# Black Hole formation from a null fluid in extended Palatini gravity

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# Main motivations

- General Relativity: static (Schwarzschild,RN,Kerr) or dynamic (Vaidya) black hole solutions.
- f(R) theories: same solutions as in GR $\Longleftrightarrow$ constant curvature.
- This degeneracy of the theories makes difficult to pull out new physics from it.
- When adding $Q = R_{\mu\nu}R^{\mu\nu}$ to the Lagrangian $\rightarrow$ f(R,Q) theories.
  - Static case: research done on charged BH. (Olmo-Rubiera.2012)
  - Dynamic case: not studied yet. $\rightarrow$ new physics?

Purpose: treating dynamically the formation of black holes (Schwarzschild) with pulses of null radiation in GR and extending this study to f(R,Q) Palatini theories.

# Index

- Differential geometry. Metric and connection: independent?.
- Palatini gravity: theories of modified gravity. f(R,Q).
- BH formation. Poisson-Israel formalism. Dynamic case (Vaidya).
  - General Relativity.
  - Palatini.
- Conclusions.

# Differential geometry

- Formulated on a differentiable manifold M.
- We can define a tangent vectorial space at any point p.
- Covariant derivative (afine connection).

$$\nabla_\mu W^\lambda = \frac{\partial W^\lambda}{\partial x^\mu} + W^\nu \Gamma^\lambda_{\mu\nu}$$

→ Connection coefficients.

$$\nabla_\mu e_\nu = \Gamma^\lambda_{\mu\nu} e_\lambda$$

# Differential geometry

- Quantities with an intrinsical geometrical meaning:

$$T(e_\mu, e_\nu) = \left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right) e_\lambda$$ Torsion tensor.

$$R(e_\mu, e_\nu) e_\lambda = \left(\partial_\mu \Gamma^\sigma_{\nu\lambda} - \partial_\nu \Gamma^\sigma_{\mu\lambda} + \Gamma^\alpha_{\nu\lambda}\Gamma^\sigma_{\mu\alpha} - \Gamma^\alpha_{\mu\lambda}\Gamma^\sigma_{\nu\alpha}\right) e_\sigma$$ Curvature tensor

- Riemannian geometry. A metric is introduced.
  - Condition: $\nabla_\mu g_{\alpha\beta} = 0$

$$2\Gamma^\lambda_{(\mu\nu)} + \left(T^\rho_{\nu\sigma} g_{\rho\mu} + T^\rho_{\mu\sigma} g_{\rho\nu}\right) g^{\sigma\lambda} = g^{\lambda\rho}\left(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right) \equiv 2L^\lambda_{\mu\nu}$$

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$\Gamma$: Levi-Civita connection (Christoffel symbols)

Metric and connection are both fundamental objects of the geometry of the manifold and they both are independent!!
Palatini!!

# Palatini Gravity

- f(R,Q) theories of modified gravity. Metric and connection indepentent.

$$S[g_{\mu\nu},\Gamma,\psi_m]=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,f(R,Q)+S_m[g_{\mu\nu},\psi_m] \qquad f(R,Q)=R+l_p^2(aR^2+R_{\mu\nu}R^{\mu\nu})$$

- Variation of the action:

$$\left\{\begin{array}{ll}\delta g^{\mu\nu}: & f_R R_{\mu\nu}-\frac{f}{2}g_{\mu\nu}+2f_Q R_{\mu\alpha}R^{\alpha}_{\nu}=\kappa^2 T_{\mu\nu} \\ \delta\Gamma^{\alpha}_{\beta\gamma}: & \nabla_\beta\left[\sqrt{-g}\left(f_R g^{\mu\nu}+2f_Q R^{\mu\nu}\right)\right]=0\end{array}\right\} \qquad f_X=\frac{\partial f}{\partial X}$$

$$\xrightarrow{f_Q=0}\left\{\begin{array}{l} f_R R_{\mu\nu}-\frac{f}{2}g_{\mu\nu}=\kappa^2 T_{\mu\nu} \\ \nabla_\beta\left[\sqrt{-g}\,f_R g^{\mu\nu}\right]=0\end{array}\right\} \xrightarrow{f_R=1} \left\{\begin{array}{l} R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=\kappa^2 T_{\mu\nu} \\ \nabla_\beta\left[\sqrt{-g}\,g^{\mu\nu}\right]=0\end{array}\right\}$$

Palatini f(R) — General Relativity

# Solving for the connection

## f(R,Q) theories:

- We define the matrix $\hat{P}$ whose components are $P_\mu^\nu \equiv R_{\mu\alpha} g^{\alpha\nu}$

  $$2 f_Q \hat{P}^2 + f_R \hat{P} - \frac{f}{2}\hat{I} = \kappa^2 \hat{T}$$ $\hat{I}$ $\hat{T}$ matrix representations of $\delta_\mu^\nu$ $T_\mu^\nu$

- The connection equation can be solved by assuming another rank 2 metric tensor. Its Levi-Civita connection will coincide with the independent connection.

$$\nabla_\beta \left[ \sqrt{-g} \left( f_R g^{\mu\nu} + 2 f_Q R^{\mu\nu} \right) \right] = \nabla_\beta \left[ \sqrt{-h} h^{\mu\nu} \right] = 0 \rightarrow \hat{h}^{-1} = \frac{\hat{g}^{-1}\hat{\Sigma}}{\sqrt{\det \hat{\Sigma}}} \qquad \hat{\Sigma} = \left( f_R \hat{I} + 2 f_Q \hat{P} \right)$$

$$\hat{P}\left( 2 f_Q \hat{P} + f_R \hat{I} \right) - \frac{f}{2}\hat{I} = \kappa^2 \hat{T} \Rightarrow \hat{P}\hat{\Sigma} = \frac{f}{2}\hat{I} + \kappa^2 \hat{T}$$

$$\boxed{R_\mu^\nu(h) = \frac{1}{\sqrt{\det \hat{\Sigma}}} \left( \frac{f}{2}\delta_\mu^\nu + \kappa^2 T_\mu^\nu \right)}$$

# Solving for the connection

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$$\boxed{R_\mu^\nu(h) = \frac{1}{\sqrt{\det\hat{\Sigma}}}\left(\frac{f}{2}\delta_\mu^\nu + \kappa^2 T_\mu^\nu\right)}$$

- The auxiliar metric satisfies 2nd order eqs.
- The geometry is caracterized by the matter sources P=P(T).

# BH formation.

- Schwarzschild BH (simplest case): vacuum, spherical symmetry.

| | $J = 0$ | $J \neq 0$ |
|---|---|---|
| $Q = 0$ | Schwarzschild | Kerr |
| $Q \neq 0$ | Reissner–Nordström | Kerr–Newman |

- Its dynamics will be studied according to Poisson & Israel's [1] formalism.

$$ds^2 = g_{ab} dx^a dx^b + r^2(x) d\Omega^2$$

- Spherical symmetry.
- Treatment by sectors (2+2).

(1):E.Poisson and W. Israel, PRD 41, 1796 (1990)

# BH formation. Poisson-Israel. General Relativity.

- Obtaining our field equations.

$$\left\{\begin{array}{ll} {}^4\Gamma^a_{bc} = \Gamma^a_{bc} & \Gamma^\theta_{a\theta} = \Gamma^\phi_{a\phi} = \frac{r_{,a}}{r} \\ \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta & \Gamma^\phi_{\theta\phi} = \cot\theta \\ \Gamma^a_{\theta\theta} = \sin^{-2}\theta\Gamma^a_{\phi\phi} = -rr'^a & \end{array}\right\} \longrightarrow \left\{\begin{array}{l} {}^4R_{ab} = R_{ab} - \frac{2}{r} r_{;ab} \\ R_{\theta\theta} = 1 - (r'^a r_{,a} + r\Box r) = \sin^{-2}\theta R_{\phi\phi} \\ {}^4R = R + \frac{2}{r^2}[1 - 2r\Box r - r'^a r_{,a}] \end{array}\right\}$$

$$\Box\psi = g^{ab}\psi_{;ab}$$

# BH formation. Poisson-Israel. General Relativity.

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$$\Box\psi = g^{ab}\psi_{;ab}$$

$$ {}^4G_{ab} = -\frac{1}{r^2}[2rr_{;ab} + (1 - 2r\Box r - r^{,a} r_{,a})g_{ab}]$$

$$G_{\theta\theta} = \sin^{-2}\theta\, G_{\phi\phi} = r\Box r - \frac{1}{2} r^2 R$$

General equations for any spherically symmetric metric!

# BH formation.
# Poisson-Israel. General Relativity.

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$2rr_{;ab} + (1 - 2r\Box r - r^{,a} r_{,a}) g_{ab} = -8\pi r^2 T_{ab} \quad (1)$$

$$r\Box r - \frac{1}{2} r^2 R = 8\pi r^2 P \quad (2)$$

**Evolution equations:**

- Manipulating (1)

$$r_{;ab} + d g_{ab} = -4\pi r (T_{ab} - g_{ab} T)$$

- Manipulating (2)

$$R - 2\partial_r d = 8\pi (T - 2P)$$

$$\rightarrow \quad m_{,a} = 4\pi r^2 r_{,b} (T_a^b - \delta_a^b T)$$

# BH formation.
# Poisson-Israel. General Relativity.

- Matter source: null fluid. $l_\mu l^\mu = 0$ We impose P=0.

$$T_{ab} = \rho l_a l_b \Rightarrow T = 0$$

$$m_{,a} = 4\pi r^2 T_a^b r_{,b}$$

- Incoming flux of particles: Vaidya metric.

$$ds^2 = -f dv^2 + 2dvdr + r^2 d\Omega^2 \qquad f(v,r) = 1 - \frac{2m(v)}{r} \qquad x^a = \{v, r\}$$

$v = t + r^*$ $\quad (u = t - r^*)$ (null radial coord.)

$r^* = \int dr / f$ Eddington-Finkelstein

- Non-trivial solutions when a=v and b=r. $\left(T_v^r = \rho\right)$ Using the conserv. eq. $\nabla_\mu T^{\mu\nu} = 0$ and double null coordinates, we derive that $r^2\rho$ is indep. of u.

$$m_{,v} = 4\pi r^2 \rho = L(v) \Rightarrow m(v) = \int_{v_1}^{v_2} L(v) dv \qquad \text{Ej.} \quad L(v) = m_0 \delta(v - v_0) \longrightarrow m(v) = m_0$$

$$ds^2 = -\left(1 - \frac{2\int_{v_1}^{v_2} L(v) dv}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

- Pulses contribute to the mass.
- They modify the event horizon.

# BH formation.
# Poisson-Israel. Palatini formalism.

- Analogous procedure as previously in GR!

$$d\tilde{s}^2 = h_{ab}dx^a dx^b + \tilde{r}^2(x)d\Omega^2 \;\Longrightarrow\; \left\{ \begin{array}{l} {}^4G_{ab} = -\frac{1}{\tilde{r}^2}[2\tilde{r}\tilde{r}_{;ab} + (1 - 2\tilde{r}\Box\tilde{r} - \tilde{r}^{,a}\tilde{r}_{,a})h_{ab}] \\ G_{\theta\theta} = \sin^{-2}\theta G_{\phi\phi} = \tilde{r}\Box\tilde{r} - \frac{1}{2}\tilde{r}^2 R \end{array} \right\}$$

- Reminder: In f(R,Q): $\hat{P} \to \hat{\Sigma} \to R_\mu^\nu(h)$

$$\left\{ \begin{array}{c} 2f_Q\hat{P}^2 + f_R\hat{P} - \frac{f}{2}\hat{I} = \kappa^2\hat{T} \\ \hat{\Sigma} = \left(f_R\hat{I} + 2f_Q\hat{P}\right) \\ f(R,Q) = R + l_P^2(aR^2 + R_{\mu\nu}R^{\mu\nu}) \end{array} \right\}$$

**J.M-A, G. J. Olmo, D. Rubiera-Garcia. PRD 86, 104010 (2012)**

$$R = 0\,,\; Q = 0 \qquad R_\mu^\nu(h) = \frac{1}{\sqrt{\det\hat{\Sigma}}}\left(\frac{f}{2}\delta_\mu^\nu + \kappa^2 T_\mu^\nu\right) = \kappa^2\rho l_\mu l^\nu$$

$$\Longrightarrow\; G_\mu^\nu(h) = \kappa^2\rho l_\mu l^\nu$$

- Similar to GR but for the auxiliar metric h.
- Metric g: complicated source. Simpler treatment in Palatini.

# BH formation.
# Poisson-Israel. Palatini formalism.

- Evolution equations:

$$\left\{ \begin{array}{l} \tilde{r}_{;ab} + \tilde{d}h_{ab} = -4\pi\tilde{r}T_{ab} \\ R = 2\partial_{\tilde{r}}\tilde{d} = \dfrac{4\tilde{m}}{\tilde{r}^3} \\ \boxed{\tilde{m}_{,a} = 4\pi\tilde{r}^2 T_a^b \tilde{r}_{,b}} \end{array} \right\}$$

$r = \tilde{r}$

a=v, b=r

$T_v^r = \rho$

$$\tilde{m}_{,v} = 4\pi r^2 \rho = L(v) \longrightarrow \boxed{\tilde{f} = f + 2l_p^2\kappa^2\rho} \longrightarrow m = \tilde{m} + 2rl_p^2\kappa^2\rho \quad m = m(r,v)$$

- Ingoing Vaidya metric

$$d\tilde{s}^2 = -\tilde{f}dv^2 + 2dvd\tilde{r} + \tilde{r}^2 d\Omega^2 \qquad \tilde{f} = 1 - \frac{2\tilde{m}(v)}{\tilde{r}}$$

$$g_{\mu\nu} = \frac{h_{\alpha\nu}\Sigma_\mu^\alpha}{\sqrt{\det\hat{\Sigma}}}$$

$$g_{\mu\nu} = \begin{pmatrix} -f & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} = \begin{pmatrix} \Sigma_v^\alpha h_{\alpha v} & \Sigma_v^\alpha h_{\alpha r} & 0 & 0 \\ \Sigma_r^\alpha h_{\alpha v} & 0 & 0 & 0 \\ 0 & 0 & \tilde{r}^2 & 0 \\ 0 & 0 & 0 & \tilde{r}^2\sin^2\theta \end{pmatrix}$$

$$h_{\mu\nu} = g_{\mu\nu} - 2l_p^2\kappa^2\rho l_\mu l_\nu$$

$$\boxed{ds^2 = -\left(1 - \frac{2\int_{v_1}^{v_2} L(v)dv}{r} - \frac{L(v)}{2\pi\rho_p r^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2}$$

$$\rho_p = \frac{1}{l_p^2\kappa^2} \sim 10^{96} kg/m^3$$

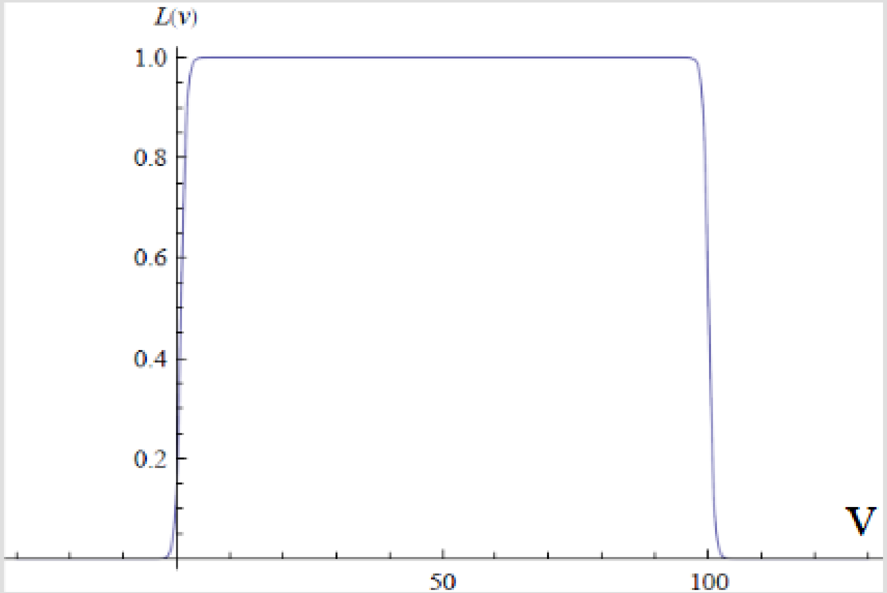
$$q_{eff}^2(v) = \frac{L(v)}{2\pi\rho_p}$$

# BH formation.
# Poisson-Israel. Palatini formalism.

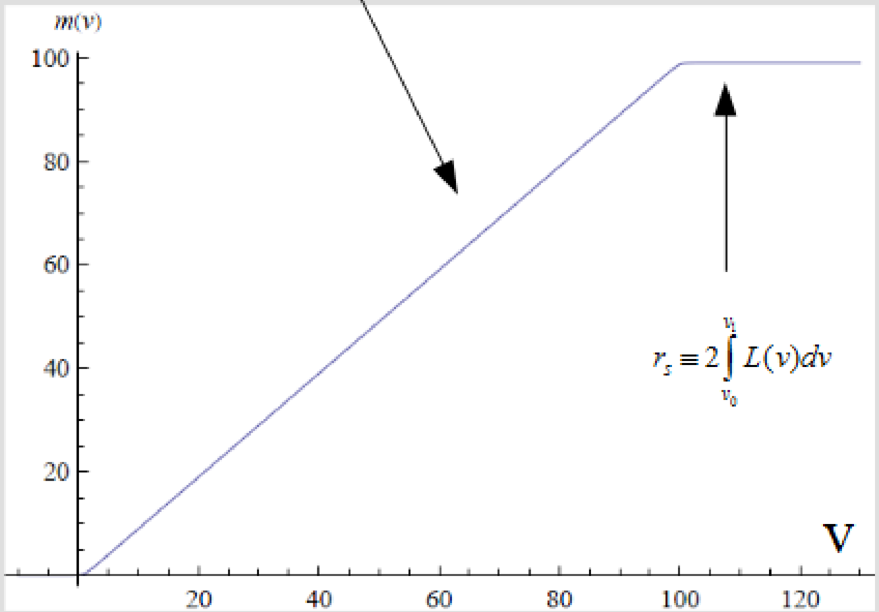
- Generation and perturbations by finite pulses of radiation. Horizons.

$$L(v) = \begin{cases} 0 & \text{if } v < v_0, \\ \frac{1}{4}(1+\tanh(v-1))(1-\tanh(v-100)) & \text{if } v \in [v_0, v_1] \\ 0 & \text{if } v > v_1. \end{cases}$$

$$r_+ = \int_{v_0}^{v_1} L(v)dv + \sqrt{\left(\int_{v_0}^{v_1} L(v)dv\right)^2 + q_{eff}^2(v)} \qquad q_{eff}^2(v) = \frac{L(v)}{2\pi\rho_p}$$



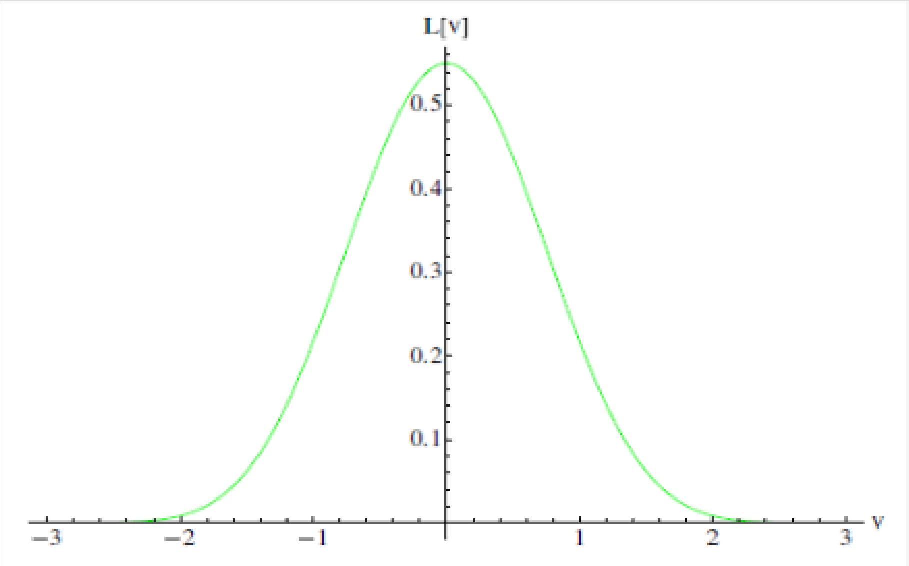

Pulse profile



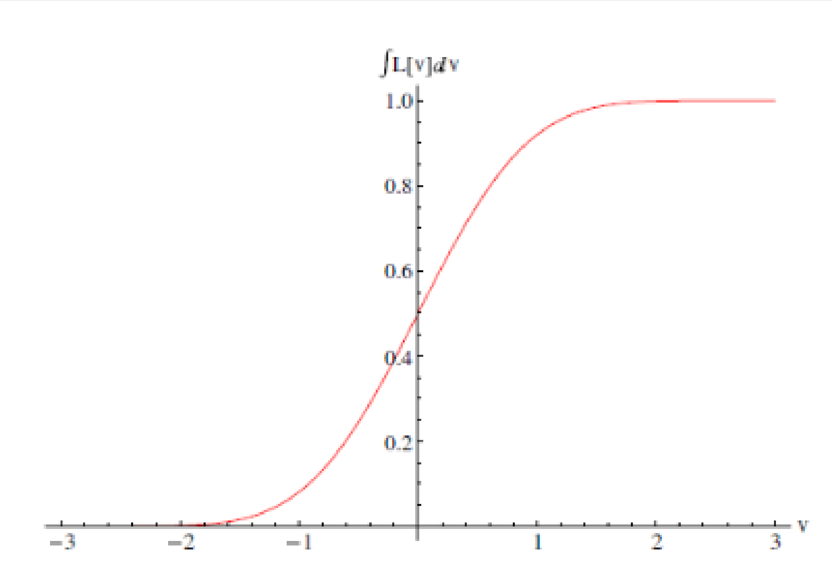

Black hole evolution

# BH formation.
# Poisson-Israel. Palatini formalism.

$$L = \frac{\varepsilon}{240}\left(|v-3|^5 - 6|v-2|^5 + 15|v-1|^5 - 20|v|^5 + 15|v+1|^5 - 6|v+2|^5 + |v+3|^5\right)$$




Pulse profile.
Compact support perturbation.




Growth of the BH mass function

# Conclusions

- We have studied the dynamics of the formation of a Schwarzschild type BH in GR using Poisson-Israel's formalism.
- The versatility of these equations has allowed us to easily extend the dynamical problem in GR to our f(R,Q) Palatini scenario. Eventually, we obtain an analytical and exact solution for our problem.
- We find that the resulting space-time is formally that of a Reissner-Nordström black hole but with an effective charge carrying the wrong sign in front of it (corrections at Planck scale).

$$ds^2 = -\left(1 - \frac{2\int_{v_1}^{v_2} L(v)dv}{r} - \frac{L(v)}{2\pi\rho_p r^2}\right)dv^2 + 2dvdr + r^2 d\Omega^2$$

- Relevant information for the BH evaporation mechanism. Recovery of ingoing quantum information via Hawking radiation?
- Research in progess: extension to more complex cases (ingoing+outgoing fluxes → (new?) mass inflation problem, non-singular RN BH, etc)

Thanks!

# f(R) field equations

- Metric formalism

$$f_R R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu} \Box f_R = \kappa^2 T_{\mu\nu}$$

$$3\Box f_R + R f_R - 2f = \kappa^2 T$$

T=0 ↓

$$R = R_0 = \frac{2f(R_0)}{f_R(R_0)}$$

$$\Rightarrow \quad R_{\mu\nu} = \frac{\kappa^2}{f_R(R_0)} T_{\mu\nu} + \frac{R_0}{4} g_{\mu\nu}$$

- Palatini formalism

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{\kappa^2}{f_{\Re}} T_{\mu\nu} - \frac{\Re f_{\Re} - f}{2 f_{\Re}} g_{\mu\nu} - \frac{3}{2 f_{\Re}^2} \left[ \partial_\mu f_{\Re} \partial_\nu f_{\Re} - \frac{1}{2} g_{\mu\nu} (\partial f_{\Re})^2 \right] + \frac{1}{f_{\Re}} \left( \nabla_\mu \nabla_\nu f_{\Re} - g_{\mu\nu} \Box f_{\Re} \right)$$

$$\Re f_{\Re} - 2f = \kappa^2 T$$

# BH Formation.
# Poisson-Israel. General Relativity.

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

$$2r r_{;ab} + (1 - 2r\Box r - r^{,a} r_{,a}) g_{ab} = -8\pi r^2 T_{ab} \quad (1)$$

$$r\Box r - \frac{1}{2} r^2 R = 8\pi r^2 P \quad (2)$$

- Manipulating (1)

Ansatz (hypersurface r=constant) : $r^{,a} r_{,a} = g^{ab} r_{,a} r_{,b} \equiv f \equiv 1 - \frac{2m}{r}$

$$r_{;ab} + \left(\frac{m}{r^2} - \Box r\right) g_{ab} = -4\pi r T_{ab} \rightarrow r_{;ab} - (d + \Box r) g_{ab} = -4\pi r T_{ab} \rightarrow \boxed{r_{;ab} + d g_{ab} = -4\pi r (T_{ab} - g_{ab} T)}$$

$$d \equiv -\frac{1}{2}\partial_r f = -\frac{m}{r^2}$$

$$\Box r = -2d + 4\pi r T$$

Tracing

- Manipulating (2)

$$\Box r - \frac{1}{2} r R = 8\pi r P \rightarrow R + \frac{4}{r} d = R - \frac{4m}{r^3} = 8\pi (T - 2P) \rightarrow \boxed{R - 2\partial_r d = 8\pi (T - 2P)}$$

$$\Rightarrow \quad f_{,a} = -\frac{2}{r} m_{,a} + \frac{2m}{r^2} r_{,a} = 2 g^{bc} r_{,b} r_{;ac} \rightarrow \boxed{m_{,a} = 4\pi r^2 r_{,b} (T_a^b - \delta_a^b T)}$$

# BH Formation.
# Poisson-Israel. Palatini formalism.

- Analogous procedure as in GR!

$$d\tilde{s}^2 = h_{ab}dx^a dx^b + \tilde{r}^2(x)d\Omega^2 \quad \longrightarrow \quad \left\{ \begin{array}{l} {}^4G_{ab} = -\frac{1}{\tilde{r}^2}[2\tilde{r}\tilde{r}_{;ab} + (1 - 2\tilde{r}\Box\tilde{r} - \tilde{r}^{,a}\tilde{r}_{,a})h_{ab}] \\ G_{\theta\theta} = \sin^{-2}\theta G_{\phi\phi} = \tilde{r}\Box\tilde{r} - \frac{1}{2}\tilde{r}^2 R \end{array} \right\}$$

$$x^\alpha = (x^a, \theta, \phi) \text{ con } a = 1,2$$

- Reminder:
  - In f(R,Q): $\hat{P} \to \hat{\Sigma} \to R_\mu^\nu(h)$

$$\left\{ \begin{array}{c} 2f_Q\hat{P}^2 + f_R\hat{P} - \frac{f}{2}\hat{I} = \kappa^2\hat{T} \\ \hat{\Sigma} = \left(f_R\hat{I} + \boxed{2f_Q\hat{P}}\right) \end{array} \right\}$$

Null fluid

$$2f_Q\left(\hat{P} + \frac{f_R}{4f_Q}\hat{I}\right)^2 = \underbrace{\left(\frac{f}{2} + \frac{f_R^2}{8f_Q}\right)\hat{I} + \kappa^2\rho l_\mu l^\nu}_{} = B^2$$

Non-diagonal in {v,r}!!

# BH Formation.
# Poisson-Israel. Palatini formalism.

- Ansatz $B_\mu^\alpha = \lambda\delta_\mu^\nu + 2\lambda\Omega l_\mu l^\nu \longrightarrow B_\mu^\alpha B_\alpha^\nu = \lambda^2\delta_\mu^\nu + 2\lambda\Omega l_\mu l^\nu$ $\quad \lambda^2 \equiv \frac{f}{2} + \frac{f_R^2}{8 f_Q}$ $\quad \Omega \equiv \frac{\kappa^2\rho}{2\lambda}$

$$(3) \Longrightarrow \sqrt{2 f_Q}\left(\hat{P} + \frac{f_R}{4 f_Q}\hat{I}\right) = \lambda\delta_\mu^\nu + \frac{\kappa^2\rho}{2\lambda} l_\mu l^\nu \longrightarrow \hat{\Sigma} = \frac{f_R}{2}\hat{I} + \sqrt{2 f_Q}\hat{B}$$

$$\boxed{f(R,Q) = R + l_p^2(aR^2 + Q)} \qquad R = -\kappa^2 T \longrightarrow \begin{Bmatrix} R = 0 \\ f_R = 1 \end{Bmatrix} \longrightarrow f(R,Q) = l_p^2 Q \longrightarrow f_Q = l_p^2$$

- Tracing (3) we get Q=0.

$$\hat{\Sigma} = \frac{f_R}{2}\hat{I} + \sqrt{2 f_Q}\hat{B} = \delta_\mu^\nu + 2 l_p^2\kappa^2\rho l_\mu l^\nu \qquad \det\hat{\Sigma} = \varepsilon_{abcd}\Sigma_0^a\Sigma_1^b\Sigma_2^c\Sigma_3^d = 1$$

$$R_\mu^\nu(h) = \frac{1}{\sqrt{\det\hat{\Sigma}}}\left(\frac{f}{2}\delta_\mu^\nu + \kappa^2 T_\mu^\nu\right) = \kappa^2\rho l_\mu l^\nu \Longrightarrow G_\mu^\nu(h) = \kappa^2\rho l_\mu l^\nu$$

The geometry is characterized by the soruces of matter!!