

Perturbative and Non Perturbative Physics at zero and finite temperature: Polyakov loop, Condensates and Holography

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Issues

- 1 Introduction
- 2 Field Theory
 - Perturbative: Effective Action at $T > T_c$
 - Phase Transition: Polyakov-Chiral Quark Models
 - OPE-breaking effects and Condensates
- 3 Holography
 - Equation of State of QCD
 - Heavy quark physics
- 4 Hydrodynamics of Relativistic Fluids
 - Chiral Magnetic and Vortical Effects
 - Kubo Formulae
 - Fluid/Gravity Correspondence

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Background

- E. Megías (**Juan de la Cierva Researcher**).
- **2001**: Degree in Physics (Theoretical Physics), U. Granada “2nd National Award”.
- **Research experience**:
 - **Jun 2006**: PhD Physics (U.Granada) “Exceptional award”.
 - **Apr 2007 - Mar 2009**: Postdoct MICINN and Fulbright (BNL, USA).
 - **Apr 2009 - Sep 2010**: Postdoct Humboldt Foundation (Heidelberg, Germany).
 - **Oct 2010 - Sep 2011**: Postdoct CPAN (IFT CSIC/UAM, Madrid).
 - **Oct 2011 - Present**: Postdoct Juan de la Cierva (UAB and IFAE, Barcelona).
- **Participation in 17 research projects**:
 - 1 **Dynamics of the Hadronic Systems**: BFM2002-03218, FIS2005-00810, FQM-0225, RII3-CT-2004-406078, ...
 - 2 **Information theory**: FQM-481, P06-FQM-01735 (Junta Andalucía).
 - 3 **String Theory and phenomenology**: FPA-2009-07908.
 - 4 **High Energy Physics**: FPA2008-01430, FPA2011-25948.
 - 5 ...

Outline

- **Quantum Chromodynamics** → Fundamental Theory of Strong Interactions.
- **Nuclear matter under extreme conditions** → High Temperatures and Densities: Big Bang, neutron stars, heavy ion colliders.
- **Methods:**
 - Perturbation theory.
 - Lattice.
 - Semiclassical methods (instantons).
 - **Holography** [Maldacena '98]:
Duality: weakly coupled gravity \leftrightarrow strongly coupled field theory.
- **Field Theory:**
 - 1 Effective Field Theories.
 - 2 OPE-breaking effects and Condensates.
 - 3 Transport properties of the QGP (Quark Gluon Plasma).
- **Holography:**
 - 1 Holographic description of the equation of state of QGP.
 - 2 Holographic description of heavy quarks in a plasma.
 - 3 Transport properties of the QGP.

Finite Temperature Field Theory

Finite Temperature Field Theory \equiv Quantum Field Theory + Statistical Mechanics. Partition function:

$$Z = \text{Tr} e^{-\beta H} = \int \mathcal{D}A_\mu \mathcal{D}q_i e^{-\int d^4x \mathcal{L}_{\text{QCD}}}, \quad \beta = \frac{1}{k_B T}.$$

- Gluons are periodic in time: $A_\mu(x_0 + \beta, \vec{x}) = A_\mu(x_0, \vec{x})$.
- Quarks are antiperiodic in time: $q_i(x_0 + \beta, \vec{x}) = -q_i(x_0, \vec{x})$.

There is a phase transition in QCD:

- $T < T_c$: **confined phase**
- $T > T_c$: **unconfined phase**

No Chiral Sym ($\langle \bar{q}q \rangle \neq 0$)

Chiral Sym ($\langle \bar{q}q \rangle = 0$)

$\mathbb{Z}(3)$ Sym ($\langle \Omega \rangle = 0$)

No $\mathbb{Z}(3)$ Sym ($\langle \Omega \rangle \neq 0$)

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Perturbative: Effective Action at $T > T_c$

[Pisarski '00], [Diakonov and Oswald '04], [Megías, Ruiz Arriola and Salcedo, PRD '04]

- The Quark-Gluon Plasma behaves classically at very high temperatures \implies Quantum fluctuations are small.

We use the background field approach (valid $T > T_c$):

$$A_\mu \longrightarrow \underbrace{A_\mu}_{\text{Classical}} + \underbrace{a_\mu}_{\text{Quantum Fluctuation}}$$

One loop effective action $T > T_c$:

$$\Gamma[A] = -\frac{\mu^{-2\epsilon}}{2g_0^2} \text{Tr}(F_{\mu\nu}^2) + \Gamma_q[A] + \Gamma_g[A] =: \int_0^\beta dx_0 \int d^3\mathbf{x} \mathcal{L}(x)$$

where

$$\Gamma_{q,g}[A] = -\text{Tr} \log(\mathcal{O}_{q,g})$$

Compute in a derivative expansion (heat kernel) **Megías PRD'04**:

$$\Gamma_{q,g}[A] = \text{tr}[f_0(\Omega) + f_E(\Omega)\vec{E}_i^2 + f_B(\Omega)\vec{B}_i^2 + f_{E_2}(\Omega)\vec{E}_i^4 + \dots]$$

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Phase Transition: Polyakov-Chiral Quark Models

- **Chiral Quark Models:** Describe the dynamics of QCD at low energies. Based on the chiral symmetry.
Example: Nambu–Jona-Lasinio model (1961).

$$\begin{aligned} \mathcal{L}_{NJL} = & \bar{q}(\not{\partial} + \hat{M}_0)q + \frac{1}{2a_s^2} \sum_{a=0}^{N_f^2-1} ((\bar{q}\lambda_a q)^2 + (\bar{q}\lambda_a \gamma_5 q)^2) \\ & - \frac{1}{2a_v^2} \sum_{a=0}^{N_f^2-1} ((\bar{q}\lambda_a \gamma_\mu q)^2 + (\bar{q}\lambda_a \gamma_\mu \gamma_5 q)^2) \end{aligned}$$

- **Polyakov-Chiral Quark Models:** Coupling of the Polyakov loop in the chiral quark models [Fukushima '04], [Megías PRD'06], [Ratti '06]:

$$\not{\partial} \longrightarrow \not{\partial} + \gamma^0 A_0$$

Compute the effective action: quarks at 1 loop, mesons at tree level.

$$\mathcal{L}(x) = \sum_n \text{tr}[f_n(\Omega(x)) \mathcal{O}_n(x)], \quad \Omega(\vec{x}, x_0) = \mathcal{T} e^{-\int_{x_0}^{x_0+\beta} dx'_0 A_0(\vec{x}, x'_0)}$$

Phase Transition: Polyakov-Chiral Quark Models

For any observable: $\langle \mathcal{O} \rangle^* = \frac{1}{Z} \int DUD\Omega e^{-\Gamma_G[\Omega]} e^{-\Gamma_Q[U,\Omega]} \mathcal{O}$.

- Beyond mean field: Integration in Color Group $\int \mathcal{D}\Omega$.
- Analytically \longrightarrow Expand the exponents and compute correlation functions of Polyakov loops:

$$\int D\Omega \text{tr}_c \Omega(\vec{x}) \text{tr}_c \Omega^{-1}(\vec{y}) = e^{-\sigma |\vec{x} - \vec{y}|/T}.$$

$$L \equiv \left\langle \frac{1}{N_c} \text{tr}_c \Omega \right\rangle \underset{\text{Low } T}{\sim} \mathcal{O}(e^{-M/T}),$$

$$\frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \underset{\text{Low } T}{\sim} 1 + \mathcal{O}(e^{-2M/T}) + \mathcal{O}(e^{-N_c M/T}).$$

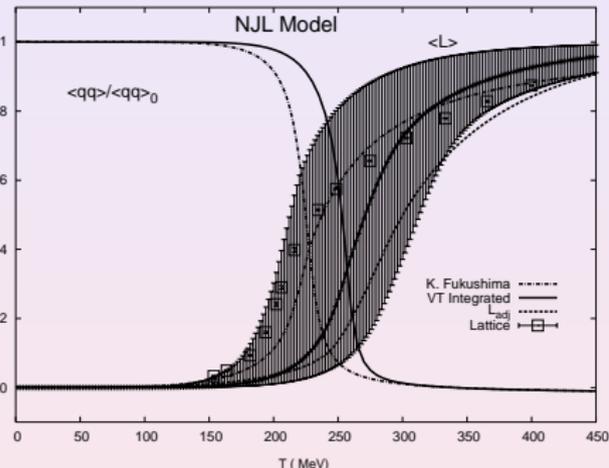
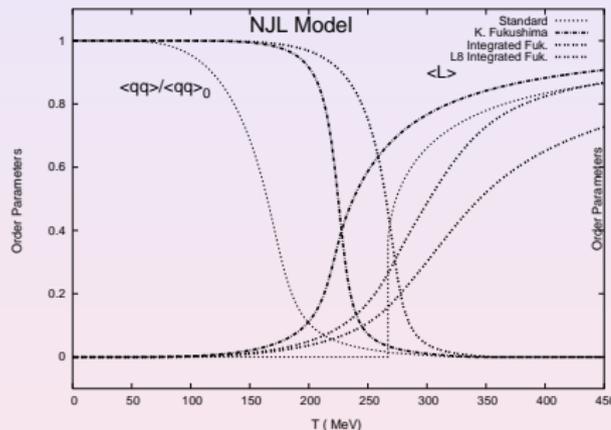
Taking into account the quark binding effects:

$$\mathcal{O}_q^* = \mathcal{O}_q + \sum_{m_\pi} \mathcal{O}_{m_\pi} \frac{1}{N_c} e^{-m_\pi/T} + \sum_B \mathcal{O}_B e^{-M_B/T} + \dots$$

[Megías, Ruiz Arriola and Salcedo, PRD'06]

Phase Transition: Polyakov-Chiral Quark Models

● Numerically:



Polyakov “cooling” : The condensate does not change at low temperatures.

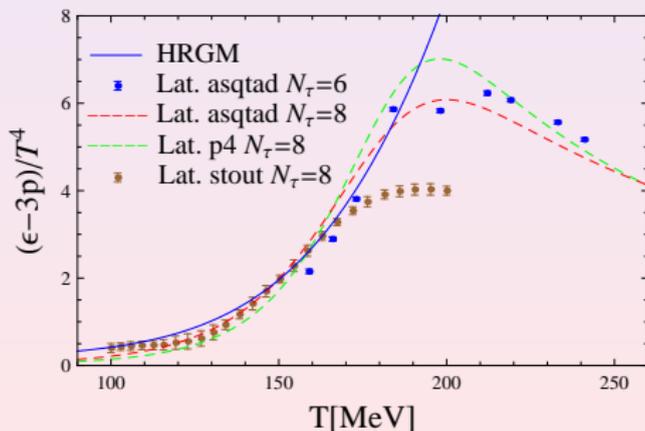
[Megías, Ruiz Arriola and Salcedo, PRD'06]

Hadron Resonance Gas Model ($T < T_c$)

[Hagedorn '85], [Agasian '01], [Tawfik '05].

Hadron Resonance Gas Model for the Equation of State of QCD:

$$\frac{1}{V} \log Z = - \int \frac{d^3 p}{(2\pi)^3} \sum_{\alpha} \zeta_{\alpha} g_{\alpha} \log \left(1 - \zeta_{\alpha} e^{-\sqrt{p^2 + M_{\alpha}^2}/T} \right), \quad \zeta_{\alpha} = \pm 1$$



Free gas of hadrons: Mesons and baryons with light quarks.

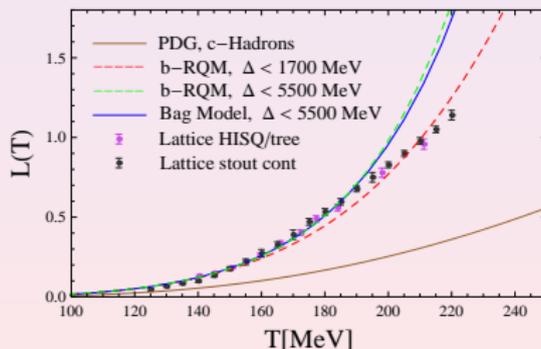
Hadron Resonance Gas Model ($T < T_c$)

[Megías, Ruiz Arriola and Salcedo, PRL'12]

- Hadron Resonance Gas Model for the Polyakov loop:

$$L(T) \simeq \frac{1}{2N_c} \left(\sum_q g_q e^{-\Delta_q/T} + \sum_{q,q'} g_{q,q'} e^{-\Delta_{q,q'}/T} + \dots \right), \quad \Delta_q \equiv \lim_{m_h \rightarrow \infty} (M_{hq} - m_h).$$

Heavy-light system \equiv heavy quark + light dynamical quarks q, q' \rightarrow
Hadrons with a heavy quark $h\bar{q} \equiv$ Meson, $hq q \equiv$ Baryon, ...



- PDG states are not enough \implies Include much more states:
Relativized quark model (RQM) Isgur et al. '85, or **Bag Model**.

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Dimension Two Gluon Condensate and Polyakov loop

$$D_{\mu\nu}^{ab}(p^2) = \frac{\delta^{ab}\delta_{\mu\nu}}{k^2} \rightarrow \frac{\delta^{ab}\delta_{\mu\nu}}{k^2} \left(1 + \frac{m_G^2}{k^2} \right) \quad \text{Narison \& Zakharov '99}$$

At finite temperature:

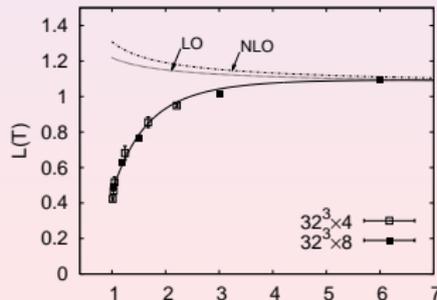
$$D_{00}(\vec{k}) = \frac{1}{\vec{k}^2 + m_D^2} \left(1 + \frac{m_G^2}{\vec{k}^2 + m_D^2} \right) \quad \text{[Megías et al. JHEP'06]}$$

$$\langle A_{0,a}^2 \rangle = (N_c^2 - 1) T \int \frac{d^3k}{(2\pi)^3} D_{00}(\vec{k}) = \underbrace{-\frac{2}{\pi} m_D T}_{\sim T^2} + \underbrace{\frac{m_G^2 T}{\pi m_D}}_{\sim T^0} \equiv \langle A_{0,a}^2 \rangle^{\text{Pert}} + \langle A_{0,a}^2 \rangle^{\text{NP}}$$

Polyakov loop ($T > T_c$):

$$L = \left\langle \frac{1}{N_c} \text{Tr}_c e^{igA_{0,a}/T} \right\rangle \rightarrow e^{-g^2 \langle A_{0,a}^2 \rangle / (4N_c T^2)} + \mathcal{O}(g^6)$$

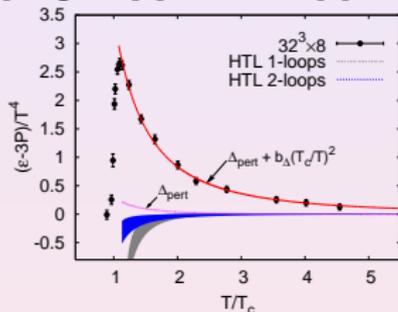
$$g^2 \langle A_{0,a}^2 \rangle^{\text{NP}} = (0.84(6) \text{ GeV})^2$$



OPE-breaking effects and Condensates

- At zero and finite temperature.
- Non-local operators.
- Possible role of the dimension two condensate.

[Burgio'98], [Zakharov'01], [Dorokhov'03], [Rakow'06], [Megías'06 '07 '09], [Pineda'08'09]



$$b_{\Delta} \sim \langle A_{0,a}^2 \rangle_{\text{NP}} \quad [\text{Megías PRD'09}]$$

$$L(T) = L_{\text{PT}}(T) \exp \left[-\frac{g^2 \langle A_{0,a}^2 \rangle_{\text{NP}}}{4N_c T^2} \right] \quad [\text{Megías JHEP'06}]$$

Dimension two gluon condensate from a fit of [Megías et al. PRD'07'09]:

Observable	$g^2 \langle A_{0,a}^2 \rangle_{\text{NP}}$
Polyakov loop	$(3.22 \pm 0.07 T_c)^2$
Heavy $\bar{q}q$ free energy	$(3.33 \pm 0.19 T_c)^2$
Trace Anomaly	$(2.86 \pm 0.24 T_c)^2$

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Holography

- AdS/CFT correspondence: Duality Gravity theory \leftrightarrow Field theory.
- Applications to strongly coupled plasmas:

1. Equation of state:

$$S_{\text{Black Hole}}(T) = \frac{\mathcal{A}(r_{\text{horizon}})}{4G_D} \longleftrightarrow S_{\text{QCD}}(T)$$

[Andreev et al. '06 '07 '09], [Thorlacius et al, '07], [Kiritsis et al. '08 '09], [Kajantie et al. '09], [Megías, Pirner, Veschgini, PRD'11, PLB'11].

$$S_{5D} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) - \frac{1}{8\pi G_5} \int_{\partial M} d^4x \sqrt{-h} K$$

Finite temperature solutions (E. Kiritsis et al. JHEP (2009) 033):

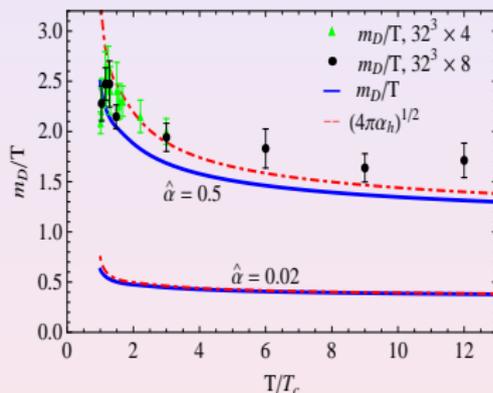
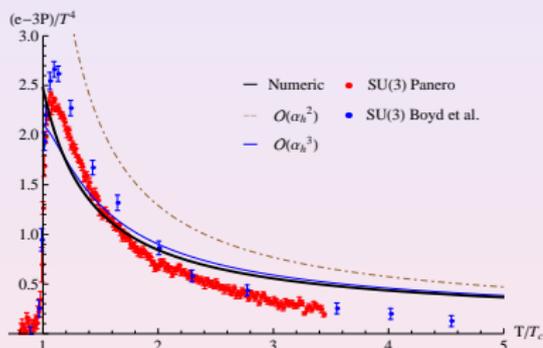
- Black hole solution (deconfined phase):

$$ds_{\text{BH}}^2 = b^2(z) \left[-f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]$$

Holography

Equation of State and Debye Mass

(Megías, Pirner, Veschini, PRD83(2011), PLB96(2011))



$$\frac{p(T)}{T^4} = \frac{\pi^3 \ell^3}{16G_5} \left(1 - \frac{4}{3} \beta_0 \alpha_h + \frac{2}{9} (4\beta_0^2 - 3\beta_1) \alpha_h^2 + \dots \right), \quad \alpha_h \simeq \frac{1}{\beta_0 \log(\pi T/\Lambda)}$$

$$\frac{m_D}{T} \underset{T \rightarrow \infty}{\simeq} (4\pi \alpha_h)^{1/2} \left(1 - \frac{2}{9} \beta_0 \alpha_h + \mathcal{O}(\alpha_h^2) \right)$$

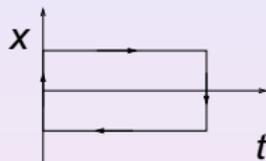
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Holography

2. **Heavy quark free energy**: Consider a rectangular Wilson loop:

$$W(C) = \exp \left(ig \int_C A_\mu dx^\mu \right)$$

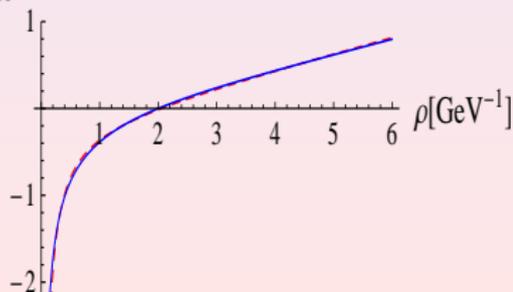


It is related to the potential $V_{q\bar{q}}(R)$ acting between charges q and \bar{q} :

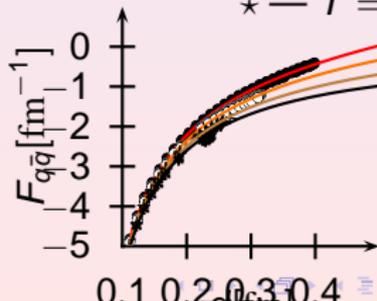
$$\langle W(C) \rangle \underset{t \rightarrow \infty}{\sim} \exp(-t \cdot V_{q\bar{q}}(R)) \simeq e^{-S_{\text{Nambu-Goto}}}$$

At zero and finite temperature. [Megías et al. NPB'10].

$V_{qq}(\rho)$ [GeV]



- — $T = 98 \text{ MeV}$
- ★ — $T = 154 \text{ MeV}$

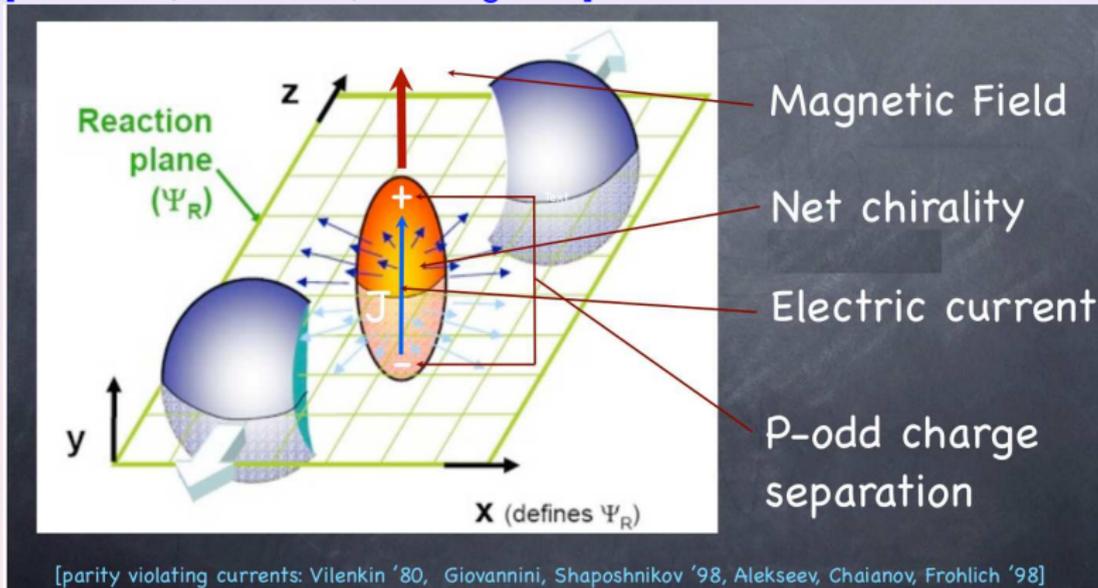


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The Chiral Magnetic Effect and Vortical Effect

[Kharzeev, McLerran, Warringa '07]



- Strong Magnetic field induces a P-odd charge separation \implies
 \implies Electric current: $\vec{J} = \sigma_B \vec{B}$.
- A vortex induces also an Electric current: $\vec{J} = \sigma_V \vec{\omega}$.

Hydrodynamics

Constitutive Relations:

$$\langle T^{\mu\nu} \rangle = \underbrace{(\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}}_{\text{Ideal Hydro}} + \underbrace{\langle T^{\mu\nu} \rangle_{\text{diss+anom}}}_{\text{Dissipative+Anomalous}},$$

$$\langle J^\mu \rangle = \underbrace{nu^\mu}_{\text{Ideal Hydro}} + \underbrace{\langle J^\mu \rangle_{\text{diss+anom}}}_{\text{Dissipative+Anomalous}}.$$

- $\frac{\eta}{s} \simeq \frac{1}{4\pi}$ Supported from RHIC and LHC.
- $\vec{J} = \sigma_B \vec{B} + \sigma_V \vec{\omega}$ Chiral Magnetic and Vortical Effects.
- Two methods: Kubo formulas, Fluid/Gravity correspondence.

[Son,Surowka], [Eling,Neiman,Oz], [Erdmenger et al.], [Banerjee et al.], [Loganayagam], [Kharzeev, Yee], [Sadovyeve et al.], [Landsteiner, Megías, Pena-Benitez, PRL'11, JHEP'11].

- **Dissipative effects**: shear, bulk and energy loss.
- **Anomalous effects**: Chiral Magnetic and Vortical effects \rightarrow chiral anomaly and **gauge-gravitational anomalies** (Megías et al, PRL107 '11, JHEP1109 '11).

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Kubo Formulae

$$\sigma_B = \lim_{k_c \rightarrow 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a J^b \rangle |_{\omega=0}, \quad \sigma_V = \lim_{k_c \rightarrow 0} \frac{i}{2k_c} \sum_{a,b} \epsilon_{abc} \langle J^a T^{0b} \rangle |_{\omega=0}.$$

- In Field Theory [Landsteiner, Megías and Pena-Benitez, PRL'11]:

$$(\sigma_V)_a = \underbrace{\frac{1}{8\pi^2} \sum_{b,c} d_{abc} \mu^b \mu^c}_{\text{Chiral Anomaly}} + \underbrace{\frac{T^2}{24} \text{tr}(T_a)}_{\text{Gauge-Gravitational Anomaly}}$$

- In Holography (Gauge and Gauge-Gravitational Chern Simons):

[Megías et al. JHEP '11]

$$S = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left[R + 2\Lambda - \frac{1}{4} F_{MN} F^{MN} \right. \\ \left. + \epsilon^{MNPQR} A_M \left(\frac{\kappa}{3} F_{NP} F_{QR} + \lambda R^A{}_{BNP} R^B{}_{AQR} \right) \right] + S_{GH} + S_{CSK}$$

Same result at weak and strong coupling!

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Fluid/Gravity Correspondence

[Erdmenger et al.], [Bhattacharyya et al.], [Banerjee et al.].

- Boosted black branes:

$$ds^2 \equiv g_{AB} dx^A dx^B = -r^2 f(r) u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu - 2u_\mu dx^\mu dr$$

where

$$f(r) = 1 - \frac{M}{r^4} + \frac{Q^2}{r^6}, \quad P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu,$$

$$A_r = 0, \quad A_\mu = A_\mu^{(b)} - \frac{\sqrt{3}Q}{r^2} u_\mu.$$

- $M(x)$, $Q(x)$, $u_\mu(x)$, $A_\mu^{(b)}(x)$ are slowly varying in x^μ . This means $\partial X \ll X/\ell_{mfp}$
- Then one can compute the fields in a derivative expansion:

$$g_{AB} = g_{AB}^{(0)} + g_{AB}^{(1)} + g_{AB}^{(2)} + \dots$$

$$A_M = A_M^{(0)} + A_M^{(1)} + A_M^{(2)} + \dots$$

Fluid/Gravity Correspondence: result up to 1st order

[Landsteiner, Megías, Pena-Benitez, '12]

- Constitutive relations:

$$\begin{aligned}\langle T_{\mu\nu} \rangle_{\text{anom}} &= (\sigma_B^\epsilon)_F (B_\mu u_\nu + B_\nu u_\mu) + (\sigma_V^\epsilon)_F (\omega_\mu u_\nu + \omega_\nu u_\mu), \\ \langle J_\mu \rangle_{\text{anom}} &= (\sigma_B)_F B_\mu + (\sigma_V)_F \omega_\mu.\end{aligned}$$

- Transport coefficients in three different frames (definition of u_μ):

Conductivities	Laboratory rest frame	Landau frame	Eckart frame
$(\sigma_B)_F$	$\sigma_B = \frac{\mu}{4\pi^2}$	$\sigma_B - \frac{n}{\epsilon+P} \sigma_B^\epsilon$	0
$(\sigma_V)_F$	$\sigma_V = \frac{\mu^2}{8\pi^2} + \frac{T^2}{24}$	$\sigma_V - \frac{n}{\epsilon+P} \sigma_V^\epsilon$	0
$(\sigma_B^\epsilon)_F$	$\sigma_B^\epsilon = \sigma^V$	0	$\sigma_B^\epsilon - \frac{\epsilon+P}{n} \sigma_B$
$(\sigma_V^\epsilon)_F$	$\sigma_V^\epsilon = \frac{\mu^3}{12\pi^2} + \frac{\mu T^2}{12}$	0	$\sigma_V^\epsilon - \frac{\epsilon+P}{n} \sigma_V$

- Currently computing up to 2nd order. Much more terms!

[Landsteiner, Megías, Pena-Benitez (in preparation)]

Contribution

- **Theoretical:**
 - Development of a wide variety of methods:
 - In Field Theory.
 - In Holography.
 - Disentanglement *Perturbative - Non Perturbative* contributions.
 - Role of condensates.
 - Role of anomalies in transport theory.
- **Phenomenological:**
 - Heavy ion physics (RHIC and LHC):
 - Static properties: equation of state, ...
 - Dissipative and anomalous transport properties.
 - Application to heavy quark physics → heavy quarkonium spectrum.

Collaborators and International Visibility

Spanish Institutions:

- **UAB & IFAE, Barcelona:** A. Pomarol (Full Prof.) , A. Pineda (Prof.), O. Pujolàs (RyC), Pedro J. Silva (Assoc. Prof.).
- **U. Granada:** E. Ruiz Arriola, L.L. Salcedo (Full Professors).
- **IFT CSIC/UAM, Madrid:** K. Landsteiner (Prof.), Melgar (PhD Stud), F. Pena-Benitez (PhD Student).

International Institutions:

- **ITP Heidelberg (Germany):** H.J. Pirner (Full Prof.), K. Veschgini (PhD Student).
- **BNL (USA):** D. Kharzeev (Full Prof.)
- **Valparaíso (Chile):** E. Levin (Prof.)

International Conferences:

- **QCD and High Energy Physics:** QCDx, Quark Matter, ICHEP, Quark Confinement, Light Cone, ...
 - **String Theory:** Eu.Work. on ST, Int.Conf. on ST, Iberian Strings, ...
- 37 publications; 571 citations; h-index=12; 44 conferences;
14 invited seminars; 1 awarded project (3 years, 93.333 euros).