A comparative study of Dirac and Majoarana ultrahigh-energy neutrino oscillations in an interstellar magnetic field

Daniil Abeyadira[1], **Konstantin Kouzakov**[1], **and Alexander Studenikin**[1,2]
1. Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia
2. Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia
Correspondence: abeiadira.dl17@physics.msu.ru; kouzakov@gmail.com; studenik@srd.sinp.msu.ru

Introduction

One of the important developments in the field of neutrino astrophysics is a search for ultrahigh-energy (UHE) cosmic neutrinos (even above PeV–EeV energies). These neutrinos are believed to be produced by reactions of UHE cosmic rays composed of protons and nuclei and are expected to provide information about cosmic accelerators and the high-energy, distant universe. The neutrino massiveness supports the assumption that neutrinos have nonzero electromagnetic characteristics [1]: even though neutrinos are generally believed to be electrically neutral particles they can still have nonzero magnetic moments. This means that the propagation of the UHE cosmic neutrinos can be influenced by the presence of magnetic fields due to the effect of spin oscillations [2]. In this work we focus on the difference between the Dirac and Majorana neutrino propagation in an interstellar magnetic field. Specifically, we examine the differences in the flavor and spin oscillation patterns of the Dirac and Majorana neutrinos.

Dirac neutrino

We limit ourselves to the case of two Dirac neutrino physical states, ν_1 and ν_2, with masses m_1 and m_2. For treating neutrino evolution in the presence of a uniform magnetic field B and homogeneous matter in the ultrarelativistic limit, we employ a four-component basis of the helicity states $\nu_{1,s=\pm1}$ and $\nu_{2,s=\pm1}$. The Schrödinger-like evolution equation is then given by

$$i\frac{d}{dt}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\\ \nu_{2,s=1}\\ \nu_{2,s=-1}\end{pmatrix} = H_{\rm eff}\begin{pmatrix}\nu_{1,s=1}\\ \nu_{1,s=-1}\\ \nu_{2,s=1}\\ \nu_{2,s=-1}\end{pmatrix}, \qquad H_{\rm eff} = H_{\rm vac} + H_{\rm mat} + H_B. \tag{1}$$

The effective Hamiltonian $H_{\rm eff}$ consists of the vacuum $H_{\rm vac}$ and interaction part H_B, corresponding to the neutrino interaction with a magnetic field (since the matter density in the interstellar space is $n_e \lesssim 10^6$ cm^{-3}, we neglect $H_{\rm mat}$). In the mass representation, the vacuum Hamiltonian and Hamiltonian of the neutrino interaction with a magnetic field acquire the forms

$$H^m_{vac} = \omega\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \qquad H^m_B = \begin{pmatrix}-\mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{11}B_\perp & -\mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp\\ \mu_{11}B_\perp & \mu_{11}\frac{B_\parallel}{\gamma_{11}} & \mu_{12}B_\perp & \mu_{12}\frac{B_\parallel}{\gamma_{12}}\\ -\mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{12}B_\perp & -\mu_{22}\frac{B_\parallel}{\gamma_{22}} & \mu_{22}B_\perp\\ \mu_{12}B_\perp & \mu_{12}\frac{B_\parallel}{\gamma_{12}} & \mu_{22}B_\perp & \mu_{22}\frac{B_\parallel}{\gamma_{22}}\end{pmatrix}, \tag{2}$$

where $\omega = \frac{\Delta m^2}{4E_\nu}$, $\Delta m^2 = m_2^2 - m_1^2$, E_ν is the neutrino energy, $B_\parallel$ and $B_\perp$ are the parallel and transverse magnetic-field components with respect to the neutrino velocity, and μ_{jk} $(j,k=1,2)$ are magnetic moments in the mass representation. Here γ_1 and γ_2 are the Lorentz factors of the massive neutrinos, and $\frac{1}{\gamma_{12}} = \frac{1}{2}\left(\frac{1}{\gamma_1}+\frac{1}{\gamma_2}\right)$. The evolution equation (1) in the masss basis leads to a homogeneous system of first-order linear differential equations, which is equivalent to a fourth-order homogeneous linear differential equation and can be solved analytically [2].

We neglect the neutrino interaction with the longitudinal magnetic-field component, setting $(\mu/\gamma)B_\parallel = 0$. The latter is justified by large γ values for UHE neutrinos. Also in this study, the transition magnetic moments of Dirac neutrinos are zeroed, i.e., $\mu_{12} = \mu_{21} = 0$. If the initial state of the neutrino is $\nu^f(0) = \nu^L_\mu$, the flavor-change probability $P^D_{\nu^L_\mu\to\nu^L_e}$ for the Dirac neutrino propagating in vacuum is determined by the following formula:

$$P^D_{\nu^L_\mu\to\nu^L_e} = \sin^2 2\theta \sin^2\left(\frac{\pi x}{L_{vac}}\right)\cos^2\left(\frac{\pi x}{L_B}\right), \tag{3}$$

where x is the neutrino propagation distance, $L_{vac} = \frac{4\pi E_\nu}{\Delta m^2}$ and $L_B = \frac{\pi}{\mu_\nu B}$ are vacuum and magnetic oscillation lengths respectively. In the case of unequal diagonal magnetic moments $\mu_{11} \neq \mu_{22}$ the probability acquires the form

$$P^D_{\nu^L_\mu\to\nu^L_e} = \frac{1}{4}\sin^2 2\theta\left[\cos^2\left(\frac{\pi x}{L_{B_1}}\right) + \cos^2\left(\frac{\pi x}{L_{B_2}}\right) - 2\cos\left(\frac{\pi x}{L_{B_1}}\right)\cos\left(\frac{\pi x}{L_{B_2}}\right)\cos\left(\frac{2\pi x}{L_{vac}}\right)\right], \tag{4}$$

where L_{B_1} and L_{B_2} are the magnetic oscillation lengths for neutrinos with different magnetic moments μ_{11} and μ_{22}. For spin oscillations, the transition probability $P_{\nu^L\to\nu^R}$ in the case of equal magnetic moments is determined by the formula:

$$P^D_{\nu^L\to\nu^R} = \sin^2\left(\frac{\pi x}{L_B}\right). \tag{5}$$

In the case of different magnetic moments $\mu_{11} \neq \mu_{22}$ one has

$$P^D_{\nu^L\to\nu^R} = \sin^2\left(\frac{\pi x}{L_{vac}}\right)\left[\cos^2\theta\sin\left(\frac{\pi x}{L_{B_1}}\right) - \sin^2\theta\sin\left(\frac{\pi x}{L_{B_2}}\right)\right] + \cos^2\theta\left[\sin^2\theta\sin\left(\frac{\pi x}{L_{B_2}}\right) + \cos^2\theta\sin\left(\frac{\pi x}{L_{B_1}}\right)\right] + \frac{1}{4}\sin^2 2\theta\left[\sin^2\left(\frac{\pi x}{L_{B_1}}\right) + \sin^2\left(\frac{\pi x}{L_{B_2}}\right) - 2\sin\left(\frac{\pi x}{L_{B_1}}\right)\sin\left(\frac{\pi x}{L_{B_2}}\right)\cos\left(\frac{2\pi x}{L_{vac}}\right)\right]. \tag{6}$$

Majorana neutrino

When considering Majorana neutrinos, the main difference in comparison with Dirac neutrinos consists not only in the interaction of the right-handed Majorana neutrinos with matter, but also in the general properties of their magnetic moments μ^M_{ij}. The matrix of magnetic moments for Majorana neutrinos is antisymmetric and Hermitian, so that in the discussed case of two neutrino mixing, the diagonal magnetic moments vanish $\mu^M_{11} = \mu^M_{22} = 0$, and the transition magnetic moments are opposite in sign and are purely imaginary: $\mu^M_{12} = -\mu^M_{21} = -(\mu^M_{12})^*$. Since the magnetic moments take purely imaginary values, they can be parameterized using a putative magnetic moment μ_ν as follows: $\mu^M_{12} = \pm i\mu_\nu$. The probabilities of flavor and spin oscillations for Majorana neutrinos are then given by

$$P^M_{\nu^L_\mu\to\nu^L_e} = \left(\frac{\tilde{L}}{L_{vac}}\right)^2\sin^2 2\theta\sin^2\left(\frac{\pi x}{\tilde{L}}\right), \qquad P^M_{\nu^L\to\nu^R} = \left(\frac{\tilde{L}}{L_B}\right)^2 sin^2\left(\frac{\pi x}{\tilde{L}}\right), \tag{7}$$

where $\tilde{L} = \frac{L_{vac}L_B}{\sqrt{L^2_{vac}+L^2_B}}$. In contrast to the Dirac case, both probabilities oscillate with the same period.

Dirac vs Majorana

We present the results for the probabilities of flavor and spin oscillations of Dirac and Majorana neutrinos at three different energies: $E_\nu = 10^{18}$ eV, $E_\nu = 10^{20}$ eV and $E_\nu = 10^{22}$ eV. The magnetic field strength is set to the value $B = 2.93\,\mu$G [3]. The putative magnetic moment value is chosen to be $\mu_\nu = 2.2\times10^{-12}\mu_B$ that corresponds to the upper astrophysical limit obtained from observations of red giants CL (90 %) [4]. The squared mass difference is taken from solar neutrino measurements, $\Delta m^2 = \Delta m^2_{sol} = 7.53\times10^{-5}$ eV [5]. All numerical calculations were performed in the case when the initial state of the neutrino is $\nu^f(0) = \nu^L_\mu$.

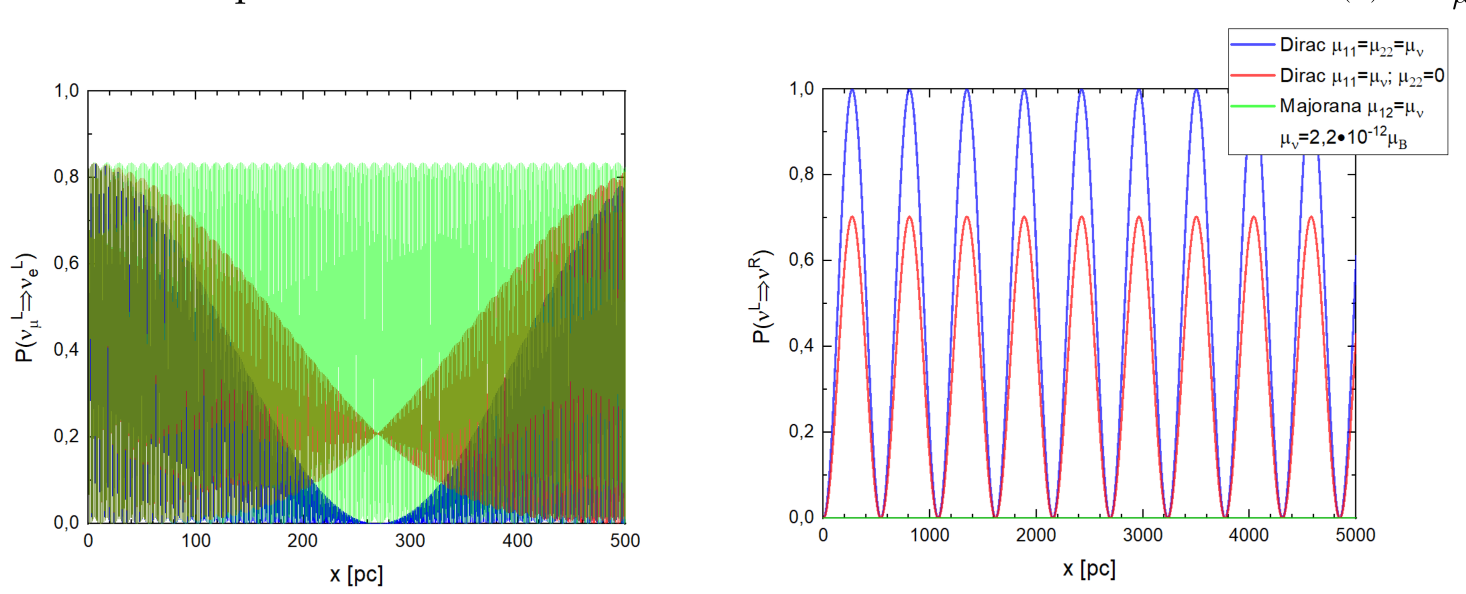

Figure 1: The neutrino flavor-change (left panel) and spin-flip (right panel) probabilities as functions of the distance x traveled by an 1-EeV neutrino interacting with an interstellar magnetic field.

The behavior of flavor-change probability for Dirac neutrinos is governed by the L_B to L_{vac} relation. If the neutrino energy is $E_\nu = 10^{18}$ eV, for Dirak neutrinos the length of the vacuum oscillations is much less than the length of the magnetic oscillations $L_B \gg L_{vac}$, and therefore the probability exhibits fast oscillations with the period L_{vac} which are modulated by a slowly changing envelope curve that depends on the μ_ν value. The spin-flip probability for Dirac neutrinos is affected only by the value of L_B.

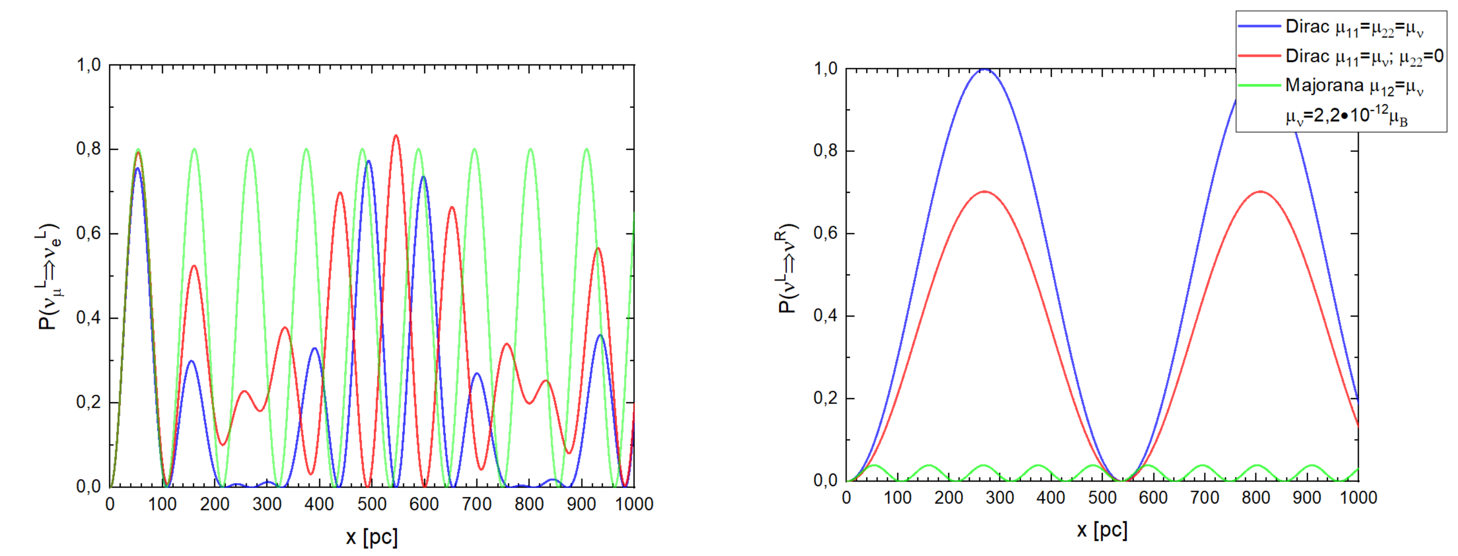

Figure 2: The same as in Fig. 1, but when $E_\nu = 100$ EeV.

For Majorana neutrinos both flavor-change and spin-flip probabilities oscillate with the same period $\tilde{L}$, but are proportional to the $\frac{\tilde{L}}{L_{vac}}$ and $\frac{\tilde{L}}{L_B}$ ratios for flavor-change and spin-flip probabilities respectively. Thus, for neutrino energies of the order of $E_\nu \sim 10$ ZeV, the probability of flavor oscillations practically vanishes.

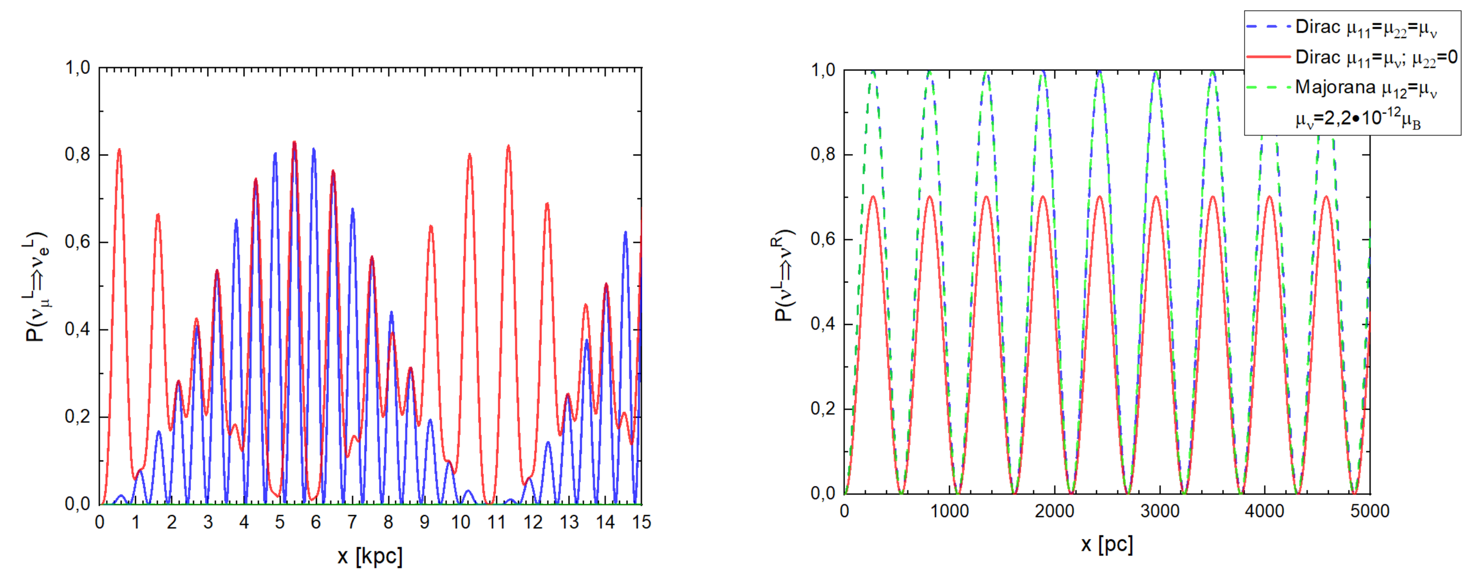

Figure 3: The same as in Fig. 1, but when $E_\nu = 10$ ZeV.

Summary

- The effective Schrödinger equation was solved for both Dirac and Majorana neutrinos with a nonzero magnetic moment propagating in an interstellar magnetic field.
- The probabilities of neutrino spin and flavor oscillations were derived.
- The numerical results for these probabilities were presented at neutrino energies below, around and above the Greisen-Zatsepin-Kuzmin limit ($E_\nu = 10^{18}$ eV, $E_\nu = 10^{20}$ eV and $E_\nu = 10^{22}$ eV, respectively). The marked differences between the Dirac and Majorana cases are outlined.

Acknowledgements

The work is supported by the Interdisciplinary Scientific and Educational School of Moscow University "Fundamental and Applied Space Research" and by the Russian Foundation for Basic Research under grant no. 20-52-53022-GFEN-A

References

[1] C. Giunti and A. Studenikin, *Neutrino electromagnetic interactions: A window to new physics*, Rev. Mod. Phys. **87**, 531 (2015), arXiv:1403.6344 [hep-ph]

[2] P. Kurashvili, K. A. Kouzakov, L. Chtorlishvili, and A. I. Studenikin, *Spin-flavor oscillations of ultrahigh-energy cosmic neutrinos in interstellar space: The role of neutrino magnetic moments*, Phys. Rev. D **96**, 103017 (2017), arXiv:1711.04303 [hep-ph]

[3] E. J. Zirnstein, J. Heerikhuisen, H. O. Funsten, G. Livadiotis, D. J. McComas, and N. V. Pogorelov, *Local interstellar magnetic field determined from the interstellar boundary explorer ribbon*, Astrophys. J. Lett. **818**, L18 (2016), arXiv:1612.02339v1 [physics.space-ph]

[4] S. Arceo-Dıaz, K.-P. Schröder, K. Zuber, D. Jack, and E. E. Bricio Barrios, *Constraint on the axion-electron coupling constant and the neutrino magnetic dipole moment using the tip-RGB luminosity of 50 globular clusters*, arXiv:1910.10568v1 [astro-ph.SR]

[5] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)