17th International Conference on Topics in Astroparticle and Underground Physics (26 August - 3 September, 2021)

Collective Neutrino Oscillations in Moving and Polarized Matter

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1 Introduction

Future large volume detectors, such as JUNO, Hyper-Kamiokande and DUNE, open a new era in studies of supernova and presupernova neutrinos [1, 2]. Astrophysical neutrinos would give important information for the explosion mechanism of collapse-driven supernovae. Our present study deals with the neutrino evolution in astrophysical environments. In most papers devoted to the collective neutrino oscillations, one usually considers transitions between flavour states without change of helicity. The authors of [3, 4] for the first time studied collective neutrino oscillations accounting for transitions between states with different helicity engendered by interaction of the neutrino magnetic moment with a magnetic field. Later, the problem of spin oscillations in collective effects was studied by different groups (see, for example, [5] and references therein). In [6] the effect of spin oscillations engendered by transversally moving matter was included in collective neutrino oscillations. In [7, 8] collective neutrino oscillations were studied in the case of neutrino nonstandard interactions. Below we summarize all effects that can lead to the neutrino spin precession in collective neutrino oscillations in the presence of moving and polarized matter.

2 Neutrino evolution in astrophysical environment

Consider two flavor neutrinos with two possible helicities $\nu_f = (\nu_e^-, \nu_x^-, \nu_e^+, \nu_x^+)$, where ν_x stands for ν_μ or ν_τ. Then the neutrino system is described by the density matrix (we follow the formalism developed in [3, 4])

$$\rho = \begin{pmatrix} \rho_\nu & X \\ X^\dagger & \rho_{\bar\nu} \end{pmatrix}, \tag{1}$$

where ρ and $\rho_{\bar\nu}$ are the usual 2×2 flavour density matrices. In the case of Dirac neutrinos, the matrices describe an active and a sterile neutrino, respectively. In the case of Majorana neutrinos, $\rho_{\bar\nu}$ describes an antineutrino. The external magnetic field and the transversally moving matter leads to coupling between ρ and $\rho_{\bar\nu}$ (see [10] and references therein) and, therefore, we should also consider non-diagonal matrices X:

$$X = \begin{pmatrix} \rho_{\nu_e\bar\nu_e} & \rho_{\nu_e\bar\nu_x} \\ \rho_{\nu_x\bar\nu_e} & \rho_{\nu_x\bar\nu_x} \end{pmatrix}. \tag{2}$$

The evolution of ρ is governed by the Liouville-von Neumann master equation

$$i\frac{d\rho}{dt} = [H, \rho], \tag{3}$$

where H is the Hamiltonian that describes the external environment. It consists of five parts

$$H = H_{vac} + H_B + H_{mat} + H_\zeta^f + H_{\nu\nu}, \tag{4}$$

where H_{vac} is the vacuum Hamiltonian, H_B is the Hamiltonian that accounts for the magnetic field, H_{mat} is the matter potential for electrons and neutrons moving in an arbitrary direction, H_ζ^f is the Hamitonian that accounts for the matter polarization.
The neutrino-neutrino interaction potential, $H_{\nu\nu}$, is

$$H_{\nu\nu} = \sqrt{2}G_F n_\nu \int dE \left[G^\dagger(\rho(E) - \rho(E)^{c*})G + \frac{1}{2}G^\dagger \operatorname{Tr}\left[(\rho(E) - \rho(E)^{c*})G\right] \right], \tag{5}$$

with G_F being the Fermi coupling constant and n_ν being a neutrino density profile. The density matrix ρ^c is defined in the same way as in [5]

$$\rho^c = \begin{pmatrix} \rho_{\bar\nu} & X^* \\ X^T & \rho_\nu \end{pmatrix}. \tag{6}$$

The dimensionless matrix G is

$$G = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{7}$$

The Hamiltonian that accounts for the neutrino magnetic moment interaction with parallel and perpendicular components of the magnetic field $B = B_{||} + B_\perp$ has the following form:

$$H_B^D = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\mu_{ee}B_\perp e^{i\phi} & -\mu_{ex}B_\perp e^{i\phi} \\ \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & \left(\frac{\mu}{\gamma}\right)_{xx} B_{||} & -\mu_{ex}B_\perp e^{i\phi} & -\mu_{xx}B_\perp e^{i\phi} \\ -\mu_{ee}B_\perp e^{-i\phi} & -\mu_{ex}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} \\ -\mu_{ex}B_\perp e^{-i\phi} & -\mu_{xx}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{xx} B_{||} \end{pmatrix}, \tag{8}$$

where ϕ is the angle between $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$.
If the neutrino is a Majorana particle, the diagonal magnetic moments are zero in both flavour and mass basis. Therefore, the Hamiltonian for a Majorana neutrino in a magnetic field is expressed as

$$H_B^M = i\mu\cos 2\theta \begin{pmatrix} 0 & \frac{1}{\gamma_{12}}B_{||} & 0 & -B_\perp e^{i\phi} \\ -\frac{1}{\gamma_{12}}B_{||} & 0 & B_\perp e^{i\phi} & 0 \\ 0 & -B_\perp e^{-i\phi} & 0 & -\frac{1}{\gamma_{12}}B_{||} \\ B_\perp e^{-i\phi} & 0 & \frac{1}{\gamma_{12}}B_{||} & 0 \end{pmatrix}. \tag{9}$$

The magnetic moments in the flavour basis $\mu_{\alpha\beta}$ are expressed in terms of the magnetic moments μ_{ij} in the mass basis as

$$\begin{aligned} \mu_{ee} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \\ \mu_{ex} &= \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11})\sin 2\theta, \\ \mu_{xx} &= \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta, \end{aligned} \tag{10}$$

(for the neutrino interaction with $B_\perp$) and

$$\begin{aligned} \left(\frac{\mu}{\gamma}\right)_{ee} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{ex} &= \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{xx} &= \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \end{aligned} \tag{11}$$

(for the neutrino interaction with $B_{||}$). Here we have introduced the following notations:

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}, \quad \gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_\beta^{-1}\right), \quad \tilde\gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_\beta^{-1}\right). \tag{12}$$

The Hamiltonian that accounts for the electron and neutron matter is

$$H_{mat}^D = \frac{G_F}{2\sqrt{2}} \begin{pmatrix} 2(2n_e - n_n)\left(1 - v_{||}\right) & 0 & (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} \\ 0 & -2n_n\left(1 - v_{||}\right) & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{xx} \\ (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & 0 & 0 \\ (2n_e - n_n)v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{xx} & 0 & 0 \end{pmatrix}, \tag{13}$$

where n_n and n_e are the neutron and electron density profiles $n_{n(e)} = \frac{n^0_{n(e)}}{\sqrt{1-v^2}}$, the electron and neutron velocity is $v = v_{||} + v_\perp$ and

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{xx} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{ex} = \frac{\sin 2\theta}{\tilde\gamma_{21}}. \tag{14}$$

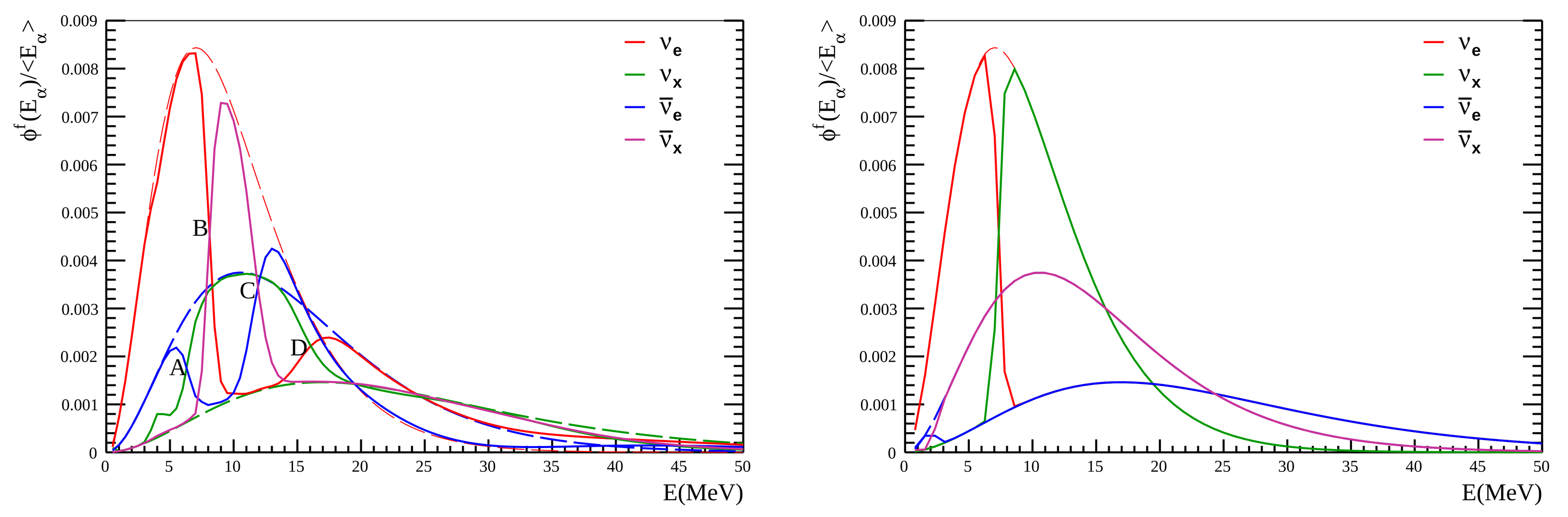
The matter polarization is taken into account by the following Hamiltonian in the flavour basis:

$$H_\zeta^f = n_e\frac{G_F}{2\sqrt{2}} \begin{pmatrix} 0 & (\frac{\eta}{\gamma})_{ee}\zeta_\perp & 0 & (\frac{\eta}{\gamma})_{e\mu}\zeta_\perp \\ (\frac{\eta}{\gamma})_{ee}\zeta_\perp & 2\zeta_{||} & (\frac{\eta}{\gamma})_{e\mu}\zeta_\perp & 0 \\ 0 & (\frac{\eta}{\gamma})_{e\mu}\zeta_\perp & 0 & (\frac{\eta}{\gamma})_{\mu\mu}\zeta_\perp \\ (\frac{\eta}{\gamma})_{e\mu}\zeta_\perp & 0 & (\frac{\eta}{\gamma})_{\mu\mu}\zeta_\perp & 2\zeta_{||} \end{pmatrix}, \tag{15}$$

where $\zeta_{||}$ and $\zeta_\perp$ are the longitudinal and transversal polarization of the matter with respect to the direction of neutrino propagation..

3 The numerical solution for transversally moving matter

The numerical solution for the collective neutrino oscillations accounting for the transversal matter current is presented in fig. 1.



a) b)

Figure 1: Initial (dashed, $r = 50$ km) and final (solid, $r = 200$ km) supernova neutrino spectra for different neutrino species as a function of the neutrino energy for the normal (left) and inverted (right) mass ordering in the case of Majorana neutrinos. The average energies of ν_e, $\bar\nu_e$ and ν_x are taken as 10 MeV, 15 MeV, and 24 MeV, respectively. Note that the bulb model for the supernova neutrino emission and single-angle approximation have been used in the numerical calculation, and A, B, C, D in the left panel are cross points for illustration.

4 Acknowledgments

This research has been supported by the Interdisciplinary Scientific and Educational School of Moscow University "Fundamental and Applied Space Research" and also by the Russian Foundation for Basic Research under Grant No. 20-52-53022-GFEN-a. The work of KS is also supported by the RFBR under grant No. 20-32-90107 and by the BASIS?Foundation No. 20-2-2-3-1.

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