Dijet Production at Large Rapidity Separation in $\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory

M. Angioni,[1] G. Chachamis,[2] J. D. Madrigal,[3] and A. Sabio Vera[3,4]

[1]*Dipartimento di Fisica, Università di Firenze, Italy*
[2]*Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland*
[3]*Instituto de Física Teórica UAM/CSIC & UAM, E-28049 Madrid, Spain*
[4]*KITP, University of California, Santa Barbara, Santa Barbara, California 93106, USA*

(Received 4 July 2011; published 1 November 2011)

Ratios of azimuthal angle correlations between two jets produced at large rapidity separation are studied in the $\mathcal{N} = 4$ maximally supersymmetric Yang-Mills (MSYM) theory. It is shown that these observables, which directly prove the $SL(2, C)$ symmetry present in gauge theories in the Regge limit, exhibit an excellent perturbative convergence. They are compared to those calculated in QCD for different renormalization schemes concluding that the momentum-substraction scheme with the Brodsky-Lepage-Mackenzie scale-fixing procedure captures the bulk of the MSYM results.

DOI: 10.1103/PhysRevLett.107.191601 PACS numbers: 11.10.−z, 11.55.Jy, 25.75.−q

Introduction.—$\mathcal{N} = 4$ maximally supersymmetric Yang-Mills (MSYM) theory is considered "the harmonic oscillator of the 21st century" (In a similar fashion, black holes can be thought of as the "hydrogen atom" of quantum gravity [1] since it seems to be, in the planar limit, a solvable theory in 4 dimensions. The simplicity of its scattering amplitudes [2] and the AdS/CFT duality [3] support this reasoning. In this work, it is considered as a theoretical laboratory to establish links with QCD. It is known that MSYM contributions to QCD amplitudes provide the "highest degree of trascendentality" terms. This is a key ingredient to calculate all-orders anomalous dimensions using integrability [4,5]. MSYM observables have been proposed in [6]. Energy flow in terms of correlation functions of the energy-momentum tensor is an example [7]. In this work the study of ratios of azimuthal angle correlations in inclusive dijet production when the two tagged jets are well separated in rapidity, first introduced in [8], is carried out. Note that, in order to further explore the AdS/CFT correspondence, it is important to identify scaling laws in the weak coupling limit of MSYM theory and try to find their gravitational counterparts. A key ingredient of the duality is conformal invariance and the observables chosen in this Letter capture the bulk of this symmetry present in the plane transverse to the colliding particles at high energies.

In the section of this Letter on azimuthal angle correlation ratios in MSYM theory and QCD, dijet production in the Balitsky-Fadin-Kuraev-Lipatov (BFKL) framework [9] at next-to-leading order [10] (NLO) in QCD (already known [8,11]) and MSYM theory (a new calculation) in the minimal subtraction renormalization scheme ($\overline{\text{MS}}$) is introduced. In the section on physical renormalization schemes and BLM procedure, the momentum-subtraction (MOM) scheme [12] with Brodsky-Lepage-Mackenzie (BLM) scale fixing [13] is applied to QCD to challenge the statement that it captures the conformal contributions to all orders (see [14] for a recent discussion). If this is correct it should give a similar result to that in MSYM theory. In the section on comparing MSYM theory with QCD results, it is shown that this is indeed the case, in particular, for the ratios of azimuthal angle correlations.

Azimuthal angle correlation ratios in MSYM theory and QCD.—The configuration under study is that of Mueller-Navelet jets [8,15], where two forward jets with similar transverse momenta $\boldsymbol{p}_{1,2}^2$ are produced with a relative rapidity separation Y, and a relative azimuthal angle ϑ. If $x_{1,2}$ are the fractions of longitudinal momentum from the parent hadrons carried by the partons generating the jets then $Y \simeq \ln(x_1 x_2 s/\sqrt{\boldsymbol{p}_1^2 \boldsymbol{p}_2^2})$. For large rapidity separation, $\mathcal{O}((\lambda Y)^n)$ terms must be resummed to all orders, with λ being the 't Hooft coupling in MSYM theory and $\bar{\alpha}_s \equiv \alpha_s N_c/\pi$ in QCD. This Regge limit, where $s \gg \sqrt{\boldsymbol{p}_1^2 \boldsymbol{p}_2^2}$, is treated using the BFKL formalism.

QCD azimuthal angle correlation ratios are insensitive to parton distribution functions at large Y [8]; hence, the focus will be on partonic cross sections. These are written as a convolution of the partonic cross section with, for simplicity, LO jet vertices $\Phi_{\text{jet}_i}(\boldsymbol{q}, \boldsymbol{p}_i) \simeq \Phi^{(0)}_{\text{jet}_i}(\boldsymbol{q}, \boldsymbol{p}_i) = \Theta(\boldsymbol{q}^2 - \boldsymbol{p}_i^2)$:

$$\hat{\sigma}(\boldsymbol{p}_1, \boldsymbol{p}_2, Y) = \int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \Phi_{\text{jet}_1}(\boldsymbol{q}_1, \boldsymbol{p}_1) \times \frac{d\hat{\sigma}}{d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2} \Phi_{\text{jet}_2}(\boldsymbol{q}_2, \boldsymbol{p}_2), \quad (1)$$

$\boldsymbol{p}_i^2$ is the resolution scale for the transverse momentum of the jet. Considering the Green function, f, at NLO, the gluon-gluon differential partonic cross section is $d\hat{\sigma}/(d^2\boldsymbol{q}_1 d^2\boldsymbol{q}_2) = (\pi^2 \bar{\alpha}_s^2/2) f(\boldsymbol{q}_1, \boldsymbol{q}_2, Y)/(\boldsymbol{q}_1^2 \boldsymbol{q}_2^2)$. Using $f(\boldsymbol{q}_1, \boldsymbol{q}_2, Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} \tilde{f}(\boldsymbol{q}_1, \boldsymbol{q}_2, \omega)$ the BFKL integral equation reads $\omega \tilde{f}(\boldsymbol{q}_1, \boldsymbol{q}_2, \omega) = \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2) + \int d^2\boldsymbol{\kappa} \mathcal{K}(\boldsymbol{q}_1, \boldsymbol{\kappa}) \tilde{f}(\boldsymbol{\kappa}, \boldsymbol{q}_2, \omega)$. At LO accuracy:

$$\tilde{f}(\boldsymbol{q}_1, \boldsymbol{q}_2, \omega) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi^2} \frac{(\boldsymbol{q}_1^2/\boldsymbol{q}_2^2)^{i\nu}}{\sqrt{\boldsymbol{q}_1^2 \boldsymbol{q}_2^2}} \frac{e^{in(\vartheta_1 - \vartheta_2)}}{\omega - \bar{\alpha}_s \chi_0(|n|, \nu)}, \quad (2)$$

where $\chi_0(n, \nu) = 2\psi(1) - \psi(\frac{1+n}{2} + i\nu) - \psi(\frac{1+n}{2} - i\nu)$.

The transverse momentum operator $\hat{q}|\boldsymbol{q}_i\rangle = \boldsymbol{q}_i|\boldsymbol{q}_i\rangle$, $\langle \boldsymbol{q}_1|\hat{\mathbf{1}}|\boldsymbol{q}_2\rangle = \delta^{(2)}(\boldsymbol{q}_1 - \boldsymbol{q}_2)$, introduces the basis of eigenfunctions $|n, \nu\rangle$ in Eq. (2), satisfying $\langle n', \nu'|n, \nu\rangle = \delta(\nu - \nu')\delta_{n,n'}$, as $\langle \boldsymbol{q}|n, \nu\rangle = \frac{1}{\pi\sqrt{2}}(\boldsymbol{q}^2)^{i\nu - 1/2} e^{in\vartheta}$. At NLO in QCD [4,8]:

$$\langle n, \nu|\hat{\mathcal{K}}|\nu', n'\rangle = \bar{\alpha}_{s,\overline{\rm MS}} \Bigg\{ \chi_0\left(|n'|, \frac{1}{2} + i\nu'\right)\left[1 - \frac{\bar{\alpha}_{s,\overline{\rm MS}}\beta_0}{8N_c}\left(i\frac{\partial}{\partial\nu} - i\frac{\partial}{\partial\nu'} - 2\ln\mu^2\right)\right] + \bar{\alpha}_{s,\overline{\rm MS}}\chi_1\left(|n'|, \frac{1}{2} + i\nu'\right) + i\frac{\bar{\alpha}_{s,\overline{\rm MS}}\beta_0}{8N_c}\left[\frac{\partial}{\partial\nu'}\chi_0\left(|n'|, \frac{1}{2} + i\nu'\right)\right]\Bigg\}\delta_{n,n'}\delta(\nu - \nu'), \quad (3)$$

where χ_1 [with $\nu = i(\frac{1}{2} - \gamma)$] is of the form ($\Omega$ can be found in [4])

$$\chi_1(n, \gamma) = \left(4 - \pi^2 + \frac{5\beta_0}{N_c}\right)\frac{\chi_0(n, \gamma)}{12} + \frac{3}{2}\zeta(3) - \frac{\beta_0}{8N_c}\chi_0^2(n, \gamma) + \Omega(n, \gamma) - \frac{\pi^2\cos(\pi\gamma)}{4\sin^2(\pi\gamma)(1 - 2\gamma)} \times \left[\left(3 + \left(1 + \frac{N_f}{N_c^3}\right)\frac{2 + 3\gamma(1 - \gamma)}{(3 - 2\gamma)(1 + 2\gamma)}\right)\delta_{n,0} - \left(1 + \frac{N_f}{N_c^3}\right)\frac{\gamma(1 - \gamma)}{2(3 - 2\gamma)(1 + 2\gamma)}\delta_{n,2}\right]. \quad (4)$$

In MSYM theory the absence of running of the coupling, now named λ, leads to [4]

$$\langle n, \nu|\hat{\mathcal{K}}_{\rm MSYM}|\nu', n'\rangle = \lambda\left[\chi_0\left(|n'|, \frac{1}{2} + i\nu'\right) + \lambda\chi_1^{\rm MSYM}\left(|n'|, \frac{1}{2} + i\nu'\right)\right] \times \delta_{n,n'}\delta(\nu - \nu'), \quad (5)$$

$$\chi_1^{\rm MSYM}(n, \gamma) = \frac{[1 - \zeta(2)]}{12}\chi_0(|n|, \gamma) + \frac{3}{2}\zeta(3) + \Omega(|n|, \gamma). \quad (6)$$

(MSYM calculations are presented in the $\overline{\rm MS}$ scheme, other renormalization approaches give very similar results.)

The differential cross section in azimuthal angle $\phi = \vartheta_1 - \vartheta_2 - \pi$, with ϑ_i for each jet, is [8]

$$\frac{d\hat{\sigma}(\mathbf{p}_{1,2}^2, Y)}{d\phi} = \frac{\pi^2\bar{\alpha}_s^2}{4\sqrt{\mathbf{p}_1^2\mathbf{p}_2^2}}\sum_{n=-\infty}^{\infty} e^{in\phi}\mathcal{C}_n(Y), \quad (7)$$

$$\mathcal{C}_n(Y) = \int_{-\infty}^{\infty}\frac{d\nu}{2\pi}\frac{e^{\bar{\alpha}_s(\mathbf{p}^2)Y(\chi_0(|n|,\nu) + \bar{\alpha}_s(\mathbf{p}^2)(\chi_1(|n|,\nu) - ((\beta_0)/(8N_c)(\chi_0(|n|,\nu))/(1/4+\nu^2))))}}{\frac{1}{4} + \nu^2}, \quad (8)$$

where $\mathbf{p}_1^2 \simeq \mathbf{p}_2^2 \simeq \mathbf{p}^2$ has been taken. In the MSYM case, with $\beta_0 = 0$, these are

$$\mathcal{C}_n(Y) = \int_{-\infty}^{\infty}\frac{d\nu}{2\pi}\frac{e^{\lambda Y(\chi_0(|n|,\nu) + \lambda\chi_1^{\rm MSYM}(|n|,\nu))}}{\frac{1}{4} + \nu^2}. \quad (9)$$

The total cross section is $\hat{\sigma}(\mathbf{p}_{1,2}^2, Y) = \frac{\pi^3\bar{\alpha}_s^2}{2\sqrt{\mathbf{p}_1^2\mathbf{p}_2^2}}\mathcal{C}_0(Y)$. To study higher conformal spins n, which is one of the main goals of this work, ratios of azimuthal angle correlations are introduced:

$$\langle\cos(m\phi)\rangle = \frac{\mathcal{C}_m(Y)}{\mathcal{C}_0(Y)}, \quad \mathcal{R}_{m,n}(Y) \equiv \frac{\langle\cos(m\phi)\rangle}{\langle\cos(n\phi)\rangle} = \frac{\mathcal{C}_m(Y)}{\mathcal{C}_n(Y)}. \quad (10)$$

(The discrete parameter n, in elastic scattering, is the conformal spin of a SL(2,C) symmetry [16]. When studying observables sensitive to the azimuthal angle, this Möbius invariance is under scrutiny.) MSYM coefficients of Eq. (9) will be denoted by $\mathcal{C}_n^{\rm MSYM}$. For QCD, when computed as in Eq. (8), $\mathcal{C}_n^{\rm NLL}$ will be used, keeping $\mathcal{C}_n^{\rm LL}$ for LO. The scale invariant coefficient $\mathcal{C}_n^{\rm SI}$ are given by putting $\beta_0 = 0$ in Eq. (8). The 't Hooft limit of $N_c \to \infty$ with $\alpha_s N_c$ fixed, will also be investigated.

Physical renormalization schemes and BLM procedure.—The NLO BFKL kernel [10] has collinear instabilities [17]. Resummation of the leading collinear contributions improves its convergence [18]. Short-range in rapidity correlations also stabilize the expansion [19]. Another approach is that in [20] which, instead of using a $\overline{\rm MS}$ scheme with arbitrary renormalization scale, uses a physical scheme like MOM [12] with optimal scale set by the BLM procedure [13]. In this way the NLO corrections have a milder behavior and the Pomeron intercept has a weak dependence on the hard scale, leading to scale invariance. This is a good motivation to compare this approach with MSYM theory, which enjoys four-dimensional conformal invariance.

The transition from $\overline{\rm MS}$ to MOM is equivalent, at LO, to [12] $\alpha_{\rm MOM} = \alpha_{\overline{\rm MS}}(1 + T_{\rm MOM}\alpha_{\overline{\rm MS}}/\pi)$, where $T_{\rm MOM}$ has the gauge parameter ξ: $T_{\rm MOM} = T_{\rm MOM}^{\rm conf} + T_{\rm MOM}^{\beta}$, $T_{\rm MOM}^{\rm conf} = (N_c/8)(17I/2 + \xi(3/2)(I-1) + \xi^2(1 - I/3) - \xi^3/6)$, $T_{\rm MOM}^{\beta} = -(\beta_0/2)(1 + 2I/3)$, with $I \simeq 2.3439$. At NLO this is equivalent to $\mu \to \bar{\mu} = \mu \exp(-T_{\rm MOM}/2\beta_0)$. A suitable choice of renormalization scheme and scale should render higher order coefficients small [21]. A physically motivated scheme is BLM, where the coupling redefinition absorbs charge renormalization corrections in such a way that the coefficients of the perturbative series are identical to those of the conformally invariant theory with $\beta = 0$. The BLM scheme was applied to the BFKL description of the $\gamma^*\gamma^*$ cross section [20]. To enhance the effect of the BLM scheme in gluon dominated processes, it is appropriate to use a physical scheme for non-Abelian interactions, such as MOM, based on the 3-gluon vertex [12] or the Υ scheme based on $\Upsilon \to ggg$ decay. This procedure is not free from complications since it has an unnaturally high scale at $\nu = 0$ [22].

Here the BLM setting for dijet production is explored for the Pomeron intercept and, very importantly, also for azimuthal correlations. To generalize it to $n \neq 0$ conformal spins one writes

$$\omega_{\overline{\rm MS}}(\boldsymbol{q}^2, n, \nu) = \chi_0(n, \nu)\frac{\alpha_{\overline{\rm MS}}(\boldsymbol{q}^2)N_c}{\pi} \times \left(1 + r_{\overline{\rm MS}}(n, \nu)\frac{\alpha_{\overline{\rm MS}}(\boldsymbol{q}^2)}{\pi}\right). \quad (11)$$

The NLO coefficient $r_{\overline{\rm MS}}$ is decomposed into β-dependent and conformal (β-independent) parts:

$$r_{\overline{\rm MS}}(n, \nu) = r_{\overline{\rm MS}}^{\beta}(n, \nu) + r_{\overline{\rm MS}}^{\rm conf}(n, \nu),$$
$$r_{\overline{\rm MS}}^{\beta}(n, \nu) = -\frac{\beta_0}{4}\left(\frac{\chi_0(n, \nu)}{2} - \frac{5}{3}\right), \quad (12)$$

$$r_{\overline{\rm MS}}^{\rm conf}(n, \nu) = -\frac{N_c}{4\chi_0(n, \nu)}\left\{\frac{(\pi^2 - 4)}{3}\chi_0(n, \nu) - 6\zeta(3) + \frac{\pi^2}{2\nu}\,{\rm sech}(\pi\nu)\tanh(\pi\nu) - \left[\psi''\left(\frac{n+1}{2} + i\nu\right) + \psi''\left(\frac{n+1}{2} - i\nu\right) - 2\phi\left(n, \frac{1}{2} + i\nu\right) - 2\phi\left(n, \frac{1}{2} - i\nu\right)\right] \times \left[\left(3 + \left(1 + \frac{N_f}{N_c^3}\right)\left(\frac{3}{4} - \frac{1}{16(1+\nu^2)}\right)\right)\delta_{n,0} - \left(1 + \frac{N_f}{N_c^3}\right)\left(\frac{1}{8} - \frac{3}{32(1+\nu^2)}\right)\delta_{n,2}\right]\right\}. \quad (13)$$

The NLO BFKL intercept in the MOM scheme, at the optimal BLM scale can be written as

$$\omega^{\rm MOM}(\boldsymbol{q}_{\rm BLM}^{2\rm MOM}, n, \nu) = \frac{\alpha_{\overline{\rm MS}}(\boldsymbol{q}_{\rm BLM}^{2\rm MOM})N_c}{\pi}\chi_0(n, \nu) \times \left(1 + r^{\rm MOM}(n, \nu)\frac{\alpha_{\overline{\rm MS}}(\boldsymbol{q}_{\rm BLM}^{2\rm MOM})}{\pi}\right), \quad (14)$$

where $r_{\rm MOM}(n, \nu) = r_{\overline{\rm MS}}(n, \nu) + T_{\rm MOM}$. If $r_{\rm MOM}^{\beta}(n, \nu) = r_{\overline{\rm MS}}^{\beta}(n, \nu) + T_{\rm MOM}$ then

$$\boldsymbol{q}_{\rm BLM}^{2\rm MOM}(n, \nu) = \boldsymbol{q}^2\exp\left(-\frac{4r_{\rm MOM}^{\beta}(n, \nu)}{\beta_0}\right) = \boldsymbol{q}^2\exp\left(\frac{1}{2}\chi_0(n, \nu) + \frac{1 + 4I}{3}\right). \quad (15)$$

Comparing MSYM theory with QCD results.—In Fig. 1 (left) the intercept of Eq. (14) for $n = 0$ in the MOM scheme with BLM is confronted with the MSYM intercept and the results at LO and NLO with no BLM scale fixing. The MSYM coupling is between $\lambda = \bar{\alpha}_s(\mathbf{q}^2/4)$ ($\rm MSYM_-$) and $\bar{\alpha}_s(4\mathbf{q}^2)$ ($\rm MSYM_+$) (yellow band). This plot agrees with the $\gamma^*\gamma^*$ total cross section in [20] since the scale invariance, with respect to the photon virtualities in that case and the jet transverse momentum now, of the intercept in MOM is manifest. This intercept for the conformal invariant MSYM theory at NLO is very close to the LO one, indicating a better convergence than QCD. Results for QCD in the 't Hooft limit are also shown. It is important to note that the MOM-BLM scheme is the closest to MSYM theory of all renormalization schemes in QCD. This is natural since the BLM scheme collects the conformal contributions to the observable.

The $n = 0$ coefficient drives the cross section. The rise of C_0 with Y is shown in Fig. 1 (right). There is a faster growth of the MSYM cross section showing that the NLO real emission in MSYM theory dominates over the virtual contributions in a much stronger fashion than in QCD, for any renormalization scheme. This also indicates that the effect of introducing the extra fields in the supersymmetric multiplet increases the minijet multiplicity in the final state. In future works it will be worth looking into these details of the final state using event generator Monte Carlo techniques [23]. For small Y the QCD result in the $\overline{\rm MS}$ scheme is lower than in the MOM-BLM scheme, with the latter being closer to MSYM theory. This is consistent with a renormalization scheme which resums conformal

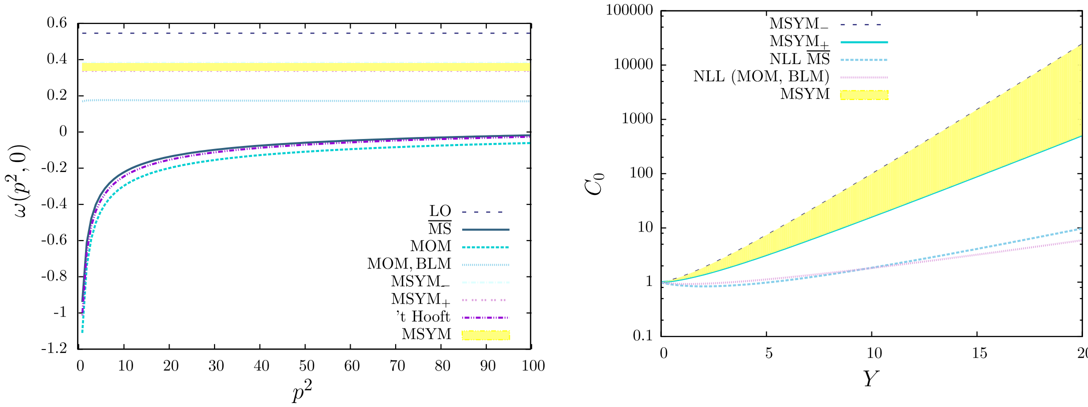

FIG. 1 (color online). Left: Intercept vs jet resolution $\mathbf{p}^2$ for different renormalization schemes in QCD and MSYM theory. Right: Growth with dijet rapidity separation of the cross section in MSYM theory and QCD.

contributions. However, from $Y \simeq 10$ the calculation in $\overline{\text{MS}}$ is now closer to the MSYM theory. This hints at the instability of the $n = 0$ component, which is very sensitive to collinear radiation, not fully included in the BFKL kernel. It is natural to predict a similar crossing of behavior at some Y for any quantity sensitive to $n = 0$. This is found when the average of $\cos(n\phi)$ is calculated as in Eq. (10) (left). Examples with a crossover of lines are plotted in Fig. 2 for $n = 1, 2$. It is also interesting to note that dijets are less correlated in the azimuthal angle in MSYM theory than in QCD, which corresponds to a higher multiplicity of parton radiation in the supersymmetric case. The main conclusion of this analysis is that to define observables only sensitive to conformal dynamics it is needed to remove the $n = 0$ contribution. One way of doing this is to use the ratios of azimuthal angle averages $\mathcal{R}_{m,n}(Y)$ in Eq. (10) (right). $\mathcal{R}_{2,1}$ and $\mathcal{R}_{3,2}$ are calculated in Fig. 3. It is important to note that all MSYM ratios are very close to those calculated in QCD, indicating that these observables capture the bulk of the conformal dynamics in QCD. Moreover, among all renormalization schemes, it is the MOM-BLM scheme which gives the closest to all MSYM ratios, independently of the separation in rapidity between the two tagged jets. Having removed the $n = 0$ dependence, the crossover of lines does not take place anymore.

Conclusions.—Dijet production has been studied when the tagged jets are largely separated in rapidity. This has been done in MSYM theory and QCD, investigating what renormalization procedures in QCD best reproduce the conformal dynamics of the MSYM theory. Ratios of azimuthal angle correlations, which are known to have an excellent perturbative convergence, capture the bulk of the conformal contributions. In QCD these ratios are insensitive to parton distribution functions and are calculated at the parton level, allowing for a direct comparison with MSYM theory. For them the results calculated in QCD with the MOM-BLM scheme are very similar to those obtained in MSYM theory. The two tagged jets are less correlated in the azimuthal angle in MSYM theory than in QCD, indicating that in MSYM theory there is a larger final state multiplicity. The fact that QCD in the BLM scheme is so close to MSYM theory for well chosen quantities brings hope that the AdS/CFT correspondence could well help when describing collider phenomenology. For future work it will be useful to study other multijet configurations and different observables at a more

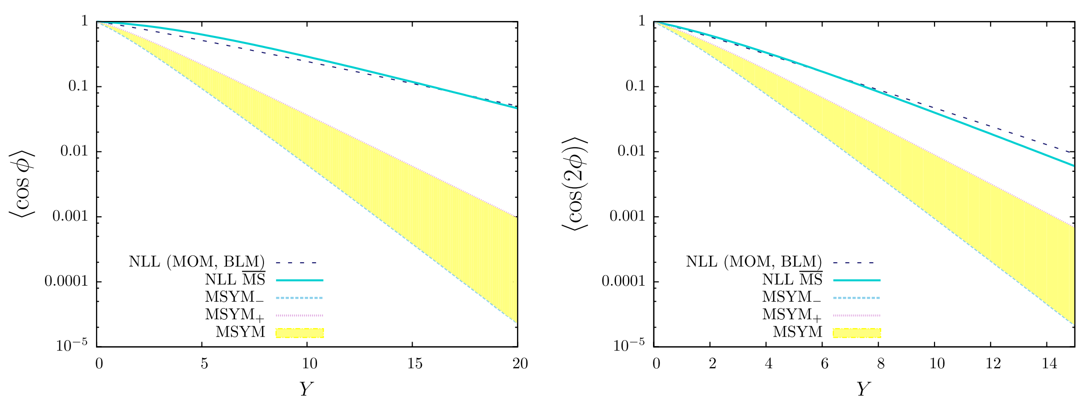

FIG. 2 (color online). Evolution of the average of $\cos\phi$ (left) and $\cos(2\phi)$ (right) with jet rapidity separation in MSYM theory and QCD for different renormalization schemes.

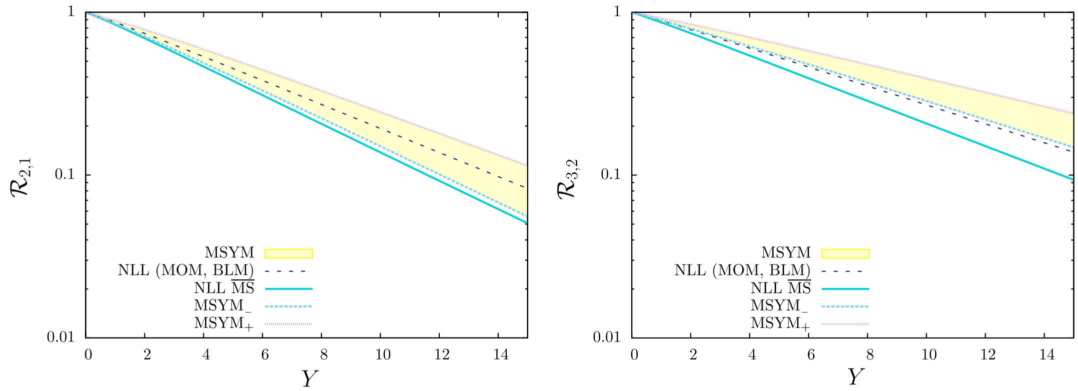

FIG. 3 (color online). Evolution of $\mathcal{R}_{2,1} = \frac{\langle\cos(2\phi)\rangle}{\langle\cos\phi\rangle}$ (left) and $\mathcal{R}_{3,2} = \frac{\langle\cos(3\phi)\rangle}{\langle\cos(2\phi)\rangle}$ (right) with jet rapidity separation in MSYM theory and QCD for different renormalization schemes.

exclusive level. It will be interesting to investigate if the kinematical window of the present work, where the center-of-mass energy dominates making the dependence on the other scales subleading and where the BLM scheme reproduces the MSYM prediction, can be broadened beyond multi-Regge kinematics.

Research supported by E. Comission [LHCPhenoNet (PITN-GA-2010-264564)] & C. Madrid (HEPHACOS ESP-1473).

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