The Curvature Perturbation from General non-Abelian Vector Fields

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Outline

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Motivation

- ζ generated by scalar fields $\Rightarrow$
  statistical homogeneity and isotropy;

- Some indications both might be
  broken:

  - alignments of low multipoles;
    broken isotropy, i.e. preferred
    direction

- New observable - statistical anisotropy

  - Can be dominant in B_ζ even if
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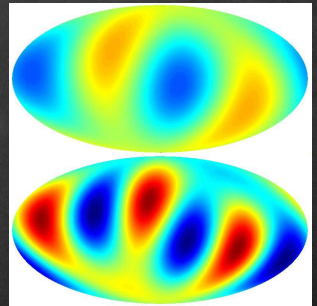- Offers a new parameter space for
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Tegmark, de Oliveira-Costa, Hamilton (2003)

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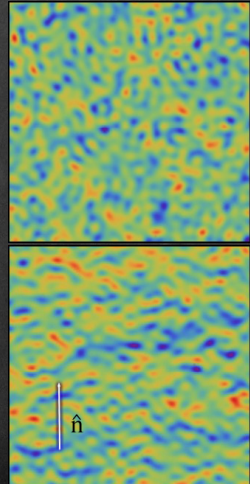- ζ generated by scalar fields $\Rightarrow$ statistical homogeneity and isotropy;
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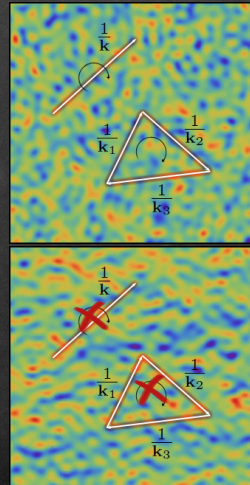- Offers a new parameter space for inflationary model building.

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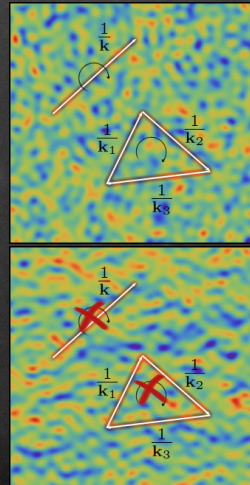- ζ generated by scalar fields $\Rightarrow$ statistical homogeneity and isotropy;
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Breaking the Conformal Invariance

- Massless $U(1)$ vector field $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
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1. Introduce "potential": $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu$
2. Non-canonical kinetic function: $\mathcal{L} = -\frac{1}{4} f(t) F_{\mu\nu} F^{\mu\nu}$

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Avoiding Large Scale Anisotropy

For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

$$T^{\nu}_{\mu} = \mathrm{diag}\left(\rho, -p, -p, +p\right)$$

- Three identical, orthogonal vector fields;
  Armendariz-Picon (2004)

- Many randomly oriented vector fields - vector inflation;
  Golovnev, Mukhanov, Vanchurin (2008)

- Vector curvaton scenario;
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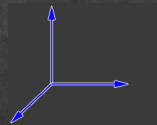For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

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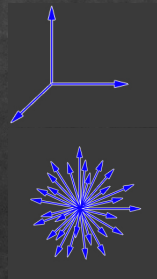

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Observational Constraints

- The power spectrum: $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\text{iso}} \left[1 + g_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}} \right)^2 \right]$:

  - $g_\zeta = 0.29 \pm 0.031$;   Hanson, Lewis (2009);
                                        Groeneboom, Ackerman, Wehus, Eriksen (2010);
    - preferred direction $\hat{\mathbf{n}} = (l, b) = (96, 30)$ - close to ecliptic pole;
  - $|g_\zeta| < 0.07$;   Hanson, Lewis, Challinor (2010);
  - Planck prospects: Ma, Efstathiou, Challinor (2011);
    - $\Delta g_\zeta \sim 0.01 \ (2\sigma)$;
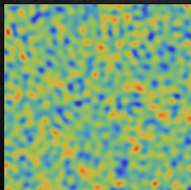   - $g_\zeta \propto k^q$ to an accuracy $\Delta q \sim 0.3 \ (1\sigma)$;

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Summary   The End-of-Inflation Scenario

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Motivation

1. The non-Abelian vector fields are ubiquitous in particle physics models;

2. Several of vector fields $\Rightarrow$ suppressed statistical and random orientation   anisotropy;

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  3. corresponds to weak coupling;
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Correlators

Using the 'In-In Formalism':

$$g_3 \left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3 \right) = - \left(2\pi \right)^3 \delta \left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 \right) \frac{2H^6}{\prod_i^3 2k_i^3} \mathcal{T}^{fgh}_{lmn} \left(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{k}}_3 \right) I \left(k_1, k_2, k_3 \right)$$

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1. Anisotropic:

$$\mathcal{T}_{lmn}^{fgh}\left(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\hat{\mathbf{k}}_3\right)\equiv W_m^b\,T_{ij}^{\mathrm{E}}\left(\hat{\mathbf{k}}_1\right)T_{nj}^{\mathrm{E}}\left(\hat{\mathbf{k}}_3\right)\left(\mathrm{f}^{abh}\mathrm{f}^{agf}+\mathrm{f}^{agh}\mathrm{f}^{abf}\right)+$$
$$+\,W_l^b T_{mj}^{\mathrm{E}}\left(\hat{\mathbf{k}}_2\right)T_{nj}^{\mathrm{E}}\left(\hat{\mathbf{k}}_3\right)\left(\mathrm{f}^{abg}\mathrm{f}^{afh}+\mathrm{f}^{afg}\mathrm{f}^{abh}\right)+$$
$$+\,W_n^b T_{lj}^{\mathrm{E}}\left(\hat{\mathbf{k}}_1\right)T_{mj}^{\mathrm{E}}\left(\hat{\mathbf{k}}_2\right)\left(\mathrm{f}^{abg}\mathrm{f}^{ahf}+\mathrm{f}^{ahg}\mathrm{f}^{abf}\right),$$

where $T_{ij}^{\mathrm{E}}\left(\hat{\mathbf{k}}\right)\equiv\delta_{ij}-\hat{k}_i\hat{k}_j$.

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2. Classical contribution dominates:

$$I = \frac{g_c^2}{f_0} \frac{k_t^7 H^{-8}}{4!} \left[6e^{4N_k}\left(\frac{1}{3} - K_1 + K_2\right) + 2e^{2N_k}\left(K_1 - 3K_2 - \frac{1}{5}\right) - \right.$$

$$\left. + \left(\gamma + N_k\right)\left(\frac{1}{5}K_1 - K_2 - \frac{1}{35}\right) + \frac{1}{300}\left(625K_2 - 137K_1 + \frac{1019}{49}\right) \right]$$

where $K_1 \equiv \frac{\sum_{i>j}^3 k_i k_j}{k_t^2}$ $K_2 \equiv \frac{\prod_i^3 k_i}{k_t^3}$, and $N_k \equiv -\ln\left(k_t \tau_{\text{end}}\right)$

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$$g_3\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3\right) = -\left(2\pi\right)^3 \delta\left(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3\right) \frac{2H^6}{\prod_i^3 2k_i^3} \mathcal{T}_{lmn}^{fgh}\left(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\hat{\mathbf{k}}_3\right) I\left(k_1,k_2,k_3\right)$$

2. Classical contribution dominates:

$$I = \frac{g_c^2}{f_0}\frac{k_t^7 H^{-8}}{4!}\left[6\mathrm{e}^{4N_k}\left(\frac{1}{3}-K_1+K_2\right) + 2\mathrm{e}^{2N_k}\left(K_1 - 3K_2 - \frac{1}{5}\right) - \right.$$

$$\left. + \left(\gamma + N_k\right)\left(\frac{1}{5}K_1 - K_2 - \frac{1}{35}\right) + \frac{1}{300}\left(625K_2 - 137K_1 + \frac{1019}{49}\right)\right]$$

where $K_1 \equiv \frac{\sum_{i>j}^3 k_i k_j}{k_t^2}$ $K_2 \equiv \frac{\prod_i^3 k_i}{k_t^3}$, and $N_k \equiv -\ln\left(k_t \tau_{\mathrm{end}}\right)$

ζ from Vector Fields
Non-Abelian Vector Fields
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The Model

The Lagrangian with local invariance under some symmetry group G:

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{1}{2}\mathrm{Tr}\left[\left(D_\mu\Phi\right)^\dagger D^\mu\Phi\right] - \tfrac{1}{4}f\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - V\left(\varphi,\Phi\right)$$

- φ - inflaton, scalar singlet;
- Φ - Higgs field;
- Covariant derivative: $D_\mu = \partial_\mu + i\lambda_A \mathbf{T}^a A^a_\mu$;
- Gauge kinetic function: $f \propto a^{-4}$, and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_c f^{abc} A^b_\mu A^c_\nu$;
- Hybrid inflation potential:

$$V\left(\varphi,\Phi\right) = \tfrac{1}{4}\lambda\left(\chi^2 - M^2\right)^2 + \tfrac{1}{2}\kappa^2\varphi^2\chi^2 + V\left(\varphi\right)$$

in the unitary gauge $\Phi = \chi\mathbf{1}$, with $\mathrm{Tr}\left[\mathbf{1}^\dagger\mathbf{1}\right] = 1$

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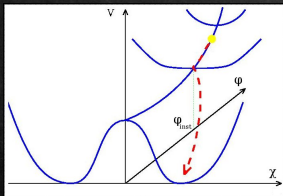
End-of-Inflation scenario

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- The effective mass of χ:
  $$m_{\text{eff}}^2 = \kappa^2\varphi^2 - \lambda M^2$$

$m_{\text{eff}} = m_{\text{eff}}\left(\mathbf{x}\right)$: 1. The function of position;
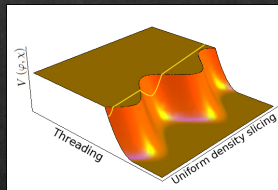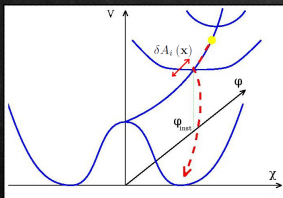2. Statistically anisotropic.

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$m_{\mathrm{eff}} = m_{\mathrm{eff}}\left(\mathbf{x}\right)$:   1. The function of position;
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The Anisotropic Spectrum

Only massive vector fields contribute to ζ and assume $W^{\bar{a}} \sim W \; \forall \bar{a}$

$$\mathcal{P}_{\zeta}\left(\mathbf{k}\right) \approx \mathcal{P}_{\zeta}^{\text{iso}}\left[1 - \frac{1}{\mathcal{N}} \sum_{\bar{a}} \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}\right)^{2}\right]$$

- Anisotropy is suppressed by the number of massive vector fields $\mathcal{N}$;

- From observational bound:

  1. $|g_{\zeta}| < 0.3$: $\mathcal{N} \geq 4$;   *Groeneboom et al. (2010)*
  2. $|g_{\zeta}| < 0.07$: $\mathcal{N} \geq 15$;   *Hanson et al. (2010)*

- Assume $SU(N) \rightarrow SU(N-1) \Rightarrow \mathcal{N} = 2N - 1$:

  1. $|g_{\zeta}| < 0.3$: $SU(3) \Rightarrow g_{\zeta} = -0.20$;
  2. $|g_{\zeta}| < 0.07$: $SU(8) \Rightarrow g_{\zeta} = -0.066$;

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The Bispectrum

$$\langle \zeta(\mathbf{k}_1)\, \zeta(\mathbf{k}_2)\, \zeta(\mathbf{k}_3) \rangle = (2\pi)\, \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B_\zeta(\mathbf{k})$$

$$B_\zeta = B_{\zeta\mathrm{self}} + B_{\zeta\mathrm{gr}}$$

1. $B_{\zeta\mathrm{self}} \propto \mathrm{all}\ \mathrm{fields} \iff$ self interactions;
2. $B_{\zeta\mathrm{gr}} \propto \mathrm{massive}\ \mathrm{fields} \iff$ non-linearity of gravity;

- Both $B_{\zeta\mathrm{self}}$ and $B_{\zeta\mathrm{gr}}$ are anisotropic;

- Isotropic parts
  $$B_{\zeta\mathrm{gr}}^{\mathrm{iso}} = -\left(\frac{g_c^2 W^2}{12 f_e H^2}\right)^{-1} B_{\zeta\mathrm{self}}^{\mathrm{iso}} = -4\pi^4 \frac{\eta}{2\mathcal{N}} \left(\frac{f_e \kappa^2 \varphi_c^2}{\lambda_A^2 W^2}\right) \frac{\sum_i k_i^3}{\prod_i k_i^3} \left(\mathcal{P}_\zeta^{\mathrm{iso}}\right)^2;$$

- $W > H \implies B_{\zeta\mathrm{gr}} \geq B_{\zeta\mathrm{self}}$;

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$$\langle \zeta\left(\mathbf{k}_1\right)\zeta\left(\mathbf{k}_2\right)\zeta\left(\mathbf{k}_3\right)\rangle = (2\pi)\,\delta\left(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3\right)B_\zeta\left(\mathbf{k}\right)$$

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- $W > H \Rightarrow B_{\zeta\mathrm{gr}} \geq B_{\zeta\mathrm{self}}$;

Outline

Summary

- Vector fields can generate or contribute to ζ;
- Generally statistically anisotropic ζ (although can be avoided);
- Present bounds:
  - $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\text{iso}} \left[1 + g_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}} \right)^2 \right]$: $|g_\zeta| < 0.3$ or $|g_\zeta| < 0.07$;
  - $f_{\text{NL}} = f_{\text{NL}}^{\text{iso}} \left[1 + \mathcal{G}_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}} \right)^2 \right]$: no bound on $\mathcal{G}_\zeta$;

Non-Abelian vector fields:

- Correlators are dominated by the classical evolution of fields;
- The anisotropy is suppressed by the number of fields generating ζ;
- Scenario in which ζ is generated through the covariant derivative term:
  $\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \text{Tr} \left[\left(D_\mu \Phi \right)^\dagger D^\mu \Phi \right] - \frac{1}{4} f \, \text{Tr} \left[F_{\mu\nu} F^{\mu\nu} \right] - V \left(\varphi, \Phi \right)$
  - $SU(3)$ or $SU(8)$ is enough to avoid WMAP bounds on g_ζ;
  - Might be observable by the Planck satellite ($\Delta g_\zeta \sim 0.01$);
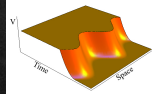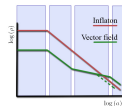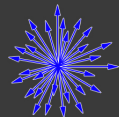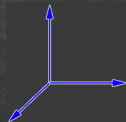
Avoiding Large Scale Anisotropy

For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

$$T^{\nu}_{\mu} = \mathrm{diag}\left(\rho, -p, -p, +p\right)$$

- Three identical, orthogonal vector fields;
  Armendariz-Picon (2004)

- Many randomly oriented vector fields - vector inflation;
  Golovnev, Mukhanov, Vanchurin (2008)

- Vector curvaton scenario;
  Dimopoulos (2006)

- End-of-inflation scenario;
  Yokoyama, Soda (2008)

Quantization

Temporal gauge: $\hat{W}_0^a = 0$;

$$\delta\hat{W}_i^a\left(\mathbf{k}\right) = \sum_{\lambda=\mathrm{L,R}}\left[e_i^\lambda\left(\hat{\mathbf{k}}\right)w\left(k,\tau\right)\hat{a}_\lambda^a\left(\mathbf{k}\right) - e_i^{\lambda*}\left(-\hat{\mathbf{k}}\right)w^*\left(k,\tau\right)\hat{a}_\lambda^{a\dagger}\left(-\mathbf{k}\right)\right]$$

- Interaction picture $\Rightarrow w = \frac{H}{\sqrt{2k^3}}\left(1-ik\tau\right)\mathrm{e}^{-ik\tau}$;

General Equations

$$\left\langle \delta W_i \left(\mathbf{k} \right) \delta W_j \left(\mathbf{k}' \right) \right\rangle = \left(2\pi \right)^3 \delta \left(\mathbf{k} + \mathbf{k}' \right) \frac{2\pi^2}{k^3} \times$$

$$\times \left[T_{ij}^{\mathrm{even}} \left(\hat{\mathbf{k}} \right) \mathcal{P}_+ \left(k \right) + i T_{ij}^{\mathrm{odd}} \left(\hat{\mathbf{k}} \right) \mathcal{P}_- + T_{ij}^{\mathrm{long}} \left(\hat{\mathbf{k}} \right) \mathcal{P}_{||} \right]$$

$$T_{ij}^{\mathrm{even}} \left(\hat{\mathbf{k}} \right) = e_i^{\mathrm{L}} \left(\hat{\mathbf{k}} \right) e_j^{\mathrm{R}} \left(\hat{\mathbf{k}} \right) + e_i^{R} \left(\hat{\mathbf{k}} \right) e_j^{\mathrm{L}} \left(\hat{\mathbf{k}} \right) = \delta_{ij} - \hat{k}_i \hat{k}_j$$

$$T^{\mathrm{odd}} \left(\hat{\mathbf{k}} \right) = i \left[e_i^{\mathrm{L}} \left(\hat{\mathbf{k}} \right) e_j^{\mathrm{R}} \left(\hat{\mathbf{k}} \right) - e_i^{R} \left(\hat{\mathbf{k}} \right) e_j^{\mathrm{L}} \left(\hat{\mathbf{k}} \right) \right] = \epsilon_{ijk} \hat{k}_k$$

$$T_{ij}^{\mathrm{long}} \left(\hat{\mathbf{k}} \right) = e_i^{||} \left(\hat{\mathbf{k}} \right) e_j^{||} \left(\hat{\mathbf{k}} \right) = \hat{k}_i \hat{k}_j$$

The In-In Formalism

$$g_3\left(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) = \left\langle 0 \left| \hat{U}^{-1} \delta \hat{W}_i^a\left(\mathbf{x}_1\right) \delta \hat{W}_j^b\left(\mathbf{x}_2\right) \delta \hat{W}_l^c\left(\mathbf{x}_3\right) \hat{U} \right| 0 \right\rangle,$$

where $\hat{U} = \exp\left\{-i \int_{\tau_0}^{\tau} \hat{H}_{\text{int}} \mathrm{d}\tau'\right\}$;

1. Tree level;
2. Third order;

$$\hat{H}_{\text{int}} \equiv a^3\left(\tau\right) \int \mathrm{d}^3\mathbf{x} \, \frac{g_c}{\sqrt{f}} \mathfrak{f}^{abc} \partial_i \delta \hat{W}_j^a \delta \hat{W}_i^b \delta \hat{W}_j^c +$$

$$+ a^4\left(\tau\right) \int \mathrm{d}^3\mathbf{x} \, \frac{1}{2} \frac{g_c^2}{f} \left(\mathfrak{f}^{abc}\mathfrak{f}^{ade} + \mathfrak{f}^{adc}\mathfrak{f}^{abe}\right) W_i^b \delta \hat{W}_j^c \delta \hat{W}_i^d \delta \hat{W}_j^e.$$

The In-In Formalism

$$g_3\left(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\right) = \left\langle 0 \left| \hat{U}^{-1} \delta \hat{W}_i^a\left(\mathbf{x}_1\right) \delta \hat{W}_j^b\left(\mathbf{x}_2\right) \delta \hat{W}_l^c\left(\mathbf{x}_3\right) \hat{U} \right| 0 \right\rangle,$$

where $\hat{U} = \exp\left\{ -i \int_{\tau_0}^{\tau} \hat{H}_{\text{int}} \mathrm{d}\tau' \right\}$;

1. Tree level;

2. Third order;

$$\hat{H}_{\text{int}} \equiv a^3\left(\tau\right) \int \mathrm{d}^3 \mathbf{x} \, \frac{g_c}{\sqrt{f}} \mathrm{f}^{abc} \partial_i \delta \hat{W}_j^a \delta \hat{W}_i^b \delta \hat{W}_j^c +$$

$$+ a^4\left(\tau\right) \int \mathrm{d}^3 \mathbf{x} \, \frac{1}{2} \frac{g_c^2}{f} \left(\mathrm{f}^{abc} \mathrm{f}^{ade} + \mathrm{f}^{adc} \mathrm{f}^{abe} \right) W_i^b \delta \hat{W}_j^c \delta \hat{W}_i^d \delta \hat{W}_j^e.$$

The Curvature Perturbation

- δN formula:

$$\zeta_{\mathrm{e}} = N_i^{\bar{a}} \delta W_i^{\bar{a}} + N_{ij}^{\bar{a}\bar{b}} \delta W_i^{\bar{a}} \delta W_j^{\bar{b}}$$

  where $N_i^{\bar{a}} = -N_{\mathrm{e}} \frac{M^{\bar{a}\bar{b}} W_i^{\bar{b}}}{\kappa^2 f_{\mathrm{e}} \varphi_{\mathrm{c}}}$ and $N_{\mathrm{e}} = \partial N / \partial \varphi_{\mathrm{c}} = \left(2 m_{\mathrm{Pl}}^2 \epsilon_{\mathrm{e}}\right)^{-\frac{1}{2}}$.

- Require $\zeta_{\mathrm{e}} \gg \zeta_{\varphi} \Rightarrow \zeta$ only from non-Abelian vector fields

The Power Spectrum

$$\langle \zeta\left(\mathbf{k}_1\right) \zeta\left(\mathbf{k}_2\right)\rangle = (2\pi)\, \delta\left(\mathbf{k}_1 + \mathbf{k}_2\right) \mathcal{P}_\zeta\left(\mathbf{k}_1\right)$$

- The general expression:

$$\mathcal{P}_\zeta\left(\mathbf{k}\right) = \mathcal{P}_+ C^2 \left(M^2\right)^{\bar{a}\bar{b}} W^{\bar{a}} W^{\bar{b}} \left[1 - \frac{\left(M^2\right)^{\bar{a}\bar{b}} \left(\mathbf{W}^{\bar{a}} \cdot \hat{\mathbf{k}}\right) \left(\mathbf{W}^{\bar{b}} \cdot \hat{\mathbf{k}}\right)}{\left(M^2\right)^{\bar{a}\bar{b}} W^{\bar{a}} W^{\bar{b}}}\right],$$

where $\left(M^2\right)^{\bar{a}\bar{b}} \equiv M^{\bar{a}\bar{c}} M^{\bar{c}\bar{b}} \sim \lambda_A^4$ and $C \equiv \frac{1}{\kappa^2 f_e} \frac{N_e}{\varphi_c}$;

  - Angular modulation!

- Assume $W^{\bar{a}} \sim W \,\, \forall \, \bar{a}$

$$\mathcal{P}_\zeta\left(\mathbf{k}\right) \approx \mathcal{P}_\zeta^{\mathrm{iso}} \left[1 - \frac{1}{\mathcal{N}} \sum_{\bar{a}} \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}\right)^2\right],$$

where $\mathcal{N}$ is the number of massive vector fields and

$$\mathcal{P}_\zeta^{\mathrm{iso}} = \lambda_A^4 \mathcal{N} \mathcal{P}_+ \left(CW\right)^2$$

Some Bounds

- Perturbation from inflaton is subdominant, i.e. $\zeta_e \gg \zeta_\varphi$:

$$\left(\frac{\lambda_A^2}{f_e} \frac{W}{\kappa^2 \varphi_c} \right) \gg \frac{\epsilon_e}{\epsilon_k} = \mathrm{e}^{-2N_e \eta}$$

- Variation of δf_e is negligible:

$$\left(\frac{M^{\bar{a}\bar{b}} W_i^{\bar{a}} W_i^{\bar{b}}}{\kappa^2 f_e \varphi_c m_{\mathrm{Pl}}} \right) \ll \epsilon_e$$

- These two bounds and $W > H$ give

The Bispectrum

$$B_{\zeta 1} = 4\pi^4 \frac{\sum_i k_i^3}{\prod_i k_i^3} \frac{g_{\text{end}}^2}{12H^2} C^3 \mathcal{P}_+^2 \left(\mathrm{f}^{ab\bar{h}} \mathrm{f}^{a\bar{g}\bar{f}} + \mathrm{f}^{a\bar{g}\bar{h}} \mathrm{f}^{ab\bar{f}} \right) M^{\bar{g}\bar{c}} M^{\bar{f}\bar{d}} M^{\bar{h}\bar{e}} \left(\mathbf{W}^b \cdot \mathbf{W}^{\bar{c}} \right) \times .$$

$$\times \left[\left(\mathbf{W}^{\bar{d}} \cdot \mathbf{W}^{\bar{e}} \right) - 2 \left(\mathbf{W}^{\bar{d}} \cdot \hat{\mathbf{k}}_1 \right) \left(\mathbf{W}^{\bar{e}} \cdot \hat{\mathbf{k}}_1 \right) + \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3 \right) \left(\mathbf{W}^{\bar{d}} \cdot \hat{\mathbf{k}}_1 \right) \left(\mathbf{W}^{\bar{e}} \cdot \hat{\mathbf{k}}_3 \right) + \mathrm{c.p.} \right]$$

$$B_{\zeta 2} = -4\pi^4 \frac{\left(\mathcal{P}_\zeta^{\text{iso}} \right)^2}{\mathcal{N} \varphi_{\text{c}} N_{\text{e}}} \left(\frac{\lambda M^2}{\lambda_A^2 W^2 / f_{\text{e}}} \right) \frac{\sum_i k_i^3}{\prod_i k_i^3} \left\{ 1 - \frac{1}{\mathcal{N}} \sum_{\bar{a}} \left[k_2^3 \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_1 \right)^2 - k_2^3 \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_3 \right)^2 \right. \right.$$

$$\left. \left. -k_2^3 \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3 \right) \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_1 \right) \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_3 \right) + \mathrm{c.p.} \right] / \sum_i k_i^3 \right\},$$