Minimal Flavour Violation; about its predictivity and origin

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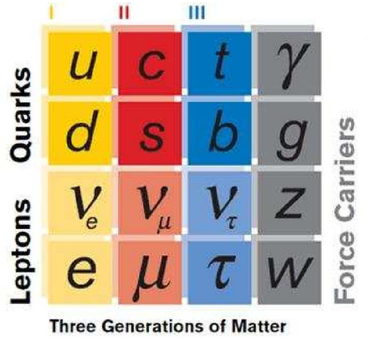
based on the work with Belén Gavela, Luca Merlo & Stefano Rigolin JHEP **1107**, 012 (2011). arXiv:1103.2915.

Outline

The Flavour Puzzle



- The matter around us is made out of the first generation.
- 3 generations allow for CP violation... which does not account for the matter-antimatter asymmetry of the universe

The Flavour Puzzle

The Flavour structure comes from the Yukawa couplings

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$-\mathcal{L}_{Yukawa} = \overline{Q}_L Y_U U_R \tilde{H} + \overline{Q}_L Y_D D_R H + \overline{Q}_L Y_E E_R H + h.c.$$

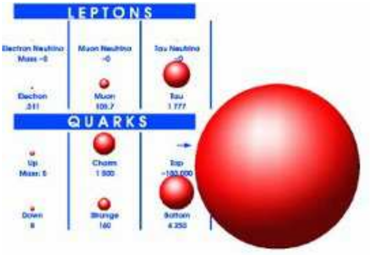
That can be parametrized as

$$Y_U = V_{CKM}^{\dagger} \mathbf{y}_U \qquad Y_D = \mathbf{y}_D \qquad Y_E = \mathbf{y}_E$$

$$\mathbf{y}_U \sim \mathrm{Diag}\left(m_u, m_c, m_t\right)$$
$$\mathbf{y}_D \sim \mathrm{Diag}\left(m_d, m_s, m_b\right)$$
$$\mathbf{y}_E \sim \mathrm{Diag}\left(m_e, m_\mu, m_\tau\right)$$

The Flavour Puzzle

The mass spectrum is strongly hierarchical



The mixing angles are small for quarks

$$V_{CKM} = \begin{pmatrix} \sim 1 & \lambda & \lambda^3 \\ \lambda & \sim 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & \sim 1 \end{pmatrix}$$

$$\lambda \sim 0.22$$

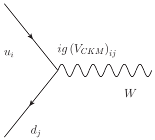
...and large for leptons

$$U_{PMNS} = \begin{pmatrix} 0.8 & 0.5 & ?0.2 \\ -0.4 & 0.5 & -0.7 \\ -0.4 & 0.5 & 0.7 \end{pmatrix}$$

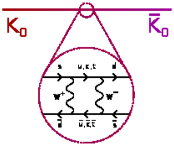
+extra phases??

Flavour Physics

The Flavour Phenomenology of the SM is very specific



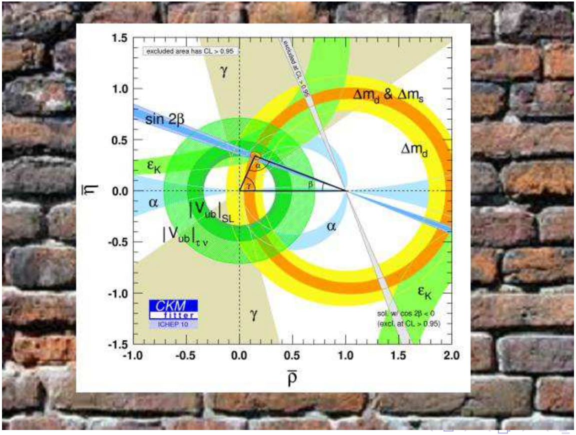
- There are no Flavour Changing Neutral Currents at tree level in the SM
- FCNC processes occur at the loop level and GIM suppressed
- CP violation 'small'; Jarlskog Inv.×GIM in SM



→ Sensitivity to new physics

The Flavour Problem

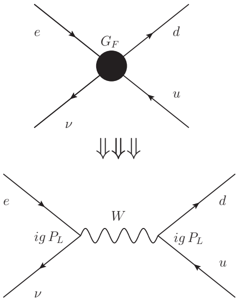
All the flavour data is $\sim$ consistent with the SM



Beyond the Standard Model; Effective Field Theory

Fermi's Theory of β decay

$$\mathcal{L}_{em} + G_F \bar{e}\gamma_\mu \nu \bar{u}\gamma^\mu d$$



$$\mathcal{L}_{em} + (g^2/M_W^2)\bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

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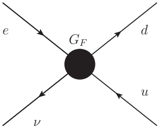

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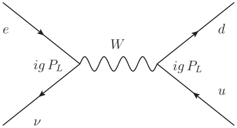
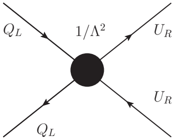


$$\mathcal{L}_{em} + (g^2/M_W^2)\bar{e}\gamma_\mu \nu_L \bar{u}\gamma^\mu d_L$$

BSM physics

$$\mathcal{L}_{SM} + \frac{1}{\Lambda}\mathcal{O}^{d=5} + \frac{1}{\Lambda^2}\mathcal{O}^{d=6} + \ldots$$



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????

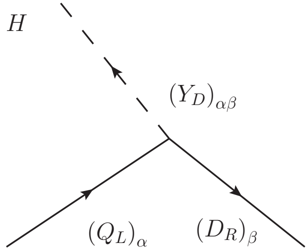
which is a valid description up to Λ

Tipically, $\Lambda > 10^{2-3}\,TeV$

Minimal Flavour Violation (MFV)

MFV is a **symmetry approach** to the flavour problem that takes the **experimental data at face value**.

The MFV hypothesis: *The Yukawa couplings are the* ***only*** *sources of flavour violation in and* ***beyond*** *the Standard Model*[1].



[1] Georgi & Chivukula 1987; D'Ambrosio, Giudice, Isidori, & Strumia, 2002; Cirigliano, Grinstein, Isidori & Wise 2005.

Minimal Flavour Violation; Realization

- Generations are distinguished by masses; in the limit of zero mass

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

the SM presents an extended symmetry group :

$$G_f = \overbrace{SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}}^{Quark} \times \ldots \qquad D_R = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix}$$

$$D_R \sim (1, 3, 1 \cdots)$$

- The Yukawa Y_U and Y_D break the symmetry, unless

$$\overline{Q}_L Y_D D_R H \qquad Y_D \sim (3, \bar{3}, 1)$$

- ...Y_U and Y_D are assigned transformation properties (spurions).

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Minimal Flavour Violation

The Effective Field Theory approach with MFV has the flavour structure fixed by formal invariance under G_f

$$\mathcal{L}_{SM} + \frac{c_{\alpha\beta}}{\Lambda_{NP}^2} \overline{Q}_\alpha \gamma^\mu Q_\beta \overline{E}_R \gamma_\mu E_R + \ldots$$

$$\text{MFV} \rightarrow c_{\alpha\beta} = (Y_U)_{\alpha\gamma} \left(Y_U^\dagger\right)_{\gamma\beta} \simeq V_{ti}^* V_{tj} y_t^2$$

$$c \sim (3 \times \bar{3}, 1, 1) = (3, 1, \bar{3}) \times (\bar{3}, 1, 3) \sim (3 \times \bar{3}, 1, 1)$$

The predictions have the Standard Model flavour structure:

$$\mathcal{A}(d^i \rightarrow d^j)_{MFV} = (V_{ti}^* V_{tj}) \mathcal{A}_{SM}^{(\Delta F=1)} \left(1 + \frac{(4\pi)^2 y_t^2 M_W^2}{\Lambda_{NP}^2}\right)$$

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Minimal Flavour Violation; it's predictivity

The strongest experimental bound sets $\Lambda_{NP} > \text{TeV} \sim \Lambda_{EW}$.
This has implications for other observables

Observable	Experiment	MFV prediction	SM prediction
β_s from $\mathcal{A}_{\rm CP}(B_s \to \psi\phi)$	[0.10, 1.44] @ 95% CL	0.04(5)*	0.04(2)
$\mathcal{A}_{\rm CP}(B \to X_s\gamma)$	$< 6\%$ @ 95% CL	< 0.02*	< 0.01
$\mathcal{B}(B_d \to \mu^+\mu^-)$	$< 1.8 \times 10^{-8}$	$< 1.2 \times 10^{-9}$	$1.3(3) \times 10^{-10}$
$\mathcal{B}(B \to X_s\tau^+\tau^-)$	–	$< 5 \times 10^{-7}$	$1.6(5) \times 10^{-7}$
$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})$	$< 2.6 \times 10^{-8}$ @ 90% CL	$< 2.9 \times 10^{-10}$	$2.9(5) \times 10^{-11}$

Isidori, Nir, Perez

MFV is not a model , does not 'predict' Λ_{NP}, it does predict relative strengths,

$$\frac{\Gamma(B_d \to \mu^+\mu^-)}{\Gamma(B_s \to \mu^+\mu^-)} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

Minimal Flavour Violation

Still one may question some aspects

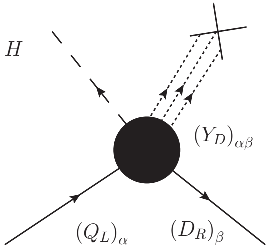
- It assumes top dominance $\left(Y_U Y_U^\dagger\right)_{ij} \simeq V_{ti}^* V_{tj} y_t^2$
- Implies no seizable up type quark transitions
- This would not be so in a 2 Higgs setup [Branco, Grimus, Lavoura,...]
- CP violation arising from V_{CKM} only
- Extend MFV with, i. e., flavour blind phases [Colangelo, Nikolidakis, Smith, Ellis, Lee, Pilaftsis,...]

This practical use of MFV in an effective field theory scheme has proven successful...

... but the formal invariance under the flavour group broken by some constant matrices Y_U and Y_D with 'transformation properties ' (!)...

The Dynamics Behind MFV

Suggests that Y_U & Y_D have a *dynamical origin* .



The Yukawa Interaction involves extra fields increasing the dimension of the 'Yukawa Operator'.

$$Y_D \sim \langle \Psi \rangle \,? \qquad Y_D = \left\langle \Psi^2 \right\rangle ? \qquad Y_D = \langle \Psi^n \rangle \,?$$

These fields acquire a v.e.v. and fix the Yukawa couplings.

The Dynamics Behind MFV

THE AIM

What we did is to construct and analyze the Potential for the fields

behind MFV

$$V(Y_D, Y_U, H)$$

With just two prescriptions

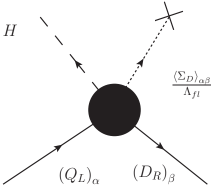
- Invariance under the SM gauge group $\rightarrow SU(3) \times SU(2) \times U(1)$
- Invariance under the flavour group $\rightarrow SU(3)_{Q_L} \times SU(3)_{D_R} \times SU(3)_{U_R}$

The Dynamics Behind MFV

Different cases shorted by EFT & Group Theory

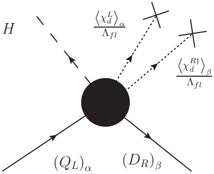
1. The Yukawas are the vev of 1 field
   $Y \sim \langle \Sigma \rangle$
   - Dim. 5 $\leftrightarrow$ Bifundamental Fields

$$\overline{Q}_L \frac{\Sigma_u}{\Lambda_f} D_R H \qquad \Sigma_d \sim (3, \bar{3}, 1)$$

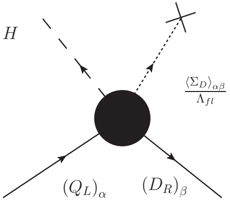


2. The Yukawas are 'composite' $Y \sim \langle \chi\chi \rangle$
   - Dim. 6 $\leftrightarrow$ Fundamental Fields

$$\overline{Q}_L \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} D_R H \qquad \chi_d^L \sim (3, 1, 1) \quad \chi_d^R \sim (1, 3, 1)$$



Dimension 5 Yukawa Operator or Bifundamental Fields



The Scalar Potential

Construction of the Potential; Bifundamental Fields

First collect the G_f and G_{gauge} invariants at hand:

$$Tr\left(\Sigma_u\Sigma_u^\dagger\right), \quad \det\left(\Sigma_u\right), \quad Tr\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right),$$
$$Tr\left(\Sigma_d\Sigma_d^\dagger\right), \quad \det\left(\Sigma_d\right), \quad \cdots$$

Feldman, Jung, Mannel

Five independent invariants for two generations.

The invariants can be expressed in terms of masses and mixing angles (2 gen.):

$$\langle\Sigma_u\rangle = \Lambda_f \cdot V^\dagger \mathrm{Diag}\{y_{u_i}\}, \qquad \langle\Sigma_d\rangle = \Lambda_f \cdot \mathrm{Diag}\{y_{d_i}\};$$

$$Tr\left(\Sigma_u\Sigma_u^\dagger\right) = \Lambda_f^2({y_u}^2 + {y_c}^2), \qquad \det\left(\Sigma_u\right) = \Lambda_f^2 y_u y_c$$
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$$Tr\left(\Sigma_u\Sigma_u^\dagger\Sigma_d\Sigma_d^\dagger\right) = \frac{1}{2}\Lambda_f^4\left[\left({y_c}^2 - {y_u}^2\right)\left({y_s}^2 - {y_d}^2\right)\cos 2\theta_c + \cdots\right]$$

Construction of the Potential; Bifundamental Fields

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Construction of the Potential; Bifundamental Fields

The most general Potential is:

$$V = \sum_{i=u,d} \left(-\mu_i{}^2\, Tr\left(\Sigma_i \Sigma_i^\dagger\right) - \tilde{\mu}_i{}^2 \det\left(\Sigma_i\right)\right) +$$

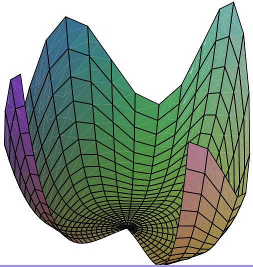
$$\sum_{i,j=u,d} \left(\lambda_{ij}\, Tr\left(\Sigma_i \Sigma_i^\dagger\right) Tr\left(\Sigma_j \Sigma_j^\dagger\right) + \tilde{\lambda}_{ij} \det\left(\Sigma_i\right) \det\left(\Sigma_j\right) + \cdots \right) \cdots$$

The minimum of the Potential is given by:

$$\frac{\partial V\left(\Sigma\left(Y_{U,D}\right)\right)}{\partial y_i} = 0 \qquad \frac{\partial V\left(\Sigma\left(Y_{U,D}\right)\right)}{\partial \theta_i} = 0$$

$$\Downarrow$$

$$y_i = y_i\left(\mu, \tilde{\mu}, \lambda\right) \qquad \theta_i = \theta_i\left(\mu, \tilde{\mu}, \lambda\right)$$



Minimum of the Potential

Bifundamental Fields

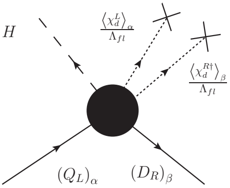
Can the actual masses and mixing fit naturally ($\mu < \Lambda_f, \lambda \sim 1$) in the minimum of the Potential?

The answer is worse than NO .

- Not only we have to input the hierarchy of masses in the potential parameters, which was to expect but $\cdots$
- To accommodate the mixing we must introduce wild fine tunings of $O(10^{-10})$ and nonrenormalizable terms of dimension 8

$$\frac{\partial V^{(4)}}{\partial \theta} \propto \left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right) \sin 2\theta = 0$$

Dimension 6 Yukawa Operator
or fundamental Fields



The Scalar Potential

Fundamental Fields

Let's first take a careful look at the Yukawa structure:

$$Y_D = \frac{\left\langle \chi_d^L \chi_d^{R\dagger} \right\rangle}{\Lambda^2} \quad ; \quad Y_U = \frac{\left\langle \chi_u^L \chi_u^{R\dagger} \right\rangle}{\Lambda^2}$$

A 'matrix' made out of 2 'vectors '.

Such a construction has rank 1 , that is
one nonvanishing eigenvalue only!

Strong Hierarchy, by construction

Fundamental Fields

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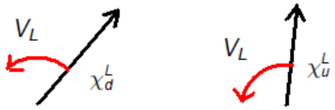
Strong Hierarchy, by construction

Fundamental Fields

- χ_u^L and χ_d^L live in the the fundamental of $SU(3)_L \subset G_f$
- A transformation of $SU(3)_L$ is a *simultaneous* 'rotation' of these vectors.



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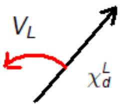
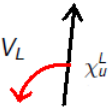
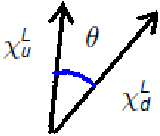




But the *relative angle* is invariant



Diagonalizing the Yukawa coupling =

finding the moduli $\sim$ masses +

the relative angle $\sim$ mixing

Fundamental Fields

We can use a G_f transformation $V = V_L \times V_{U_R} \times V_{D_R}$ to rotate the vectors to:

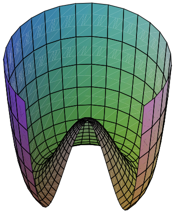
$$Y_D \xrightarrow{V} \frac{\left|\chi_d^L\right| \left|\chi_d^R\right|}{\Lambda_f^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$Y_U \xrightarrow{V} \frac{\left|\chi_u^L\right| \left|\chi_u^R\right|}{\Lambda_f^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

This means a hierarchy among the masses and an angle only by construction!

Minimum of the Potential

The explicit invariants are:



$$\chi_u^{L\dagger}\chi_u^L\,, \qquad \chi_u^{R\dagger}\chi_u^R\,, \qquad \chi_d^{L\dagger}\chi_d^L\,,$$
$$\chi_d^{R\dagger}\chi_d^R\,, \qquad \chi_u^{L\dagger}\chi_d^L = \left|\chi_u^L\right|\left|\chi_d^L\right|\cos\theta_c\,.$$

We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u\left(\chi_u^{L\dagger}\chi_u^L - \frac{\mu_u^2}{2\lambda_u}\right)^2 + \lambda_d\left(\chi_d^{L\dagger}\chi_d^L - \frac{\mu_d^2}{2\lambda_d}\right)^2$$
$$+\lambda_{ud}\left(\chi_u^{L\dagger}\chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}}\right)^2 + \cdots$$

The minimum is: $y_c^2 = \frac{\mu_u^2}{2\lambda_u\Lambda^2}\,, \quad y_s^2 = \frac{\mu_d^2}{2\lambda_d\Lambda^2}\,, \quad \cos\theta = \frac{\mu_{ud}^2\sqrt{\lambda_u\lambda_d}}{\mu_u\mu_d\lambda_{ud}}\,.$

naturally

Combination of Fundamental and Bifundamental fields

Sensible from the effective field theory point of view

$$\mathcal{L}_Y = \overline{Q}_L \left[\frac{\Sigma_d}{\Lambda_f} + \frac{\chi_d^L \chi_d^{R\dagger}}{\Lambda_f^2} \right] D_R H + \overline{Q}_L \left[\frac{\Sigma_u}{\Lambda_f} + \frac{\chi_u^L \chi_u^{R\dagger}}{\Lambda_f^2} \right] U_R \tilde{H} + h.c.$$

which points to $$\frac{\langle \Sigma \rangle}{\Lambda_f} < \frac{\langle \chi^2 \rangle}{\Lambda_f^2}$$

and suggests

$$\frac{\langle \Sigma_{u,d} \rangle}{\Lambda_f} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_{t,b} \end{pmatrix},$$

$$\frac{\left\langle \chi_d^L \chi_d^{R\dagger} \right\rangle}{\Lambda_f^2} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & 0 \end{pmatrix}, \frac{\left\langle \chi_u^L \chi_u^{R\dagger} \right\rangle}{\Lambda_f^2} \sim \begin{pmatrix} 0 & \sin\theta y_c & 0 \\ 0 & \cos\theta y_c & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

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Summary

- We constructed and analyzed the Potential of MFV.
- The symmetry strongly constrains the potential.
- To the extent that the bifundamental field realization ($Y \sim \Sigma$) does not fit the quark masses and mixing angles.
- The fundamental fields ($Y \sim \chi\chi$) scheme does fit a massive up and down quark and accommodates mixing
- Ongoing work on the lepton sector...

Thank you for your attention

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BACKUP

Minimum of the Potential

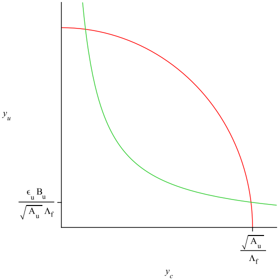
Bifundamental Fields: 2 Generations

What is the problem?

- While the masses can be fit in a fine-tuned renormalizable potential:

$$V_u^{(4)} = \lambda \left(\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger\right) - \frac{\mu^2}{2\lambda} \right)^2 + \tilde{\lambda} \left(\det\left(\Sigma_u\right) - \frac{\tilde{\mu}^2}{2\tilde{\lambda}} \right)^2 ,$$

$$\mathrm{Tr}\left(\Sigma_u \Sigma_u^\dagger\right) = \Lambda_f^2 (y_u^2 + y_c^2) ,$$
$$\det\left(\Sigma_u\right) = \Lambda_f^2 y_u y_c .$$



- the mixing , at the renormalizable level, is set to zero

$$\frac{\partial V^{(4)}}{\partial \theta} \propto \left(y_c^2 - y_u^2\right)\left(y_s^2 - y_d^2\right) \sin 2\theta = 0$$

obtaining a nonzero angle would imply a non renormalizable term of

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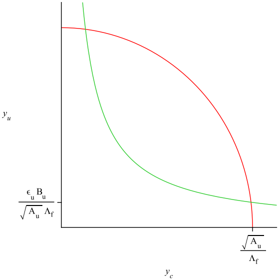

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Minimum of the Potential

Bifundamental Fields: 3 Generations

For the mixing, at the renormalizable level there is only 1 term

$$A_{ud} = \Lambda_f^4 \left(P_0 + P_{int}\right)$$

$$P_0 \equiv -\sum_{i<j} \left(y_{u_i}^2 - y_{u_j}^2\right)\left(y_{d_i}^2 - y_{d_j}^2\right) \sin^2\theta_{ij} \,,$$

$$P_{int} \equiv \sum_{i<j,k} \left(y_{d_i}^2 - y_{d_k}^2\right)\left(y_{u_j}^2 - y_{u_k}^2\right) \sin^2\theta_{ik} \sin^2\theta_{jk} +$$

$$-\left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right) \sin^2\theta_{12} \sin^2\theta_{13} \sin^2\theta_{23} +$$

$$+\frac{1}{2}\left(y_d^2 - y_s^2\right)\left(y_c^2 - y_t^2\right) \cos\delta \, \sin 2\theta_{12} \sin 2\theta_{23} \sin\theta_{13} \,,$$

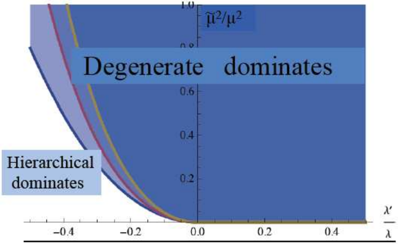
The minimum sits at no mixing $V_{CKM} = 1$ unavoidably.
The fixing of the angles would require higher order terms + huge fine tuning .

Minimum of the Potential

Bifundamental Fields: 3 Generations

In this case a small parameter region opens for a hierarchical solution $\mathbf{y}_U = \text{Diag}(0, 0, y)$

$$V^{(4)} = -\mu^2 \text{Tr}(\Sigma_u \Sigma_u^\dagger) + \tilde{\mu} \det(\Sigma_u) + \lambda \text{Tr}^2(\Sigma_u \Sigma_u^\dagger) + \lambda' \text{Tr}(\Sigma_u \Sigma_u^\dagger)^2$$



but also the cross term $g\text{Tr}(\Sigma_u \Sigma_u^\dagger)\text{Tr}(\Sigma_u \Sigma_d^\dagger)$ shall be suppressed as $g < y_b^2/y_t^2$ to prevent top-bottom degeneracy.

A more complete picture

Dimension 6 Yukawa Operator

The structure of the measured masses and mixing suggests a sequential breaking[2].

$$Y_u \equiv \frac{\langle \chi^L \rangle \left\langle \chi_u^{R\dagger} \right\rangle}{\Lambda_f^2} + \epsilon_u \frac{\langle \chi_u'^L \rangle \left\langle \chi_u'^{R\dagger} \right\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & \sin\theta\, y_c & 0 \\ 0 & \cos\theta\, y_c & 0 \\ 0 & 0 & y_t \end{pmatrix},$$

$$Y_d \equiv \frac{\langle \chi^L \rangle \left\langle \chi_d^{R\dagger} \right\rangle}{\Lambda_f^2} + \epsilon_d \frac{\langle \chi_d'^L \rangle \left\langle \chi_d'^{R\dagger} \right\rangle}{\Lambda_f^2} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix},$$

$$SU(3)^3 \xrightarrow{\chi} SU(2)^3 \xrightarrow{\chi'} \emptyset$$

[2] $SU(2)^3$ $\rightarrow$ Barbieri, Isidori, Jones-Pérez, Lodone, Straub