# Characterizing the observation bias in gravitational-wave detections and finding structured population properties

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# Observation bias in gravitational-wave detections

- Similar to other astronomical observations, gravitational-wave (GW) detections also have an observation bias
- Sources that are easier to detect are detected more frequently than their actual frequencies
- Different than other astronomical detections, detectability of GWs is not proportional to the luminosity of the source
  - Generated signal power in the GW detectors are proportional to the square of the induced distance difference between test masses.

$$\propto \int |h|^2 dt \propto (\Delta L)^2$$

  - Physical received GW power is proportional to the square of the induced oscillation speeds of the test masses

$$\propto \int \left|\frac{dh}{dt}\right|^2 dt \propto \mathrm{v}^2$$

# Where the bias comes from

- Related to having a detection criteria, e.g. signal-to-noise ratio
- Consider mass distribution

$$P(m_1|D) = \frac{P(D|m_1)P(m_1)}{P(D)}$$

Detected mass distribution

Astrophysical mass distribution

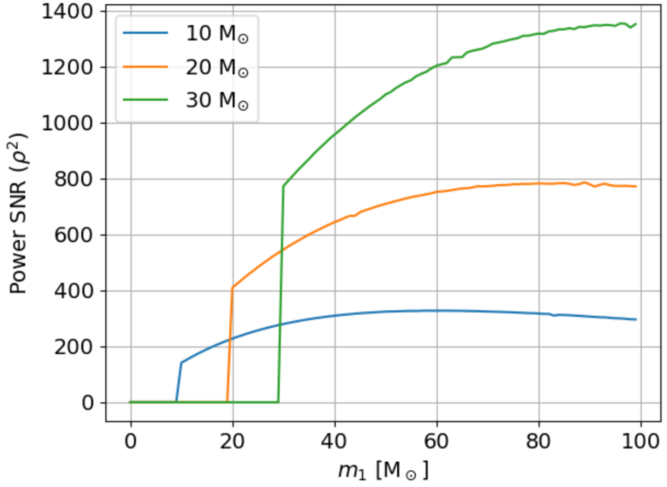
Detection probability for a given mass

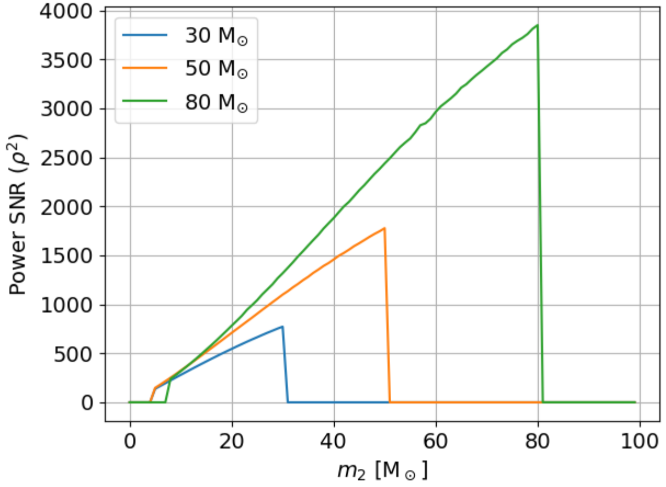
$$P(D|m_1) = \int P(D|m_1, m_2, r, \mathbf{\Omega}, \iota) P(m_2|m_1) \times P(\mathbf{\Omega})P(r)P(\iota)P(\psi)dm_2 dr d\mathbf{\Omega} d\iota d\psi$$

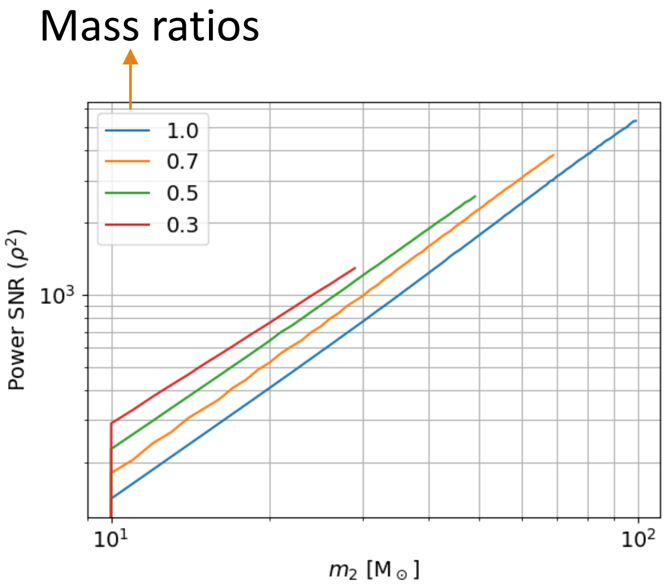
$$\begin{aligned} P(D|m_1, m_2, r, \mathbf{\Omega}, \iota, \psi) \\ &= P(E(m_1, m_2, r, \mathbf{\Omega}, \iota, \psi) > \rho_{th}^2) \\ &= \Theta(E(m_1, m_2, r, \mathbf{\Omega}, \iota, \psi) - \rho_{th}^2) \end{aligned}$$

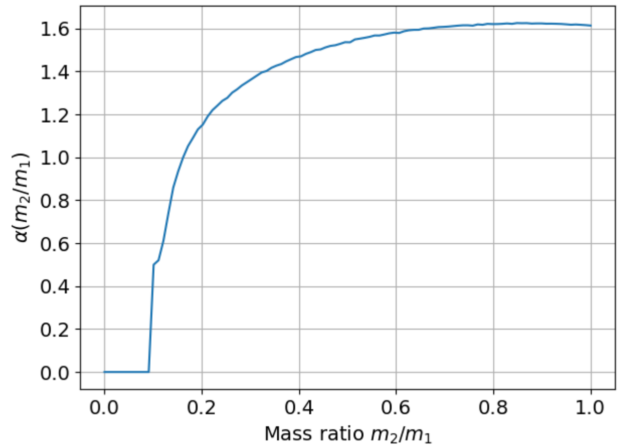
Step function depending on the generated SNR

# Mass dependency of SNR




No analytical from on component mass dependency!

For constant mass ratio, SNR is found to have a power-law dependency on the total mass. Exponent depends on the mass ratio $\propto (m_1 + m_2)^{\alpha(\frac{m2}{m1})}$

# Finding the bias - static universe

Network SNR's dependency on various parameters can be written as

$$E(m_1, m_2, r, \mathbf{\Omega}, \iota) = \frac{\sum_{i=1}^{N_d} E_{0,i}(m_1, m_2) f_i(\mathbf{\Omega}, \iota, \psi)}{r^2/r_0^2}$$

For $P(r) \propto r^2$, the integral for the observed distribution exactly reduces to

$$P(m_1|D) = \frac{P(m_1) \int E_0(m_1, m_2)^{3/2} P(m_2|m_1) dm_2}{\int P(m_1) E_0(m_1, m_2)^{3/2} P(m_2|m_1) dm_2 dm_1}$$

It can be used as a good approximation for low-redshift observations, e.g. binary neutron stars

# Finding the bias - expanding universe

$$E(m_1, m_2, r, \mathbf{\Omega}, \iota) = \frac{\sum_i E_{0,i}(m_1(1+z(r)), m_2(1+z(r))) f_i(\mathbf{\Omega}, \iota, \psi)}{r^2/r_0^2} = \frac{\sum_i E_{0,i}(m_1, m_2)(1+z(r))^{\alpha(m_2/m_1)} f_i(\mathbf{\Omega}, \iota, \psi)}{r^2/r_0^2}$$

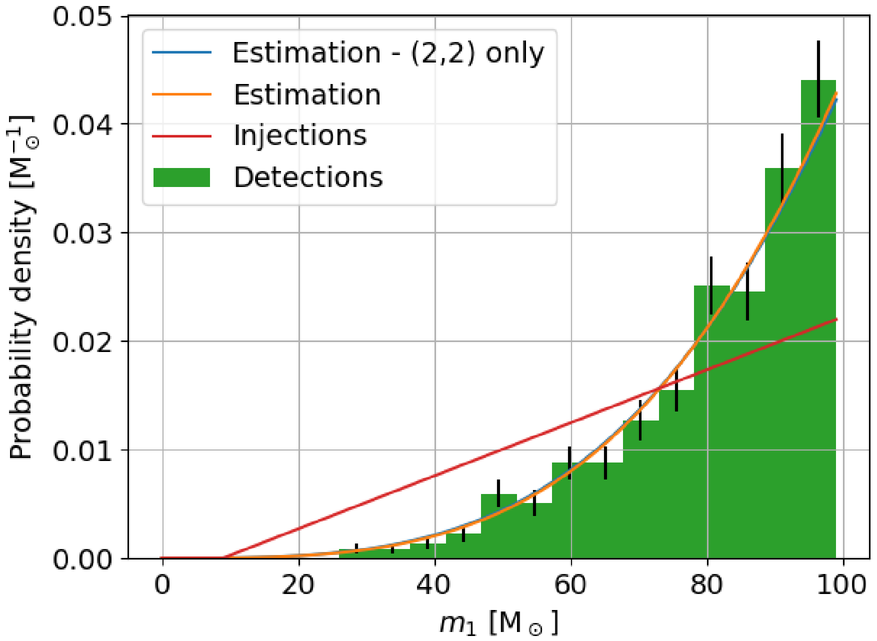
Observed masses are affected by the redshift

Using the power-law relationship we found for constant mass ratios

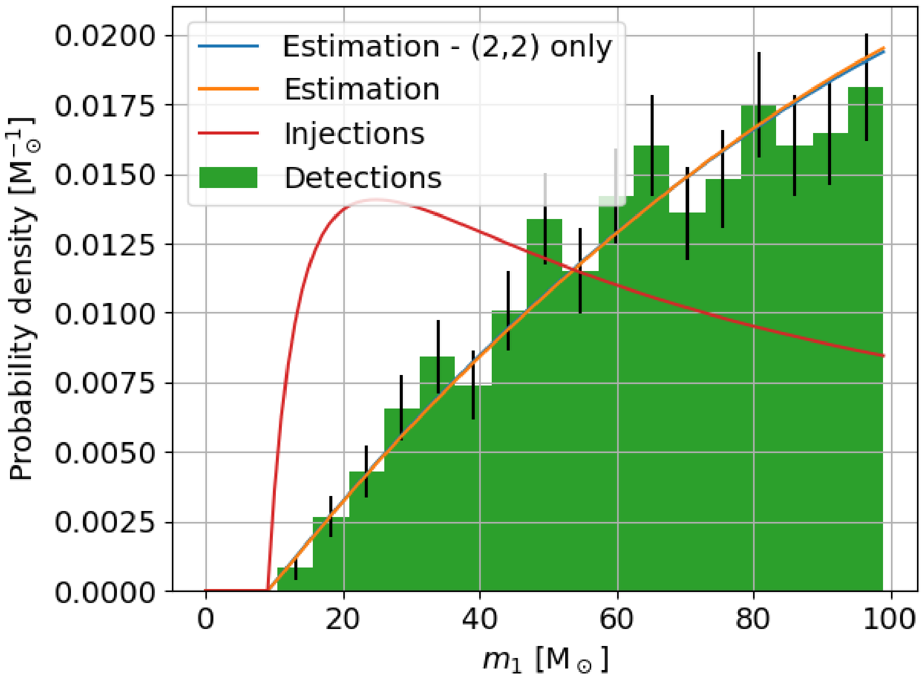
Due to convoluted relationship between masses and distance, integrals for observed distributions doesn't get simplified more

Still need to calculate SNR only for one reference distance (huge gain from the most computationally expensive part)

# Examining with injections




Static universe, calculation with reduced simple integral




Expanding universe

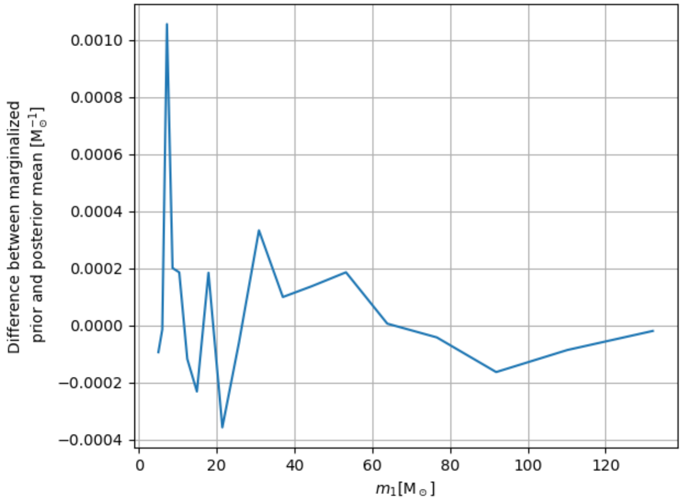
**Calculating SNR for each configuration once yields ~$10^6$ times faster computation than simulations**

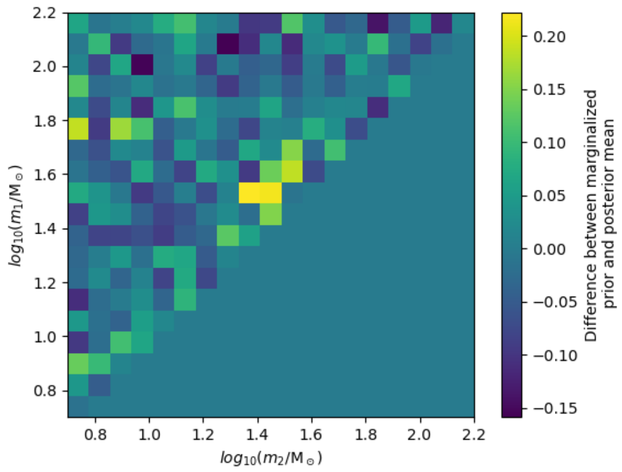
# Finding structure in the astrophysical distributions from observations without model

Probability for a distribution can be calculated as (hierarchical inference)

$$P(f(m)|p_1(m), p_2(m) \dots) \propto \frac{P(f(m))}{\left(\int f(m') \sum_{i=1}^{N_d} \Delta t_i P_i(det|m')dm'\right)^N} \prod_{i=1}^{N} \int f(m')p_i(m')P_i(\det|m')dm'$$

- Assume histogram type of distributions. Uniform prior on the distributions.
- Differences between the *mean* posterior distribution and *mean* prior distribution show peaks and dips at several points.



# Conclusion

- Worked out the statistics for observation bias, found simpler relationships with the physical insights from SNR’s dependencies
- Derived expressions agree with simulations, with a million times cheaper computation expense
- Made model independent estimates for astrophysical distributions, found extra structures that were not found with modelled inferences (agrees with the findings of Tiwari & Fairhurst 2021 ApJL)
- Example scripts are at https://github.com/dveske/observation-bias-gw