey

$$\ddot{R} = \frac{\ddot{a}}{a}R - \frac{G\mathcal{M}(r)}{R^2}$$

where

$$\mathcal{M}(r) = 4\pi \int_0^R dR\, R^2 \rho_{total}$$

$$\rightarrow \quad R_c = \left(\frac{3\mathcal{M}}{4(3w+1)\pi\rho_{DE}}\right)^{1/3}$$

(or other expressions in modified gravity) $\rightarrow w_{\text{dark energy}}$.

Test gravity or the equation of state of dark energy in the ΛCDM model.

Turnaround surface without spherical symmetry

Allow for arbitrarily large deviations from spherical symmetry (but small perturbations of the FLRW metric). The key idea consists of identifying the turnaround surface with an equipotential surface of the local metric perturbation potential with the special property that, if a test particle initially sits on this surface at rest with respect to it, it remains on this surface. There is a unique critical surface $\mathcal{S}^*$ on which the two opposing forces balance. On any other closed surface $\mathcal{S}$ nearby, particles will collapse (if $\mathcal{S}$ lies inside $\mathcal{S}^*$) or disperse (if $\mathcal{S}$ lies outside $\mathcal{S}^*$).

Emphasis shifts from the *size* of the turnaround surface to the surface itself.

Timelike geodesics in the perturbed FLRW universe

Spacetime metric in the conformal Newtonian gauge is

$$ds^2 = a^2(\eta)\left\{-(1+2\Phi)\,d\eta^2 + (1-2\Phi)\left[dr^2 + r^2 d\Omega^2_{(2)}\right]\right\}$$

where η = FLRW conformal time, $\Phi(x^i)$ = Newtonian perturbation.
Test particles lying on $\mathcal{S}^*$ follow timelike geodesics with proper time τ and 4-tangents $u^\mu = dx^\mu/d\tau$ satisfying

$$\frac{du^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta}\, u^\alpha u^\beta = 0\,,$$

that gives

$$
\begin{aligned}
\frac{du^0}{d\tau} &= \frac{1}{a\,(2\Phi+1)}\left\{a_{,\eta}\left[2\left(r^2(u^3)^2\sin^2\theta + r^2(u^2)^2 + (u^1)^2 - (u^0)^2\right)\Phi\right.\right. \\
&\quad -r^2(u^3)^2\sin^2\theta - r^2(u^2)^2 \\
&\quad \left.\left.-(u^1)^2 - (u^0)^2\right] - 2u^0 a\left(u^3\Phi_{,\varphi} + u^2\Phi_{,\theta} + u^1\Phi_{,r}\right)\right\} \, ,
\end{aligned}
$$

$$
\begin{aligned}
\frac{du^1}{d\tau} &= \frac{1}{a(2\Phi-1)}\left\{2u^1u^0a_{,\eta}(1-2\Phi) + a\left[\left(r^2(u^2)^2 + r^2(u^3)^2\sin^2\theta\right.\right.\right. \\
&\quad \left.-(u^1)^2 + (u^0)^2\right)\Phi_{,r} - 2u^3u^1\Phi_{,\varphi} - 2u^2u^1\Phi_{,\theta} \\
&\quad \left.\left.+2r\left((u^3)^2\sin^2\theta + (u^2)^2\right)\Phi - r(u^3)^2\sin^2\theta - r(u^2)^2\right]\right\} \, ,
\end{aligned}
$$

$$\frac{du^2}{d\tau} = \frac{1}{r^2 a(2\Phi - 1)} \left\{ 2r^2 u^2 u^0 a_{,\eta} (1 - 2\Phi) + a \Big[- 2r^2 u^2 u^3 \Phi_{,\varphi} \right.$$

$$+ \Big(- r^2 (u^2)^2 + r^2 (u^3)^2 \sin^2\theta + (u^1)^2 + (u^0)^2 \Big) \Phi_{,\theta}$$

$$-2r^2 u^1 u^2 \Phi_{,r} + r \Big(r(u^3)^2 \sin(2\theta) - 4u^1 u^2 \Big) \Phi$$

$$\left. - r^2 (u^3)^2 \sin\theta \cos\theta + 2ru^1 u^2 \Big] \right\} ,$$

$$\frac{du^3}{d\tau} = \frac{1}{r^2 a(2\Phi - 1)} \left\{ 2r^2 u^3 u^0 a_{,\eta} (1 - 2\Phi) \right.$$

$$+ a \Big[\Big(\left(r^2 (u^2)^2 + (u^1)^2 + (u^0)^2 \right) \csc^2\theta - r^2 (u^3)^2 \Big) \Phi_{,\varphi}$$

$$+ 2ru^3 \left(-ru^1 \Phi_{,r} - ru^2 \Phi_{,\theta} + ru^2 \cot\theta + u^1 \right)$$

$$\left. - 4ru^3 (ru^2 \cot\theta + u^1) \Phi \Big] \right\} .$$

The four-tangent is

$$u^\mu = u^\mu_{(0)} + \delta u^\mu = \left(u^0_{(0)} + \delta u^0, \delta \boldsymbol{u} \right) = \left(\frac{1}{a} + \delta u^0, \delta \boldsymbol{u} \right)$$

and the normalization $u^\mu u_\mu = -1$ yields $\delta u^0 = -\Phi/a$ to first order. Assuming

$$\mathcal{O}(\delta u^1) = \mathcal{O}(\delta u^2) = \mathcal{O}(\delta u^3)$$

we have

$$\mathcal{O}(u^i) = \mathcal{O}(\delta u^i) = \mathcal{O}(\Phi) \, , \quad i = 1, 2, 3$$

and

$$\boxed{\frac{d(\delta u^i)}{d\tau} + \frac{2a_{,\eta}}{a^2} \, \delta u^i + g^{ij} \partial_j \Phi = 0 \, .} \tag{1}$$

General definition of turnaround surface

In spherical symmetry, the turnaround sphere is an equipotential surface of Φ but this is not sufficient to identify it. We require the extra property that, if test particles initially lay on this surface and have zero initial velocity *with respect to it*, they remain on this surface as the latter evolves.

These dust particles, and this surface, are not comoving with the FLRW background: they are slowed down by the attraction of the mass inside the turnaround surface, expand more slowly.

Still not sufficient to identify the turnaround surface because many timelike geodesics cross the turnaround surface: we further restrict to timelike geodesics that initially have zero velocity with respect to this surface. Since they satisfy the 2nd order geodesic equation, assigning their initial position (on Σ_{t_0}) and initial velocity (at rest on Σ_{t_0}) specifies them completely.

These massive test particles stay on Σ_{t_0} initially and at all later times.
Finally we require that, on this surface, the Newtonian attraction balances the cosmic expansion.

Formally, the **turnaround surface** Σ_t at (comoving) time t is a 2D closed, simply connected surface that is an equipotential surface of the perturbation Φ such that:

i) The time evolution of the surface is such that the three-dimensional components of the tangent to the timelike geodesics crossing Σ_t are *locally* proportional to the gradient $\nabla\Phi$ (and therefore perpendicular to Σ_t in the three-dimensional sense):

$$u^i|_{\Sigma_t} = \sigma(t)\, g^{ij}\partial_j\Phi|_{\Sigma_t} \,.$$

ii) A dust particle initially comoving with the surface remains on this surface Σ_{t_0}. *i.e.*, if

$$u^i|_{\Sigma_{t_0}} = \sigma(t_0)\, g^{ij}\partial_j\Phi|_{\Sigma_{t_0}} \,,$$

then at $t > t_0$ its 3-velocity satisfies

$$u^i|_{\Sigma_t} = \sigma(t)\, g^{ij}\partial_j\Phi|_{\Sigma_t} \,.$$

iii) In an unperturbed FLRW universe, the (purely radial) acceleration of a massive test particle is $\ddot{r} = \ddot{a}r/a$; on the initial surface Σ_{t_0} (assumed to be convex), the acceleration of a massive test particle normal to this surface vanishes because the local attraction $-\nabla\Phi$ balances exactly the force per unit mass due to the cosmic expansion *at that point* $\frac{\ddot{a}(t_0)}{a(t_0)}\,\boldsymbol{x}_\perp$ (on Σ_{t_0} there is no sideways acceleration due to the local perturbation because Σ_{t_0} is an equipotential surface of Φ). Or: if $\boldsymbol{x}$ is a point on Σ_{t_0} and $\boldsymbol{n} = \left.\frac{\nabla\Phi}{|\nabla\Phi|}\right|_{\Sigma_{t_0}}$ is the normal to Σ_{t_0}, we impose

$$-\nabla\Phi = \frac{\ddot{a}(t_0)}{a(t_0)}\,\boldsymbol{x}_\perp \quad \text{on} \;\; \Sigma_{t_0}\,,$$

with $\boldsymbol{x}_\perp \equiv (\boldsymbol{x}\cdot\boldsymbol{n})\,\boldsymbol{n}$, which implies

$$-|\nabla\Phi|^2 = \frac{\ddot{a}(t_0)}{a(t_0)}\,\boldsymbol{x}\cdot\nabla\Phi \qquad \text{on}\;\Sigma_{t_0}\,.$$

Now the general result (1) reads

$$\frac{d(\delta u^i)}{dt} + 2H\delta u^i + g^{ij}\partial_j\Phi = 0\,,$$

since $u^0_{(0)} = d\eta/d\tau = d\eta/dt = 1/a$ to order $\mathcal{O}(\Phi^0)$, Eq. (1) leads to

$$\delta u^i|_{\Sigma_t} = \frac{a^2(t_0)}{a^2(t)}\,\delta u^i|_{\Sigma_{t_0}} - \frac{1}{a^2(t)}\int_{t_0}^{t} h^{ij}(x^\alpha(t'))\partial_j\Phi(x^\alpha(t'))\,dt'\,.$$

Astronomical observations of the turnaround radius only span a small redshift interval near the time when the light was emitted $\rightarrow$ linearize $a(t)$ and the integral to 1st order in $t - t_0$ $\rightarrow$

$$\begin{aligned} \delta u^i|_{\Sigma_t} &= \left[(1 - 2H(t_0)\epsilon)\,\sigma(t_0) - \frac{\epsilon}{a^2(t_0)} \right] h^{ij}\partial_j \Phi|_{\Sigma_{t_0}} + \mathcal{O}(H^2(t_0)\epsilon^2) \\ &= \left[\sigma(t_0) - \left(2\sigma(t_0) H(t_0) + \frac{1}{a^2(t_0)} \right) \epsilon \right] h^{ij}\partial_j \Phi|_{\Sigma_{t_0}} + \mathcal{O}(H^2(t_0)\epsilon^2) \,. \end{aligned}$$

This equation reproduces known formulas in the spherical case and for small deviations from sphericity

$+$

definition generalized to scalar-tensor gravity
in A. Giusti & VF, *Phys. Rev. D* 103 (2021) 044049

OUTLOOKS

- Turnaround radius is an opportunity to test gravity and/or the ΛCDM model. Need to move beyond spherical approximation (theory) $+$ get a handle on error (astronomy).
- Given definition of turnaround surface for arbitrary deviations from spherical symmetry.
- The application to small deviations from spherical symmetry reproduces previous results obtained with a completely different method (the splitting of the Hawking mass contained in the turnaround surface into local $+$ cosmological parts).
- Now need numerical implementation by astronomers for specific cosmic structures ...

THANK YOU

Supported by