The Curvature Perturbation from General non-Abelian Vector Fields

Mindaugas Karčiauskas

Universidad de Granada

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Outline

1. ζ from Vector Fields
   - Motivation
   - The Curvature Perturbation From Vector Fields
2. Non-Abelian Vector Fields
   - The Set Up
   - Correlators in the In-In Formalism
   - The End-of-Inflation Scenario
3. Summary

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- ζ generated by scalar fields $\Rightarrow$ statistical homogeneity and isotropy;
- Some indications both might be broken:
  - alignments of low multipoles
    broken isotropy, i.e. preferred direction;
- New observable - statistical anisotropy
  - Can be dominant in B_ζ even if subdominant in $\mathcal{P}_\zeta$;
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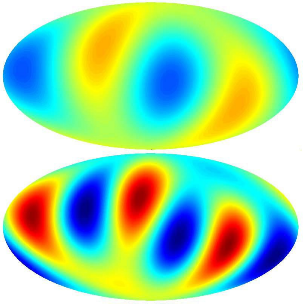

Tegmark, de Oliveira-Costa, Hamilton (2003)

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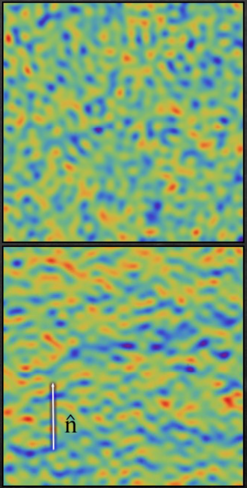

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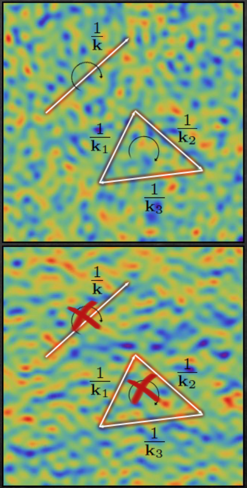

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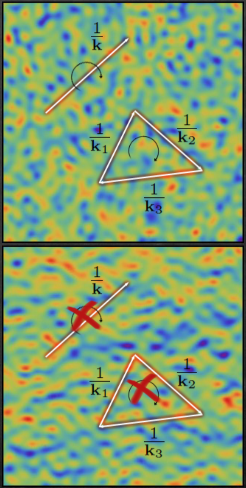

Breaking the Conformal Invariance

- Massless $U(1)$ vector field is conformally invariant; $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

1. Introduce "potential": $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu$
2. Non-canonical kinetic function: $\mathcal{L} = -\frac{1}{4}f(t)F_{\mu\nu}F^{\mu\nu}$

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Avoiding Large Scale Anisotropy

For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

$$T_\mu^\nu = \mathrm{diag}\,(\rho, -p, -p, +p)$$

- Three identical, orthogonal vector fields;
  Armendariz-Picon (2004)
- Many randomly oriented vector fields - vector inflation;
  Golovnev, Mukhanov, Vanchurin (2008)
- Vector curvaton scenario;
  Dimopoulos (2006)
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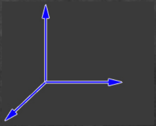

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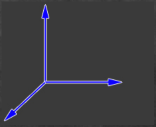

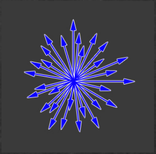

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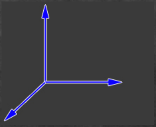

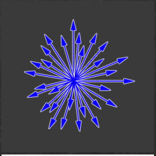

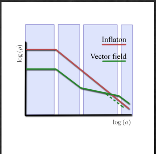

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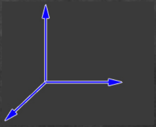

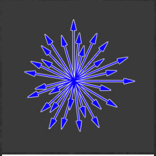

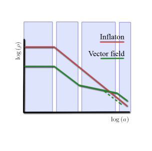

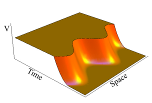

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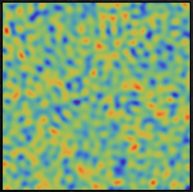

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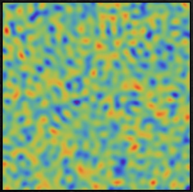
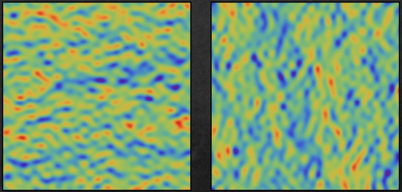


$\mathcal{P}_{\rm L} \neq / = \mathcal{P}_{\rm R} \neq / = \mathcal{P}_{||}$



Observational Constraints

- The power spectrum: $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\text{iso}} \left[1 + g_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}} \right)^2 \right]$:
  1. $g_\zeta = 0.29 \pm 0.031$: *Hanson, Lewis (2009); Groeneboom, Ackerman, Wehus, Eriksen (2010);*
     - preferred direction $\hat{\mathbf{n}} = (l, b) = (96, 30)$ - close to ecliptic pole;
  2. $|g_\zeta| < 0.07$: *Hanson, Lewis, Challinor (2010);*
  - Planck prospects: *Ma, Efstathiou, Challinor (2011);*
     - $\Delta g_\zeta \sim 0.01 \ (2\sigma)$;
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- If g_ζ is due to vector fields: *MK, Dimopoulos, Lyth (2010)*
  1. The same referred direction: $\hat{\mathbf{n}} = \hat{\mathbf{m}} = \hat{\mathbf{W}}$;
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Motivation

1. The non-Abelian vector fields are ubiquitous in particle physics models;
2. Several of vector fields and random orientation $\Rightarrow$ suppressed statistical anisotropy;

The Lagrangian

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$$\mathcal{L} = -\frac{1}{4} f(t)\, F^a_{\mu\nu} F_a^{\mu\nu}, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_c \mathrm{f}^{abc} A^b_\mu A^c_\nu$$

- $f = f(t) \Leftarrow$ 1. SUGRA; 2. moduli;
- $f \propto a^{-4}$
  1. flat perturbation spectrum: *Dimopoulos, MK, Wagstaff (2010)*
  2. attractor solution if $f(t)$ is modulated by the inflaton;
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  3. corresponds to weak coupling;
     - 3.1 The physical, canonically normalized vector field: $W^a_i \equiv \sqrt{f}\frac{A^a_i}{a} \quad \Rightarrow \quad \mathcal{L} \supset \frac{{g_c}^2}{f} \mathrm{f}^{abc} \mathrm{f}^{ade} W^b_i W^c_j W^d_i W^e_j$;
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Correlators

Using the 'In-In Formalism':

$$g_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = -(2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{2H^6}{\prod_i^3 2k_i^3} \mathcal{T}^{fgh}_{lmn} \left(\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2, \hat{\mathbf{k}}_3\right) I(k_1, k_2, k_3)$$

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1. Anisotropic:

$$\begin{aligned}\mathcal{T}^{fgh}_{lmn}\left(\hat{\mathbf{k}}_1,\hat{\mathbf{k}}_2,\hat{\mathbf{k}}_3\right) \equiv{}& W^b_m T^{\mathrm{E}}_{ij}\left(\hat{\mathbf{k}}_1\right)T^{\mathrm{E}}_{nj}\left(\hat{\mathbf{k}}_3\right)\left(\mathrm{f}^{abh}\mathrm{f}^{agf}+\mathrm{f}^{agh}\mathrm{f}^{abf}\right)+\\ &+W^b_l T^{\mathrm{E}}_{mj}\left(\hat{\mathbf{k}}_2\right)T^{\mathrm{E}}_{nj}\left(\hat{\mathbf{k}}_3\right)\left(\mathrm{f}^{abg}\mathrm{f}^{afh}+\mathrm{f}^{afg}\mathrm{f}^{abh}\right)+\\ &+W^b_n T^{\mathrm{E}}_{lj}\left(\hat{\mathbf{k}}_1\right)T^{\mathrm{E}}_{mj}\left(\hat{\mathbf{k}}_2\right)\left(\mathrm{f}^{abg}\mathrm{f}^{ahf}+\mathrm{f}^{ahg}\mathrm{f}^{abf}\right),\end{aligned}$$

where $T^{\mathrm{E}}_{ij}\left(\hat{\mathbf{k}}\right) \equiv \delta_{ij} - \hat{k}_i\hat{k}_j$.

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2. Classical contribution dominates:

$$I = \frac{g_c^2}{f_0}\frac{k_t^7 H^{-8}}{4!}\left[6e^{4N_k}\left(\frac{1}{3}-K_1+K_2\right)+2e^{2N_k}\left(K_1-3K_2-\frac{1}{5}\right)-\right.$$

$$\left.+\left(\gamma+N_k\right)\left(\frac{1}{5}K_1-K_2-\frac{1}{35}\right)+\frac{1}{300}\left(625K_2-137K_1+\frac{1019}{49}\right)\right]$$

where $K_1 \equiv \frac{\sum_{i>j}^3 k_i k_j}{k_t^2}$ $\quad K_2 \equiv \frac{\prod_i^3 k_i}{k_t^3}$, and $N_k \equiv -\ln\left(k_t\tau_{\rm end}\right)$

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The Model

The Lagrangian with local invariance under some symmetry group G:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\mathrm{Tr}\left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] - \frac{1}{4}f\,\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - V(\varphi,\Phi)$$

- φ - inflaton, scalar singlet;
- Φ - Higgs field;
- Covariant derivative: $D_\mu = \partial_\mu + i\lambda_A \mathbf{T}^a A^a_\mu$;
- Gauge kinetic function: $f \propto a^{-4}$,
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$$V(\varphi,\Phi) = \frac{1}{4}\lambda\left(\chi^2 - M^2\right)^2 + \frac{1}{2}\kappa^2\varphi^2\chi^2 + V(\varphi)$$

in the unitary gauge $\Phi = \chi\mathbf{l}$, with $\mathrm{Tr}\left[\mathbf{l}^\dagger\mathbf{l}\right] = 1$

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End-of-Inflation scenario

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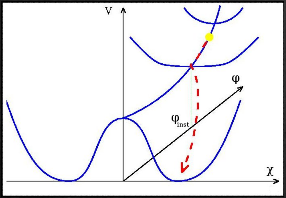
- The effective mass of χ:
  $m_{\rm eff}^2 = \kappa^2\varphi^2 - \lambda M^2$



$m_{\rm eff} = m_{\rm eff}(\mathbf{x})$:
1. The function of position;
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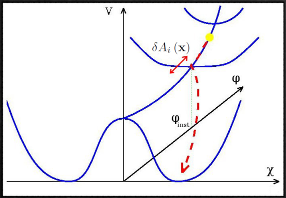

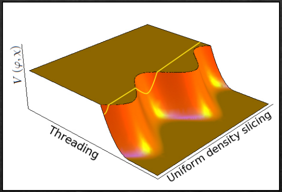

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The Anisotropic Spectrum

Only massive vector fields contribute to ζ and assume $W^{\bar{a}} \sim W \,\forall\, \bar{a}$

$$\mathcal{P}_\zeta(\mathbf{k}) \approx \mathcal{P}_\zeta^{\text{iso}} \left[1 - \frac{1}{\mathcal{N}} \sum_{\bar{a}} \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}\right)^2\right]$$

- Anisotropy is suppressed by the number of massive vector fields $\mathcal{N}$;
- From observational bound:
  1. $|g_\zeta| < 0.3$: $\mathcal{N} \geq 4$; *Groeneboom et al. (2010)*
  2. $|g_\zeta| < 0.07$: $\mathcal{N} \geq 15$; *Hanson et al. (2010)*
- Assume $SU(N) \to SU(N-1) \Rightarrow \mathcal{N} = 2N - 1$:
  1. $|g_\zeta| < 0.3$: $SU(3) \Rightarrow g_\zeta = -0.20$;
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The Bispectrum

$$\langle \zeta(\mathbf{k}_1)\,\zeta(\mathbf{k}_2)\,\zeta(\mathbf{k}_3)\rangle = (2\pi)\,\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\,B_\zeta(\mathbf{k})$$

$$B_\zeta = B_{\zeta\mathrm{self}} + B_{\zeta\mathrm{gr}}$$

1. $B_{\zeta\mathrm{self}} \propto$ all fields $\Leftarrow$ self interactions;
2. $B_{\zeta\mathrm{gr}} \propto$ massive fields $\Leftarrow$ non-linearity of gravity;

- Both $B_{\zeta\mathrm{self}}$ and $B_{\zeta\mathrm{gr}}$ are anisotropic;
- Isotropic parts
  $B^{\mathrm{iso}}_{\zeta\mathrm{gr}} = -\left(\frac{g_c^2 W^2}{12 f_e H^2}\right)^{-1} B^{\mathrm{iso}}_{\zeta\mathrm{self}} = -4\pi^4 \frac{\eta}{2\mathcal{N}}\left(\frac{f_e \kappa^2 \varphi_c^2}{\lambda_A^2 W^2}\right)\frac{\sum_i k_i^3}{\prod_i k_i^3}\left(\mathcal{P}^{\mathrm{iso}}_\zeta\right)^2;$
- $W > H \;\Rightarrow\; B_{\zeta\mathrm{gr}} \geq B_{\zeta\mathrm{self}};$

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Outline

1. ζ from Vector Fields
   - Motivation
   - The Curvature Perturbation From Vector Fields
2. Non-Abelian Vector Fields
   - The Set Up
   - Correlators in the In-In Formalism
   - The End-of-Inflation Scenario
3. Summary

Summary

- Vector fields can generate or contribute to ζ;
- Generally statistically anisotropic ζ (although can be avoided);
- Present bounds:
  - $\mathcal{P}_\zeta = \mathcal{P}_\zeta^{\text{iso}} \left[1 + g_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}\right)^2\right]$: $|g_\zeta| < 0.3$ or $|g_\zeta| < 0.07$;
  - $f_{\text{NL}} = f_{\text{NL}}^{\text{iso}} \left[1 + \mathcal{G}_\zeta \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{n}}\right)^2\right]$: no bound on $\mathcal{G}_\zeta$;

Non-Abelian vector fields:

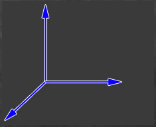
- Correlators are dominated by the classical evolution of fields;
- The anisotropy is suppressed by the number of fields generating ζ;
- Scenario in which ζ is generated through the covariant derivative term:
  $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\text{Tr}\left[\left(D_\mu\Phi\right)^\dagger D^\mu\Phi\right] - \frac{1}{4}f\,\text{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] - V\left(\varphi, \Phi\right)$
  - $SU(3)$ or $SU(8)$ is enough to avoid WMAP bounds on g_ζ;
  - Might be observable by the Planck satellite ($\Delta g_\zeta \sim 0.01$);

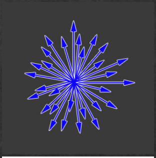
Avoiding Large Scale Anisotropy

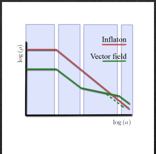
For massless or light $U(1)$ vector field $\mathbf{W} = (0, 0, W)$:

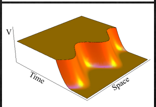
$$T_\mu^\nu = \mathrm{diag}\left(\rho, -p, -p, +p\right)$$

- Three identical, orthogonal vector fields;
  Armendariz-Picon (2004)
- Many randomly oriented vector fields - vector inflation;
  Golovnev, Mukhanov, Vanchurin (2008)
- Vector curvaton scenario;
  Dimopoulos (2006)
- End-of-inflation scenario;
  Yokoyama, Soda (2008)









Quantization

Temporal gauge: $\hat{W}_0^a = 0$;

$$\delta\hat{W}_i^a(\mathbf{k}) = \sum_{\lambda=\mathrm{L,R}}\left[e_i^\lambda\left(\hat{\mathbf{k}}\right) w(k,\tau)\,\hat{a}_\lambda^a(\mathbf{k}) - e_i^{\lambda *}\left(-\hat{\mathbf{k}}\right) w^*(k,\tau)\,\hat{a}_\lambda^{a\dagger}(-\mathbf{k})\right]$$

- Interaction picture $\Rightarrow w = \frac{H}{\sqrt{2k^3}}(1 - ik\tau)\,\mathrm{e}^{-ik\tau}$;

General Equations

$$\left\langle \delta W_i \left(\mathbf{k}\right) \delta W_j \left(\mathbf{k}'\right) \right\rangle = (2\pi)^3 \, \delta\left(\mathbf{k}+\mathbf{k}'\right) \frac{2\pi^2}{k^3} \times$$

$$\times \left[T_{ij}^{\rm even}\left(\hat{\mathbf{k}}\right) \mathcal{P}_+\left(k\right) + i T_{ij}^{\rm odd}\left(\hat{\mathbf{k}}\right) \mathcal{P}_- + T_{ij}^{\rm long}\left(\hat{\mathbf{k}}\right) \mathcal{P}_{||} \right]$$

$$T_{ij}^{\rm even}\left(\hat{\mathbf{k}}\right) = e_i^{\rm L}\left(\hat{\mathbf{k}}\right) e_j^{\rm R}\left(\hat{\mathbf{k}}\right) + e_i^{R}\left(\hat{\mathbf{k}}\right) e_j^{\rm L}\left(\hat{\mathbf{k}}\right) = \delta_{ij} - \hat{k}_i \hat{k}_j$$

$$T^{\rm odd}\left(\hat{\mathbf{k}}\right) = i \left[e_i^{\rm L}\left(\hat{\mathbf{k}}\right) e_j^{\rm R}\left(\hat{\mathbf{k}}\right) - e_i^{R}\left(\hat{\mathbf{k}}\right) e_j^{\rm L}\left(\hat{\mathbf{k}}\right) \right] = \epsilon_{ijk} \hat{k}_k$$

$$T_{ij}^{\rm long}\left(\hat{\mathbf{k}}\right) = e_i^{||}\left(\hat{\mathbf{k}}\right) e_j^{||}\left(\hat{\mathbf{k}}\right) = \hat{k}_i \hat{k}_j$$

The In-In Formalism

$$g_3\left(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3\right)=\left\langle 0\left|\hat{U}^{-1}\delta\hat{W}_i^a\left(\mathbf{x}_1\right)\delta\hat{W}_j^b\left(\mathbf{x}_2\right)\delta\hat{W}_l^c\left(\mathbf{x}_3\right)\hat{U}\right|0\right\rangle,$$

where $\hat{U}=\exp\left\{-i\int_{\tau_0}^{\tau}\hat{H}_{\rm int}{\rm d}\tau'\right\}$;

1. Tree level;
2. Third order;

$$\hat{H}_{\rm int}\equiv a^3\left(\tau\right)\int{\rm d}^3\mathbf{x}\,\frac{g_{\rm c}}{\sqrt{f}}{\rm f}^{abc}\partial_i\delta\hat{W}_j^a\delta\hat{W}_i^b\delta\hat{W}_j^c+$$
$$+a^4\left(\tau\right)\int{\rm d}^3\mathbf{x}\,\frac{1}{2}\frac{g_{\rm c}^2}{f}\left({\rm f}^{abc}{\rm f}^{ade}+{\rm f}^{adc}{\rm f}^{abe}\right)W_i^b\delta\hat{W}_j^c\delta\hat{W}_i^d\delta\hat{W}_j^e.$$

The In-In Formalism

$$g_3(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \left\langle 0 \left| \hat{U}^{-1} \delta\hat{W}_i^a(\mathbf{x}_1)\, \delta\hat{W}_j^b(\mathbf{x}_2)\, \delta\hat{W}_l^c(\mathbf{x}_3)\, \hat{U} \right| 0 \right\rangle,$$

where $\hat{U} = \exp\left\{-i \int_{\tau_0}^{\tau} \hat{H}_{\rm int} \mathrm{d}\tau'\right\}$;

1. Tree level;
2. Third order;

$$\hat{H}_{\rm int} \equiv a^3(\tau) \int \mathrm{d}^3\mathbf{x}\, \frac{g_{\rm c}}{\sqrt{f}} \mathrm{f}^{abc} \partial_i \delta\hat{W}_j^a \delta\hat{W}_i^b \delta\hat{W}_j^c + \\ + a^4(\tau) \int \mathrm{d}^3\mathbf{x}\, \frac{1}{2} \frac{g_{\rm c}^2}{f} \left(\mathrm{f}^{abc}\mathrm{f}^{ade} + \mathrm{f}^{adc}\mathrm{f}^{abe}\right) W_i^b \delta\hat{W}_j^c \delta\hat{W}_i^d \delta\hat{W}_j^e .$$

The Curvature Perturbation

- δN formula:

$$\zeta_{\rm e} = N_i^{\bar{a}} \delta W_i^{\bar{a}} + N_{ij}^{\bar{a}\bar{b}} \delta W_i^{\bar{a}} \delta W_j^{\bar{b}}$$

where $N_i^{\bar{a}} = -N_{\rm e} \frac{M^{\bar{a}\bar{b}} W_i^{\bar{b}}}{\kappa^2 f_{\rm e} \varphi_{\rm c}}$ and $N_{\rm e} = \partial N / \partial \varphi_{\rm c} = \left(2 m_{\rm Pl}^2 \epsilon_{\rm e}\right)^{-\frac{1}{2}}$.

- Require $\zeta_{\rm e} \gg \zeta_\varphi \Rightarrow \zeta$ only from non-Abelian vector fields

The Power Spectrum

$$\langle \zeta(\mathbf{k}_1)\,\zeta(\mathbf{k}_2)\rangle = (2\pi)\,\delta(\mathbf{k}_1+\mathbf{k}_2)\,\mathcal{P}_\zeta(\mathbf{k}_1)$$

- The general expression:

$$\mathcal{P}_\zeta(\mathbf{k}) = \mathcal{P}_+ C^2 \left(M^2\right)^{\bar{a}\bar{b}} W^{\bar{a}} W^{\bar{b}} \left[1 - \frac{\left(M^2\right)^{\bar{a}\bar{b}} (\mathbf{W}^{\bar{a}}\cdot\hat{\mathbf{k}})(\mathbf{W}^{\bar{b}}\cdot\hat{\mathbf{k}})}{(M^2)^{\bar{a}\bar{b}} W^{\bar{a}} W^{\bar{b}}}\right],$$

where $\left(M^2\right)^{\bar{a}\bar{b}} \equiv M^{\bar{a}\bar{c}} M^{\bar{c}\bar{b}} \sim \lambda_A^4$ and $C \equiv \frac{1}{\kappa^2 f_e}\frac{N_e}{\varphi_c}$;

  - Angular modulation!

- Assume $W^{\bar{a}} \sim W \;\forall\, \bar{a}$

$$\mathcal{P}_\zeta(\mathbf{k}) \approx \mathcal{P}_\zeta^{\rm iso}\left[1 - \frac{1}{\mathcal{N}}\sum_{\bar{a}}\left(\hat{\mathbf{W}}^{\bar{a}}\cdot\hat{\mathbf{k}}\right)^2\right],$$

where $\mathcal{N}$ is the number of massive vector fields and

$$\mathcal{P}_\zeta^{\rm iso} = \lambda_A^4 \mathcal{N} \mathcal{P}_+ (CW)^2$$

Some Bounds

- Perturbation from inflaton is subdominant, i.e. $\zeta_{\rm e} \gg \zeta_\varphi$:

$$\left(\frac{\lambda_A^2}{f_{\rm e}}\frac{W}{\kappa^2\varphi_{\rm c}}\right) \gg \frac{\epsilon_{\rm e}}{\epsilon_k} = {\rm e}^{-2N_{\rm e}\eta}$$

- Variation of $\delta f_{\rm e}$ is negligible:

$$\left(\frac{M^{\bar{a}\bar{b}}W_i^{\bar{a}}W_i^{\bar{b}}}{\kappa^2 f_{\rm e}\varphi_{\rm c} m_{\rm Pl}}\right) \ll \epsilon_{\rm e}$$

- These two bounds and $W > H$ give

The Bispectrum

$$B_{\zeta 1} = 4\pi^4 \frac{\sum_i k_i^3}{\prod_i k_i^3} \frac{g_{\rm end}^2}{12H^2} C^3 \mathcal{P}_+^2 \left(\mathfrak{f}^{ab\bar{h}} \mathfrak{f}^{a\bar{g}\bar{f}} + \mathfrak{f}^{a\bar{g}\bar{h}} \mathfrak{f}^{ab\bar{f}} \right) M^{\bar{g}\bar{c}} M^{\bar{f}\bar{d}} M^{\bar{h}\bar{e}} \left(\mathbf{W}^b \cdot \mathbf{W}^{\bar{c}} \right) \times .$$
$$\times \left[\left(\mathbf{W}^{\bar{d}} \cdot \mathbf{W}^{\bar{e}} \right) - 2 \left(\mathbf{W}^{\bar{d}} \cdot \hat{\mathbf{k}}_1 \right) \left(\mathbf{W}^{\bar{e}} \cdot \hat{\mathbf{k}}_1 \right) + \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3 \right) \left(\mathbf{W}^{\bar{d}} \cdot \hat{\mathbf{k}}_1 \right) \left(\mathbf{W}^{\bar{e}} \cdot \hat{\mathbf{k}}_3 \right) + \text{c.p.} \right]$$

$$B_{\zeta 2} = -4\pi^4 \frac{\left(\mathcal{P}_\zeta^{\rm iso} \right)^2}{\mathcal{N} \varphi_{\rm c} N_{\rm e}} \left(\frac{\lambda M^2}{\lambda_A^2 W^2 / f_{\rm e}} \right) \frac{\sum_i k_i^3}{\prod_i k_i^3} \left\{ 1 - \frac{1}{\mathcal{N}} \sum_{\bar{a}} \left[k_2^3 \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_1 \right)^2 - k_2^3 \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_3 \right)^2 - \right. \right.$$
$$\left. \left. -k_2^3 \left(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_3 \right) \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_1 \right) \left(\hat{\mathbf{W}}^{\bar{a}} \cdot \hat{\mathbf{k}}_3 \right) + \text{c.p.} \right] \Big/ \sum_i k_i^3 \right\} ,$$