Cosmology of modified Chaplygin gas under the purview of f(T) gravity

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Introduction

In 1998, accumulating observational data from distant type Ia supernovae, two teams [1,2] independently presented compelling evidence in support of accelerated expansion of the universe. Prior to their discovery, the universe was believed to have a slowed down expansion due to the effect of gravity and the cosmologists were so certain of the slow-down that they termed the parameter quantifying the second derivative of the expansion, q_0, the deceleration parameter. In this work, we investigate the cosmological application of modified Chaplygin gas (MCG) interacting with pressureless dark matter (DM) in the f(T) modified gravity framework, where T is the torsion scalar in teleparallelism. The interaction term has been chosen proportional to the MCG density with positive coupling constant. In the Einstein general relativity (GR) framework, the interacting MCG has been found to have equation of state (EoS) parameter behaving like quintessence. However, the f(T) gravity reconstructed via the interacting MCG has been found to have EoS crossing the phantom boundary of −1. Thus, one can generate a quintom-like EoS from an interacting MCG model in flat universe in the modified gravity cosmology framework. The reconstructed f(T) model has been found to interpolate between dust and ΛCDM. Stability of the reconstructed f(T) has been investigated and it has been observed that the model is stable against gravitational perturbation. Cosmological evolution of primordial perturbations has also been investigated and the self-interacting potential has been found to increase with cosmic time and the squared speed of sound has been found to be non-negative.

1. A. G. Riess et al., Astron. J. 116 (1998) 1009.
2. S. Perlmutter et al., Astrophys. J. 517 (1999) 565.

Cosmological Dynamics of Viscous MCG-EHRDE

$$3H^2=\rho,\qquad 2\dot{H}+3H^2=-p-\Pi,\qquad \dot{\rho}+3H(\rho+p)=-3H\Pi.$$

$$p_{\rm eff}=p+\Pi$$

$$\xi=\xi_0+\xi_1 H+\xi_2(\dot{H}+H^2).$$

Density of EHRDE is $\rho_R=3(\alpha H^2+\beta\dot{H}).$

The scale factor is reconstructed as $a(t)=\left(1+\frac{H_0 t}{\beta}(\alpha-1)\right)^{\frac{\beta}{\alpha-1}}$

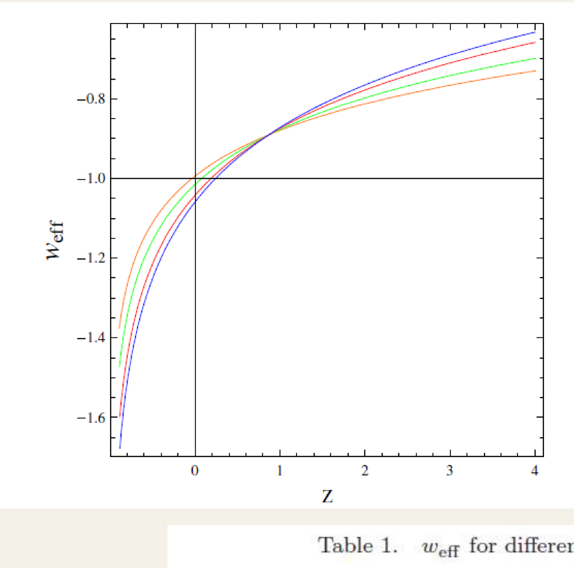
We assume that the cosmological fluid obeys MCE EoS

$$p_c=\gamma\rho_c-\frac{B}{\rho_c^n},$$

$0\le\gamma\le 1$ and $0\le n\le 1$
B is a positive constant

Effective EoS comes out to be

$$w_{\rm eff}=-3^{-1-n}B\left(a^{\frac{2-2\alpha}{\beta}}H_0^2\right)^{-1-n}+\gamma-\frac{a^{\frac{-1+\alpha}{\beta}}\xi_0}{H_0}-\xi_1-\frac{a^{\frac{1-\alpha}{\beta}}H_0(1-\alpha+\beta)\xi_2}{\beta}.$$



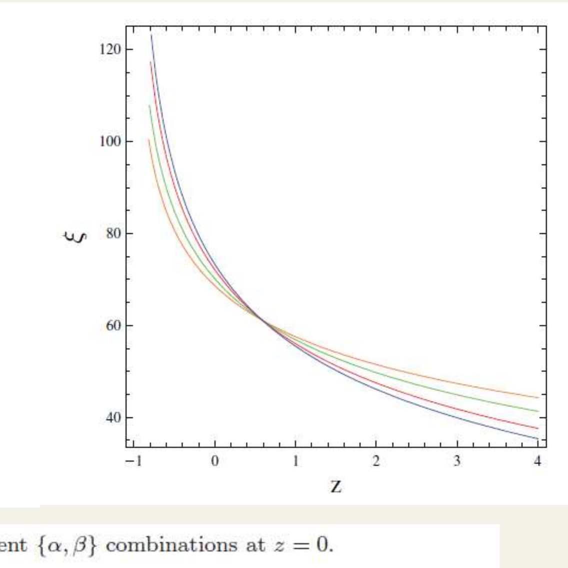


Table 1. $w_{\rm eff}$ for different $\{\alpha,\beta\}$ combinations at $z=0$.

EoS parameter	{0.980, 0.320}	{0.976, 0.322}	{0.971, 0.322}	{0.968, 0.321}
$w_{\rm eff}$	−0.9938	−1.0148	−1.0420	−1.0588

Interacting MCG

In the interacting scenario, the conservation equation takes the form

$$a\rho_c'+3\rho_c\left(1+A-\frac{B}{\rho_c^{1+\alpha}}\right)=3\delta\rho_c,$$

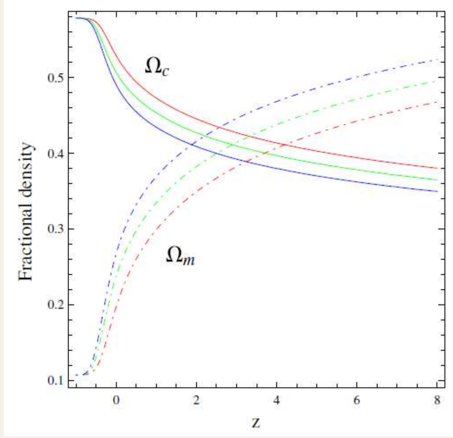
$$a\rho_m'+3\rho_m=-3\delta\rho_c,$$

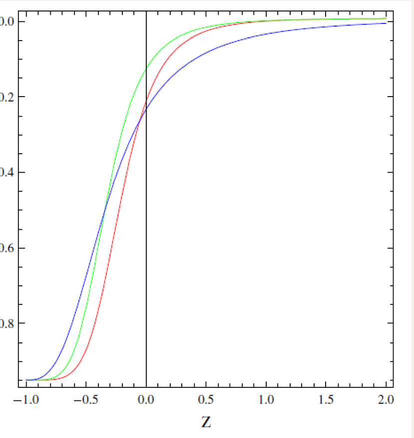
The upper prime implies derivative with respect to scale factor.
The MCG density is reconstructed in the interacting scenario as

$$\rho_c=\left(\frac{B+a^{-3(1+\alpha)(1+A-\delta)}e^{(1+\alpha)(1+A-\delta)C_1}}{1+A-\delta}\right)^{\frac{1}{1+\alpha}}.$$

And EoS is reconstructed to

$$w_c=A-\frac{B(1+A-\delta)}{B+a^{-3(1+\alpha)(1+A-\delta)}e^{(1+\alpha)(1+A-\delta)C_1}}.$$





Interacting MCG in f(T) gravity

In f(T) gravity we have.

$$\rho_T=\frac{1}{2}(2Tf_T-f-T),\qquad w_T=\frac{p_T}{\rho_T}=-1+\frac{4\dot{H}(2Tf_{TT}+f_T-1)}{2Tf_T-f-T}.$$

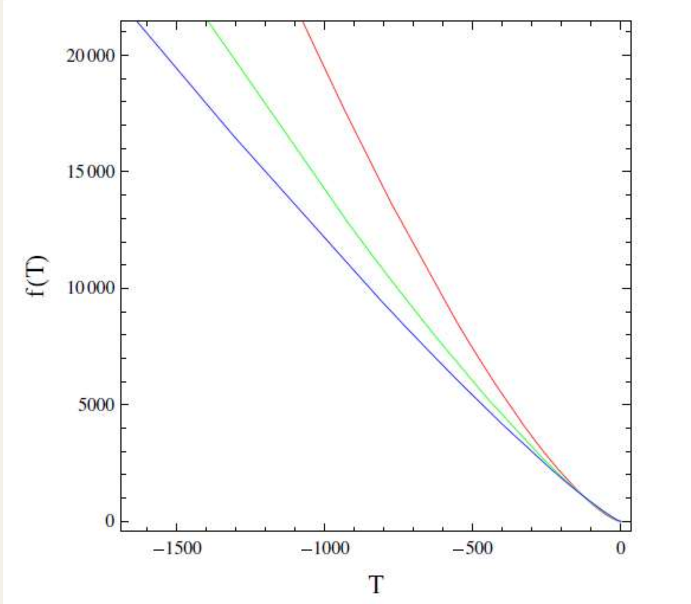
$$p_T=-\frac{1}{2}(-8\dot{H}Tf_{TT}+(2T-4\dot{H})f_T-f+4\dot{H}-T)$$

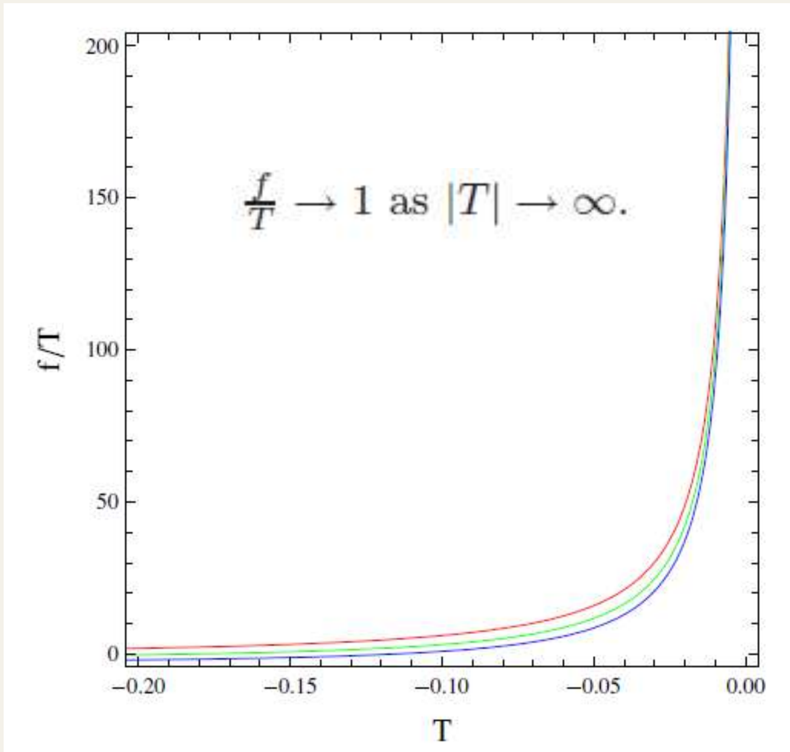
Considering correspondence between MCG and density contribution due to f(T) gravity we have

$$t\frac{df(t)}{dt}+f(t)=\frac{6h_0^2}{t^2}-2\left(\frac{B+(a_0t^{h_0})^{-3(1+\alpha)(1+A-\delta)}e^{(1+\alpha)(1+A-\delta)C_1}}{1+A-\delta}\right)^{\frac{1}{1+\alpha}}$$

Solving we get

$$\begin{aligned}f(T(t))&=\mathcal{F}(t)\\&=\frac{-6h_0^2+C_1t}{t^2}+\frac{2}{-1+3h_0(1+A-\delta)}\\&\quad\times\left(e^{C_1(1+\alpha)(1+A-\delta)}+B(a_0t^{h_0})^{3(1+\alpha)(1+A-\delta)}\right)^{-\frac{1}{1+\alpha}}\\&\quad\times\left(e^{C_1(1+\alpha)(1+A-\delta)}+a_0^3Bt^{3h_0}(a_0t^{h_0})^{3(\alpha+(A-\delta)(1+\alpha))}\right)^{\frac{1}{1+\alpha}}\\&\times\left(1+a_0^3Be^{-C_1(1+\alpha)(1+A-\delta)}t^{3h_0}(a_0t^{h_0})^{3(\alpha+(A-\delta)(1+\alpha))}\right)^{-\frac{1}{1+\alpha}}\\&\times\left(\frac{B+e^{C_1(1+\alpha)(1+A-\delta)}(a_0t^{h_0})^{-3(1+\alpha)(1+A-\delta)}}{1+A-\delta}\right)^{\frac{1}{1+\alpha}}\\&\times 2F1\left[\frac{-3+\frac{1}{h_0(1+A-\delta)}}{3(1+\alpha)},-\frac{1}{1+\alpha},\frac{3\alpha+\frac{1}{h_0(1+A-\delta)}}{3(1+\alpha)},\right.\\&\left.-a_0^3Be^{-C_1(1+\alpha)(1+A-\delta)}t^{3h_0}(a_0t^{h_0})^{3(A+\alpha+A\alpha-(1+\alpha)\delta)}\right],\end{aligned}$$

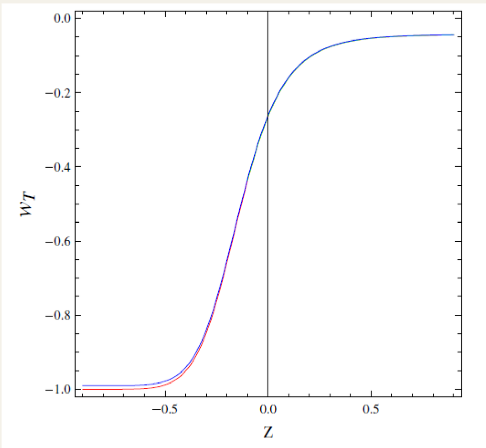




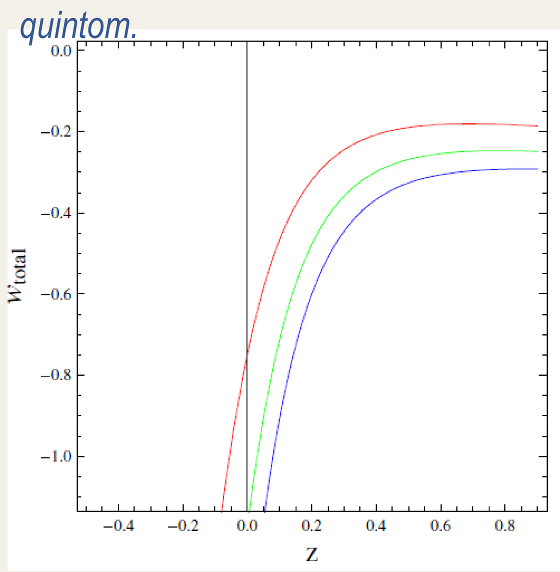
Reconstructed EoS parameter is

$$w_T=-1+\frac{2e^{(1+A)C_1(1+\alpha)}t(a_0t^{h_0})^{3(1+\alpha)\delta}(1+A-\delta)}{(a_0^3Be^{C_1(1+\alpha)\delta}t^{3h_0}(a_0t^{h_0})^{3(A+\alpha+A\alpha)}+e^{(1+A)C_1(1+\alpha)}(a_0t^{h_0})^{3(1+\alpha)\delta})}\times\frac{\left(\frac{Ba_0^3+e^{C_1(1+\alpha)(1+A-\delta)}t^{-3h_0}(a_0t^{h_0})^{-3(A+\alpha+A\alpha-\delta-\alpha\delta)}}{a_0^3(1+A-\delta)}\right)^{\frac{1}{1+\alpha}}}{\left(-C_1+C_2+2t\left(\frac{Ba_0^3+e^{C_1(1+\alpha)(1+A-\delta)}t^{-3h_0}(a_0t^{h_0})^{-3(A+\alpha+A\alpha-\delta-\alpha\delta)}}{a_0^3(1+A-\delta)}\right)^{\frac{1}{1+\alpha}}\right)}.$$

Reconstructed EoS parameters of f(T)



We observed that the reconstructed EoS parameter due to torsion contribution *wT ≥ −1 i.e. it behaves like quintessence. However, wtotal = pT/(ρT+ρm)* shows a clear transition *from wtotal > −1 to wtotal < −1, i.e. from* quintessence to phantom *at z ≈ 0.05. Thus, wtotal behaves like quintom.*



Cosmological evolution of primordial perturbations

We take the matter component to be a canonical scalar field φ with a Lagrangian in the form

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi).$$

In order to understand the evolution of scalar-sector metric perturbations, we use the perturbed equation of motion for the gravitational potential Φ. The complete form of the equation of motion of one Fourier mode Φk is

$$\ddot{\Phi}_k+\alpha\dot{\Phi}_k+\mu^2\Phi_k+c_s^2\frac{k^2}{a^2}\Phi_k=0$$

with

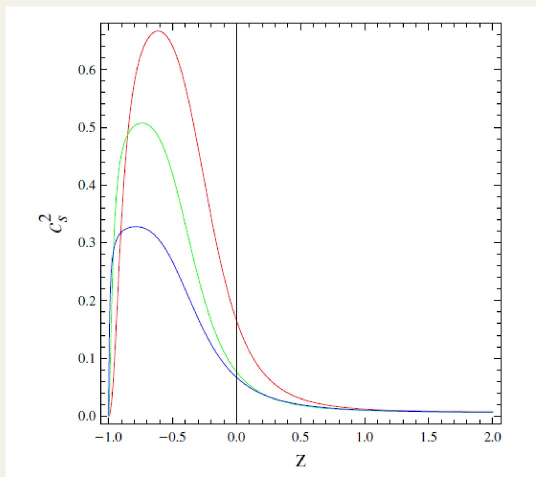
$$\alpha=7H+\frac{2V_\phi}{\dot{\phi}}-\frac{36H\dot{H}(f_{TT}-4H^2f_{TTT})}{1+f_T-12H^2f_{TT}},$$

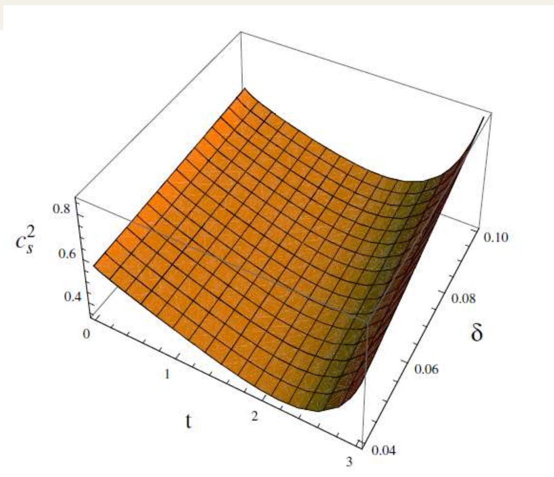
$$\mu^2=6H^2+2\dot{H}+\frac{2HV_\phi}{\dot{\phi}}-\frac{36H^2\dot{H}(f_{TT}-4H^2f_{TTT})}{1+f_T-12H^2f_{TT}},$$

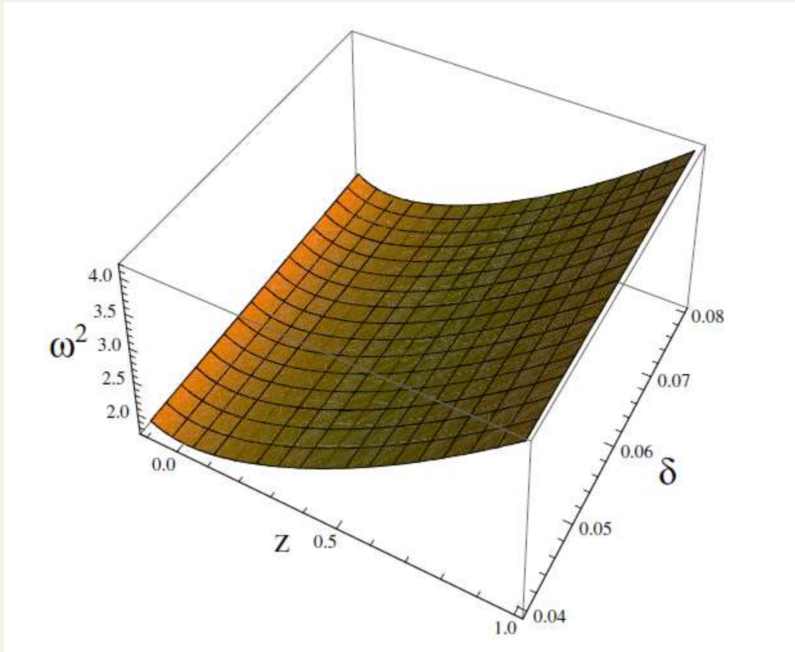
$$c_s^2=\frac{1+f_T}{1+f_T-12H^2f_{TT}}.$$

The scalar field background evolution is

$$\ddot{\phi}+3H\dot{\phi}+V_\phi=0$$







Conclusion

We observed that f/T ⟶ 1 as |T| ⟶ ∞, which is compatible with the primordial nucleosynthesis and CMB constraints.

In this reconstruction phase, we computed both the reconstructed EoS for torsion contribution wT as well as wtotal. We observed that although the wT behaved like quintessence, wtotal showed a clear transition from quintessence to phantom i.e. behaved like quintom. We studied the statefinder diagnostics and observed that for the reconstructed f(T) model it is possible to attain the ΛCDM fixed point and the model interpolated between dust and ΛCDM phase of the universe.

We examined if the pure gravitational sector is stable under this reconstruction scheme and we observed that defining the perturbed torsion scalar from as T = T0 + T1 and subsequently studying a perturbation parameter ω^2 for its time evolution for a range of coupling constant δ, we observed that ω^2 is positive throughout

Acknowledgement

The author acknowledges financial support under the Council of Scientific and Industrial Research (CSIR) (Govt. of India) Grant No. 03(1420)/18/EMR-II