

Dirac and Majorana neutrino oscillations in magnetized moving and polarized matter

**Umsalimat Abdullaeva¹, Vadim Shakhov¹, Alexander Studenikin^{1,2},
Alexander Tsvirov¹**

1. Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia

2. Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia

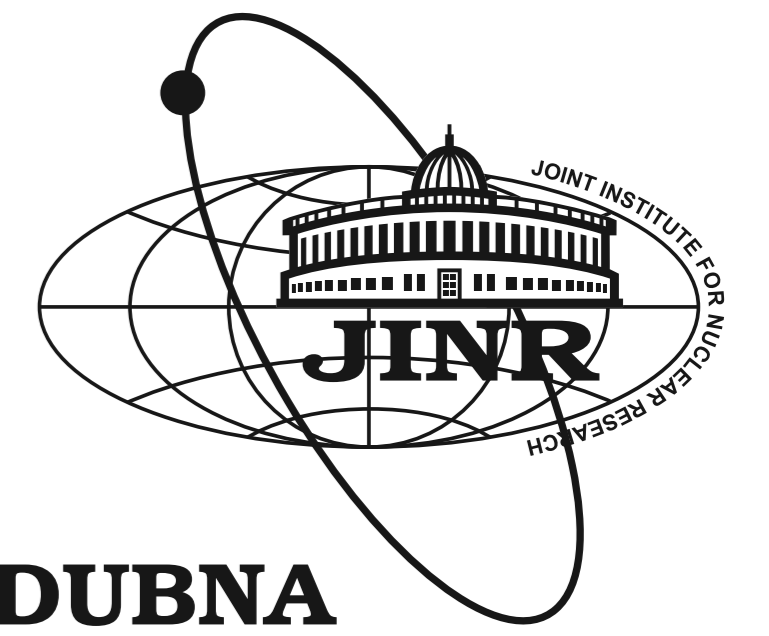
aleksander.tsvirov@yandex.ru; schakhov.vv15@physics.msu.ru; Abdullaeva.um@gmail.com; studenik@srd.sinp.msu.ru

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The history of neutrino spin oscillations in transversal matter currents and transversally polarized matter

For many years, until 2004, it was believed that a neutrino helicity precession and the corresponding spin oscillations can be induced by the neutrino magnetic interactions with an external electromagnetic field that provided the existence of the transversal magnetic field component B_{\perp} in the particles rest frame. A new and very interesting possibility for neutrino spin (and spin-flavour) oscillations engendered by the neutrino interaction with matter background was proposed and investigated for first time in [?]. It was shown [?] that neutrino spin oscillations can be induced not only by the neutrino interaction with a magnetic field, as it was believed before, but also by neutrino interactions with matter in the case when there is a transversal matter current or matter polarization. This new effect has been explicitly highlighted in [?]:

"The possible emergence of neutrino spin oscillations owing to neutrino interaction with matter under the condition that there exists a nonzero transverse current component or matter polarization is the most important new effect that follows from the investigation of neutrino-spin oscillations in Section 4. So far, it has been assumed that neutrino-spin oscillations may arise only in the case where there exists a nonzero transverse magnetic field in the neutrino rest frame."

For historical notes reviewing studies of the discussed effect see in [?, ?, ?]. It should be noted that the predicted effect exist regardless of a source of the background matter transversal current or polarization (that can be a background magnetic field, for instance). Note that the existence of the discussed effect of neutrino spin oscillations engendered by the transversal matter current and matter polarization and its possible impact in astrophysics have been confirmed in a series of papers [?, ?, ?, ?]. In our recent paper [?, ?] we have developed a consistent quantum treatment of the neutrino spin and spinflavor oscillations engendered by the transversal matter currents. The presence of the transversal and longitudinal magnetic fields as well as the longitudinal matter currents are accounted for. The developed treatment [?] also allows to account neutrino nonstandard interactions. In addition, different possibilities for the resonance amplification of these kind of neutrino spin and spin-flavor oscillations are considered. Here below [?] we consider the quantum treatment to take into account the matter polarization in neutrino oscillations. Also [?] we consider the case of Majorana neutrino.

Neutrino spin oscillations $\nu_e^L \Leftarrow (j_{\perp}, \zeta_{\perp}) \Rightarrow \nu_e^R$ engendered by transversal matter current and polarization: semiclassical treatment

Following the discussion in [?] consider, as an example, an electron neutrino spin precession in the case when neutrinos with the Standard Model interaction are propagating through moving and polarized matter composed of electrons (electron gas) in the presence of an electromagnetic field given by the electromagnetic-field tensor $F_{\mu\nu} = (\mathbf{E}, \mathbf{B})$. To derive the neutrino spin oscillation probability in the transversal matter current we use the generalized Bargmann-Michel-Telegdi equation that describes the evolution of the three-dimensional neutrino spin vector \mathbf{S} ,

$$\frac{d\mathbf{S}}{dt} = \frac{2}{\gamma} [\mathbf{S} \times (\mathbf{B}_0 + \mathbf{M}_0)], \quad (1)$$

where the magnetic field \mathbf{B}_0 in the neutrino rest frame is determined by the transversal and longitudinal (with respect to the neutrino motion) magnetic and electric field components in the laboratory frame,

$$\mathbf{B}_0 = \gamma \left(\mathbf{B}_{\perp} + \frac{1}{\gamma} \mathbf{B}_{\parallel} + \sqrt{1 - \gamma^{-2}} \left[\mathbf{E}_{\perp} \times \frac{\boldsymbol{\beta}}{\beta} \right] \right) \quad (2)$$

$\gamma = (1 - \beta^2)^{-1/2}$, $\boldsymbol{\beta}$ is the neutrino velocity. The matter term \mathbf{M}_0 in Eq. (??) is also composed of the transversal $\mathbf{M}_{0\perp}$ and longitudinal $\mathbf{M}_{0\parallel}$ parts.

$$\mathbf{M}_0 = \mathbf{M}_{0\parallel} + \mathbf{M}_{0\perp} \quad (3)$$

$$\mathbf{M}_{0\parallel} = \gamma \beta \frac{n_0}{\sqrt{1 - v_e^2}} \left\{ \rho_e^{(1)} \left(1 - \frac{\mathbf{v}_e \boldsymbol{\beta}}{1 - \gamma^{-2}} \right) - \frac{\rho_e^{(2)}}{1 - \gamma^{-2}} \left\{ \zeta_e \beta \sqrt{1 - v_e^2} + \left(\zeta_e \mathbf{v}_e \frac{(\boldsymbol{\beta} \mathbf{v}_e)}{1 + \sqrt{1 - v_e^2}} \right) \right\} \right\}, \quad (4)$$

$$\mathbf{M}_{0\perp} = -\frac{n_0}{\sqrt{1 - v_e^2}} \mathbf{v}_{e\perp} \left(\rho_e^{(1)} + \rho_e^{(2)} \frac{(\boldsymbol{\beta} \mathbf{v}_e)}{1 + \sqrt{1 - v_e^2}} \right) + \zeta_{e\perp} \rho_e^{(2)} \sqrt{1 - v_e^2} \quad (5)$$

Here $n_0 = n_e \sqrt{1 - v_e^2}$ is the invariant number density of matter given in the referenceframe for which the total speed of matter is zero. The vectors \mathbf{V}_e , and ζ_e ($0 \leq |\zeta_e|^2 \leq 1$) denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ calculated within the extended Standard Model supplied with SU(2)-singlet right-handed neutrino ν_R are respectively, $\rho_e^{(1)} = \frac{G_F}{2\sqrt{2}\mu}$, $\rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}$, where $G_F = G_F (1 + 4 \sin^2 \theta_W)$.

For neutrino evolution between two neutrino states $\nu_e^L \Leftarrow \nu_e^R$ in presence of the magnetic field and moving matter we get [?] the following equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} = \mu \begin{pmatrix} \frac{1}{2} |\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}| & |\mathbf{B}_{\perp} + \frac{1}{2} \mathbf{M}_{0\perp}| \\ |\mathbf{B}_{\perp} + \frac{1}{2} \mathbf{M}_{0\perp}| & -\frac{1}{2} |\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}| \end{pmatrix} \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix}. \quad (6)$$

Thus, the probability of the neutrino spin oscillations in the adiabatic approximation is given by

$$P_{\nu_e^L \rightarrow \nu_e^R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2 \frac{\pi x}{L_{\text{eff}}}, \quad \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad (7)$$

$$L_{\text{eff}} = \frac{\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}},$$

where

$$E_{\text{eff}} = \mu |\mathbf{B}_{\perp} + \frac{1}{2} \mathbf{M}_{0\perp}|, \quad \Delta_{\text{eff}} = \frac{\mu}{\gamma} |\mathbf{M}_{0\parallel} + \mathbf{B}_{0\parallel}|. \quad (8)$$

From this it follows [1] that even in the absence of the transversal magnetic field the neutrino spin oscillations can appear due to the transversal matter current and matter polarization.

Neutrino spin oscillations $\nu_e^L \Leftarrow (j_{\perp}, \zeta_{\perp}) \Rightarrow \nu_e^R$ engendered by transversal matter currents and transversal matter polarization: quantum treatment

Here below we continue our studies of the effect of neutrino spin evolution induced by the transversal matter currents and transversal matter polarization develop a consistent derivation of the effect based on the direct calculation of the spin evolution effective Hamiltonian in the case when a neutrino is propagating in polarized moving matter.

Consider two flavour neutrinos with two possible helicities $\nu_f = (\nu_e^+, \nu_e^-, \nu_{\mu}^+, \nu_{\mu}^-)^T$ in moving matter composed of electrons. Also we consider that neutrino moves along electron current and the polarization of the neutrino directed against the magnetic field direction. The neutrino interaction Lagrangian reads

$$L_{\text{int}} = -f^{\mu} \sum_i \bar{\nu}_i(x) \gamma_{\mu} \frac{1 - \gamma_5}{2} \nu_i(x) = -f^{\mu} \sum_i \bar{\nu}_i(x) \gamma_{\mu} \frac{1 - \gamma_5}{2} \nu_i(x), \quad (9)$$

where $f^{\mu} = \frac{G_F}{2\sqrt{2}} (1 + 4 \sin^2 \theta_W) j_e^{\mu} - \lambda_e^{\mu}$, $j_e^{\mu} = n_0(1, \mathbf{v})$ - matter current, $\lambda_e^{\mu} = n(\zeta_e, \mathbf{v}_e)$, $\zeta_e \sqrt{1 - v_e^2} + \mathbf{v}_e \frac{(\boldsymbol{\zeta}_e \mathbf{v}_e)}{1 + \sqrt{1 - v_e^2}}$ - polarization vector, $l = e, \mu$ indicates the neutrino flavour and $i = 1, 2$ indicates the neutrino mass state. Each of the flavour neutrinos is a superposition of the neutrino mass states,

$$\nu_e^{\pm} = \nu_1^{\pm} \cos \theta + \nu_2^{\pm} \sin \theta, \quad \nu_{\mu}^{\pm} = -\nu_1^{\pm} \sin \theta + \nu_2^{\pm} \cos \theta. \quad (10)$$

The neutrino evolution equation in the flavour basis is

$$i \frac{d}{dt} \nu_f = \left(H_0^{eff} + \Delta H_{\text{weak}}^{eff} + \Delta H_{\zeta} \right) \nu_f, \quad (11)$$

where the effective Hamiltonian consists of the vacuum part, weak interaction with matter and interaction with matter polarization parts:

$$H^{eff} = H_0^{eff} + \Delta H_{\text{weak}}^{eff} + \Delta H_{\zeta}. \quad (12)$$

ΔH_{ζ}^{eff} can be expressed as

$$\Delta H_{\zeta}^{eff} = \begin{pmatrix} \Delta_{ee}^{++} & \Delta_{ee}^{+-} & \Delta_{e\mu}^{++} & \Delta_{e\mu}^{+-} \\ \Delta_{ee}^{-+} & \Delta_{ee}^{--} & \Delta_{e\mu}^{-+} & \Delta_{e\mu}^{--} \\ \Delta_{\mu e}^{++} & \Delta_{\mu e}^{+-} & \Delta_{\mu\mu}^{++} & \Delta_{\mu\mu}^{+-} \\ \Delta_{\mu e}^{-+} & \Delta_{\mu e}^{--} & \Delta_{\mu\mu}^{-+} & \Delta_{\mu\mu}^{--} \end{pmatrix} \quad (13)$$

where

$$\Delta_{kl}^{ss'} = \langle \nu_k^s | H_{\zeta}^{eff} | \nu_l^{s'} \rangle \quad (14)$$

$k, l = e, \mu, s, s' = \pm$. In evaluation of $\Delta_{kl}^{ss'}$ we have first introduced the neutrino flavour states ν_k^s and $\nu_l^{s'}$ as superpositions of the mass states $\nu_{1,2}^{\pm}$. Then, using the exact free neutrino mass states spinors,

$$\nu_{\alpha}^s = C_{\alpha} \begin{pmatrix} u_{\alpha}^s \\ \frac{\sigma_{\mu\nu} v_{\alpha}^s}{E_{\alpha} + m_{\alpha}} \end{pmatrix} \sqrt{\frac{E_{\alpha} + m_{\alpha}}{2E_{\alpha}}} \exp(i\mathbf{p}_{\alpha} \mathbf{x}) \quad (15)$$

where the two-component spinors define neutrino helicity states, and are given by

$$u_{\alpha}^+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_{\alpha}^- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (16)$$

for the typical term $\Delta_{\alpha\alpha'}^{ss'}$ = $\langle \nu_{\alpha}^s | \Delta H_{\zeta}^{eff} | \nu_{\alpha'}^{s'} \rangle$, that by fixing proper values of α, s, α' and s' it reproduces all of the elements of the neutrino evolution Hamiltonian ΔH_{ζ}^{eff} that accounts for the effect of matter polarization, we obtain,

$$\Delta_{\alpha\alpha'}^{ss'} = G_n \left\{ u_{\alpha}^{sT} \left[\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \zeta_{\parallel} + \begin{pmatrix} 0 & \gamma_{\alpha}^{-1} \\ \gamma_{\alpha'}^{-1} & 0 \end{pmatrix} \zeta_{\perp} \right] u_{\alpha'}^{s'} \right\} \delta_{\alpha\alpha'}, \quad (17)$$

where ζ_{\parallel} and ζ_{\perp} are the longitudinal and transversal polarization of the matter, $G = \frac{G_F}{2\sqrt{2}}$, $n = \frac{n_0}{\sqrt{1 - v_e^2}}$ and

$$\gamma_{\alpha\alpha'}^{-1} = \frac{1}{2} (\gamma_{\alpha}^{-1} + \gamma_{\alpha'}^{-1}), \quad \tilde{\gamma}_{\alpha\alpha'}^{-1} = \frac{1}{2} (\gamma_{\alpha}^{-1} - \gamma_{\alpha'}^{-1}), \quad \gamma_{\alpha}^{-1} = \frac{m_{\alpha}}{E_{\alpha}}. \quad (19)$$

so that the effective interaction Hamiltonian in the flavour basis has the following structure,

$$H_{\zeta}^{eff} = nG \begin{pmatrix} 0 & \left(\frac{\eta}{\gamma}\right)_{ee} \zeta_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} \zeta_{\perp} \\ \left(\frac{\eta}{\gamma}\right)_{ee} \zeta_{\perp} & 2\zeta_{\parallel} & \left(\frac{\eta}{\gamma}\right)_{e\mu} \zeta_{\perp} & 0 \\ 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} \zeta_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} \zeta_{\perp} \\ \left(\frac{\eta}{\gamma}\right)_{e\mu} \zeta_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} \zeta_{\perp} & 2\zeta_{\parallel} \end{pmatrix}. \quad (20)$$

Here we introduce the following formal notations:

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2 \theta}{\gamma_{11}} + \frac{\sin^2 \theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{\mu\mu} = \frac{\sin^2 \theta}{\gamma_{11}} + \frac{\cos^2 \theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{e\mu} = \frac{\sin 2\theta}{\gamma_{21}}. \quad (21)$$

For the part of the Hamiltonian responsible for weak interaction one can obtain

$$H_{\text{weak}}^{eff} = n\tilde{G} \begin{pmatrix} 0 & \left(\frac{\eta}{\gamma}\right)_{ee} v_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_{\perp} \\ \left(\frac{\eta}{\gamma}\right)_{ee} v_{\perp} & 2(1 - v_{\parallel}) & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_{\perp} & 0 \\ 0 & \left(\frac{\eta}{\gamma}\right)_{e\mu} v_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} v_{\perp} \\ \left(\frac{\eta}{\gamma}\right)_{e\mu} v_{\perp} & 0 & \left(\frac{\eta}{\gamma}\right)_{\mu\mu} v_{\perp} & 2(1 - v_{\parallel}) \end{pmatrix}, \quad (22)$$

where $\tilde{G} = \frac{G_F}{2\sqrt{2}} (1 + 4 \sin^2 \theta_W)$.

The flavor neutrino evolution Hamiltonian in the magnetic field H_B^f can be calculated the same way, we just should start from the neutrino electromagnetic interaction Lagrangian $L_{EM} = \frac{1}{2} \mu_{\alpha\alpha'} \bar{\nu}_{\alpha'} \sigma_{\mu\nu} \nu_{\alpha} F^{\mu\nu}$

$$H_B^f = \begin{pmatrix} -\left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & -\mu_{ee} B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & -\mu_{e\mu} B_{\perp} \\ -\mu_{ee} B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} & -\mu_{e\mu} B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} \\ -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & -\mu_{e\mu} B_{\perp} & -\left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} & -\mu_{\mu\mu} B_{\perp} \\ -\mu_{e\mu} B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_{\parallel} & -\mu_{\mu\mu} B_{\perp} & \left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_{\parallel} \end{pmatrix}, \quad (23)$$

where

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}} \cos^2 \theta + \frac{\mu_{22}}{\gamma_{22}} \sin^2 \theta + \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta, \quad (24)$$

$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_{12}} \cos 2\theta + \frac{1}{2} \left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}} \right) \sin 2\theta,$$

$$\left(\frac{\mu}{\gamma}\right)_{\mu\mu} = \frac{\mu_{11}}{\gamma_{11}} \sin^2 \theta + \frac{\mu_{22}}{\gamma_{22}} \cos^2 \theta - \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta,$$

$$\mu_{ee} = \mu_{11} \cos^2 \theta + \mu_{22} \sin^2 \theta + \mu_{12} \sin 2\theta,$$

$$\mu_{e\mu} = \mu_{12} \cos 2\theta + \frac{1}{2} (\mu_{22} - \mu_{11}) \sin 2\theta, \quad (25)$$

$$\mu_{\mu\mu} = \mu_{11} \sin^2 \theta + \mu_{22} \cos^2 \theta - \mu_{12} \sin 2\theta.$$

So we can clearly see that in addition to the matter current and the magnetic field, the neutrino oscillations also can be influenced by the matter polarization.

Probability of neutrino spin oscillations $\nu_e^L \Leftarrow (j_{\perp}, \zeta_{\perp}, B_{\perp}) \Rightarrow \nu_e^R$

Consider two states of neutrino (ν_e^L, ν_e^R). The corresponding oscillations are governed by the evolution equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} + n[\tilde{G}(1 - v_{\parallel}) + G\zeta_{\parallel}] & -\mu_{ee} B_{\perp} + \left(\frac{\eta}{\gamma}\right)_{ee} n[\tilde{G}v_{\perp} + G\zeta_{\perp}] \\ -\mu_{ee} B_{\perp} + \left(\frac{\eta}{\gamma}\right)_{ee} n[\tilde{G}v_{\perp} + G\zeta_{\perp}] & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} - n[\tilde{G}(1 - v_{\parallel}) + G\zeta_{\parallel}] \end{pmatrix} \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} \quad (26)$$

For the oscillation $\nu_e^L \Leftarrow (j_{\perp}, \zeta_{\perp}, B_{\perp}) \Rightarrow \nu_e^R$ probability we get (??) with

$$E_{\text{eff}} = \left| \mu_{ee} B_{\perp} - \left(\frac{\eta}{\gamma}\right)_{ee} n[\tilde{G}v_{\perp} + G\zeta_{\perp}] \right| \quad (27)$$

$$\Delta_{\text{eff}} = \left| \left(\frac{\mu}{\gamma}\right)_{ee} B_{\parallel} + n[\tilde{G}(1 - v_{\parallel}) + G\zeta_{\parallel}] \right|.$$

This formulas can be useful in further investigations of neutrino oscillations in extremal environments.

The case of Majorana neutrino

We also consider a Majorana neutrino that propagates in moving media composed of neutrons in the presence of magnetic field. In the mass basis for Majorana neutrinos the magnetic moment has only nondiagonal components, so that

$$\mu_{Maj} = \begin{pmatrix} 0 & i\mu \\ -i\mu & 0 \end{pmatrix}. \quad (28)$$

For the corresponding contribution to the neutrino effective Hamiltonian we obtain

$$H_B^f(Maj) = i\mu \cos 2\theta \begin{pmatrix} 0 & 0 & -\frac{1}{\gamma_{12}} B_{\parallel} & -B_{\perp} \\ 0 & 0 & -B_{\perp} & \frac{1}{\gamma_{12}} B_{\parallel} \\ \frac{1}{\gamma_{12}} B_{\parallel} & B_{\perp} & 0 & 0 \\ B_{\perp} & -\frac{1}{\gamma_{12}} B_{\parallel} & 0 & 0 \end{pmatrix}. \quad (29)$$

It's clearly seen that in the Majorana neutrino case the magnetic field can generate only spin-flavour oscillations. In the case of unpolarized matter the corresponding oscillation probability $P_{\nu_e^L \rightarrow \nu_e^R}$ is determined by (??) with

$$E_{\text{eff}} = \sqrt{\left(\mu B_{\perp} \sin \phi \cos 2\theta + 2 \left(\frac{\eta}{\gamma}\right)_{e\mu} \tilde{G} n v_{\perp} \right)^2 + (\mu B_{\perp} \cos \phi \cos 2\theta)^2}, \quad (30)$$

$$\Delta_{\text{eff}} = \left| \frac{\Delta M^2 \cos 2\theta}{4p_0^2} - 2\tilde{G}n(1 - v_{\parallel}) \right|,$$

where $\Delta M = \frac{\Delta m^2 \cos 2\theta}{4p_0}$ and ϕ is an angle between \mathbf{B}_{\parallel} and \mathbf{v}_{\perp} .

From the comparison of the probability of oscillations for the case of Majorana and Dirac neutrinos (see [?]), it follows that these two cases differ in the neutrino mixing efficiency. Also, the conditions for resonances in these two cases are realized at different densities of the matter.

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