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Spin and spin-flavor oscillations due to neutrino charge radii interaction with an external environment

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Neutrino effective Hamiltonian

It is believed that the running and forthcoming terrestrial neutrino experiments will be sensitive to the neutrino charge radius [1] that is one of the neutrino fundamental electromagnetic characteristics [2] predicted [3] to be non-zero even in the Standard Model. In this work we continue our studies [4] on neutrino oscillations accounting for diagonal and non-diagonal neutrino charge radii. We consider spin and spin-flavor neutrino oscillations in an extreme astrophysical environment and on the basis of exact expressions for the corresponding neutrino oscillation probabilities we study conditions for possible new neutrino resonances engendered by the neutrino charge radii. We consider two flavour neutrinos with two possible helicities $\nu_f = (\nu_e^-, \nu_x^-, \nu_e^+, \nu_x^+)$. To reach this goal we perform calculations analogous to those of [2,6]. we use the calculations that are analogous to those performed in [6, 5]. In the mass basis the matrix elements of the effective interaction Hamiltonian, describing the electromagnetic interactions of a neutrino field ν are given by

$$H_J^{(m)fi} = \lim_{q\to 0} \frac{1}{T} \frac{\langle \nu_f(p_f, h_f)| \int d^4x \mathcal{H}_J |\nu_i(p_i, h_i)\rangle}{\langle \nu(p,h)|\nu(p,h)\rangle}, \quad (1)$$

where $q = p_i - p_f$ is the transition moment, T is the normalization time and

$$\langle \nu_f(p_f, h_f)| \mathcal{H}_J |\nu_i(p_i, h_i)\rangle = \bar{u}_f(p_f, h_f) \Lambda_\mu^{fi}(q) \frac{1}{q^2} u_i(p_i, h_i) J^\mu_{EM} e^{-iqx}. \quad (2)$$

Here $J^\mu_{EM} = e(n_f, n_f \mathbf{v}_f)$ is the electric current of fermions f (protons or electrons) and Λ_μ^{fi} is the neutrino electromagnetic vertex that can be decomposed according to [2]. Here below, we are interested only in the charge radii and anapole form factors

$$\Lambda_\mu^{fi}(q) = (\gamma_\mu - q_\mu \gamma_\nu q^\nu/q^2) \left[f_Q^{fi}(q^2) + f_A^{fi}(q^2) q^2 \gamma_5 \right], \quad (3)$$

where $f_Q^{fi}(q^2)$ and $f_A^{fi}(q^2)$ are charge and anapole form factors in mass basis. The neutrino charge radii is determined by the second term in the expansion of the neutrino charge form factor in series of powers of q^2

$$f_Q(q^2) = f_Q(0) + q^2 \left. \frac{df_Q(q^2)}{dq^2} \right|_{q^2=0} + \ldots \quad (4)$$

where the charge radii is given by $\langle r^2 \rangle = 6 \left. \frac{df_Q(q^2)}{dq^2} \right|_{q^2=0}$. In the present work we consider zero neutrino millicharge $f_Q(0) = 0$. Then the electromagnetic vertex accounted for the charge radii and the anapole form factors is expressed as

$$\Lambda_\mu^{fi}(q) = (q^2 \gamma_\mu - q_\mu \gamma_\nu q^\nu) \left[\frac{\langle r^2 \rangle^{fi}}{6} + f_A^{fi} \gamma_5 \right]. \quad (5)$$

This vertex gives the following effective interaction Hamiltonian

$$H_J^{(m)fi} = \frac{1}{\sqrt{E_f E_i}} \bar{u}_f(p_f, h_f) \gamma_\mu \left[\frac{\langle r^2 \rangle^{fi}}{6} + f_A^{fi} \gamma_5 \right] u_i(p_i, h_i) J^\mu_{EM}(x). \quad (6)$$

Substituting spinors for neutrino free states into equation (6) we get

$$H_J^{(m)fi} = 2\chi^{(h_f)\dagger} \left\{ \boldsymbol{J}_\parallel^{EM} \left(\frac{\langle r^2 \rangle^{fi}}{6} + f_A^{fi} \sigma_3 \right) + \boldsymbol{J}_\perp^{EM} \left[\left(\sigma_1 \gamma_{fi}^{-1} \cos\chi + \sigma_2 \gamma_{fi}^{-1} \sin\chi \right) f_A^{fi} + \left(i\sigma_1 \tilde{\gamma}_{fi}^{-1} \sin\chi - i\sigma_2 \tilde{\gamma}_{fi}^{-1} \cos\chi \right) \frac{\langle r^2 \rangle^{fi}}{6} \right] \right\} \chi^{(h_i)}, \quad (7)$$

where $\boldsymbol{J}_\parallel^{EM}$ is the longitudinal to the neutrino moving component of the electric current and $\boldsymbol{J}_\perp^{EM}$ is the transversal component and χ is the angle between x-axis and $\boldsymbol{J}_\perp^{EM}$. The gamma factors are given by

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}, \quad \gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_\beta^{-1} \right), \quad \tilde{\gamma}_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_\beta^{-1} \right). \quad (8)$$

Now consider contribution of the longitudinal $\boldsymbol{J}_\parallel^{EM}$ and the transversal $\boldsymbol{J}_\perp^{EM}$ components of the electric current separately.

Longitudinal electric current $J_\parallel^{EM}$ contribution

In the flavour basis $\nu_f = (\nu_e^L, \nu_x^L, \nu_e^R, \nu_x^R)$ the corresponding part of the interaction Hamiltonian is given by

$$H_{J_\parallel}^{(f)} = 2J_\parallel^{EM} \begin{pmatrix} \frac{\langle r^2 \rangle^{ee}}{6} - f_A^{ee} & \frac{\langle r^2 \rangle^{ex}}{6} - f_A^{ex} & 0 & 0 \\ \frac{\langle r^2 \rangle^{ex}}{6} - f_A^{ex} & \frac{\langle r^2 \rangle^{xx}}{6} - f_A^{xx} & 0 & 0 \\ 0 & 0 & \frac{\langle r^2 \rangle^{ee}}{6} + f_A^{ee} & \frac{\langle r^2 \rangle^{ex}}{6} + f_A^{ex} \\ 0 & 0 & \frac{\langle r^2 \rangle^{ex}}{6} + f_A^{ex} & \frac{\langle r^2 \rangle^{xx}}{6} + f_A^{xx} \end{pmatrix}, \quad (9)$$

where

$$\begin{aligned} \langle r^2 \rangle^{ee} &= \langle r^2 \rangle^{11} \cos^2\theta + \langle r^2 \rangle^{22} \sin^2\theta + \langle r^2 \rangle^{12} \sin 2\theta, \\ \langle r^2 \rangle^{xx} &= \langle r^2 \rangle^{11} \sin^2\theta + \langle r^2 \rangle^{22} \cos^2\theta - \langle r^2 \rangle^{12} \sin 2\theta, \\ \langle r^2 \rangle^{ex} &= \langle r^2 \rangle^{12} \cos 2\theta + \frac{1}{2}\left(\langle r^2 \rangle^{22} - \langle r^2 \rangle^{11} \right) \sin 2\theta, \\ f_A^{ee} &= f_A^{11} \cos^2\theta + f_A^{22} \sin^2\theta + \langle r^2 \rangle^{12} \sin 2\theta, \\ f_A^{xx} &= f_A^{11} \sin^2\theta + f_A^{22} \cos^2\theta - f_A^{12} \sin 2\theta, \\ f_A^{ex} &= f_A^{12} \cos 2\theta + \frac{1}{2}\left(f_A^{22} - f_A^{11} \right) \sin 2\theta. \end{aligned} \quad (10)$$

It can be seen that $\boldsymbol{J}_\parallel^{EM}$ is able to affect on flavour neutrino oscillations.

Transversal electric current $J_\perp^{EM}$ contribution

In the flavour neutrino basis the part of the Hamiltonian with $\boldsymbol{J}_\perp^{EM}$ is

$$H_{J_\perp}^{(f)} = 2J_\perp^{EM} \begin{pmatrix} 0 & 0 & \left(\frac{f_A}{\gamma}\right)_{ee} e^{i\chi} & \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{i\chi} \\ 0 & 0 & \left[-\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{i\chi} & \left(\frac{f_A}{\gamma}\right)_{xx} e^{i\chi} \\ \left(\frac{f_A}{\gamma}\right)_{ee} e^{-i\chi} & \left[-\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{-i\chi} & 0 & 0 \\ \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{-i\chi} & \left(\frac{f_A}{\gamma}\right)_{xx} e^{-i\chi} & 0 & 0 \end{pmatrix}, \quad (11)$$

where

$$\begin{aligned} \left(\frac{f_A}{\gamma}\right)_{ee} &= \frac{f_A^{11}}{\gamma_{11}} \cos^2\theta + \frac{f_A^{22}}{\gamma_{22}} \sin^2\theta + \frac{f_A^{12}}{\gamma_{12}} \sin 2\theta, \\ \left(\frac{f_A}{\gamma}\right)_{xx} &= \frac{f_A^{11}}{\gamma_{11}} \sin^2\theta + \frac{f_A^{22}}{\gamma_{22}} \cos^2\theta - \frac{f_A^{12}}{\gamma_{12}} \sin 2\theta, \\ \left(\frac{f_A}{\gamma}\right)_{ex} &= \frac{f_A^{12}}{\gamma_{12}} \cos 2\theta + \frac{1}{2}\left(\frac{f_A^{22}}{\gamma_{22}} - \frac{f_A^{11}}{\gamma_{11}} \right) \sin 2\theta. \end{aligned} \quad (12)$$

From this expression one can see that neutrino charge radius and anapole form factor interaction with transversal electric current can generate neutrino spin and spin-flavour oscillations.

Effect of moving matter

The effect of moving matter on neutrino propagation is described by the effective Hamiltonian [6]

$$H_{mat} = \frac{G_F}{2\sqrt{2}} \begin{pmatrix} 2(2n_e - n_n)(1 - v_{||}) & 0 & (2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & (2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} \\ 0 & -2n_n (1 - v_{||}) & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{xx} \\ (2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & 0 & 0 \\ (2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp \left(\frac{\eta}{\gamma}\right)_{xx} & 0 & 0 \end{pmatrix}, \quad (13)$$

where n_n and n_e are the neutron and electron density profiles, $v = v_{||} + v_\perp$ is the matter velocity and

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{xx} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{ex} = \frac{\sin 2\theta}{\tilde{\gamma}_{21}}. \quad (14)$$

The Hamiltonian that accounts for the neutrino magnetic moment interaction with longitudinal $B_{||}$ and transversal components of the magnetic field $B_\perp$ has the following form

$$H_B = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\mu_{ee} B_\perp e^{i\phi} & -\mu_{ex} B_\perp e^{i\phi} \\ \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & \left(\frac{\mu}{\gamma}\right)_{xx} B_{||} & -\mu_{ex} B_\perp e^{i\phi} & -\mu_{xx} B_\perp e^{i\phi} \\ -\mu_{ee} B_\perp e^{-i\phi} & -\mu_{ex} B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} \\ -\mu_{ex} B_\perp e^{-i\phi} & -\mu_{xx} B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{xx} B_{||} \end{pmatrix}, \quad (15)$$

where ϕ is the angle between $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$. Neutrino magnetic moments in flavour basis $\mu_{\alpha\beta}$ are expressed through the magnetic moments μ_{ij} that are introduced for the mass states

$$\begin{aligned} \mu_{ee} &= \mu_{11} \cos^2\theta + \mu_{22} \sin^2\theta + \mu_{12} \sin 2\theta, \\ \mu_{ex} &= \mu_{12} \cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11}) \sin 2\theta, \\ \mu_{xx} &= \mu_{11} \sin^2\theta + \mu_{22} \cos^2\theta - \mu_{12} \sin 2\theta, \end{aligned} \quad (16)$$

$$\begin{aligned} \left(\frac{\mu}{\gamma}\right)_{ee} &= \frac{\mu_{11}}{\gamma_{11}} \cos^2\theta + \frac{\mu_{22}}{\gamma_{22}} \sin^2\theta + \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{ex} &= \frac{\mu_{12}}{\gamma_{12}} \cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}} \right) \sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{xx} &= \frac{\mu_{11}}{\gamma_{11}} \sin^2\theta + \frac{\mu_{22}}{\gamma_{22}} \cos^2\theta - \frac{\mu_{12}}{\gamma_{12}} \sin 2\theta, \end{aligned} \quad (17)$$

This Hamiltonians let one to write the neutrino oscillation probabilities for the different neutrino states. Here below we will consider two oscillation probabilities.

Neutrino spin oscillations $\nu_e^L \leftrightarrow \nu_e^R$

Consider two neutrino states (ν_e^L, ν_e^R). The corresponding oscillations are governed by the evolution equation

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix} = \left[\begin{pmatrix} \frac{G_F}{\sqrt{2}}(2n_e - n_n)(1 - v_{||}) & \frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} \\ \frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} & 0 \end{pmatrix} + \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\mu_{ee} B_\perp e^{i\phi} \\ -\mu_{ee} B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} \end{pmatrix} + \right.$$
$$\left. + \begin{pmatrix} -2J_\parallel^{EM} f_A^{ee} & 2J_\perp^{EM} \left(\frac{f_A}{\gamma}\right)_{ee} e^{i\chi} \\ 2J_\perp^{EM} \left(\frac{f_A}{\gamma}\right)_{ee} e^{-i\chi} & 2J_\parallel^{EM} f_A^{ee} \end{pmatrix} \right] \begin{pmatrix} \nu_e^L \\ \nu_e^R \end{pmatrix}. \quad (18)$$

For the oscillation $\nu_e^L \leftrightarrow \nu_e^R$ probability we get

$$P_{\nu_e^L \to \nu_e^R}(x) = \sin^2 2\theta_{\text{eff}} \sin^2 \frac{\pi x}{L_{\text{eff}}}, \quad \sin^2 2\theta_{\text{eff}} = \frac{E_{\text{eff}}^2}{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}, \quad L_{\text{eff}} = \frac{\pi}{\sqrt{E_{\text{eff}}^2 + \Delta_{\text{eff}}^2}}, \quad (19)$$

where E^2_{eff} and Δ^2_{eff} are expressed in terms of the elements H_{ij} of the effective Hamiltonian:

$$E_{eff}^2 = 4|H_{12}|^2 = 4\left[\frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ee} - \mu_{ee} B_\perp \cos\phi + \right.$$
$$\left. + 2J_\perp^{EM} \left(\frac{f_A}{\gamma}\right)_{ee} \cos\chi \right]^2 + 4\left[\mu_{ee} B_\perp \sin\phi - 2J_\perp^{EM} \left(\frac{f_A}{\gamma}\right)_{ee} \sin\chi \right]^2, \quad (20)$$

$$\Delta_{eff}^2 = (H_{11} - H_{22})^2 = \left[\frac{G_F}{\sqrt{2}}(2n_e - n_n)(1 - v_{||}) + 2\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} - 4J_\parallel^{EM} f_A^{ee} \right]^2. \quad (21)$$

Neutrino spin-flavour oscillations $\nu_e^L \leftrightarrow \nu_x^R$

Now we consider the neutrino oscillations $\nu_e^L \leftrightarrow \nu_x^R$ where index $x = \mu$ or τ

$$i\frac{d}{dt}\begin{pmatrix} \nu_e^L \\ \nu_x^R \end{pmatrix} = \left[\frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \frac{G_F}{\sqrt{2}}(2n_e - n_n)(1 - v_{||}) & \frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} \\ \frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} & 0 \end{pmatrix} + \right.$$
$$\left. + \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\mu_{ex} B_\perp e^{i\phi} \\ -\mu_{ex} B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{xx} B_{||} \end{pmatrix} + \begin{pmatrix} 2J_\parallel^{EM} \left(\frac{\langle r^2 \rangle^{ee}}{6} - f_A^{ee} \right) & 2J_\perp^{EM} \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{i\chi} \\ 2J_\perp^{EM} \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] e^{-i\chi} & 2J_\parallel^{EM} \left(\frac{\langle r^2 \rangle^{xx}}{6} + f_A^{xx} \right) \end{pmatrix} \right] \begin{pmatrix} \nu_e^L \\ \nu_x^R \end{pmatrix}. \quad (22)$$

The oscillation $\nu_e^L \leftrightarrow \nu_x^R$ probability we get (19) where now

$$E_{eff}^2 = 4\left[\frac{G_F}{2\sqrt{2}}(2n_e - n_n) v_\perp \left(\frac{\eta}{\gamma}\right)_{ex} - \mu_{ex} B_\perp \cos\phi + 2J_\perp^{EM} \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] \cos\chi \right]^2 +$$
$$+ 4\left[\mu_{ex} B_\perp \sin\phi - J_\perp^{EM} \left[\tilde{\gamma}_{12}^{-1} \frac{\langle r^2 \rangle^{12}}{6} + \left(\frac{f_A}{\gamma}\right)_{ex} \right] \sin\chi \right]^2, \quad (23)$$

$$\Delta_{eff}^2 = \left[-\frac{\Delta m^2}{2E} \cos\theta + \frac{G_F}{\sqrt{2}}(2n_e - n_n)\left(1 - v_{||}\right) + \right.$$
$$\left. + \left[\left(\frac{\mu}{\gamma}\right)_{ee} + \left(\frac{\mu}{\gamma}\right)_{xx} \right] B_{||} + J_\parallel^{EM} \left(\frac{\langle r^2 \rangle^{ee} - \langle r^2 \rangle^{xx}}{6} - f_A^{ee} + f_A^{xx} \right) \right]^2. \quad (24)$$

Thus, we have obtained an exact expression for the neutrino oscillation probability accounting for the impact of the neutrino charge radii and anapole form factors to neutrino oscillations in the presence of moving matter and magnetic field.

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