Screening vs. gevolution:

in chase of a perfect cosmological simulation code

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Both competing relativistic approaches to the N-body simulation of the large-scale structure formation cover the whole space (except for its small portion where gravity is strong) as the smallness of energy-momentum fluctuations (generating the inhomogeneous gravitational field) is not demanded.

Do the “gevolution” and “screening” codes produce different or almost identical results? Which code runs faster?

gevolution

J. Adamek, D. Daverio, R. Durrer and M. Kunz, General relativity and cosmic structure formation, Nature Phys. 12 (2016) 346; arXiv:1509.01699 [astro-ph.CO]

J. Adamek, D. Daverio, R. Durrer and M. Kunz, gevolution: a cosmological N-body code based on General Relativity, JCAP 07 (2016) 053; arXiv:1604.06065 [astro-ph.CO]

Perturbed Friedmann-Lemaître-Robertson-Walker metric in the Poisson gauge:

$$ds^2 = a^2 \left[-(1+2\Psi)d\tau^2 - 2B_i dx^i d\tau + (1-2\Phi)\delta_{ij} dx^i dx^j\right] , \quad i,j = 1,2,3$$

scale factor

conformal time

comoving coordinates

We disregard tensor modes.

scalar perturbations

vector perturbation

$$\delta^{ij} B_{i,j} = 0$$

Einstein equations:

$$(1+4\Phi)\Delta\Phi - 3\mathcal{H}\Phi' + 3\mathcal{H}^2(\chi - \Phi) + \frac{3}{2}\delta^{ij}\Phi_{,i}\Phi_{,j} = -4\pi G a^2 \left(T_0^0 - \overline{T_0^0}\right)$$

$$-\frac{1}{4}\Delta B_i - \Phi'_{,i} - \mathcal{H}(\Phi_{,i} - \chi_{,i}) = 4\pi G a^2 T_i^0$$

gravitational constant

$$\left(\delta_k^i \delta_l^j - \frac{1}{3}\delta^{ij}\delta_{kl}\right)\left[B'_{(i,j)} + 2\mathcal{H}B_{(i,j)} + \chi_{,ij} - 2\chi\Phi_{,ij} + 2\Phi_{,i}\Phi_{,j} + 4\Phi\Phi_{,ij}\right]$$

$$= \quad 8\pi G a^2 \left(\delta_{ik}T_l^i - \frac{1}{3}\delta_{kl}T_i^i\right)$$

symmetrization over ***i, j***

$\Delta \equiv \delta^{ij}\partial^2/\partial x^i \partial x^j$ **is the Laplacian in comoving coordinates;**

$\mathcal{H} \equiv a'/a$ **, with prime denoting the derivative with respect to conformal time;**

$\chi \equiv \Phi - \Psi$ **represents the difference between scalar modes.**

Components of the energy-momentum tensor for a system of point-like particles:

$$T_0^0 = -\frac{1}{a^4}\sum_n \delta(\mathbf{r}-\mathbf{r}_n)\sqrt{q_n^2+m_n^2a^2}\left(1+3\Phi+\frac{q_n^2}{q_n^2+m_n^2a^2}\Phi\right)$$

$$T_i^0 = \frac{1}{a^4}\sum_n \delta(\mathbf{r}-\mathbf{r}_n)(q_n)_i(1+2\Phi+\chi) \qquad q_n^2 \equiv \delta^{ij}(q_n)_i(q_n)_j$$

$$T_j^i = \frac{\delta^{ik}}{a^4}\sum_n \delta(\mathbf{r}-\mathbf{r}_n)\frac{(q_n)_j(q_n)_k}{\sqrt{q_n^2+m_n^2a^2}}\left(1+4\Phi+\frac{m_n^2a^2}{q_n^2+m_n^2a^2}\Phi\right)$$

comoving radius-vectors masses momenta

Equation of motion:

$$\frac{d(q_n)_i}{d\tau} = -\sqrt{q_n^2+m_n^2a^2}\left[\left(1+\frac{q_n^2}{q_n^2+m_n^2a^2}\right)\Phi_{,i}-\chi_{,i}+\frac{\delta^{jk}(q_n)_kB_{j,i}}{\sqrt{q_n^2+m_n^2a^2}}\right]$$

Screening

M. Eingorn, First-order cosmological perturbations engendered by point-like masses, Astrophys. J. 825 (2016) 84; arXiv:1509.03835 [gr-qc]

R. Brilenkov and M. Eingorn, Second-order cosmological perturbations engendered by point-like masses, Astrophys. J. 845 (2017) 153; arXiv:1703.10282 [gr-qc]

M. Eingorn, N.D. Guran and A. Zhuk, Analytic expressions for the second-order scalar perturbations in the ΛCDM Universe within the cosmic screening approach, Phys. Dark Univ. 26 (2019) 100329; arXiv:1903.09024 [gr-qc]

E. Canay and M. Eingorn, Duel of cosmological screening lengths, Phys. Dark Univ. 29 (2020) 100565; arXiv:2002.00437 [gr-qc]

Ψ, Φ, B_i → **first order of smallness**

Nonrelativistic matter in the focus of attention → $\Psi = \Phi$

χ → **second-order quantity**

Einstein equations:

$$\overline{T_0^0} = -\overline{\rho}/a^3$$

$$\Delta\Phi - 3\mathcal{H}\Phi' - 3\mathcal{H}^2\Phi = -4\pi G a^2 \left(T_0^0 - \overline{T_0^0}\right)$$

$$T_0^0 - \overline{T_0^0} = -\frac{1}{a^3}\delta\rho - \frac{3}{a^3}\overline{\rho}\Phi$$

$$-\frac{1}{4}\Delta B_i - \Phi'_{,i} - \mathcal{H}\Phi_{,i} = 4\pi G a^2 T_i^0$$

$$T_i^0 = \frac{1}{a^4}\sum_n \delta(\mathbf{r} - \mathbf{r}_n)(q_n)_i$$

$$\rho = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n) \qquad \delta\rho \equiv \rho - \overline{\rho}$$

↓ **mass density**

The linear "screening" equation for Φ is much simpler than the nonlinear "gevolution" counterpart, and admits the exact analytical solution.

Equation of motion: $$\frac{d(q_n)_i}{d\tau} = -m_n a \Phi_{,i}$$

$$\chi \equiv \Phi^{(2)} - \Psi^{(2)}$$

second-order scalar perturbations

Instead of computing χ at each iteration step, one can calculate this and other second-order quantities only at redshifts of interest, thereby saving valuable computational time (which may be already substantially reduced owing to modification of equations).

$\Phi_{\mathrm{gevolution}} = \Phi_{\mathrm{screening}}$ + **higher-order admixture**

Friedmann equation for the scale factor: $$-\frac{3\mathcal{H}^2}{a^2} = 8\pi G\left(\overline{T_0^0} + \ldots\right)$$

additional contributions from radiation (treated as homogeneous) and the cosmological constant (for the concordance ΛCDM model)

Simulations

Boxes of sizes 280, 336, 560, 980, 1680 Mpc/h with 1 Mpc/h resolution;
280, 560, 1120, 2016, 2800 Mpc/h with 2 Mpc/h resolution.

Initial redshift: 100
Values of standard cosmological parameters:

$$\Omega_b h^2 = 0.02242 \qquad h = 0.6766$$
$$\Omega_c h^2 = 0.11933 \qquad n_s = 0.9665$$
$$A_s = 2.105 \times 10^{-9}$$

We treat baryons similarly to cold dark matter.
In addition, we neglect radiation in both codes, with no effect on the conclusions.

In order to generate initial conditions, we use the code CLASS:

D. Blas, J. Lesgourgues and T. Tram, The Cosmic Linear Anisotropy Solving System (CLASS) II: Approximation schemes, JCAP 07 (2011) 034; arXiv:1104.2933 [astro-ph.CO]

Definitions of the power spectra:

$$4\pi k^3 \langle \Phi(\mathbf{k}, z)\Phi(\mathbf{k}', z)\rangle = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')P_\Phi(k, z)$$

$$4\pi k^3 \langle B_i(\mathbf{k}, z)B_j(\mathbf{k}', z)\rangle = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')P_{ij}P_B(k, z)\,, \quad P_{ij} \equiv \delta_{ij} - \frac{k_i k_j}{k^2}$$

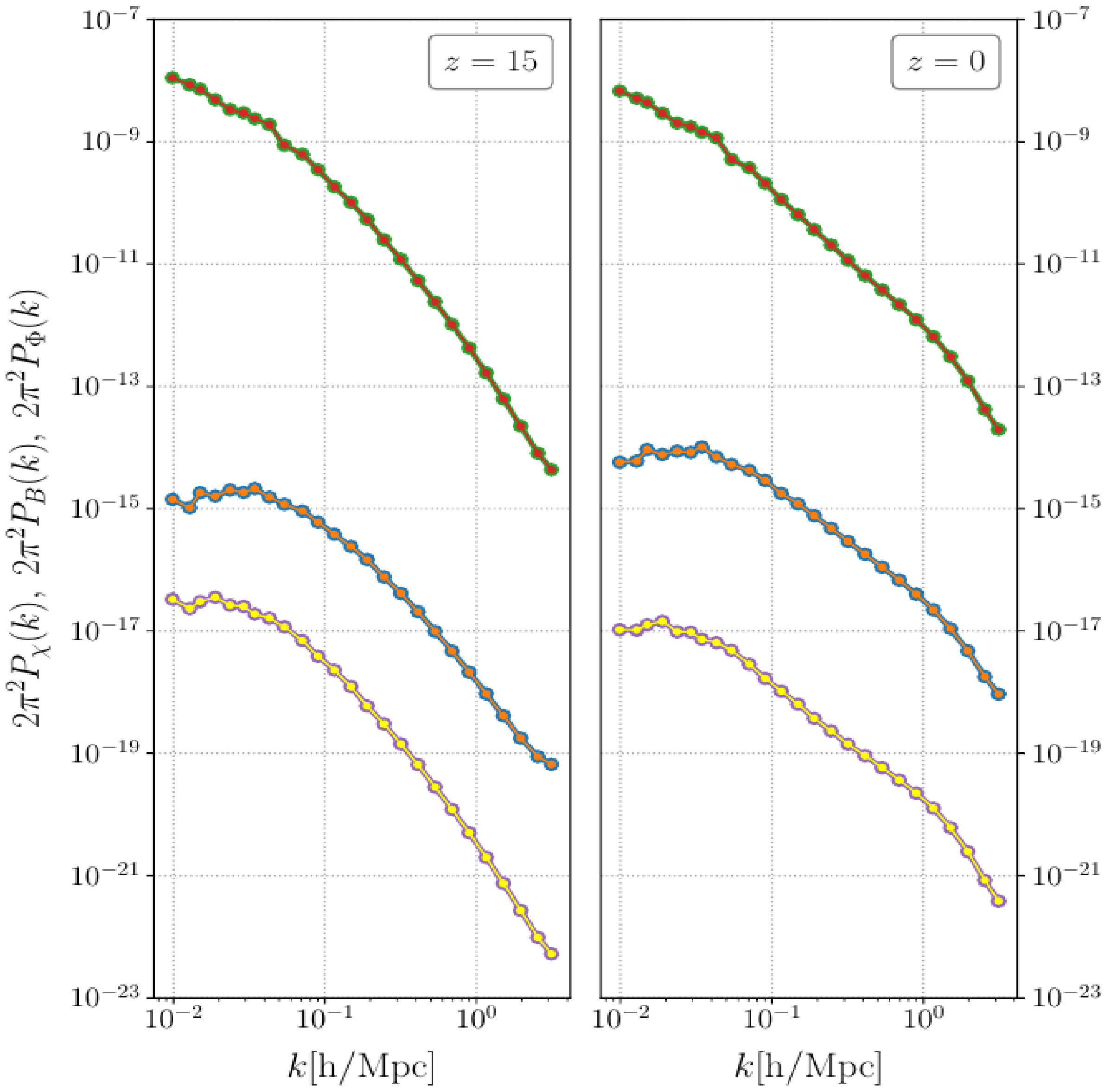
$$4\pi k^3 \langle \chi(\mathbf{k}, z)\chi(\mathbf{k}', z)\rangle = (2\pi)^3\delta(\mathbf{k} - \mathbf{k}')P_\chi(k, z)$$

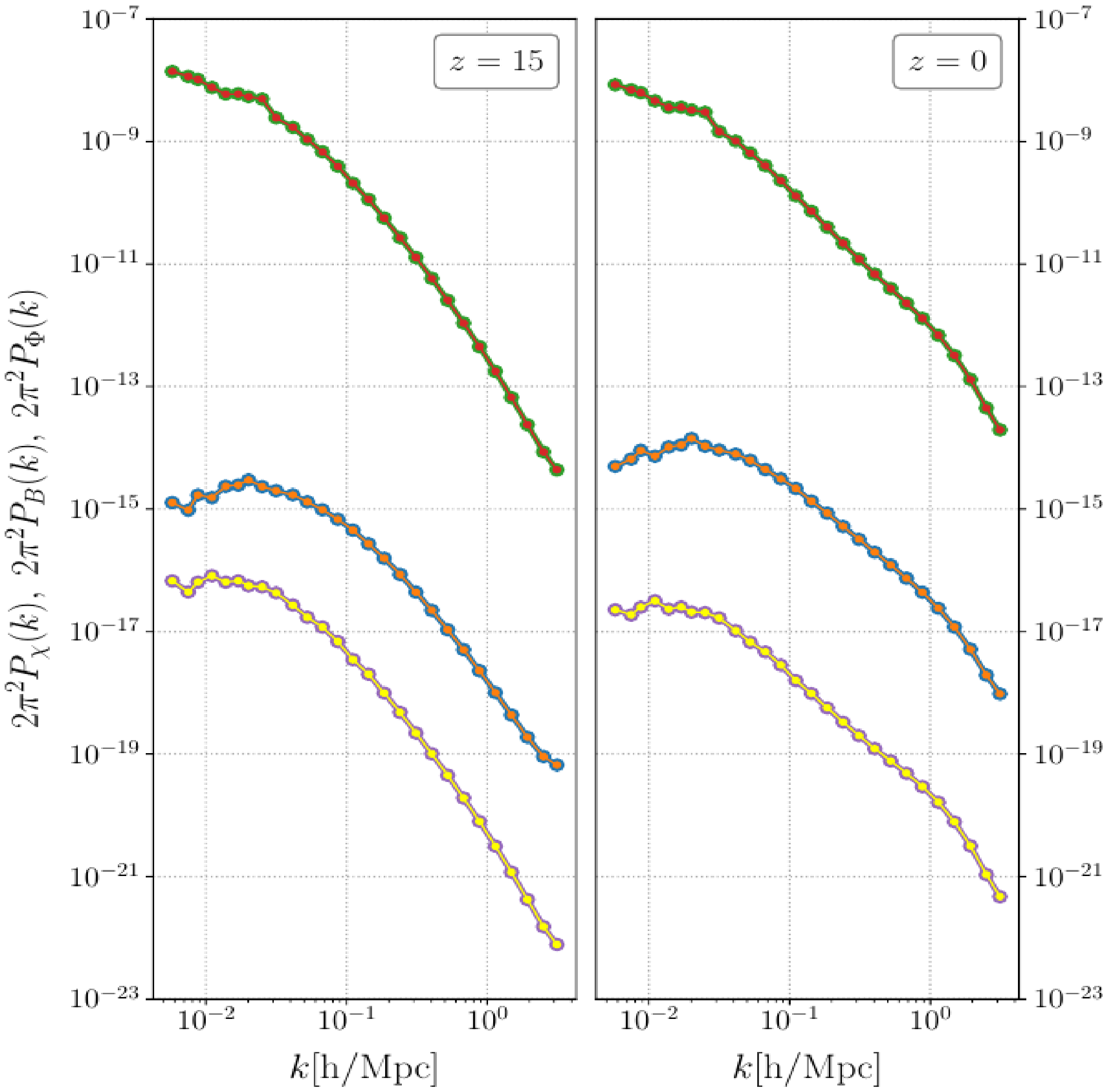
Relative deviation:

$$\Delta P \equiv \left| \frac{P_{\mathrm{screening}} - P_{\mathrm{gevolution}}}{P_{\mathrm{gevolution}}} \right|$$

We also compare the computational time (in CPU hours) for the "gevolution" and "screening" approaches by estimating the relative deviation

$$\Delta t \equiv \left| \frac{t_{\mathrm{s}} - t_{\mathrm{g}}}{t_{\mathrm{g}}} \right|$$



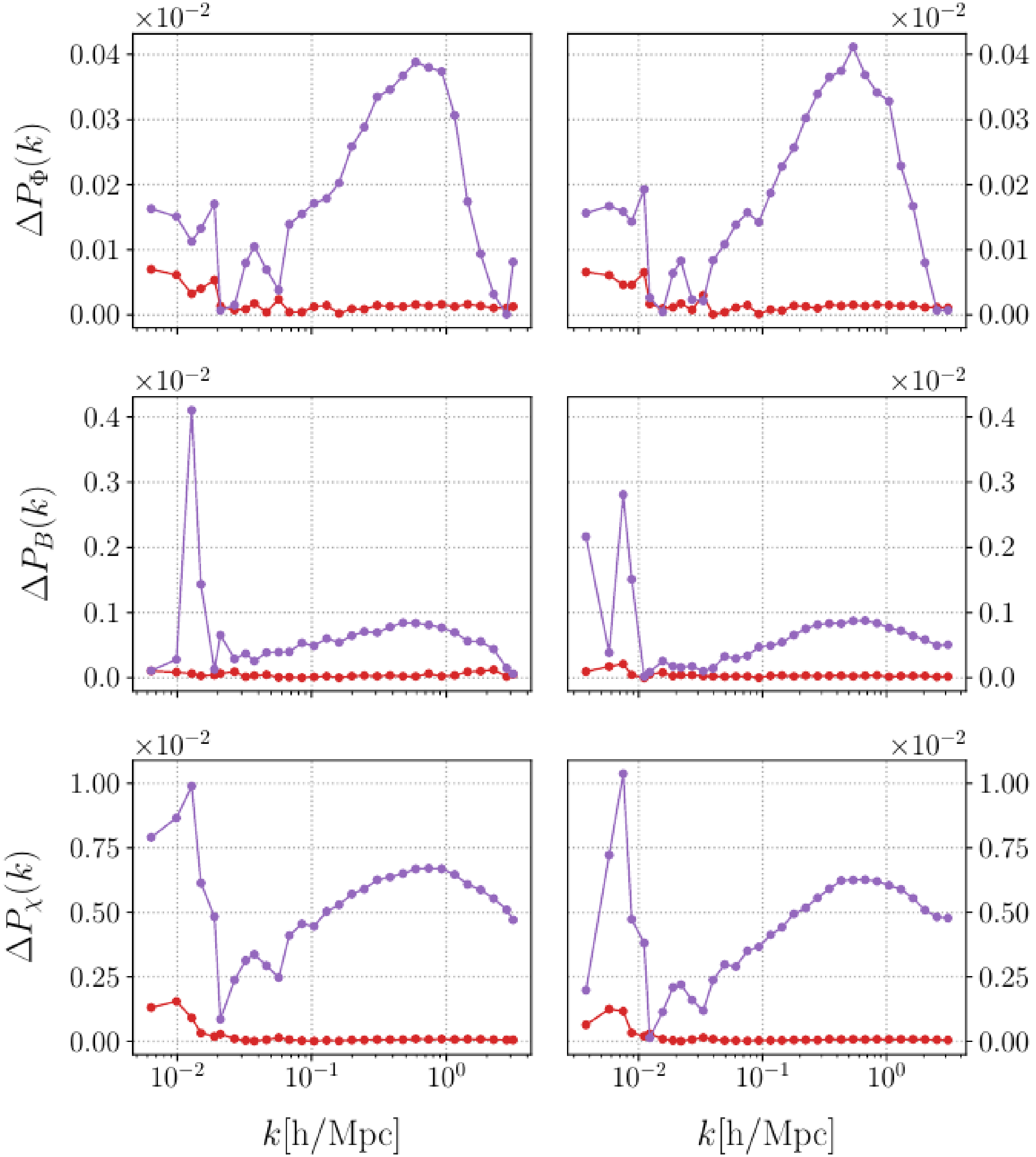


980 Mpc/h

1680 Mpc/h

“gevolution”: green, blue and purple curves in the background

“screening”: red, orange and yellow curves in the foreground



Left: **980 Mpc/h** **Purple:** **z = 0**

Right: **1680 Mpc/h** **Red:** **z = 15**

1 Mpc/h resolution

L [Mpc/h]	t_g	t_s	Δt [%]
280	6.2	3.8	38.7
336	11.5	7.1	38.3
560	47.1	29.2	38.0
980	294.9	180.9	38.7
1680	1551.1	939.6	39.4

2 Mpc/h resolution

L [Mpc/h]	t_g	t_s	Δt [%]
280	0.8	0.5	37.5
560	4.3	2.7	37.2
1120	34.8	21.2	39.1
2016	216.2	132.1	38.9
2800	563.9	347.0	38.5

Conclusion

The results of “gevolution” and “screening” simulations remarkably coincide.

It is natural to expect that the code using simpler equations consumes less computational time. Indeed, the simpler “screening” code saves almost 40% of CPU hours. Since the smaller the computational time, the cheaper the project costs, this is a definite advantage of the “screening” approach.

Additionally, for the fixed allotted time, the faster “screening” code makes it possible to simulate a larger box or to probe higher resolution.

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Thank You!

Meow.