Higgs mass implications on the stability of the electroweak vacuum

J. Elias-Miró

Universitat Autònoma de Barcelona (UAB)
Institut de Física d'Altes Energies (IFAE)

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Work based on collaboration with:

J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia
+ A. Riotto + H.M. Lee + G.Degrassi, S. Di Vita
[hep-ph/1112.3022], [hep-ph-1203.0237], [hep-ph/1205.6497]

For recent related work see:

- M. Holthausen, K.S. Lim, M. Lindner [hep-ph/1112.2415]
- F. Bezrukov, M.Y. Kalmykov, B.A. Kniehl, M. Shaposhnikov [hep-ph/1205.2893]
- ...

The Higgs sector of the SM

It is the part of the theory from which we have less experimental information.

Interestingly, most of the theoretical problems of the SM arise from the Higgs sector.

The Higgs sector of the SM

- Severe fine tuning
  $m^2 \sim \Lambda^2$

$$-\mathcal{L}_{int} = -m^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 + Y^{ij}\Psi_L^i\Psi_R^j\Phi$$

- Instability for $\lambda < 0$
  or loss of perturbativity for$\lambda > 4\pi$
- Puzzling hierarchical structure +
  huge splitting $m_t/m_e = 3 \times 10^5$

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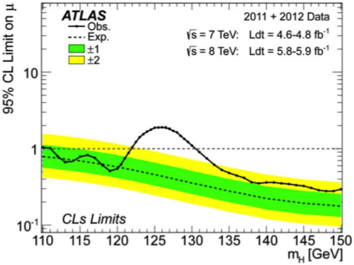
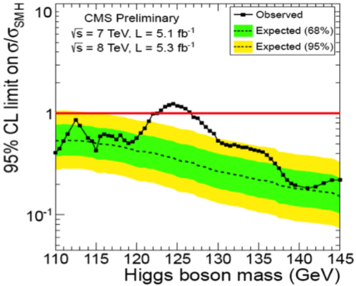


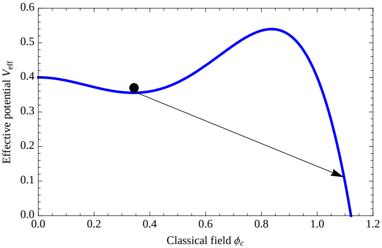


for $M_h \approx 125$ GeV, $\lambda = \dfrac{M_h^2}{2v^2} \approx 0.129$

Assume the SM up to very high energies,
is it a consistent model?

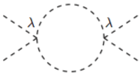
For large field values, $V_{eff} \approx \frac{1}{4}\lambda_{eff}(\phi)\phi^4$.

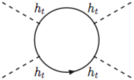
If $\lambda_{eff} \approx \lambda < 0$ at some high energy scale Λ_I, the Electroweak (EW) minimum at $\phi = v \approx 246$ GeV of the Higgs potential is unstable.



But, can λ become negative? Yes, two main competing effects:

$$\mu \frac{d\lambda(\mu)}{d\log(\mu)} = (\ \#\ \lambda^2 \ + \ldots \ -\#\ h_t^4 \ + \ldots) + \ldots$$

- 

, makes λ grow
- 

, makes λ decrease.

$h_t(v) = \sqrt{2} M_t / v$ and $\lambda(v) = M_h^2/(2v^2)$.

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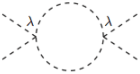
- 

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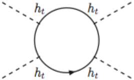
- 

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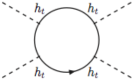
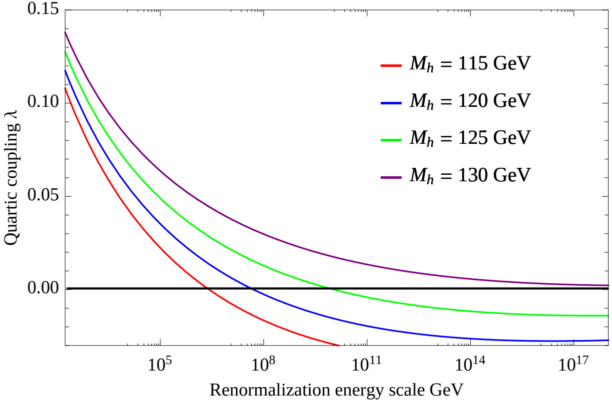
- 

, makes λ decrease.

$h_t(v) = \sqrt{2} M_t / v$ and $\lambda(v) = M_h^2/(2v^2)$.

For large field values $V_{eff} \sim \dfrac{\lambda(\phi)}{4}\phi^4$.

$\lambda(\mu)$ for $M_t = 173.1 GeV$:



Can the SM be ruled out (and claim that new physics must come in before the instability scale Λ_I) if there is a minimum deeper than the EW minimum at some field value ϕ_I ($\approx \Lambda_I$)?

- No, the SM can still be consistent if the lifetime of the unstable EW vacuum is much longer than the age of the universe.

We need to compute

$$\text{Decay Probability}=\Gamma/(\Delta x^3 \Delta t) \times V_u \ , \tag{1}$$

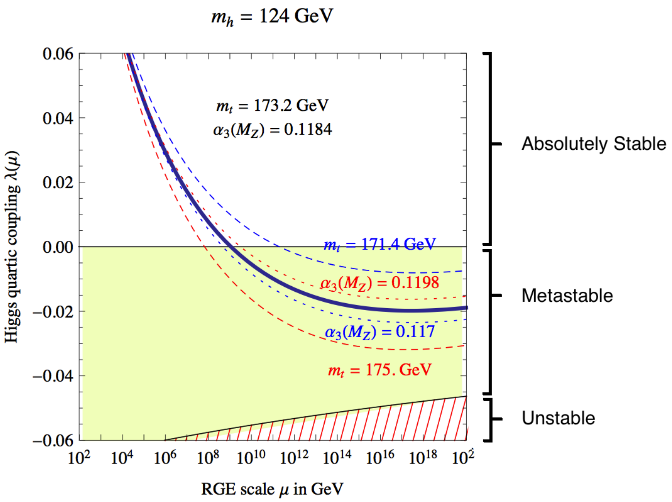
where $V_u \approx e^{409} \times [246 \text{ GeV}]^{-4}$ is the past light cone and $\Gamma =$ Decay Rate.

For $V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4$,

$$\Gamma/V \approx max\left(\phi^4 \exp[-\frac{8\pi^2}{3|\lambda_{eff}(\phi)|}]\right) \ , \tag{2}$$

where $\lambda_{eff}(\phi) < 0$. When $p << 1$, the lifetime of the EW vacuum is much longer than the age of the universe.

S. Coleman Phys. Rev. D. **15** 10 (1977)



Recall that $\lambda_{efff} = \lambda + \mathcal{O}(1\text{-loop})$

State-of-the-art of the NNLO calculation:

- 2-loop V_{eff}:

    Ford, Jack, Jones [hep-ph/011.1190]

- 3-loop RGEs:

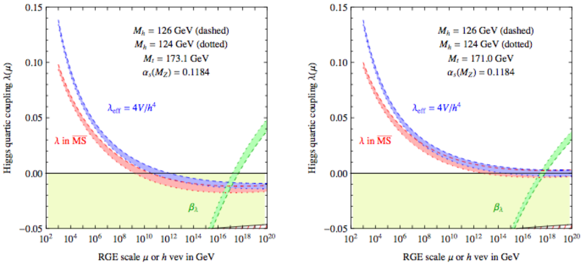
    Chetyrkin, Zoller [hep-ph/1205.2892]

    ...

- 2-loop matching in $\lambda(M_h^2)$, $h_t(M_t)$:

    Shaposhnikov et. al [hep-ph/1205.2893]

    Degrassi et. al [hep-ph/1205.6497]



Putting all the NNLO ingredients together, we estimate an overall theory error on M_h of ± 1.0 GeV (see section 3). Our final results for the condition of absolute stability up to the Planck scale is

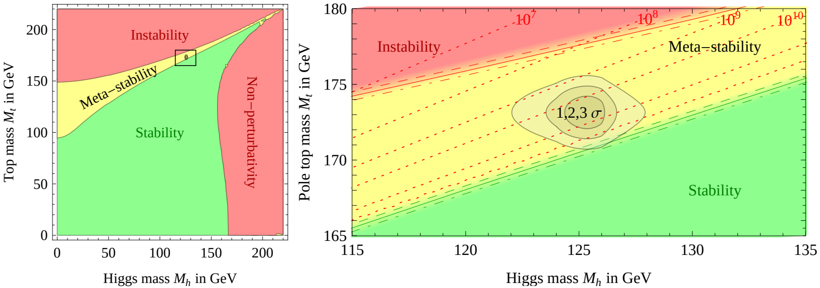
$$M_h\ [\text{GeV}] > 129.4 + 1.4\left(\frac{M_t\ [\text{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\text{th}}\,. \qquad (2)$$

Combining in quadrature the theoretical uncertainty with the experimental errors on M_t and α_s we get

$$M_h > 129.4 \pm 1.8\ \text{GeV}. \qquad (3)$$

From this result we conclude that vacuum stability of the SM up to the Planck scale is excluded at 2σ (98% C.L. one sided) for $M_h < 126$ GeV.

A Higgs mass of ~ 125 GeV is a very special value



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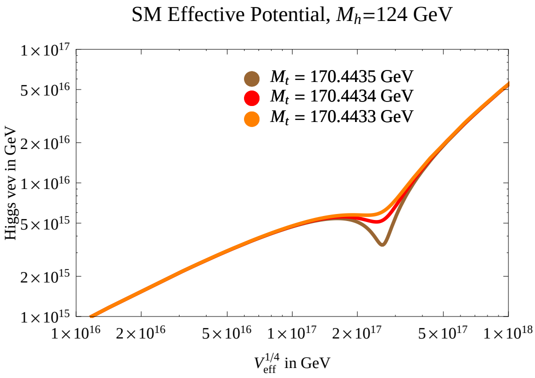
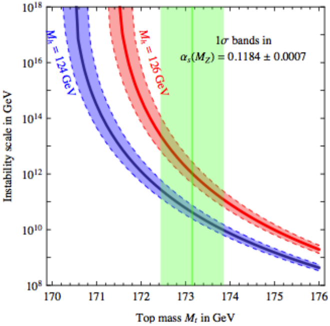
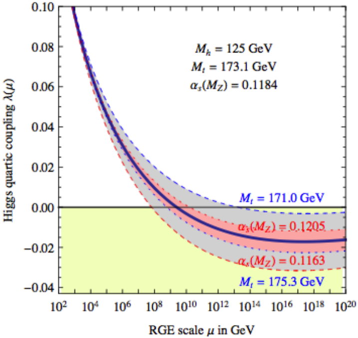
In the absence of BSM physics, some people like $\lambda|_{M_{Planck}} = 0$, or doing inflation in the following plateau ($\lambda \approx \beta_\lambda \approx 0$)



but...


1σ bands in
$\alpha_s(M_Z) = 0.1184 \pm 0.0007$
$M_h = 124$ GeV
$M_h = 126$ GeV
Instability scale in GeV
Top mass M_t in GeV

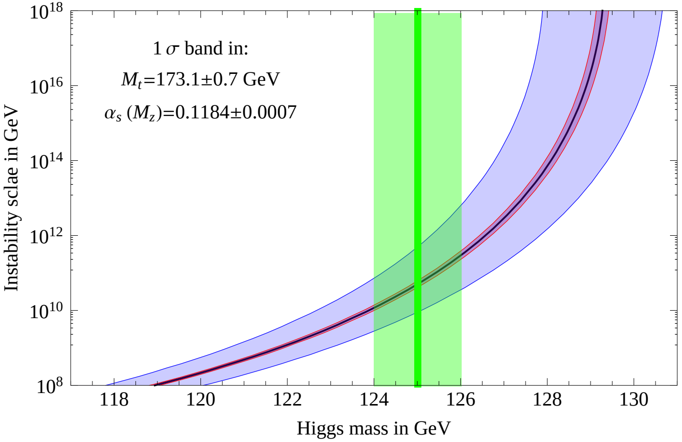

$M_h = 125$ GeV
$M_t = 173.1$ GeV
$\alpha_s(M_Z) = 0.1184$
$M_t = 171.0$ GeV
$\alpha_s(M_Z) = 0.1205$
$\alpha_s(M_Z) = 0.1163$
$M_t = 175.3$ GeV
Higgs quartic coupling $\lambda(\mu)$
RGE scale μ in GeV

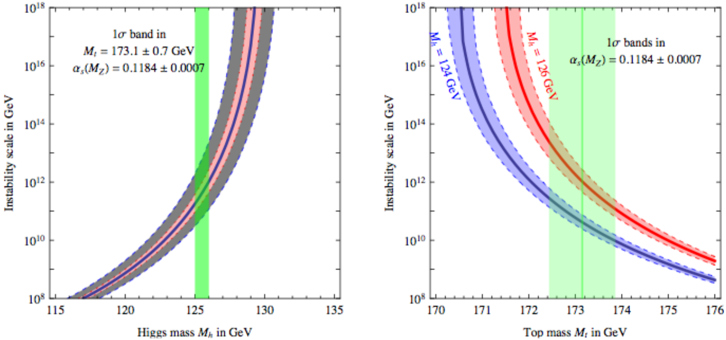
Conclusions

- From metastabilty considerations, a SM Higgs with $M_h \sim 125$ GeV does not imply an strict upper bound on the scale of new physics. The SM is such a good model that admits a theoretical extrapolation up to M_{Planck} without any consistency problem.
- The Higgs quartic coupling becomes very small. Very unlikely becomes zero at the Planck scale. However $\lambda(M_{Planck}) \approx 0$
- Intriguing situation: our vacuum sits just in between absolute stability and metastability.

I hope this analysis will be soon invalidated by nature, due to new physics coming in close to the EW scale...

Thank you for your attention!


1 σ band in:
M_t=173.1±0.7 GeV
α_s (M_z)=0.1184±0.0007
Instability sclae in GeV
10^{18}
10^{16}
10^{14}
10^{12}
10^{10}
10^{8}
118
120
122
124
126
128
130
Higgs mass in GeV



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$$\lambda_{\rm eff}(h) = e^{4\Gamma(h)}\left\{\lambda(h) + \frac{1}{(4\pi)^2}\sum_p N_p \kappa_p^2 (r_p - C_p)\right.$$

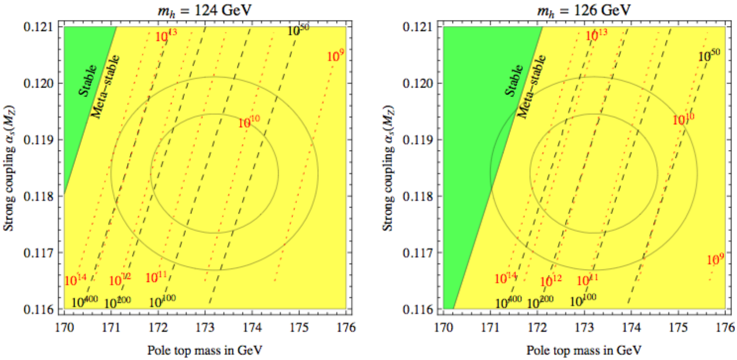
$$+$$

$$
\begin{aligned}
\delta\lambda_{\rm eff} \;=\;& \kappa^2 \Bigg\{ \frac{g^6}{48}\left[-30r_w^2 - 18r_{t/w}r_{(t-w)^2/(tw)} + 532r_w + 144r_{z/w} - 598 + 12\pi^2\right) \\
&+ \frac{g^4G^2}{96}\left[397 - 32r_{t/z}^2 + 126r_{z/w}^2 + 66r_z^2 + 27r_w^2 - 232r_z - 138r_w + 160\frac{\pi^2}{3}\right] \\
&+ \frac{g^4y_t^2}{24}\left(-27r_w^2 + 27r_{t/w}r_{(t-w)^2/(tw)} - 100r_t - 128r_z + 36r_w + 333 + 9\pi^2\right) \\
&- \frac{g^2G^4}{96}\left[219r_z^2 - 40r_{t/z}^2 + 21r_{w/z}^2 - 730r_z + 6r_w + 715 + 200\frac{\pi^2}{3}\right] \\
&+ \frac{2}{3}G^2y_t^4\left(3r_t^2 - 8r_t + 9\right) - \frac{G^6}{192}\left(34r_{t/z}^2 - 273r_z^2 + 3r_{w/z}^2 + 940r_z - 961 - 206\frac{\pi^2}{3}\right) \\
&+ \frac{G^4y_t^2}{48}\left[27\left(r_{t/z}^2 - r_z^2\right) - 68r_t - 28r_z + 189\right] + \frac{5}{3}g^2G^2y_t^2\left(2r_t + 4r_z - 9\right) \\
&- \frac{3y_t^6}{2}\left(3r_t^2 + 2r_{t/w}r_{(t-w)/t} - 16r_t + 23 + \frac{\pi^2}{3}\right) + \frac{3}{4}\left(g^6 - 3g^4y_t^2 + 4y_t^6\right){\rm Li}_2[w/t] \\
&+ \frac{y_t^2}{48}\left[\left(14G^2 - 160g^2 + 128\frac{g^4}{G^2}\right)y_t^2 + 17G^4 - 40g^2G^2 + 32g^4\right]\xi_{11zt} \\
&+ \frac{g^2}{192}\left[3G^4 + 4\left(12G^2 - 51g^2 - 36\frac{g^4}{G^2}\right)g^2\right]\xi_{11zw}\Bigg\} \;,
\end{aligned}
$$

where $\xi_{11xy} = \xi(1,1,x/y)$,

$$
r_p \equiv \ln[\kappa_p e^{2\gamma(h)}] \;,\quad r_{t/w} \equiv \ln[\kappa_t/\kappa_w] \;,\quad r_{(t-w)/t} \equiv \ln[(\kappa_t - \kappa_w)/\kappa_w] \;,
$$

and so on.


m_h = 124 GeV
m_h = 126 GeV
Strong coupling $\alpha_s(M_Z)$
Pole top mass in GeV
Stable
Meta–stable
0.116
0.117
0.118
0.119
0.120
0.121
170
171
172
173
174
175
176
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
10^{50}
10^{100}
10^{200}
10^{400}