Warm inflation and the primordial spectrum

Inflation & CMB

Cold inflation/ Warm inflation

Primordial spectrum from warm inflation:
Dissipation + shear viscosity

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MBG, A. Berera, R. Ramos JCAP07 (2011)030

Expanding Universe

Flatness problem

$$\Omega_T = 1 \Rightarrow \Omega_T(t_{nucl}) - 1 \approx 10^{-16}$$

Horizon problem

The observable Universe was larger than the **particle horizon** at LSS

Inflation

Early period of accelerated expansion

$$\ddot{a} > 0 : \quad P < -\rho/3$$

Super-horizon perturbations?

Too small sub-horizon (**causal**) perturbations

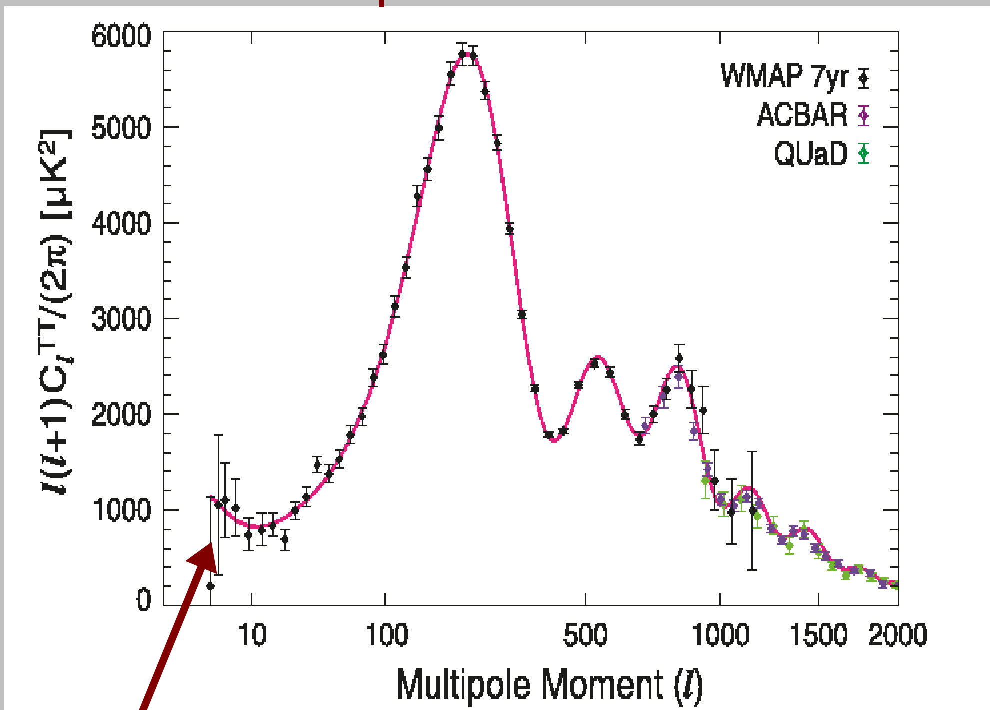
Unwanted relics

monopoles, moduli, gravitinos,...

Starobinsky ‘80; Guth PRD23 ‘81; Albrecht, Steinhardt PRL48 ‘82; Linde PLB108 '82

Cosmic **M**icrowave **B**ackground **R**adiation

WMAP: astro-ph/1001.4538



Parameter	WMAP-year	WMAP+BAO+H$_0$ Mean
$\Delta^2_{\mathcal{R}}(k_0{}^a)$	$(2.43 \pm 0.11) \times 10^{-9}$	$(2.441^{+0.088}_{-0.092}) \times 10^{-9}$
n_s	0.963 ± 0.014	0.963 ± 0.012
Gravitational Wave	$r < 0.20$	$r < 0.24$
Running Index (r=0)	$-0.084 < dn_s/d\ln k < 0.020$	$-0.061 < dn_s/d\ln k < 0.017$
Gaussianity: Local	$-10 < f_{NL}^{\rm local} < 74$	
Gaussianity:Equilateral	$-214 < f_{NL}^{\rm equil} < 266$	
Gaussianity:Ortoghonal	$-410 < f_{NL}^{\rm orthog} < 6$	

$^a k_0 = 0.002$ Mpc^{-1}.

Tensors?

Gaussian?

Largest observable scale O(3000 Mpc):

Primordial spectrum: ~adiabatic, scale-invariant, gaussian

$$P_R = \Delta_R^2(k_0)\left(\frac{k}{k_0}\right)^{n_s-1}$$

(R= comoving curvature)

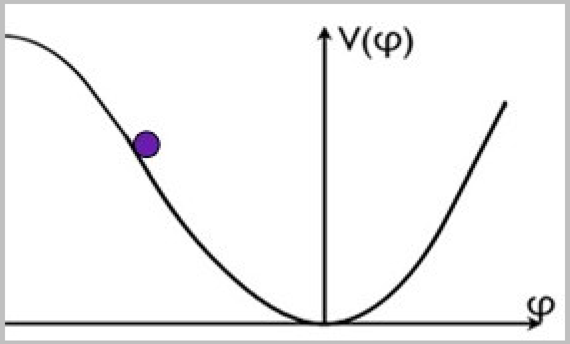
Inflation: **amplitude (input)** **Spectral index (output)**

Slow Roll Inflation

Scalar field rolling down its (flat) potential

$$P = \dot{\phi}^2/2 - V(\phi) \approx -V(\phi)$$

negative pressure



"Flat" potential

The curvature and the slope smaller than the (Hubble) expansion rate H

Kinetic energy << potential energy $H^2 = V/3m_P^2$

Slow-roll parameters

$$|\eta_\phi| = m_P^2 \left|\frac{V''}{V}\right| < 1 \qquad \epsilon_\phi = \frac{m_P^2}{2}\left(\frac{V'}{V}\right)^2 < 1$$

curvature **slope**

Inflation → Reheating → Radiation → Matter

Inflaton interacts with other particles

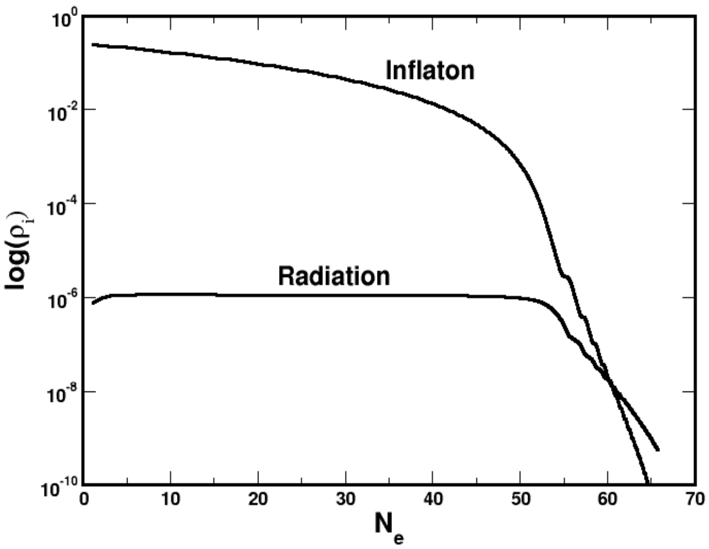
"Cold" inflation

Interactions negligible during Inflation $\xrightarrow{\text{Reheating}}$ Radiation

<u>"Warm" inflation</u>

Inflaton decay into light d. of f. $\longrightarrow$ "Dissipation" $\longrightarrow$ Radiation

$$\rho = \rho(\phi) + \rho_R, \quad \rho_R \ll \rho(\phi)$$



A (small) fraction of the vacuum energy is converted into radiation during inflation

$$\ddot{\phi} + (3H + \Upsilon)\dot{\phi} + V_\phi = 0$$

$$\dot{\rho}_R + 4H\rho_R = \Upsilon \dot{\phi}^2 \quad \text{"Source term"}$$

"Decay" into light dof= extra friction

Moss PLB154,'85; Yokoyama & Maeda PLB207 '88

Extra friction term: $Q = \Upsilon/(3H)$

- $Q \ll 1$, $T \ll H$ → Standard **C**old **I**nflation
- $Q < 1$, $T > H$ → **W**eak **D**issipative Regime

Standard slow-roll: $3H\dot{\phi} \simeq -V_\phi$, $4H\rho_r \simeq \Upsilon \dot{\phi}^2$

- $Q > 1$, $T > H$ → **S**trong **D**issipative Regime

Slow-roll: $3H(1+Q)\dot{\phi} \simeq -V_\phi$, $4H\rho_r \simeq \Upsilon \dot{\phi}^2$

$|\eta_\phi| < (1+Q)$, $\epsilon_\phi < (1+Q)$, $\beta_\Upsilon < (1+Q)$, $\delta_T < 1$

Larger ϕ mass (no "η" problem)

Smaller ϕ values / scale of inflation

$\beta_\Upsilon = m_P^2 (\Upsilon_\phi V_\phi)/(\Upsilon V)$

$\delta_T = T V_{T\phi}/V_\phi$

Fluctuations & primordial spectrum

Thermal fluctuations become the source for the adiabatic perturbations

(inflaton) System Environment (light d. of f., integrated out)

transfer of energy

Dissipation Υ

fluctuation force ξ

$$\delta\ddot{\phi}_k + (3H + \Upsilon)\,\delta\dot{\phi}_k + \dot{\phi}\,\delta\Upsilon + \left(\frac{k^2}{a^2} + V_{\phi\phi}\right)\delta\phi_k = \xi_k$$

Fluctuation-Dissipation rel.:

$$\langle \xi(k)\,\xi(k') \rangle = 2(H + \Upsilon)\,T\,a^{-3}(2\pi)^3\,\delta^{(3)}(k - k')\,\delta(t - t')$$

Berera & Fang PRL74 (1995); Taylor& Berera PRD62 (2000);
Oliveira & Joras, PRD64 (2001); Hall, Moss &Berera PRD69 (2004)

Primordial spectrum: $P_R \simeq \left(\frac{H}{\dot{\phi}}\right)^2 P_{\delta\phi}$

R is constant after horizon crossing (freeze-out)

	Cold inflation	**Warm inflation ($\Upsilon(\phi, H)$)**
Freeze-out	$k_F = k/a = H$	$k_F = k/a(t_F) > H$ (extra friction)
Field spectrum	$P_{\delta\phi} \simeq \left(\frac{k_F}{2\pi}\right)^2 \simeq \left(\frac{H_F}{2\pi}\right)^2$	$P_{\delta\phi} \simeq \int_{k<k_F} \frac{d^3k}{(2\pi)^3} \frac{n_B}{\omega_k} \simeq \frac{k_F T}{2\pi^2}$ Thermal spectrum
Primordial spectrum	$P_R \simeq \frac{1}{2\epsilon_\phi}\left(\frac{H}{2\pi m_P}\right)^2$	$P_R \simeq C(1+Q)^{5/2}\left(\frac{T}{H}\right)(P_R)_{Q=0}$
Tensor-to-scalar ratio	$r = \frac{P_T}{P_R} = 16\epsilon_\phi$	$r = \frac{4\epsilon_\phi}{\pi(1+Q)^{5/2}}\left(\frac{H}{T}\right) \leq 16\epsilon_\phi$
Non gaussianity	$f_{NL} \sim O(\epsilon, \eta)$	$\lvert f_{NL}\rvert < 15 \ln(1+Q/14)$
	(Single, canonical field)	Moss &Xiong JCAP 04(2007)

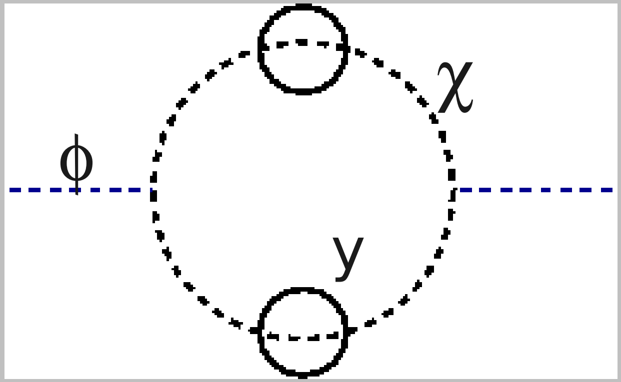
Dissipative coefficient

light fermions

$$L = \cdots - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g^2}{2} \phi^2 \chi^2 + h \chi \psi \bar{\psi} + \cdots$$

heavy $m_\chi = g\phi > H, T$

Inflaton moves down the potential, it excites χ (massive field), which decays into light dof (thermal bath)



$$\ddot{\phi} + \int d^4 x_1 \underbrace{\Sigma_\phi (x - x_1)}_{\text{self-energy}} \delta \phi (t_1) + V_\phi = 0$$

$$\Upsilon = - \int d^4 x \, \Sigma_\phi (x) t$$ Dissipative coefficient

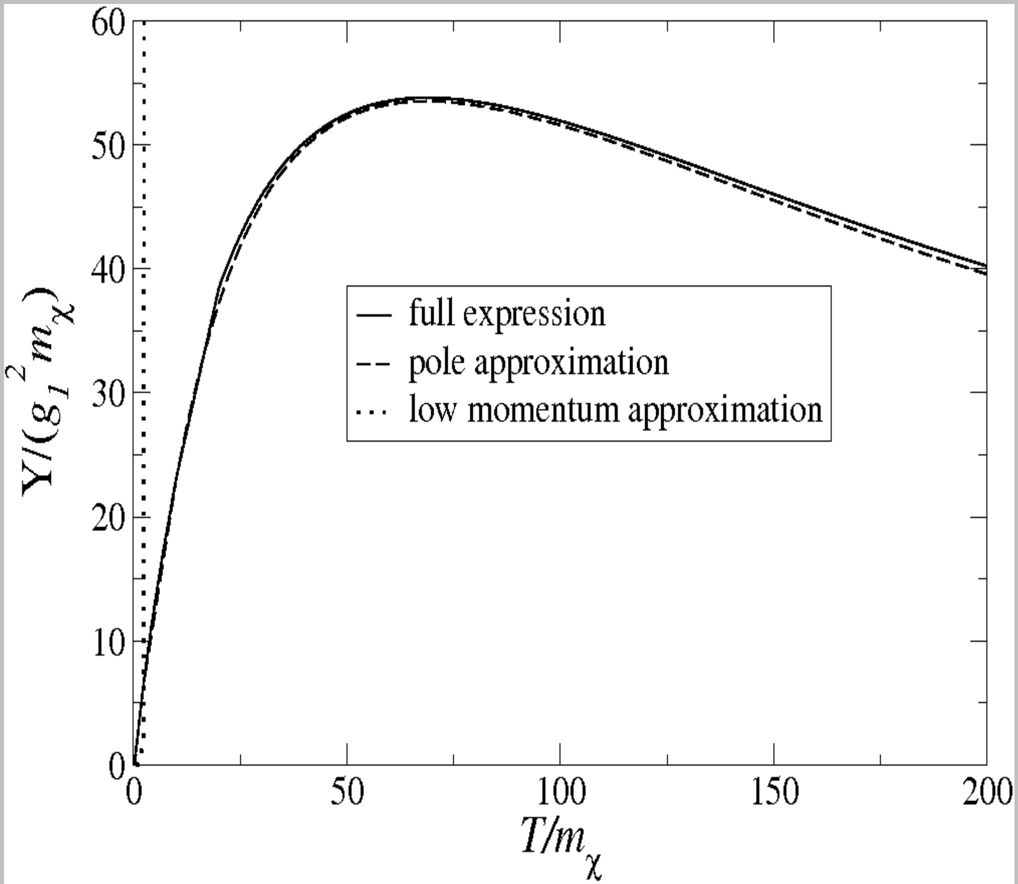
Adiabatic approx.: $\dot{\phi}/\phi, H, \dot{T}/T < \Gamma_\chi \simeq h^2 m_\chi / (8\pi)$

macroscopic microscopic

Berera and Ramos PRD ‘01, ‘03, ‘05; Hall and Moss PRD ’05

Dissipative (T-dependent) coefficient

(Adiabatic, close-to-equilibrium approx.)



T/H > 1

$\Upsilon \propto T^3/\phi^2$ Low T, $m\chi/T > 10$

$\Upsilon \propto T$ High T, $m\chi/T < 1$

$\Upsilon \propto T^{-1}$ Very high T, $m\chi/T << 1$

General: $\Upsilon \propto \left(\frac{T}{\phi}\right)^c \phi$ c=-1,0,1,3

Moss& Xiong hep-ph/0603266; MBG, Berera & Ramos, 1008.1929

Dissipation & Viscosity

- The radiation bath is expected to depart from a perfect fluid due to particle production

(u_σ=fluid flow velocity)

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p\, g^{\mu\nu} + \pi_b (u^\mu u^\nu + g^{\mu\nu}) + \pi^{\mu\nu}$$

Adiabatic pressure $p \sim T^4,$

Bulk vis. $\pi_b \simeq -3\zeta_b \theta$ $(\theta = \text{expansion rate})$

Shear vis. $\pi_{\mu\nu} \simeq -2\zeta_s \sigma_{\mu\nu}$ $(\sigma_{\mu\nu} = \text{shear})$

- Bulk vis. can affect both background & fluctuations dynamics $\pi_b < 0$

-More e-folds of inflation -Few percent effect on the primordial spectrum

Mimoso, Nunes, Pavón PRD73 '06; Del Campo, Herrera, Pavón PRD75 '07;
Del Campo et al. 1007.0103

- Shear vis. enters in the EOM for the fluctuations

Light fields: $\zeta_b(T) \ll \zeta_s(T) \propto T^3$ Jeon & Yaffe PRD53 ('96)

We have set $\pi_b = 0$

Gauge invariant Fluctuations at linear order

Field	$\delta\phi$	$\delta\phi^{GI} = \delta\phi - (\dot{\phi}/H)\varphi$
Energy denstiy	$\delta\rho_r$	$\delta\rho_r^{GI} = \delta\rho_r - (\dot{\rho}_r/H)\varphi$
Momentum	Ψ_r	$\Psi_r^{GI} = \Psi_r - (\rho_r + p_r/H)\varphi$
Dissipation	$\delta\Upsilon$	$\delta\Upsilon^{GI} = \delta\Upsilon - (\dot{\Upsilon}/H)\varphi$
Curvature Perturbation	φ	$R = -\dfrac{H}{\rho_T + p_T}\Psi_T$

<u>Comoving curvature Perturbation</u>

Primordial spectrum: $P_R = \dfrac{k^3}{2\pi^2}\langle |R_k|^2 \rangle_\xi$

(<...>: average over noise realizations)

Evolution equations at linear order

Field EOM:

$$\delta\ddot{\phi}_k^{GI} + (3H+\Upsilon)\delta\dot{\phi}_k^{GI} + \underline{\dot{\phi}\,\delta\Upsilon^{GI}} + \left(\frac{k^2}{a^2} + V_{\phi\phi}\right)\delta\phi_k^{GI} \simeq (2(\Upsilon+H)T)^{1/2}\hat{\xi}_k$$

Radiation:

$$\delta\dot{\rho}_r^{GI} + 4H\delta\rho_r^{GI} \simeq \frac{k^2}{a^2}\Psi_r^{GI} + \underline{\dot{\phi}^2\,\delta\Upsilon^{GI}} + 2\dot{\phi}\Upsilon\delta\dot{\phi}^{GI}$$

<u>Shear parameter</u>

$$\dot{\Psi}_r^{GI} + 3H\left(1 + \frac{k^2}{a^2H^2}\bar{\zeta}_s\right)\Psi_r^{GI} \simeq -\delta\rho_r^{GI}/3 - \dot{\phi}\Upsilon\delta\phi^{GI}$$

$$\boxed{\left(\bar{\zeta}_s = \frac{4}{9}\frac{\zeta_s H}{\rho_r + p_r}\right)}$$

$$\rightarrow \boxed{\delta\Upsilon^{GI} = \frac{c}{4}\frac{\delta\rho_r^{GI}}{\rho_r} + (1-c)\frac{\delta\phi^{GI}}{\phi}} \rightarrow$$

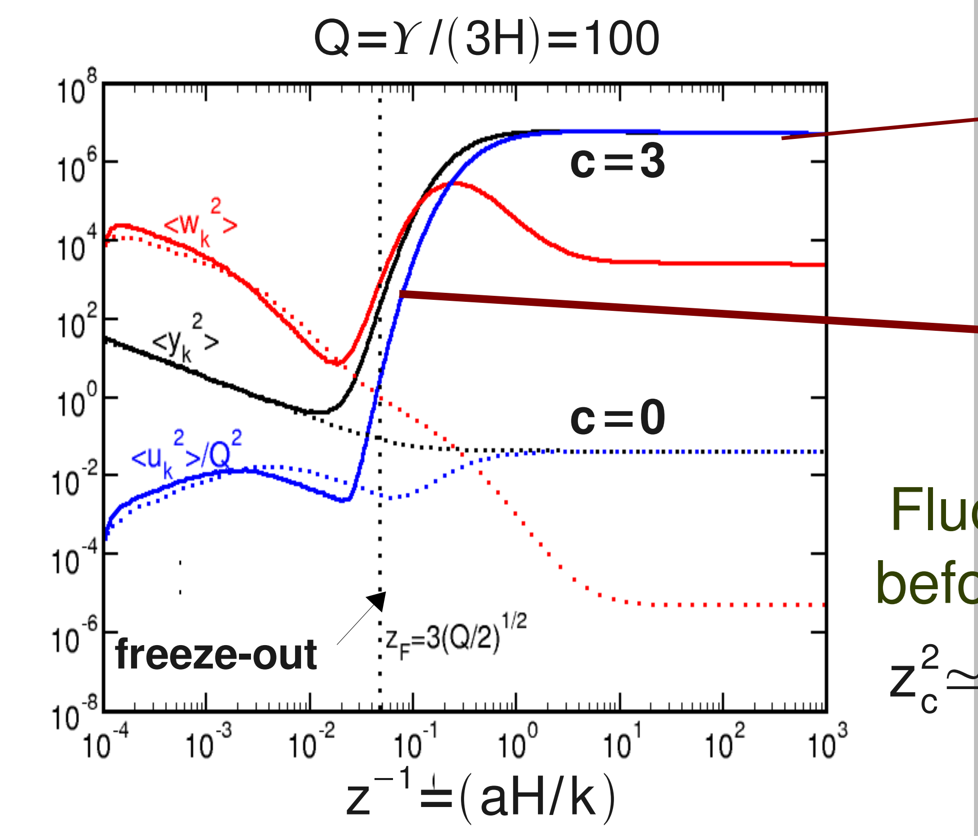
<u>Coupled system</u>

Primordial spectrum: no shear

$$y_k = \frac{k^{3/2}\,\delta\phi^{GI}}{(2(\Upsilon+H)T)^{1/2}}$$

$$w_k = \frac{k^{3/2}\,\delta\rho_r^{GI}}{(2(\Upsilon+H)T)^{1/2}\,\Upsilon\,\dot\phi}$$

$$u_k = \frac{k^{3/2}\,\Psi^{GI}}{(2(\Upsilon+H)T)^{1/2}\,\dot\phi}$$



$\Psi_r \simeq Q\,\Psi_\phi$

Growing mode !

Fluctuations coupled before freeze-out:

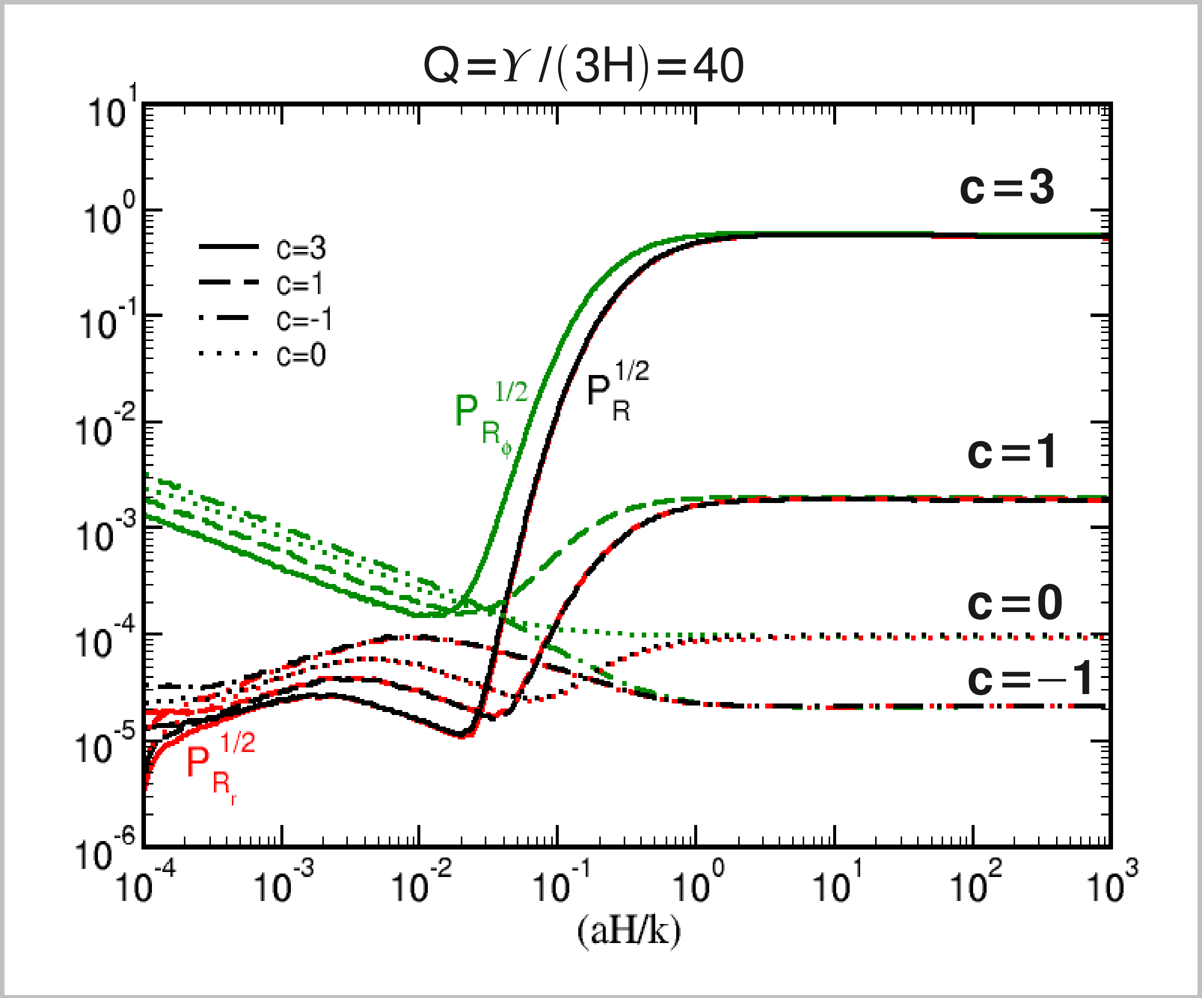
$z_c^2 \simeq 18\,Q\,c > z_F^2 = 9Q/4$

c=0: $\langle y_k^2\rangle_0 = \frac{\sqrt{3\pi}}{4}\frac{\sqrt{1+Q}}{1+3Q} \propto Q^{-1/2}$

c>0: $\langle y_k^2\rangle_c \propto Q^{(5c-1)/2}$

Graham & Moss JCAP 07(2009); MBG, Berera & Ramos JCAP 07 (2011)

Primordial spectrum: no shear



$$P_R = \frac{h_\phi}{h_T} P_{R_\phi} + \frac{h_r}{h_T} P_{R_r} \simeq P_{R_r} \simeq P_{R_\phi}$$

$$h_\phi = \rho_\phi + p_\phi = \dot{\phi}^2 \ll h_r \simeq Q h_\phi$$

$$\boxed{P_R \sim P_R(c=0) \times (1+Q)^{\frac{5c}{2}}}$$

Q < 1 : no enhancement of the spectrum ($z_c > z_F$)

Q > 1 : too large amplitude ⇒ smaller inflationary scale

⇒ $r \ll 1$

spectral index:

$$n_s - 1 = \frac{d\ln P_R}{dN_e} = (n_s - 1)_{c=0} + \frac{5c}{2} d\ln Q/dN_e \Rightarrow n_s > 1 ? (dQ/dN_e > 0)$$

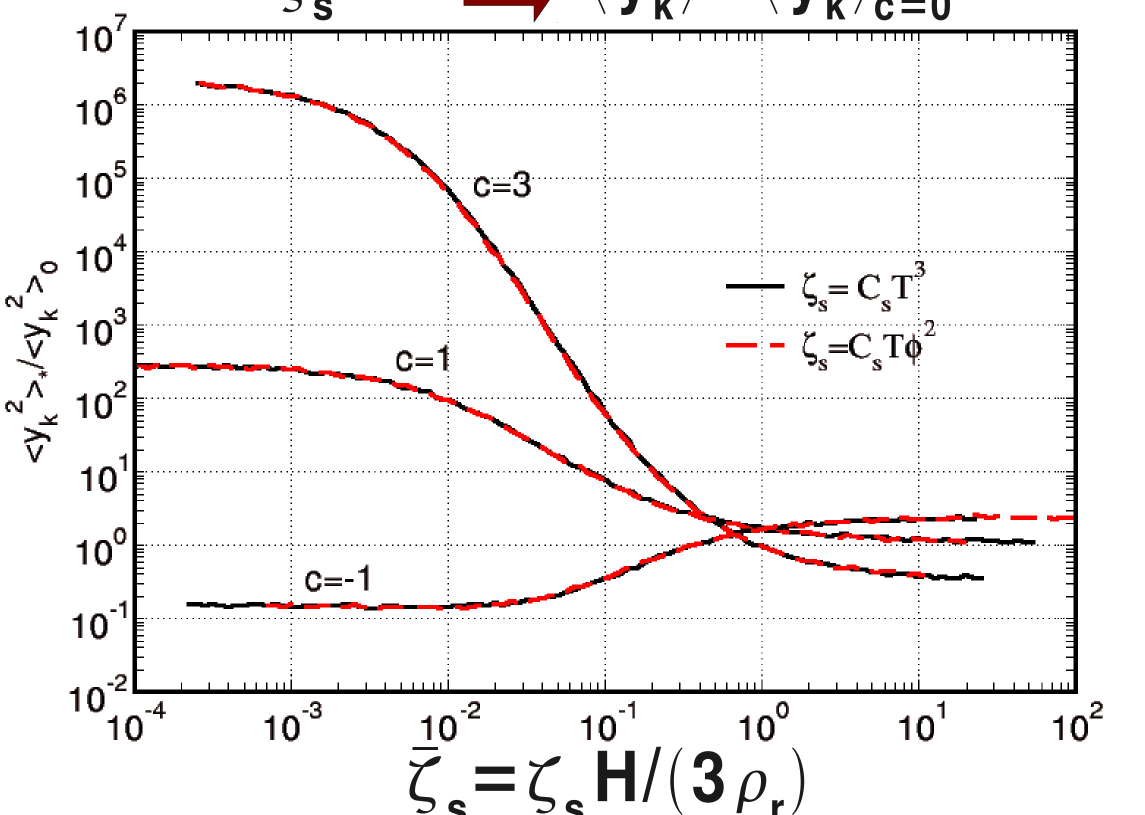
Blue tilted ?

Primordial spectrum: shear effects

Shear tends to damp the growth of the radiation fluctuations (before freeze-out), and therefore that of the field

$$\dot{\Psi}_r^{GI} + 3H\left(1+\frac{k^2}{a^2H^2}\bar{\zeta}_s\right)\Psi_r^{GI} \simeq -\delta\rho_r^{GI}/3 - \dot{\phi}\Upsilon\delta\phi^{GI}$$

$$\bar{\zeta}_s > 1 \Rightarrow \langle y_k^2\rangle \simeq \langle y_k^2\rangle_{c=0}$$



- Independent of the shear T dependence

-Susy models:

$$\Upsilon \simeq 0.1 g^4 N_\chi N_\sigma^2 T^3/\phi^2$$

$$\zeta_s \simeq 127 N_\sigma T^3/g^4$$

$$\bar{\zeta}_s > 1 \Rightarrow g^4 < 69H/T$$

Primordial spectrum: shear effects

$$P_R \sim P_R(c=0) F_Q[Q]^{F_s[\bar{\zeta}_s]}, \qquad F_Q[Q] \propto Q^{5c/2}$$

- Spectral index:

$$n_s - 1 = (n_s - 1)_{c=0} + \Delta n_s[\bar{\zeta}_s]$$

$$\Delta n_s[\bar{\zeta}_s] \simeq \frac{5c}{14Q} F_s[\bar{\zeta}](A_s \epsilon_\phi + B_s \eta_\phi) \qquad (Q>1)$$

- Chaotic (monomial) potential: $V \propto \phi^p \Rightarrow \Delta n_s[\bar{\zeta}_s] > 0$

- Hybrid (quadratic) potential: $V \simeq V_0\left(1 + \eta_\phi\left(\frac{\phi}{m_P}\right)^2\right) \Rightarrow \Delta n_s[\bar{\zeta}_s] < 0$

$$c=3,\ F_s[\bar{\zeta}_s] \simeq 1/3,\ Q>40 \Rightarrow n_s \simeq 1$$

- Hybrid (log) potential: $V \simeq V_0\left(1 + \gamma \ln\left(\frac{\phi}{m_P}\right)\right) \Rightarrow \Delta n_s[\bar{\zeta}_s] > 0$

Summary

Dissipative effects due to decaying fields can be relevant during inflation, and modify the inflationary predictions (warm inflation)

For a T dependent dissipative coefficient, the field and radiation perturbation EOM form a coupled system

The radiation backreacts on the field perturbations, and the field fluctuations are amplified before freeze-out. The larger the dissipation, the larger the growth

$$P_R \sim P_R(c=0) \times (1+Q)^{(5c-1)/2}$$

This can be counterbalanced by the dissipative effects within the radiation fluid (shear viscous pressure)

$P_R \sim P_R(c=0)$ when $\bar{\zeta}_s \geq 1$ (freeze-out value, model independent)