Lattice QCD and flavor physics

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QCD is the theory describing strong interaction. It is a non-abelian gauge theory based on the local symmetry group of colour $SU(3)$.

$$\mathcal{L}_{QCD} = \underbrace{-\frac{1}{4}G^{a}{}_{\mu\nu}G_{a}{}^{\mu\nu}}_{\mathcal{L}_{\text{gauge}}} + \underbrace{\bar{\psi}(i\not{D} - \mathcal{M})\psi}_{\mathcal{L}_{\text{fermions}}} + \underbrace{\bar{c}^{a}\,\partial^{\mu}(D_{\mu})^{ab}\,c^{b}}_{\mathcal{L}_{\text{FP}}} \underbrace{-\frac{1}{2\xi}(\partial^{\mu}A^{a}{}_{\mu})(\partial^{\nu}A^{a}{}_{\nu})}_{\mathcal{L}_{\text{GF}}}$$

with the strength field tensor and the covariant derivative :

$$G^{a}{}_{\mu\nu} = \partial_{\mu}A^{a}{}_{\nu} - \partial_{\nu}A^{a}{}_{\mu} - gf^{a}{}_{bc}A^{b}{}_{\mu}A^{c}{}_{\nu} \text{ and } D_{\mu} = \partial_{\mu} + igA^{a}{}_{\mu}\frac{\lambda_{a}}{2}$$

$\mathcal{M}$ is the fermion mass matrix, $A^{a}, a = 1, \ldots, 8$ are the gluon fields and $g^{2} = 4\pi\alpha_{s}$ is the strong coupling constant.

$\rightarrow$ The usual treatment is perturbation theory : expansion into powers of the coupling constant.

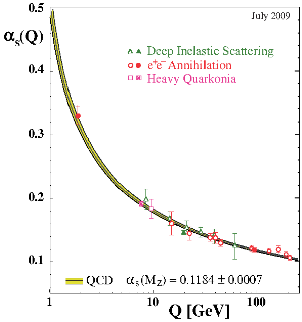
QCD objects

Fundamental objects : quarks and gluons
Observed objects at low energy :

$$\text{hadrons} \Rightarrow \text{mesons}\,(\pi, \mathrm{K}, \ldots) + \text{baryons}\,(\mathrm{n}, \mathrm{p}, \ldots)$$

$\Rightarrow$ HUGE discrepancy : not even the same particles observed than in the lagrangian.

Perturbation theory will fail certainly.



Bethke (2009)

Relative momentum transfer between two quarks :

Low-energy $\Leftrightarrow$ Strong coupling $\Leftrightarrow$ Confinement

High-energy $\Leftrightarrow$ Small coupling $\Leftrightarrow$ Asymptotic freedom

$\rightarrow$ Perturbation theory at low energy fails : need to use others methods

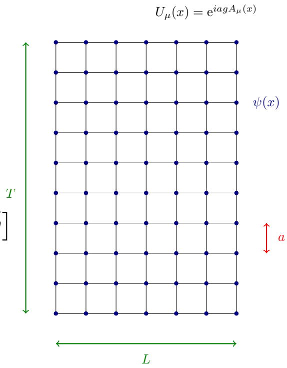
Generalities on the lattice

- Continuum infinite minkowskian spacetime $\rightarrow$ Discrete finite euclidean lattice
- Natural regularization :

  Lattice spacing $a \Rightarrow$ UV cut-off

Then : the path-integral is well-defined

$$\begin{aligned}\langle O \rangle &= \int \mathcal{D}U\, \mathcal{D}\bar{\psi}\, \mathcal{D}\psi\, \mathrm{e}^{-S_G - \int \bar{\psi}\, D[\mathcal{M}]\psi}\, O\left[U, \psi, \bar{\psi}\right] \\ &= \int \underbrace{\mathcal{D}U\, \mathrm{e}^{-S_G}\, \mathrm{Det}\left(D\left[\mathcal{M}\right]\right)}_{\text{Probability measure if } \geq 0}\, O\left[U\right]_{\mathrm{Wick}}\end{aligned}$$



$\rightarrow$ Finite number of degrees of freedom : integration can be performed numerically using Monte-Carlo methods.

Lattice QCD is not a model but it is QCD once the continuum limit $a \rightarrow 0$ and when the statistics $\rightarrow \infty$ are taken.

Degrees of freedom

$$\# \text{ d.o.f.} = \mathrm{L}^3 \times \mathrm{T} \times \underbrace{4}_{\text{Dirac}} \times \underbrace{N_c}_{\text{colors}} \times \underbrace{N_f}_{\text{flavors}}$$

For usual lattices : volume $V = 24^3 \times 48$

$\Rightarrow$ Matrices of order $\approx 10^6$.

Quenched approximation

$$\langle O \rangle = \int \mathcal{D}U \; \mathrm{e}^{-S_G} \; \mathrm{Det}\left(D\left[\mathcal{M}\right]\right) \; O\left[U\right]_{\text{Wick}}$$

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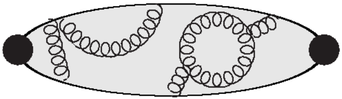
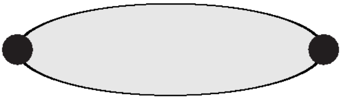
$\Rightarrow$ neglect fermionic loops.

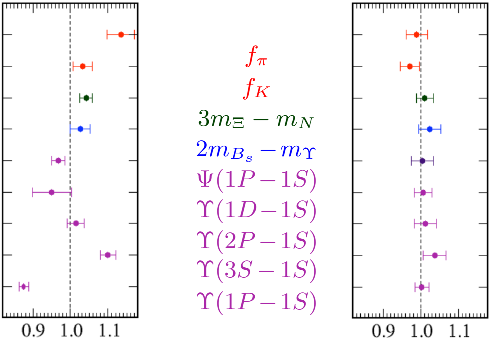




Lattice field theory : Computing observables from first principles

Fermionic determinant is the most expensive part of lattice computation, especially when the quark mass decrease.

HPQCD and UKQCD and MILC and Fermilab Lattice Collaborations (C.T.H. Davies et al.)
Phys. Rev. Lett. **92** 022001 (2004).



Ratio quenched/experiment

Ratio $N_f = 2+1$ /experiment

Now simulations with dynamical fermions

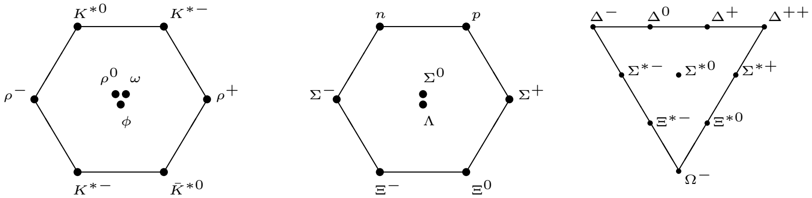
10% discrepancy.

- Statistics : error goes like $1/\sqrt{N_{conf}}$
- Discretization errors : to remove discretization errors, have to take the limit $a \to 0$
- Chiral extrapolation : necessary to reach the physical m_{ud}, it is the major source of error. Utilization of ChPT or Taylor expansion to guide.
- Finite volume effects : virtual pions interactions, effects like $\mathrm{e}^{-M_\pi L}$ and are suppressed if $M_\pi L \gtrsim 4$. Have to treat the case of resonances.
- Renormalization : perturbation theory available but better with non-perturbative methods.

Ab initio determination of the light hadron spectrum

Dürr, Fodor, Frison, Hoelbling, Hoffman, Katz, Krieg, Kurth, Lellouch, Lippert, Szabo, Vulvert [BMW Coll.],

Science **322** (2008)



→ Electromagnetic effects are about 1% of the mass, thus just isospin average is needed

- Aim
  → check that QCD indeed describes the strong interaction at low energy.
- Method
  → Postdiction of the light hadron spectrum.
  → Try to keep all the sources of systematic errors under control.

One has to extract the mass for each channel. For example, to extract M_π :

$$C_{PP}(t) = \left(\frac{a}{L}\right)^3 \sum_{\vec{x}} \left\langle \underbrace{\left[\bar{d}\gamma_5 u(x)\right]}_{\pi^-(x)} \underbrace{\left[\bar{u}\gamma_5 d(0)\right]}_{\pi^+(0)} \right\rangle \xrightarrow{0 \ll t \ll T} \frac{Z_{PP}}{2M_\pi} \mathrm{e}^{-M_\pi t}$$

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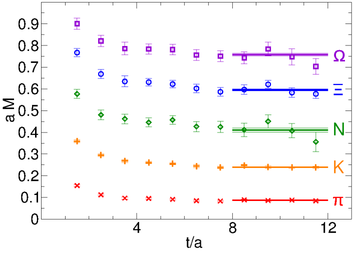

Example of effective masses for $\beta \approx 3.57$ and $M_\pi \approx 190$ MeV

Dürr, Fodor, Frison, Hoelbling, Hoffman, Katz, Krieg, Kurth, Lellouch, Lippert, Szabo, Vulvert [BMW Coll.], Science **322** (2008)

Repeat this procedure for several quark masses and several lattice spacing and then extrapolate to the physical point and take the contiuum limit.

β	$\approx a$ [fm]	am_s	am_ℓ	M_π [MeV]	$L^3 \times T$	Number of trajectories
3.3	0.125	-0.057	-0.0960	650	$16^3 \times 32$	10000
			-0.1100	510	$16^3 \times 32$	1450
			-0.1200	390	$16^3 \times 32$	4500
			-0.1233	330	$16^3 \times 64$	5000
					$24^3 \times 64$	2000
					$32^3 \times 64$	1300
			-0.1265	270	$24^3 \times 32$	2100
3.57	0.085	$0.0/-0.01$	-0.0318	510/510	$24^3 \times 64$	1650/1650
		$0.0/-0.01$	-0.0380	420/410	$24^3 \times 64$	1350/1550
		$0.0/-0.007$	-0.0440	310/310	$32^3 \times 64$	1000/1000
		$0.0/-0.007$	-0.0483	200/190	$48^3 \times 64$	500/1000
3.7	0.065	0.0	-0.0070	650	$32^3 \times 96$	1100
			-0.0130	560	$32^3 \times 96$	1450
			-0.0200	430	$32^3 \times 96$	2050
			-0.0220	390	$32^3 \times 96$	1350
			-0.0250	310	$40^3 \times 96$	1450

$\rightarrow$ First simulation down to 200 MeV.

$\rightarrow$ First time so large volumes are used.

Parametrization :

$$M_X = M_X^{(0)} + \alpha_K M_K^2 + \alpha_\pi M_\pi^2 + \textit{h.o.t.}$$

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- Cut-off on pion masses to have an idea of the contributions of *h.o.t.*

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$$M_X^{phy} \to M_X^{phy}\left[1+\gamma\alpha_s a\right] \text{ or } M_X^{phy} \to M_X^{phy}\left[1+\gamma a^2\right]$$

Extracting hadron masses

Combined chiral and **continuum** limits (II)

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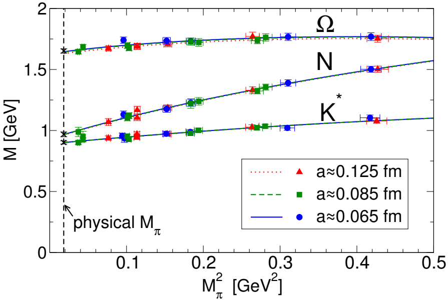

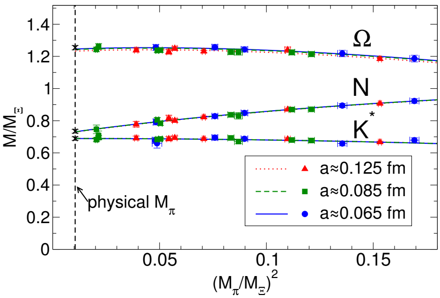

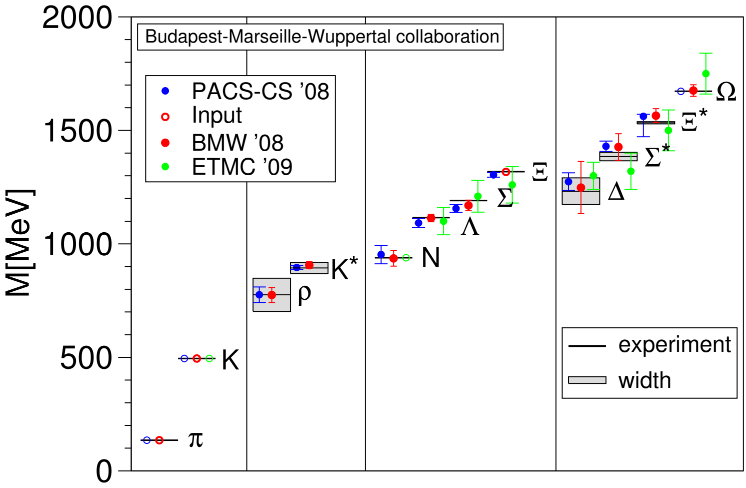
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Λ	1.116	$1.114(15)(5)$	$1.103(23)(10)$
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- obtained by freezing the individual contributions

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Decay constants on the lattice

- Aim
  - $\rightarrow$ check the unitarity of the first row of the CKM matrix
- Method
  - $\rightarrow$ Compute the ratio F_K/F_π.
  - $\rightarrow$ Use perturbative relation between $|V_{ud}|^2/|V_{us}|^2$ accurate to 0.4%.

→ This method is valid if one uses Ginsparg-Wilson valence quarks.

Semileptonic decay constant

It is defined by :

$$\langle 0| A_\mu(x) |P, \vec{p}\rangle = i\sqrt{2}\, F_P\, p_\mu\, \mathrm{e}^{\mathrm{ip\cdot x}},$$

with the axial-vector current $A_\mu(x) = q_1 \gamma^\mu \gamma^5 q_2(x)$ and the pseudoscalar density $P(x) = q_1 \gamma^5 q_2(x)$.

The divergence of this correlator gives :

$$\langle 0| \partial^\mu A_\mu(0) |P, \vec{p}\rangle = \sqrt{2}\, F_P\, M_P^2,$$

expression valid for any impulsion.
Using now the axial Ward identity :

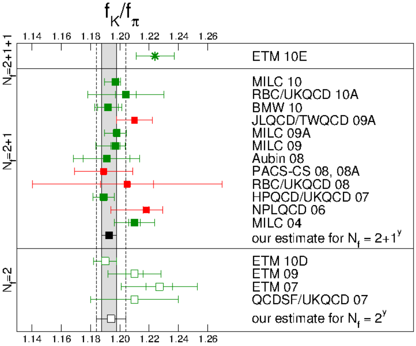
$$\partial^\mu A_\mu = (m_1 + m_2)\, P,$$

which is true if chiral symmetry is not broken, we obtain :

$$F_P = (m_1 + m_2)\, \frac{\langle 0|P(0)|P, \vec{p}\rangle}{\sqrt{2} M_P^2}.$$

FLAG = Flavianet Lattice Average Group

FLAG, Colangelo *et al.*, European Physical Journal **C** **71**, 1695 (2011).



$$\frac{F_K}{F_\pi} = 1.193(5).$$

Following the method proposed by Marciano, the ratio of $K_{\ell 2}$ and $\pi_{\ell 2}$ decays gives :

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2}\frac{M_K\left(1-m_\ell^2/M_K^2\right)^2}{M_\pi\left(1-m_\ell^2/M_\pi^2\right)^2}.$$

$\rightarrow$ known at 0.4% if $\ell=\mu$.

Using :

- $M_\pi,\ M_K$ and m_μ : known with relative precision of $3.10^{-6},\ 3.10^{-5}$ and 10^{-7} respectively
- super-allowed nuclear β decay : $|V_{ud}|=0.97452(22)$ (accuracy better than 0.03%).

$\rightarrow$ understand why a precise determination of F_K/F_π is crucial.

Using the value of the Flavianet Kaon working group :

$$\frac{|V_{us}|^2}{|V_{ud}|^2}\frac{F_K^2}{F_\pi^2} = 0.27599(59),$$

combined with the value of $F_K/F_\pi = 1.192(7)_{\rm stat}(6)_{\rm syst}$ of the B.M.W. Collaboration and $|V_{ud}| = 0.97425(22)$, one gets :

$$|V_{us}| = 0.2256(18).$$

Using also

$$|V_{ub}| = (3.93 \pm 0.36) \times 10^{-3},$$

the first row CKM unitarity relation reads :

$$|V_{ud}| + |V_{us}| + |V_{ub}| = 1.0001(9).$$

→ unitarity of the first row of the CKM matrix satisfied at the permil level : no signal of physics beyond the Standard Model.

Antonelli *et al.* [FlaviaNet Working Group on Kaon Decays], Nucl. Phys. Proc. Suppl. **181-182**, 83 (2008).

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C. Amsler *et al.* [Particle Data Group], Phys. Lett. **B** 667 (2008).

Conclusion

- Lattice QCD is a powerful tool to probe QCD at low energy.
- The era of precision is finally arrived : now simulate QCD directly at the physical point

  Dürr *et al.* [B.M.W. Collaboration], Phys. Lett. **B** 701, 265 (2011).
- All sources of systematic errors are now under control.
- Many quantities of interests can be computed : progress also in the heavy sector.