Study of local parity breaking in heavy ion collisions

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Based on

- A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.
- A. A. Andrianov, D. Espriu & X. Planells; arXiv: 1210.7712 [hep-ph]

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Outline

- Motivation of local parity breaking (LPB)
- Axial baryon charge and axial chemical potential
- Effective meson theory in a medium with LPB
  - Effective scalar/pseudosalar meson theory with μ_5
  - Vector Meson Dominance (VMD) approach to LPB
- Manifestation of LPB in heavy ion collisions (HIC)
- Conclusions

Motivation of local parity breaking

Parity is one of the well established global symmetries of strong interactions. Yet there are reasons to believe that it may be broken in a finite volume since no fundamental principle forbids spontaneous parity breaking for $\mu \neq 0$.

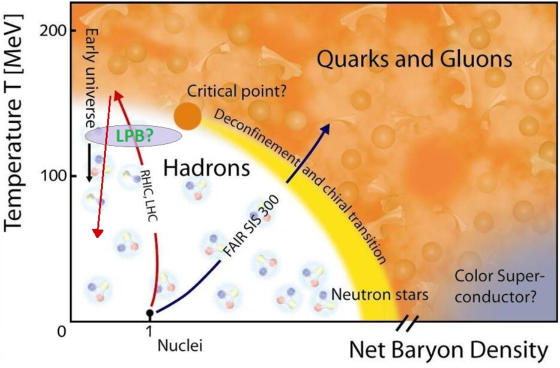
- *P- and CP-odd condensates = "pion" condensates*
  - A. Vilenkin, Phys. Rev. D22, 3080 (1980);
  - A.B. Migdal, Zh. Eksp. Teor. Fiz. 61 (1971);
  - T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974);
  - A. A. Andrianov & D. Espriu, Phys. Lett. B 663 (2008) 450;
  - A. A. Andrianov, V. A. Andrianov & D. Espriu, Phys. Lett. B 678 (2009) 416
- *Topological fluctuations*
  - D. Kharzeev, R. D. Pisarski & M. H. G. Tytgat, Phys. Rev. Lett. 81, 512 (1998);
  - K. Buckley, T. Fugleberg, & A. Zhitnitsky, Phys. Rev. Lett. 84 (2000) 4814;
  - D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008);
  - A. A. Andrianov, V. A. Andrianov, D. Espriu & X. Planells, Phys. Lett. B 710 (2012) 230.

Motivation of local parity breaking

Local large fluctuations in the topological charge probably exist in a hot environment.

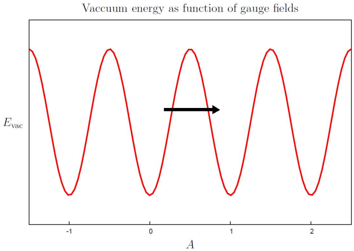
For *peripheral* heavy ion collisions they lead to the Chiral Magnetic Effect (CME): Large $\vec{B} \Rightarrow$ large $\vec{E} \Rightarrow$ charge separation.

For *central* collisions (and light quarks) they correspond to an ephemeral phase with axial chemical potential $\mu_5 \neq 0$.



Axial baryon charge and axial chemical potential

QCD has a non-trivial vacuum structure with different topological sectors.

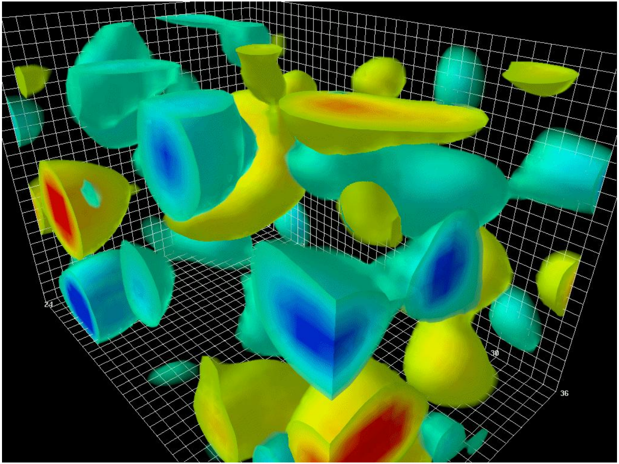


Topological charge T_5 may arise in a finite volume due to quantum fluctuations in a hot medium due to sphaleron transitions [Manton, McLerran, Rubakov, Shaposhnikov]

$$T_5 = \frac{1}{8\pi^2}\int_{\rm vol.} d^3x \varepsilon_{jkl} \mathrm{Tr}\left(G^j\partial^k G^l - i\frac{2}{3}G^j G^k G^l\right)$$

and survive for a sizeable lifetime in a heavy-ion fireball

$$\langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\rm fireball} \simeq 5 \div 10 \text{ fm}.$$



Lattice simulation of local fluctuations of the topological charge in the QCD vacuum [Leinweber].

Axial baryon charge and axial chemical potential

For the fireball lifetime one can control the value of $\langle \Delta T_5 \rangle$ introducing into the QCD Lagrangian a topological chemical potential μ_θ in a gauge invariant way via $\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5$, where

$$\Delta T_5 = T_5(t_f) - T_5(0) = \frac{1}{8\pi^2} \int_0^{t_f} dt \int_{\text{vol.}} d^3x \text{Tr} \left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right).$$

The partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2im_q J_5 = \frac{N_f}{8\pi^2} \text{Tr} \left(G^{\mu\nu} \widetilde{G}_{\mu\nu} \right)$$

predicts the induced axial charge (in the chiral limit $m_q \simeq 0$)

$$\frac{d}{dt} \left(Q_5^q - 2N_f T_5 \right) \simeq 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle$$

to be conserved during $\tau_{fireball}$.

Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For u, d quarks $1/m_q \sim 1/5\ \text{MeV}^{-1} \sim 40\ \text{fm} \gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.
- For s quark $1/m_s \sim 1/150\ \text{MeV}^{-1} \sim 1\ \text{fm} \ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \simeq 0$ due to left-right oscillations.

For u, d quarks QCD with a background topological charge leads to the generation of an axial chemical potential μ_5, conjugate to Q_5^q

$$\langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{top} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$

Axial baryon charge and axial chemical potential

A convenient way of learning about the physical processes in a HIC is by studying the leptons and photons flying out from the fireball. How is electromagnetism affected by μ_5?

- Inclusion of e.m. interaction implies a $U(1)$ anomaly

$$Q_5^q \rightarrow \tilde{Q}_5 = Q_5^q - T_5^{em}, \quad T_5^{em} = \frac{N_c}{8\pi^2} \int_{vol.} d^3x \, \varepsilon_{jkl} \, \mathrm{Tr} \left(\hat{A}^j \partial^k \hat{A}^l \right) .$$

- μ_5 is conjugated to (nearly) conserved $\tilde{Q}_5$

How does $\tilde{Q}_5$ affect the hadronic phenomenology?

Effective meson theory in a medium with LPB

- Scalar (and pseudoscalar) mesons

The scalar sector can be estimated by using the spurion technique in the Lagrangian

$$D_\nu \Longrightarrow D_\nu - i\{\mu_5 \delta_{0\nu}, \cdot\}$$

- Vector mesons

Low energy QCD can be described by Vector Meson Dominance. In this framework, the following term appears

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right]$$

with $\hat{\zeta}_\mu = \hat{\zeta}\delta_{\mu 0}$ for a spatially homogeneous and isotropic background ($\hat{}$ $\equiv$ isospin content) and $\zeta \propto \mu_5$.

Note the breaking of Lorentz symmetry in both cases.

Effective scalar/pseudosalar meson theory with μ_5

Two different cases of isospin structure for μ_5:

- Isosinglet pseudoscalar background ($T \gg \mu$) [RHIC, LHC]
- Pion-like (isotriplet) background (not considered) ($\mu \gg T$) [FAIR, NICA]

In the scalar sector we take μ_5 as the time component of some external axial-vector field

$$D_\nu \Longrightarrow D_\nu - i\{\mathbf{I}_q \mu_5 \delta_{0\nu}, \cdot\} = D_\nu - 2i\mathbf{I}_q \mu_5 \delta_{0\nu}.$$

Two new processes are likely to appear inside the fireball: the decays $\eta, \eta' \to \pi\pi$ that are strictly forbidden in QCD on parity grounds.

In a medium where parity is broken: are these processes relevant within the fireball? Can these particles reach thermal equilibrium?!

Effective Lagrangian:

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\left(D_\mu H D^\mu H^\dagger\right) + \frac{b}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right] + \frac{M^2}{2}\mathrm{Tr}\left(HH^\dagger\right)$$
$$-\frac{\lambda_1}{2}\mathrm{Tr}\left[(HH^\dagger)^2\right] - \frac{\lambda_2}{4}\left[\mathrm{Tr}\left(HH^\dagger\right)\right]^2 + \frac{c}{2}(\det H + \det H^\dagger)$$
$$+\frac{d_1}{2}\mathrm{Tr}\left[M(HH^\dagger H + H^\dagger H H^\dagger)\right] + \frac{d_2}{2}\mathrm{Tr}\left[M(H+H^\dagger)\right]\mathrm{Tr}\left(HH^\dagger\right)$$

where

$$H = \xi\Sigma\xi, \quad \xi = \exp\left(i\frac{\Phi}{2f}\right), \quad \Phi = \lambda^a\phi^a, \quad \Sigma = \lambda^b\sigma^b.$$

The v.e.v. of the neutral scalars are defined as $v_i = \langle\Sigma_{ii}\rangle$ where $i = u, d, s$, and satisfy the following gap equations:

$$M^2 v_i - 2\lambda_1 v_i^3 - \lambda_2 \vec{v}^2 v_i + c\frac{v_u v_d v_s}{v_i} = 0.$$

For further purposes we need the non-strange meson sector and η_s

$$\Phi = \begin{pmatrix} \eta_q + \pi^0 & \sqrt{2}\pi^+ & 0 \\ \sqrt{2}\pi^- & \eta_q - \pi^0 & 0 \\ 0 & 0 & \sqrt{2}\eta_s \end{pmatrix}, \Sigma = \begin{pmatrix} v_u + \sigma + a_0^0 & \sqrt{2}a_0^+ & 0 \\ \sqrt{2}a_0^- & v_d + \sigma - a_0^0 & 0 \\ 0 & 0 & v_s \end{pmatrix}$$

$$\begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

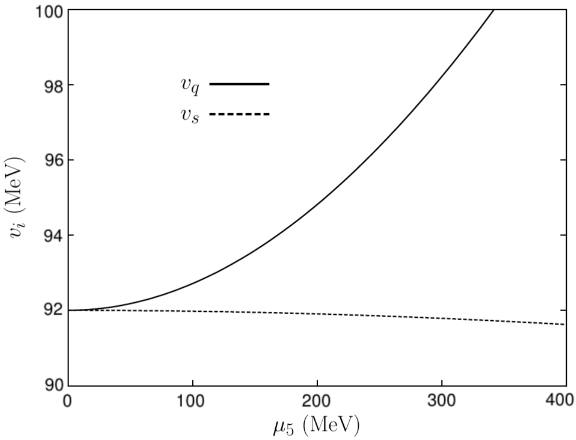
For $\mu_5 = 0$, we assume $v_u = v_d = v_s = v_0 \equiv f_\pi \approx 92$ MeV. The coupling constants (in units of MeV) are fitted to phenomenology assuming isospin symmetry via χ^2 minimization (`MINUIT`):

$$b = -3510100/m,\ M^2 = 1255600,\ c = 1252.2,\ \lambda_1 = 67.007,$$

$$\lambda_2 = 9.3126,\ d_1 = -1051.7/m,\ d_2 = 523.21/m,$$

where $m \equiv m_q = (m_u + m_d)/2$ and $m/m_s \simeq 1/25$.

Vacuum: for non-vanishing isosinglet μ_5 we impose our solutions to be $v_u = v_d = v_q \neq v_s$.



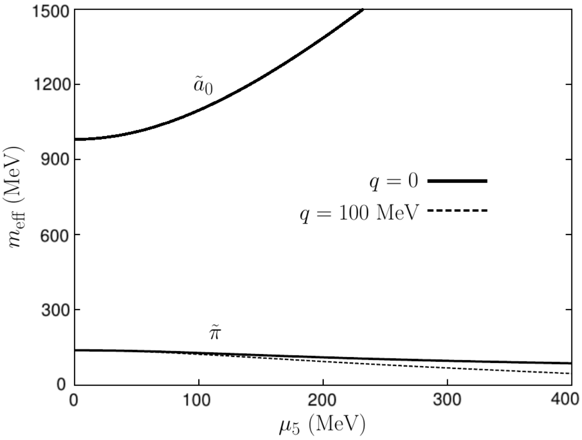
We present a simple case of mixing due to LPB in the isotriplet sector with π and a_0. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}(\partial a_0)^2 + \frac{1}{2}(\partial \pi)^2 - \frac{1}{2}m_1^2 a_0^2 - \frac{1}{2}m_2^2 \pi^2 - 4\mu_5 a_0 \dot{\pi},$$

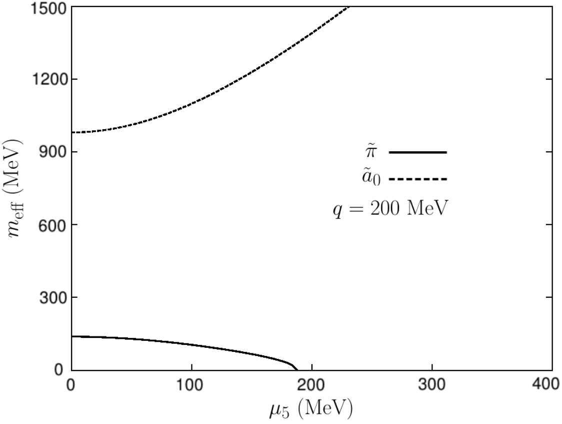
where

$$\begin{aligned} m_1^2 =& -2[M^2 - 2(3\lambda_1 + \lambda_2)v_q^2 - \lambda_2 v_s^2 \\ & - c v_s + 2(3d_1 + 2d_2) m v_q + 2d_2 m_s v_s + 2\mu_5^2] \\ m_2^2 =& \frac{2m}{v_q}\left[b + (d_1 + 2d_2)v_q^2 + d_2 v_s^2\right] \end{aligned}$$

After diagonalization in the momentum representation, the new (momentum-dependent) eigenstates are defined $\tilde{\pi}$ and $\tilde{a}_0$.



For high energies $k_0, |\vec{k}| > m_1 m_2/(4\mu_5) \equiv k^c_{\tilde{\pi}}$, in-medium $\tilde{\pi}$ goes tachyonic. Nevertheless, energies are always positive (no vacuum instabilities). $\tilde{a}_0$ mass shows an enhancement, but μ_5 has to be understood as a perturbatively small parameter.



In the isosinglet sector, we show the mixing of η, σ and η'. The kinetic and mixing terms in the Lagrangian are given by

$$\mathcal{L} = \frac{1}{2}[(\partial\sigma)^2 + (\partial\eta_q)^2 + (\partial\eta_s)^2] - \frac{1}{2}m_3^2\sigma^2 - \frac{1}{2}m_4^2\eta_q^2 - \frac{1}{2}m_5^2\eta_s^2 - 4\mu_5\sigma\dot{\eta}_q - 2\sqrt{2}cv_q\eta_q\eta_s,$$

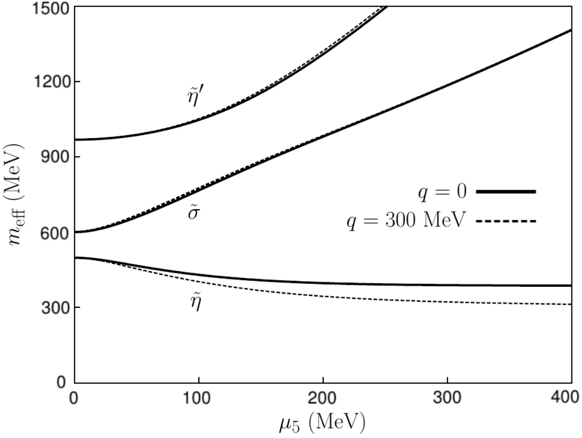
where

$$m_3^2 = -2(M^2 - 6(\lambda_1+\lambda_2)v_q^2 - \lambda_2 v_s^2 + cv_s + 6(d_1+2d_2)mv_q + 2d_2 m_s v_s + 2\mu_5^2),$$

$$m_4^2 = \frac{2m}{v_q}\left[b + (d_1+2d_2)v_q^2 + d_2 v_s^2\right] + 2cv_s,$$

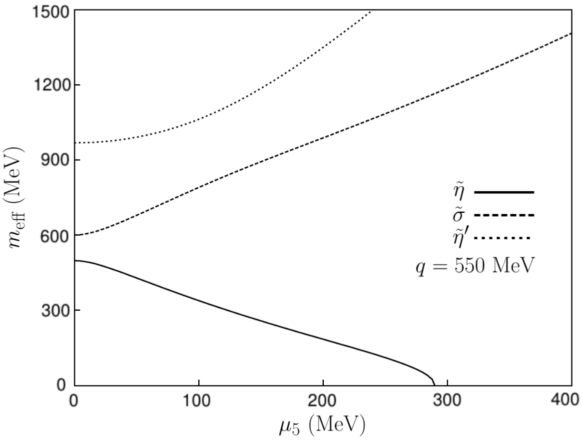
$$m_5^2 = \frac{2m_s}{v_s}[b + 2d_2 v_q^2 + (d_1+d_2)v_s^2] + \frac{cv_q^2}{v_s}.$$

After diagonalization, the new eigenstates are $\tilde{\sigma}$, $\tilde{\eta}$ and $\tilde{\eta}'$.



Again, for high energies $k_0, |\vec{k}| > k^c_{\tilde{\eta}}$ with

$k^c_{\tilde{\eta}} \equiv \frac{m_3}{4\mu_5 m_5}\sqrt{m_4^2 m_5^2 - 8c^2 v_q^2}$, in-medium $\tilde{\eta}$ goes tachyonic.



The cubic couplings used to calculate the widths $\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}' \to \tilde{\pi}\tilde{\pi}$ from the Lagrangian are given by

$$\mathcal{L}_{\sigma aa} = 2[(3d_1 + 2d_2)m - 2(3\lambda_1 + \lambda_2)v_q]\sigma \vec{a}_0^2,$$

$$\mathcal{L}_{\sigma\pi\pi} = \frac{1}{v_q^2}\left[(\partial\vec{\pi})^2 v_q - (b + 3(d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\vec{\pi}^2\right]\sigma,$$

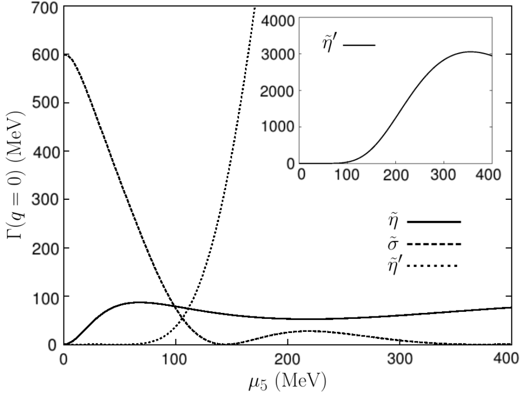
$$\mathcal{L}_{\eta a\pi} = \frac{2}{v_q^2}\vec{a}_0[\partial\eta_q\partial\vec{\pi}v_q - (b + (3d_1 + 2d_2)v_q^2 + d_2 v_s^2)m\eta_q\vec{\pi}],$$

$$\mathcal{L}_{\sigma a\pi} = -\frac{4\mu_5}{v_q}\sigma\vec{a}_0\dot{\vec{\pi}}, \quad \mathcal{L}_{\eta aa} = -\frac{2\mu_5}{v_q}\dot{\eta}_q\vec{a}_0^2, \quad \mathcal{L}_{\eta\pi\pi} = 0.$$

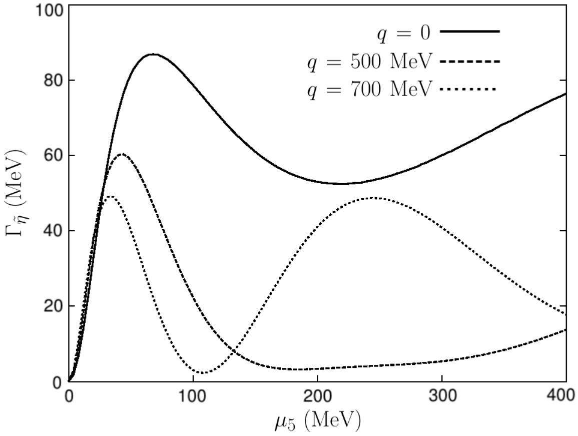
After diagonalization, one replaces the initial $\{\eta_q, \eta_s, \sigma\}$ to $\{\tilde{\eta}, \tilde{\sigma}, \tilde{\eta}'\}$ and $\{\pi, a_0\}$ to $\{\tilde{\pi}, \tilde{a}_0\}$.

The widths are firstly computed at the rest frame of the decaying particle and then with a boost.

$\tilde{\eta}$ exhibits a smooth behaviour with $\langle \Gamma_{\tilde{\eta}} \rangle \sim 60$ MeV $\leftrightarrow$ mean free path ~ 3 fm $\lesssim L_{\text{fireball}} \sim 5 \div 10$ fm. Possible thermalization!
Down to $\mu_5 \sim 100$ MeV, $\tilde{\sigma}$ width decreases and becomes stable.
The bumps seem to reflect the tachyonic nature of the decaying $\tilde{\pi}$.
$\tilde{\eta}'$ width grows up to the GeV scale (violation of unitarity).



Decay widths in the isosinglet case show strong dependences on the 3-momentum (nothing to do with Lorentz time dilatation).



Vector Meson Dominance approach to LPB

Does such a mixing of states of different parities occur in the vector/axial-vector sector?

Many models dealing with vector particles phenomenologically. If we assume that the vector mesons appear as part of a covariant derivative, no mixing term can be generated by operators of dimension 4 if μ_5 is an isosinglet. However, such a mixing is not forbidden on global symmetry grounds if μ_5 appears as the time component of an axial-vector field. This means that this coupling is very model dependent.

Vector mesons will be introduced and treated in the conventional way using VMD with no mixing of states with different parities. The only LPB effect will be the Chern-Simons term

$$\Delta\mathcal{L} \simeq \varepsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left[\hat{\zeta}_\mu V_\nu V_{\rho\sigma}\right].$$

Vector Meson Dominance bosonization:

$$\mathcal{L}_{\text{int}} = \bar{q}\gamma_\mu \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -eA_\mu Q + \frac{1}{2}g_\omega \omega_\mu \mathbb{I} + \frac{1}{2}g_\rho \rho^0_\mu \tau_3,$$

$$(V_{\mu,a}) \equiv (A_\mu,\, \omega_\mu,\, \rho^0_\mu)$$

where $Q = \frac{\tau_3}{2} + \frac{1}{6}$, $g_\omega \simeq g_\rho \equiv g \simeq 6$.

Maxwell and mass terms

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} + \rho_{\mu\nu}\rho^{\mu\nu}\right) + \frac{1}{2}V_{\mu,a}(\hat{m}^2)_{a,b}V^\mu_b$$

$$\hat{m}^2 \simeq m_V^2 \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}$$

P-odd interaction

$$\mathcal{L}_{mixing}(k) = -\frac{1}{4}\varepsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}\left(\hat{\zeta}_\mu \hat{V}_\nu \hat{V}_{\rho\sigma}\right) = \frac{1}{2}\zeta\epsilon_{jkl}\, V_{j,a}\, N_{ab}\, \partial_k V_{l,b}$$

- Isosinglet pseudoscalar background

$$(N^\theta_{ab}) \simeq \begin{pmatrix} \frac{10e^2}{9g^2} & -\frac{e}{3g} & -\frac{e}{g} \\ -\frac{e}{3g} & 1 & 0 \\ -\frac{e}{g} & 0 & 1 \end{pmatrix}, \quad \det\left(N^\theta\right) = 0$$

- Pion-like background (not considered)

$$(N^\pi_{ab}) \simeq \begin{pmatrix} \frac{2e^2}{3g^2} & -\frac{e}{g} & -\frac{e}{3g} \\ -\frac{e}{g} & 0 & 1 \\ -\frac{e}{3g} & 1 & 0 \end{pmatrix}, \quad \det\left(N^\pi\right) = 0$$

After diagonalization: the photon itself is unaffected by a singlet $\hat{\zeta}$.

Vector mesons exhibit the following dispersion relation:

$$m_{V,\epsilon}^2 = m_V^2 - \epsilon\zeta|\vec{k}|,$$

where $\epsilon = 0, \pm 1$ is the meson polarization.

The position of the poles for $\pm$ polarized mesons is moving with wave vector $|\vec{k}|$.

Massive vector mesons split into three polarizations with masses $m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2$.

This splitting unambiguously signifies LPB. Can it be measured?

Manifestation of LPB in heavy ion collisions

Must look for vector meson decays into leptons

$$\rho, \omega \to e^+ e^-.$$

The simulations are implemented with PHENIX acceptance: $|y_{ee}| < 0.35$, $|\vec{p}_t^{\,e}| > 200$ MeV, gaussian M_{ee} smearing (width=10 MeV).

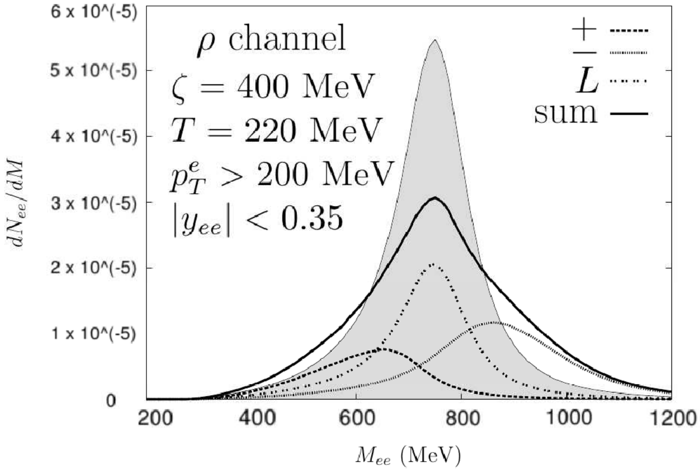
The total dilepton production also receives potential contribution from the pseudoscalar Dalitz decays

$$\eta, \eta' \to \gamma e^+ e^-,$$

and the ω Dalitz decay

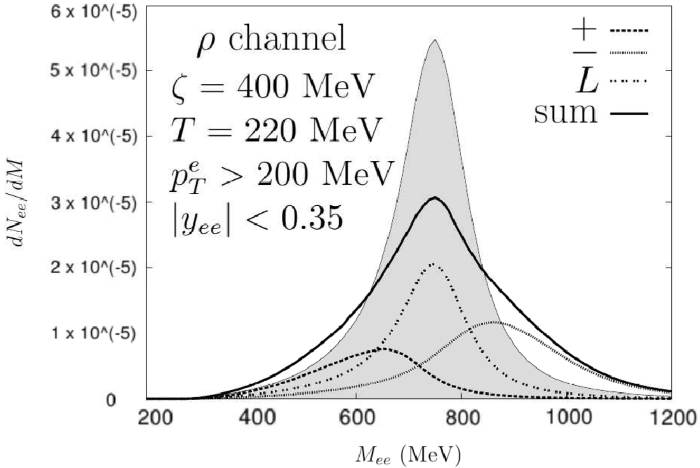
$$\omega \to \pi^0 e^+ e^-,$$

which we will not consider here.



Polarization splitting in ρ spectral function for LPB $\zeta = 400$ MeV ($\mu_5 = 290$ MeV) compared with $\zeta = 0$ (shaded region).

POLARIZATION ASYMMETRY!!



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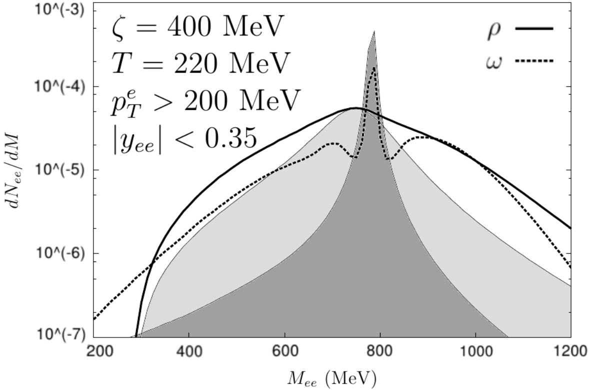
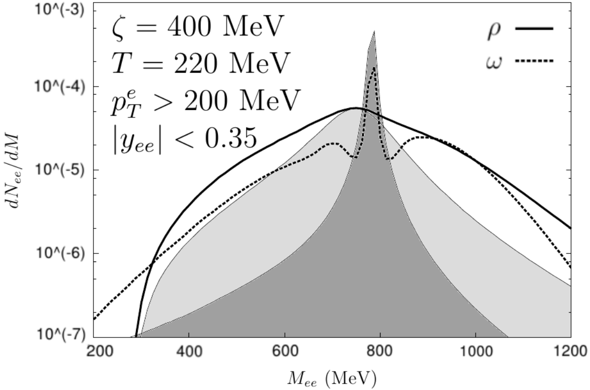


In-medium ρ and ω channels (solid and dashed line) and their vacuum contributions (light and dark shaded regions) for $\zeta = 400$ MeV. In-medium ρ is enhanced by a factor 1.8 due to $\pi\pi$ regeneration into ρ.

ENHANCEMENT OF DILEPTON YIELD!!



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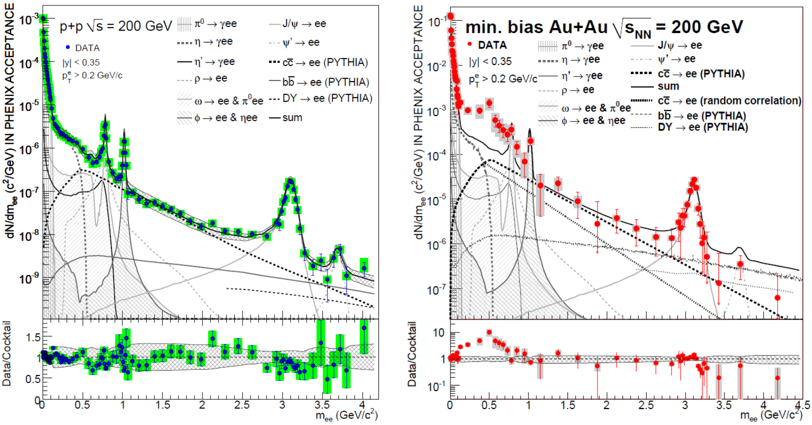
Manifestation of LPB in heavy ion collisions

PHENIX/STAR anomaly - Abnormal e^+e^- excess in central HIC



Manifestation of LPB in heavy ion collisions

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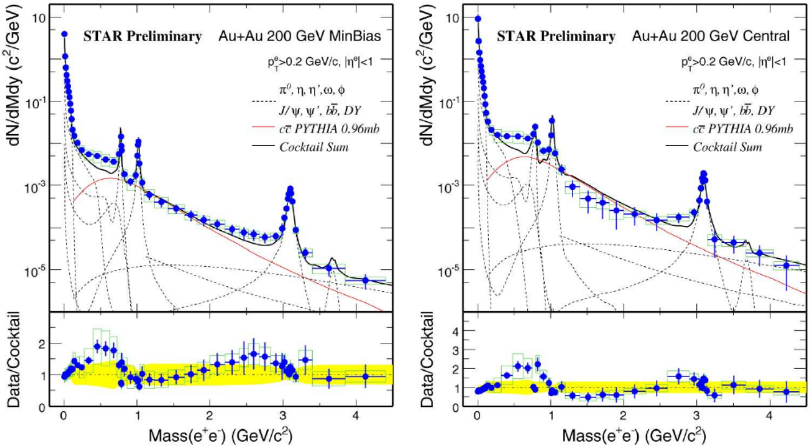

Conclusions

- LPB not forbidden by any physical principle in QCD at finite temperature/density.
- Topological fluctuations transmit their influence to hadronic physics via an axial chemical potential.
- LPB leads to unexpected modifications of the in-medium properties of scalar and vector mesons.
- LPB may help explaining the observed lepton spectrum in the LMR of PHENIX and STAR.
- *Event-by-event* measurements of the lepton polarization asymmetry may reveal in an unambiguous way the existence of LPB.
- Dalitz ω and η, η' (mixed with σ) decays and isotriplet μ_5 could be the responsible cases for the enhancement at $300 < M < 700$.

Thank you for your attention!

Backup I

Explicit formula for the simulation with acceptance correction:

$$\frac{dN}{d^4x dM} = \int d\tilde{M} \frac{1}{\sqrt{2\pi}\Delta} \exp\left[-\frac{(M-\tilde{M})^2}{2\Delta^2}\right] c_V \frac{\alpha^2}{24\pi\tilde{M}} \left(1 - \frac{n_V^2 m_\pi^2}{\tilde{M}^2}\right)^{3/2}$$

$$\times \sum_\epsilon \int_{\text{acc.}} \frac{k_t dk_t dy d^2\vec{p}_t}{|E_k p_\parallel - k_\parallel E_p|} \frac{1}{e^{\tilde{M}_t/T} - 1} P_\epsilon^{\mu\nu} \left(\tilde{M}^2 g_{\mu\nu} + 4p_\mu p_\nu\right)$$

$$\times \frac{m_{V,\epsilon}^4}{\left(\tilde{M}^2 - m_{V,\epsilon}^2\right)^2 + m_{V,\epsilon}^4 \frac{\Gamma_V^2}{m_V^2}}$$

MINUIT input in MeV

$$v_0^{\rm exp} = 92 \pm 5, \quad m_\pi^{\rm exp} = 137 \pm 5, \quad m_a^{\rm exp} = 980 \pm 50,$$
$$m_\sigma^{\rm exp} = 600 \pm 120, \quad m_\eta^{\rm exp} = 548 \pm 50, \quad m_{\eta'}^{\rm exp} = 958 \pm 100,$$
$$\Gamma_a^{\rm exp} = 60 \pm 30, \quad \Gamma_\sigma^{\rm exp} = 600 \pm 120.$$

MINUIT output versus experimental values in MeV

Magnitude	MINUIT	Exp. value	Error
v_0	92.00	92	-3.52×10^{-7}
m_π	137.84	137	6.10×10^{-3}
m_a	980.00	980	-1.26×10^{-6}
m_σ	599.99	600	-1.66×10^{-5}
m_η	497.78	548	-9.16×10^{-2}
$m_{\eta'}$	968.20	958	1.06×10^{-2}
Γ_a	60.00	60	2.04×10^{-5}
Γ_σ	600.00	600	6.81×10^{-6}

$\psi^{\rm exp} \simeq -18^\circ + \arctan\sqrt{2} \simeq 36.7^\circ$, while $\psi_{\rm MINUIT} \approx 35.46^\circ$.

μ_5-dependence of the tachyon critical energy for isotriplet $k^c_{\tilde{\pi}}$ and isosinglet case $k^c_{\tilde{\eta}}$.