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Effective field theory approach to non-relativistic neutralino dark matter pair-annihilation processes

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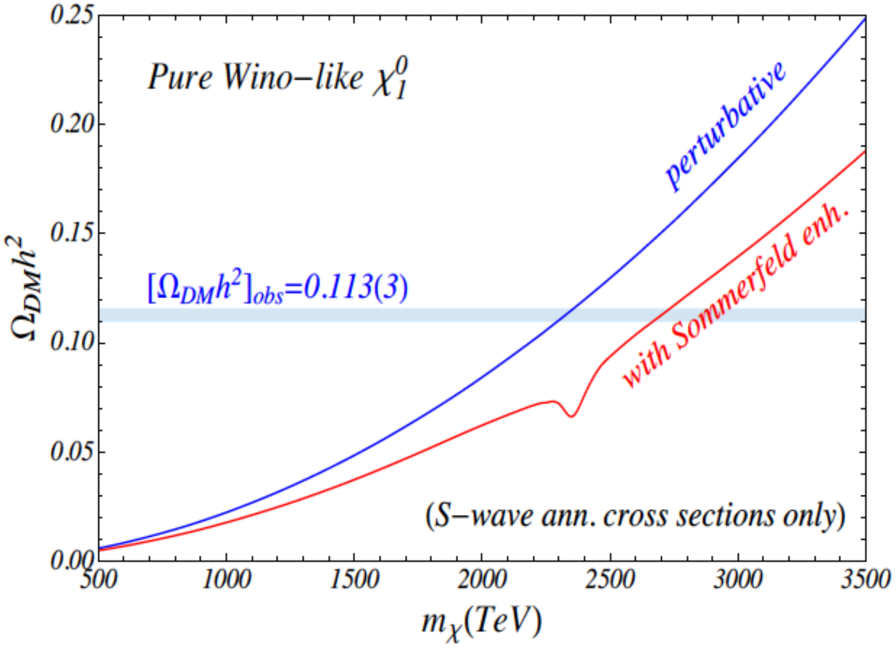
Outline

Motivation

Recent astrophysical measurements allow for the determination of the cold dark matter density with percent level accuracy: $\Omega_{\rm cdm}\, h^2 \;=\; 0.111 \pm 0.006 \; (68\% CL)$
[J. Beringer et al (PDG), (2012)]

Attractive scenario to explain the observed abundance: thermal relic of a WIMP

- Lightest neutralino ($\tilde{\chi}_1^0$) promising candidate within the MSSM
- Requirement to reproduce observed CDM abundance as $\tilde{\chi}_1^0$ relic poses strong constraints on the MSSM parameter space



Sommerfeld enhancement on the annihilation cross sections can significantly shift m_χ consistent with the experimental $\Omega_{\rm cdm} h^2$ value *[J. Hisano et al. (2007)]*

Revisit the Sommerfeld enhancement in the $\tilde{\chi}_1^0$ dark matter relic abundance calculations for arbitrary $\tilde{\chi}_1^0$ composition and including P-wave effects

II. Sommerfeld enhancement effect

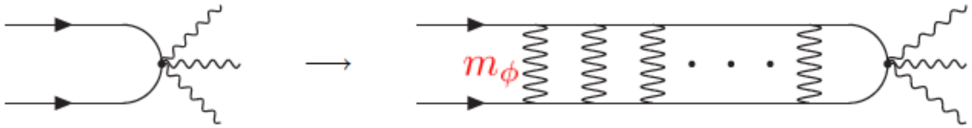
Sommerfeld enhancement for neutralinos in the MSSM (I)

Sommerfeld enhancement

Enhancement of cross sections due to distortion of incoming (outgoing) non-relativistic particles' plane wave functions in presence of long-range potential interactions

In DM annihilation this happens when the Coulomb (Yukawa) force generated by massless (massive) particle exchange between the DM particles becomes strong at small relative velocities

$\hookrightarrow v \lesssim \alpha$ and $1/m_\phi \gtrsim 1/\alpha\, m_\chi$



[J. Hisano et al. (2005/2007); M. Cirelli et al. (2007); N. Arkani-Hamed et al. (2009); R. Iengo (2009); S. Cassel (2010); T. Slatyer (2010); A. Hryczuk et al. (2010) (MSSM investigation)]

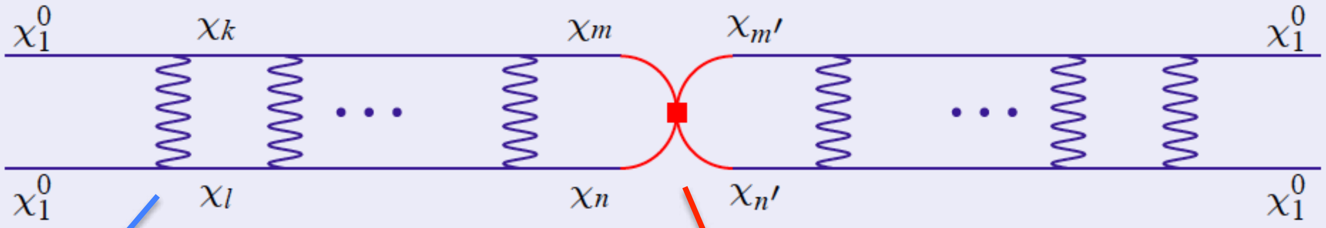
Naturally realized in the MSSM when the lightest $\tilde{\chi}^0$ has $m_{\tilde{\chi}^0} \gtrsim 1$ TeV

- Wino- or Higgsino-like $\tilde{\chi}^0$ must be relatively heavy to produce the observed dark matter density: $m_{\tilde{\chi}^0} \sim \mathcal{O}(\text{TeV}) \to m_{\tilde{\chi}^0} \gg m_W, m_Z$
- $\tilde{\chi}^0$ is non-relativistic during thermal decoupling in the early universe: $v \sim 0.2\, c$
- Mass degeneracies with slightly heavier particles in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector are generic for heavy SUSY $\to$ Co-annihilations in the relic abundance calculation

Sommerfeld enhancement for neutralinos in the MSSM (II)

Annihilation cross sections are related to the absorptive part of forward scattering amplitudes:

$$|\mathcal{M}(\vec{p})|^2 \;=\; 2\,\Im \sum$$



- Long-range effects described by potential interactions $V_{\{ij\}\{k\ell\}}$
  → t- and u-channel exchange of the MSSM gauge and Higgs bosons $[W^\pm, Z, \gamma, h^0, H^0, A^0, H^\pm]$

- Short-distance annihilation described by absorptive part of $\chi_i\chi_j \to X_A X_B \to \chi_k\chi_\ell$
  → annihilation matrix $\Gamma_{\{ij\}\{k\ell\}}$

- Need absorptive part of all (off-)diagonal reactions $\chi_i\chi_j \to X_A X_B \to \chi_k\chi_l$
  [Not included in A. Hryczuk et al. (2010) !]

- Diagonal entries of $\Gamma_{\{ij\}\{kl\}}$ are related to coefficients a and $b = b_{S\text{-wave}} + b_{P\text{-wave}}$ in

$$\sigma^{\rm tree}_{\{ij\}\to\{AB\}}\; v_{\rm rel} \;=\; a + b\,v_{\rm rel}^2 + \mathcal{O}(v_{\rm rel}^4) \qquad\qquad v_{\rm rel} = |\vec{v}_i - \vec{v}_j|$$

→ a and b may be obtained numerically from codes as DarkSUSY or micrOMEGAS

× Off-diagonal reactions not accessible

× No decomposition of $b = b_{S\text{-wave}} + b_{P\text{-wave}}$ (needed to study Sommerfeld enh. at $\mathcal{O}(v^2)$)

III. EFT approach to dark matter annihilations

Non-relativistic effective theory approach: NRMSSM

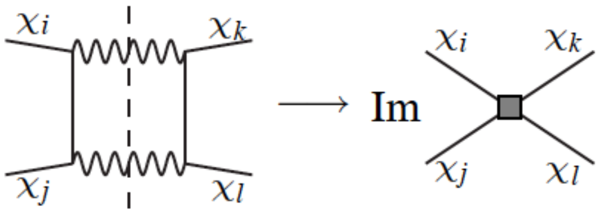
(Co-) annihilation processes of non-relativistic $\tilde{\chi}^0_i s$ and $\tilde{\chi}^\pm_i s$ are characterized by well separated scales:

- hard scale: m_i [associated with the short-distance annihilation reactions]
- potential momenta: $E_i \sim m_i \vec{v}^{\,2}$, $|\vec{p}| \sim m_i |\vec{v}|$ [associated with long-range effects]

→ Use methods of non-relativistic Effective Field Theories (EFTs) to integrate out all MSSM modes down to the scale of potential momenta

$$\mathcal{L}_{\rm eff} = \xi_i^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2}{2m_0} - \delta m_i \right) \xi_i + \int d^3\vec{r}\, [\xi_i^\dagger \xi_j^c](x,\vec{r})\, V_{\{ij\}\{kl\}}(\vec{r})\, [\xi^{c\,\dagger}_l \xi_k](x)$$

$$+ f_{\{ij\}\{kl\}}({}^{2s+1}L_J)\, \mathcal{O}_{\{ij\}\{kl\}}({}^{2s+1}L_J) + \ldots$$



- Short-distance ($\sim 1/m_i$) effects:
  Accounted for by imaginary parts of Wilson coefficients of local four-fermion operators $\mathcal{O}_{\{ij\}\{kl\}}$

$$\mathcal{O}({}^1S_0) = \xi_l^\dagger \xi_k^c \quad \xi_j^{c\,\dagger} \xi_i\,,$$

$$\mathcal{O}({}^3S_1) = \xi_l^\dagger \vec{\sigma} \xi_k^c \quad \xi_j^{c\,\dagger} \vec{\sigma} \xi_i\,, \quad \cdots\cdots$$

[see *G.T. Bodwin et al., (1995)*]

Derive purely analytical expressions for $V_{\{ij\}\{kl\}}$ and $f({}^{2s+1}L_J)_{\{ij\}\{kl\}}$ in matching the NRMMSM to the MSSM

Four-fermion operators – MSSM matching calculation

[M. Beneke, C. Hellmann and PRF (2012)]

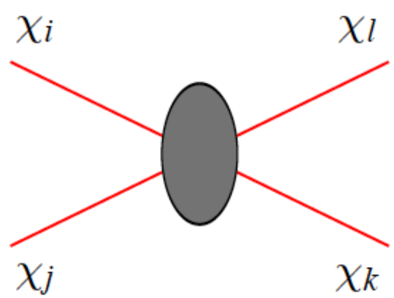
Determine absorptive part of 1-loop $\chi\chi \to X_A X_B \to \chi\chi$ scattering amplitudes

- Expansion of spin-projected $\chi\chi \to X_A X_B \to \chi\chi$ amplitudes in the non-relativistic regime ($X_A X_B$ given by all accessible standard model and higgs final states)
- Diagonal reactions $\chi_i\chi_j \to X_A X_B \to \chi_i\chi_j$ encode tree-level annihilation rates into the $X_A X_B$ state
  $\to$ Allows for checks with numerical codes (MadGraph)

Subtle point: non-relativistic expansion of off-diagonal $\chi_i\chi_j \to X_A X_B \to \chi_k\chi_l$ amplitudes

Consider center-of-mass system:

$$\sqrt{s}_{|\text{in}} = m_i + m_j + \frac{\vec{p}^{\,2}}{2\mu_{\text{red}_{ij}}} + \cdots$$



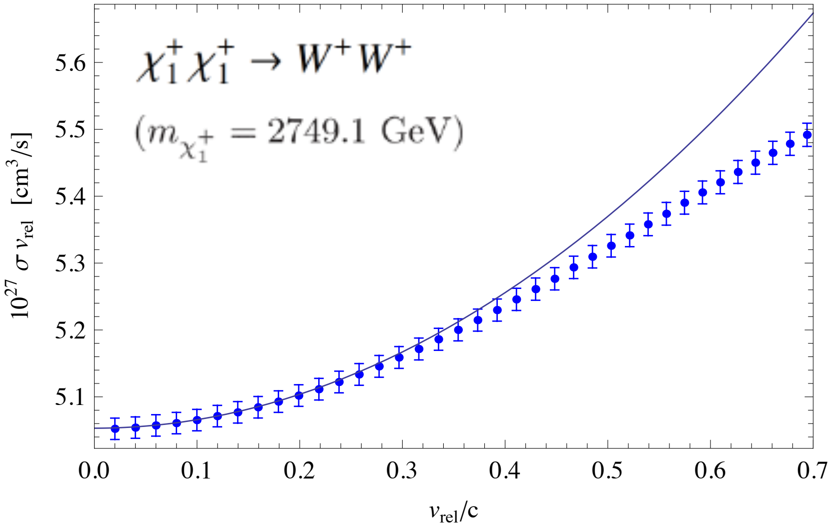
$$\sqrt{s}_{|\text{out}} = m_k + m_l + \frac{\vec{p}'^{\,2}}{2\mu_{\text{red}_{kl}}} + \cdots$$

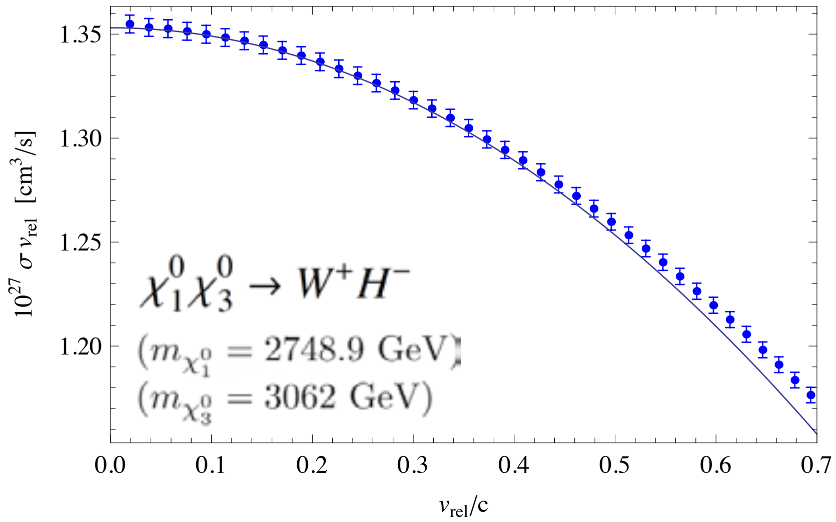
$\to$ Mass difference $(m_i + m_j) - (m_k + m_l)$ can be of order $\mathcal{O}(\vec{p}^{\,2})$ at most

Need simultaneous expansion in non-relativistic momenta and mass differences

Born-level annihilation cross section: EFT vs MadGraph

Numerical comparison of the tree-level annihilation cross section: $\sigma^{\chi\chi\to X_A X_B}\, v_{\rm rel}$





solid: $\sigma\, v_{\rm rel} = a + b\, v_{\rm vrel}^2$ computed with the EFT

dots: full $\sigma\, v_{\rm rel}$ determined numerically with MadGraph

EFT approach provides a good aproximation in the non-relativistic regime:

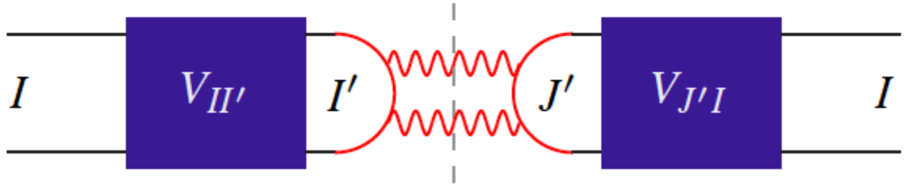
accuracy $\sim$ few percent at $v_{\rm rel} \sim 0.6$

$\to$ reliable for calculations in the early universe ($v_{\rm rel} \sim 0.4$ at χ^0-decoupling)

Long-range potential interactions: Sommerfeld factor

Consider case of N non-relativistic two-particle states of same charge:

$$I \;=\; \chi^0\chi^0,\, \chi^-\chi^+ \\ \chi^0\chi^\pm \\ \chi^\pm\chi^\pm$$



Enhancement of annihilation rate relative to corresponding Born level expression Γ_{II} is described by

$$S_I \;=\; \frac{\left[\,\vec{\psi}_I^{\,*}\,(\vec{r}{=}0)\,\right]_{J'}\,\Gamma_{J'I'}\,\left[\vec{\psi}_I\,(\vec{r}{=}0)\right]_{I'}}{\left[\vec{\psi}_I^{(0)*}\,(\vec{r}{=}0)\,\right]_{J'}\,\Gamma_{J'I'}\,\left[\vec{\psi}_I^{(0)}\,(\vec{r}{=}0)\,\right]_{I'}} \qquad \text{(leading order } S\text{-wave)}$$

- $\vec{\psi}_I, I = 1, \ldots, N$ are the regular scattering solutions ($\psi_I^{(0)}$ corresp. free solutions) of the Schrödinger equation

$$\left(-\frac{\vec{\partial}^{\,2}}{\mu_J}\,\delta_{IJ} \;+\left[\,V_{IJ}(|\vec{r}|) + \left(M_J - 2m_{\tilde{\chi}_1^0}\right)\delta_{IJ}\right]\right)\,\vec{\psi}_J \;=\; E\vec{\psi}_I$$

- $V_{IJ}(|\vec{r}|)$ calculated at leading order arising from potential $W^\pm, Z, \gamma$, and $h^0, H^0, A, H^\pm$ exchange

Annihilation cross section including long-range interactions

Determination of $\sigma v_{\rm rel}$ within the NRMSSM:

- For given SUSY spectrum identify all two particle channels in $\tilde\chi^0/\tilde\chi^\pm$ sector participating in (co-)annihilation processes during $\tilde\chi_1^0$ freeze-out (criterion: $m_{\rm channel} - 2m_{\tilde\chi_1^0} < m_{\tilde\chi_1^0}/20$)
- Solve the corresponding multi-state Schrödinger equation numerically for given V^s_{IJ} and $\Gamma_{IJ}(^{2s+1}L_J)$ for different values of the non-relativistic energy E of the respective annihilating two particle system [done separately for each of the charge sectors (neutral, single and double charged)]

Sommerfeld enhanced $\sigma_I v_{\rm rel}$ for annihilating two particle channel I, composed of χ_i and χ_j:

$$\sigma_I|\vec v_i - \vec v_j| = \sum_{^1S_0,\,^3S_1} S_I(^{2s+1}L_J)\,\Gamma_{II}(^{2s+1}L_J) + \vec p_i^{\,2}\Big[\sum_{^1P_1,\,^3P_J} S_I(^{2s+1}L_J)\,\Gamma_{II}(^{2s+1}L_J) + \sum_{^1S_0,\,^3S_1} S_I(^{2s+1}L_J)\,\Gamma^{p^2}_{II}(^{2s+1}L_J)\Big]$$

IV. Relic abundance calculation

Relic abundance calculation, standard framework

Precise method available to determine the $\tilde{\chi}^0$ relic abundance in presence of co-annihilations:

[K. Griest, D. Seckel, (1991); P. Gondolo, G. Gelmini, (1991)]

Consider the Boltzmann equation for the yield $Y \equiv \frac{n}{s}$, where s is the entropy density of the Universe

$$\frac{dY}{dx} = \frac{\langle \sigma_{\rm eff} v_{\rm rel} \rangle}{H\,x} \left(1 - \frac{x}{3\,g_{*s}} \frac{d\,g_{*s}}{d\,x} \right) s \left(Y^2 - (Y^{eq})^2 \right)$$

- $n = \sum_i n_{\chi_i}$ is the sum of all (co-)annihilating $\tilde{\chi}^0/\tilde{\chi}^\pm$ species at $\tilde{\chi}^0_1$ freeze-out
- $x = m_{\tilde{\chi}^0_1}/T$

$$\langle \sigma_{\rm eff}\, v_{\rm rel} \rangle = \sum_{i,j} \langle \sigma_{ij}\, v_{\rm rel} \rangle \frac{4}{g_{\rm eff}(x)} \left[1 + \Delta_i\right]^{3/2} \left[1 + \Delta_j\right]^{3/2} \exp\left(-x\,(\Delta_i + \Delta_j)\right)$$

- $\Delta_i = (m_{\chi_i} - m_{\tilde{\chi}^0_1})/m_{\tilde{\chi}^0_1}$
- $g_{\rm eff}(x)$ describes number of effective degrees of freedom in $\tilde{\chi}^0/\tilde{\chi}^\pm$ sector during freeze-out

$\rightarrow$ Relic abundance determined as $\Omega_{\chi_1} h^2 = \rho^0_{\chi_1}/\rho_{crit}\, h^2 = m_\chi s_0 Y_0/\rho_{crit}\, h^2$

Results for a wino-like neutralino LSP (I)

[M. Beneke, C. Hellmann and PRF (2012)]

Consider wino-like $\tilde{\chi}_1^0$ scenario with $m_{\tilde{\chi}_1^0} = 2.75$ TeV:

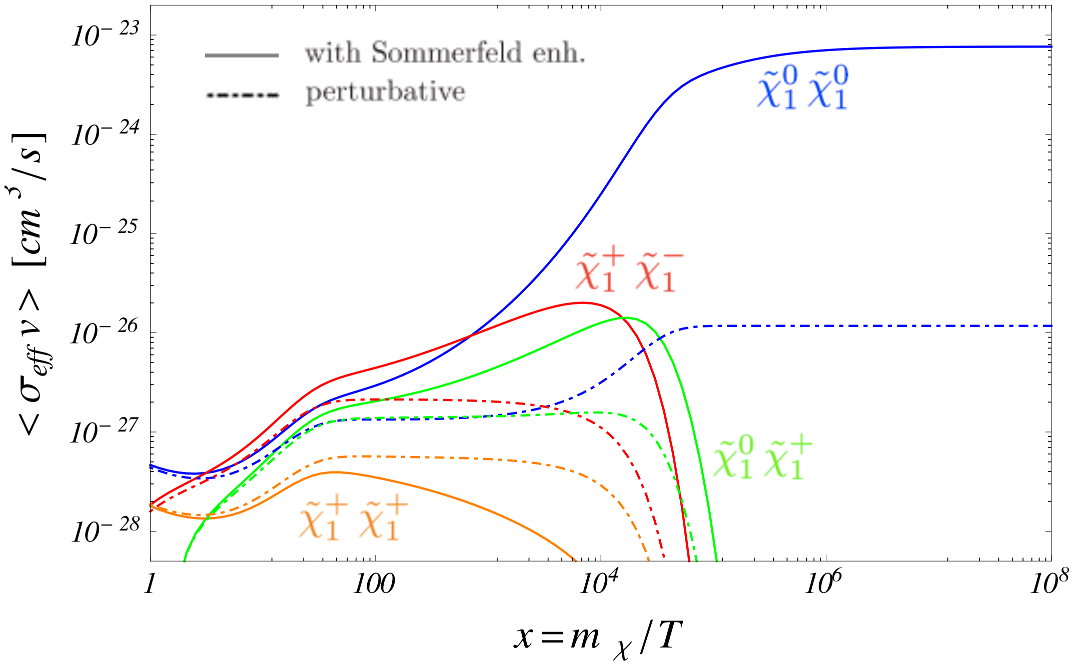
- $\tilde{\chi}_1^{\pm}$ with $\delta m_{\tilde{\chi}_1^-} = 0.21$ GeV
- next-to next-to lightest particle in $\tilde{\chi}^0/\tilde{\chi}^{\pm}$ sector with $\delta m_{\tilde{\chi}_2^0} \sim 200$ GeV

$\rightarrow$ Perform relic abundance calculation within the NRMSSM with three non-relativistic species $\tilde{\chi}_1^0, \tilde{\chi}_1^{\pm}$

Cross section calculation includes

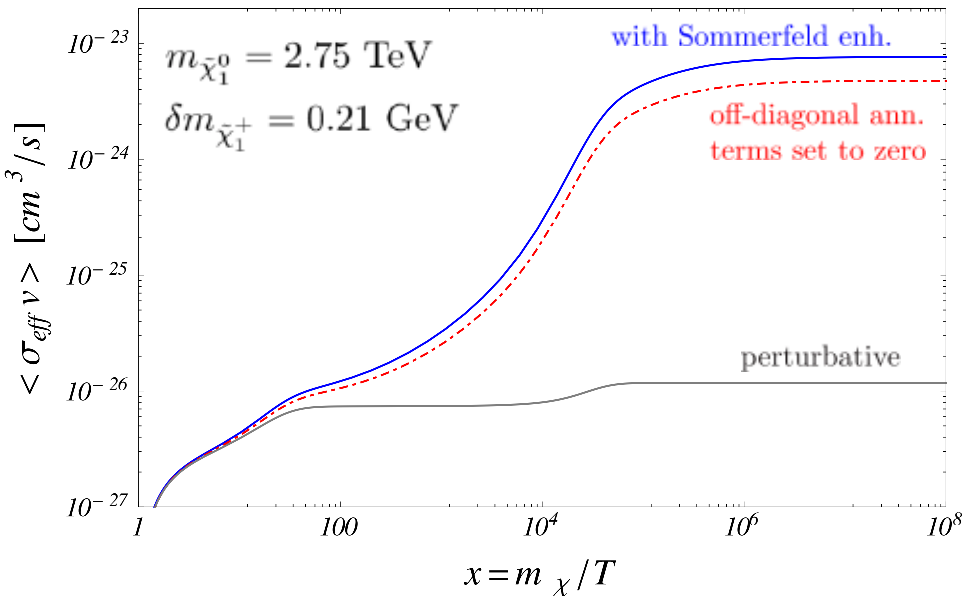
- full mixing matrix effects
- effects of $\mathcal{O}(v^2)$ S- and P- waves
- off-diagonal elements in annihilation matrix $\Gamma_{\{ij\}\{kl\}}$
- taking channel $\tilde{\chi}_1^0\tilde{\chi}_2^0$ perturbatively into account ($E \ll \delta M \ll m_{\tilde{\chi}_1^0}$)

Thermally averaged cross sections

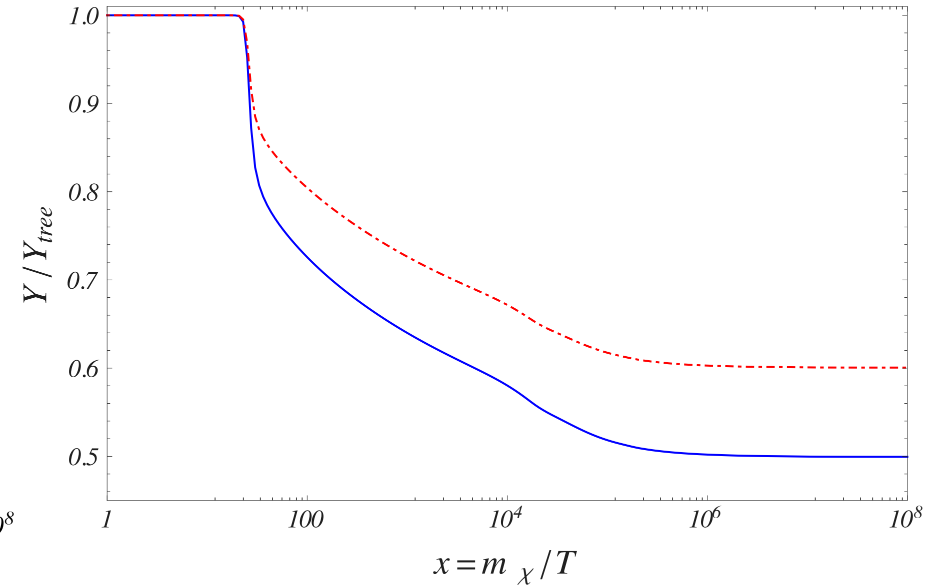


Results for a wino-like neutralino LSP (II)

Thermally averaged cross section $\langle \sigma_{\rm eff}\, v \rangle$



Evolution of yield $Y/Y_{\rm tree}$



- Since the ann. cross section is increased for low T by the Sommerfeld enhancement, the sudden freeze-out of the yield does not occur

→ The resultant dark matter abundance is reduced by $\sim 50\%$ compared to the pertubative result

(in agreement with previous investigations *[J. Hisano et al. (2007), M. Cirelli et al. (2007)]*)

- Neglecting the off-diagonal annihilation terms decreases $\langle \sigma_{\rm eff}\, v \rangle$ by a factor $\gtrsim 1.5$ at small T → thermal relic abundance $\Omega_{\rm DM}\, h^2$ increased by $\sim 20\%$

Summary

- The calculation of the thermal relic abundance of the lightest neutralino as a dark matter candidate places strong bounds on the MSSM parameter space

- Given the increasing exp. accuracy of $\Omega\, h^2$, radiative corrections to $\sigma_{\rm ann}$ have to be taken into account (Sommerfeld correction can be the dominant one!)

$\rightarrow$ Also relevant in dark matter annihilation in the present universe

- Annihilating neutralinos are non-relativistic $\rightarrow$ Non-relativistic EFT is the appropriate setup for a systematic investigation of rad. cor. to $\tilde{\chi}^0$ LSP pair annihilation

  ✓ factorization of long-range and short-distance effects

  ✓ co-annihilations with nearly mass-degenerate $\tilde{\chi}^0$'s and $\tilde{\chi}^\pm$'s

  ✓ off-diagonal reactions $\chi_i\chi_j \rightarrow X_A X_B \rightarrow \chi_k\chi_\ell$ with $\{ij\} \neq \{k\ell\}$ (new!)

  ✓ separation of $S-$ and $P-$wave annihilation rates at $\mathcal{O}(v^2)$ (new!)

for a generic MSSM parameter-space point

- A lot of work ahead: use these tools to study the impact of the Sommerfeld enhancement effect in the MSSM relic density calculations

Backup slides

Sommerfeld enhancement in today's universe

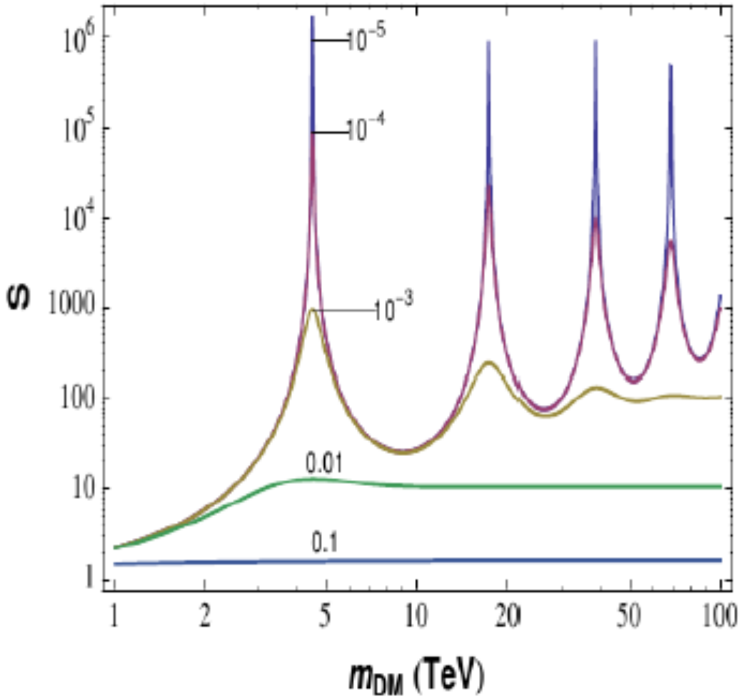
Impact of the Sommerfeld effect on DM pair annihilations today?

- Cool 'late' universe: velocities v of annihilating DM particles are typically very small

$\rightarrow$ $\langle \sigma_{\text{ann.}} v \rangle = a + b v^2 + \cdots \approx a,$ $\quad a$ contains pure s-wave contributions

- Cross section in s-wave case: σ_0 annihilation cross section without enhancement

$$\sigma = \sigma_0 \cdot S_{\ell=0}, \qquad S_{\ell=0} \equiv \frac{|\psi_+^{(E)}(\vec{r}=0)|^2}{|\psi_0(\vec{r}=0)|^2},$$



[Lattanzi, Silk; 2008]: for Yukawa type potential

$\rightarrow$ S exhibits a resonance structure *(zero energy resonances; special feature of Yukawa potential for negligible kin. energy)*

$\rightarrow$ Enhancement up to 6 orders of magnitude

$\Rightarrow$ *Orders of magnitude required to explain e.g. PAMELA data by dark matter annihilation signal*