# MRI IVIM Parameter Estimation by Model weighting

## fitting pixel selecion

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# Outline



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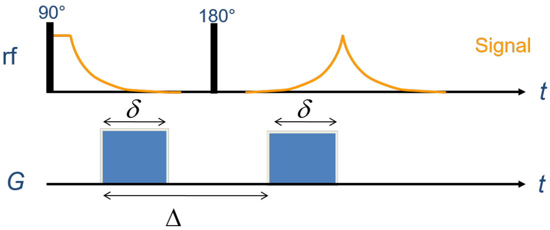
# NMR measurement of diffusion




Stejskal-Tanner equation (1965)

$$\frac{S(G,\delta,\Delta)}{S(0,\delta,\Delta)} = exp\left(\quad \gamma^2 G^2 \delta^2 D\left(\Delta \quad \delta/3\right)\right) = exp\left(\quad bD\right) \qquad (1)$$

*b*-value depends on gradient intensity and shape

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# Outline



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# Bi-exponential and more

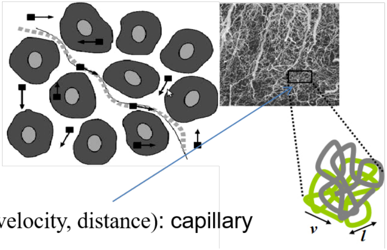
IVIM model ➔ $$\frac{S(b)}{S(0)} = (1-f)e^{-bD} + f \cdot e^{-bD^*}$$

$$\frac{S(b)}{S(0)} \approx (1-f)e^{-bD}$$

$D$: Diffusion
$D^*$: pseudo-diff, perfusion
$f$: Perfusion frac. (#$D^*$ vs. #$D^*$ )

$K$: Kurtosis, non Gaussian models.

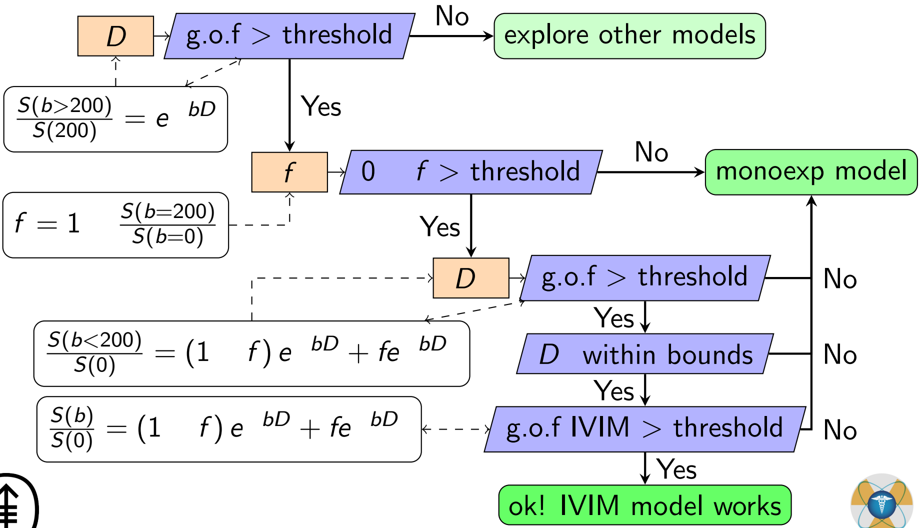
$$\ln\left(\frac{S(b)}{S(0)}\right) = -f - bD + b^2 D^2 K / 6$$




$D^*$ (velocity, distance): capillary

# Outline

# Decisions for parametric error category mapping exclusion

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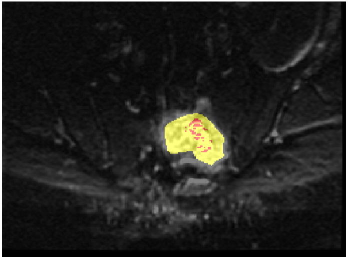
# Anatomical images



Diffusion

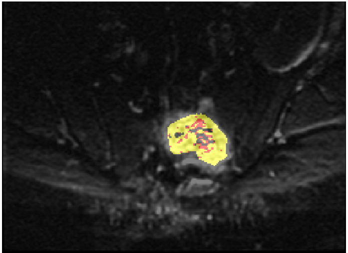
$D$ (top left), $f$ (bottom left) and $D$ (bottom right) spatial distribution

Red only physical thresholds are applied:

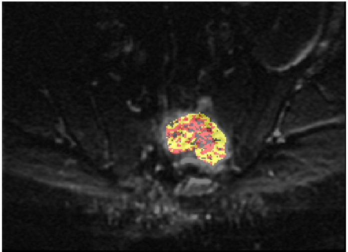
- $0 < D < 0.03mm^2/s$
- $0 < f < 1$
- $0 < D < 3mm^2/s$

Yellow pixels fill selection criteria based on g.o.f (normalized maximum absolute error [0 to 1]) and thresholds:

- $0 < D < 5$ std dev $< 0.03mm^2/s$
- $0 < f < 5$ std dev $< 1$
- $0 < D < 5$ std dev $< 3mm^2/s$



Perfusion fraction



Pseudo-diffusion

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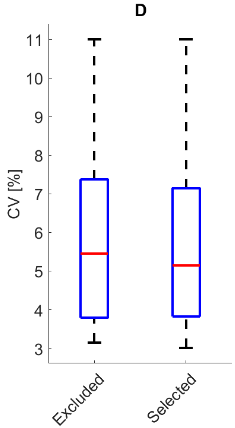
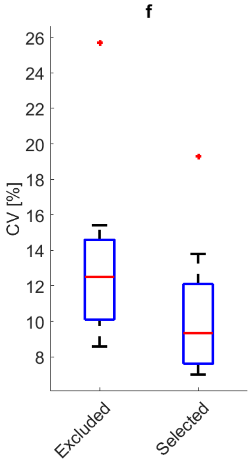
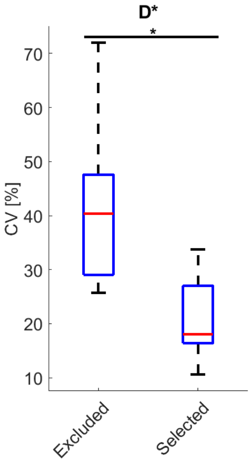
# Outline

1. MRI Diffusion
   - monoexponential decay
2. MRI Diffusion: IVIM model
   - IVIM
3. The project: parametric maps
   - Methods
4. Results
   - Anatomical images
   - Results

# Coefficient of variance of all patients

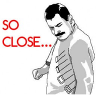


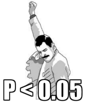




p-value 0.989

p-value 0.183




p-value 0.001



# Any questions ?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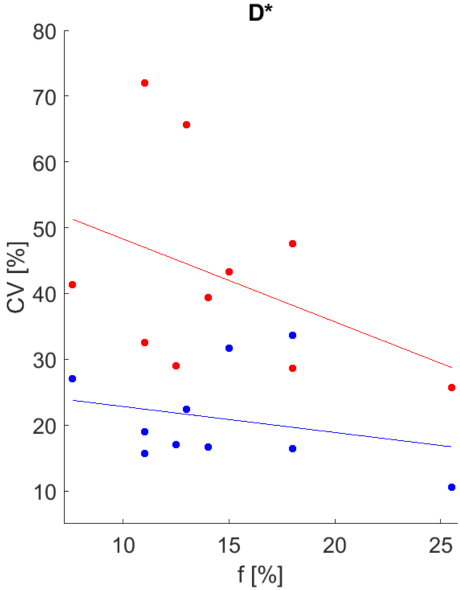
# CV of pseudo diffusion dependence on perfusion fraction




CV of $D$ vs. f average. CV of excluded data (red) and selected data (blue). Experimental data (full circle) is fitted to a linear regression.

$$S(b) = S(0)\left((1 \quad f)\, e^{-bD} + fe^{-bD}\right)$$

| Data | Excluded | Selected |
|---|---|---|
| Slope | -1.3 1 | -0.4 0.5 |
| Intercept | 61 16 | 27 8 |
| $r^2$ | 0.16 | 0.07 |
| $p$-value | 0.25 | 0.47 |
| Pearson | -0.40 | -0.26 |

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# Summary

Simulations test fitting robustness: SNR, precision and accuracy.

- Based on simulation, precision and accuracy depends on SNR ; real images, has a variety of SNR within the ROI.

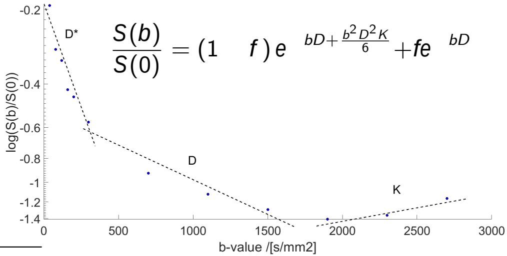
Patient images test data robustness by measuring the CV.

- Selection of pixels leads to an enhance of the bi-exponential model data reliability.
- Psuedo-diffusion fitted values benefits from selected pixels more than diffusion and perfusion fraction.
- CV of D* dependence on f, decreases if selected pixels based on residuals are considered.

Outlook

- Extend the diffusion model to kurtosis.

$$\frac{S(b)}{S(0)} = (1 \quad f)\, e^{\;bD + \frac{b^2 D^2 K}{6}} + fe^{\;bD}$$




residuals: difference between the predicted value based on the regression equation and the observed value.

# Outlook

The Akaike information criterion (AIC) is a measure of the relative quality of statistical models for a given set of data:it provides a means for model selection.

$$AIC = N \ln(SS) + 2p$$

$N$: number of data
$SS$: sum of square deviance
$p$: number of stimated parameters

$$\frac{S(b)}{S(0)} = e^{\quad bD} \tag{2}$$

$$\frac{S(b)}{S(0)} = (1 \quad f)\, e^{\quad bD+} + f e^{\quad bD} \tag{3}$$

$$\frac{S(b)}{S(0)} = e^{\quad bD+\frac{b^2D^2K}{6}} \tag{4}$$

$$\frac{S(b)}{S(0)} = (1 \quad f)\, e^{\quad bD+\frac{b^2D^2K}{6}} + f e^{\quad bD} \tag{5}$$

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# Diffusion Models

| Model | type | Gaussian | Non-Gaussian: Kurtosis |
|---|---|---|---|
| Mono-exp | | $\frac{S(b)}{S(0)} = \exp(-bD)$ | $\frac{S(b)}{S(0)} = \exp\left(-bD + b^2D^2K/6\right)$ |
| IVIM | Bi-exp | $\frac{S(b)}{S(0)} = (1-f)\exp(-bD) + f\exp(-bD^*)$ | $\frac{S(b)}{S(0)} = (1-f)\exp\left(-bD + b^2D^2K/6\right) + f\exp(-bD^*)$ |
| | ADC-approx | $\frac{S(b)}{S(0)} = \exp(-f - bD)$ | $\frac{S(b)}{S(0)} = \exp\left(-f - bD + b^2D^2K/6\right)$ |

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# IVIM: Piece-wise fitting

$$\frac{S(b)}{S(0)} = (1 \quad f)\, e^{\quad bD} + f e^{\quad bD} \qquad (6)$$

## Segmented fitting

1. Fit $D$ from high b-values; usually $b > 200 s/mm^2$.

$$\frac{S(b > 200)}{S_{int}} = e^{\quad bD} \qquad (7)$$

2. Set $f$ from the difference of the intersection of the two slopes

$$f = 1 \quad \frac{S_{int}}{S(b = 0)} \qquad (8)$$

3. Fit $D$ from small b-values; usually $b < 200 s/mm^2$.

$$\frac{S(b < 200)}{S(b = 0)} = (1 \quad f)\, e^{\quad bD} + f e^{\quad bD} \qquad (9)$$

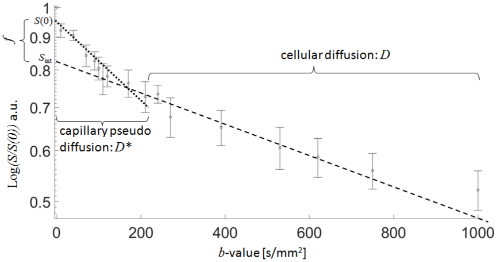


Figure: Perfusion fraction ($f$) and signal intercept ($S_{int}$) definition

Image from Koh *et al.* AJR, 160, p1352 june 2011