$O(3)$ Nonlinear Sigma Model with θ-Term on the Lattice

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III Jornadas CPAN
Barcelona, 3rd November 2011

Work in collaboration with V. Azcoiti, G. Cortese, G. Di Carlo, E. Follana, A. Vaquero

Outline

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1. Introduction: Theories with θ-Term
2. Reconstructing the θ Dependence: the Method of Scaling Transformations
3. $O(3)$ Nonlinear Sigma-Model
4. Outlook

Motivation: QCD with θ-Term and the Sign Problem

θ term in (Euclidean) QCD

$$\mathcal{L}_{\rm gauge} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - i\theta q(x)\,, \qquad q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$

Topological charge $Q = \int d^4x\, q(x)$, $Q \in \mathbb{N}$ (for finite-action configs)

θ-term breaks P and T (also with quarks with $m_{\rm quark} \neq 0$), but strong interactions are P, T-symmetric, i.e., $\theta = 0$: why? Strong CP problem

Nontrivial θ-dependence provides a solution to the $U(1)_A$ problem

QCD at $\theta = \pi$: CP breaking, 1st-order ph. tr. Dashen phenomenon

Nonperturbative problems (θ-term does not contribute to perturbation theory), require nonperturbative treatment: Lattice Gauge Theory

Simulation of a complex action: sign problem

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Analytic Continuation

The sign problem can be in principle overcome by means of analytic continuation:

1. perform the analytic continuation $\theta \to -ih$
2. simulate numerically with real h (good Boltzmann factor)
3. obtain the analytic dependence of the relevant quantity on h
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Determination of the analytic dependence on h (point 3.) affected by errors which can grow exponentially after the process of analytic continuation (point 4.)

It would be nice to perform the analytic continuation in such a way as to minimise this possibility [Azcoiti *et al.* (2003)]

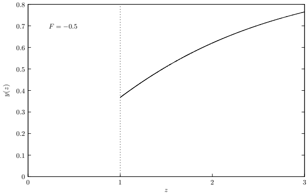
Outline

Antiferromagnetic 1D Ising model ($F < 0$) [Azcoiti *et al.* (2003),(2011)]

$$H/KT = -F\sum_{i=1}^{N} s_i s_{i+1} - \frac{h}{2}\sum_{i=1}^{N} s_i$$

Imaginary $h = i\theta$, even N $\sim$ topological term

Magnetisation expressed through $y(z) = \frac{\langle Q\rangle_h}{\tanh\frac{h}{2}}$, $Q = \frac{1}{2}\sum_{i=1}^{N} s_i$, $z = \cosh\frac{h}{2}$



$$y(z) = \frac{e^{-\frac{\mu}{2}}z}{\sqrt{e^{-\mu}z^2+1}}$$

$$\mu = \log(e^{-4F} - 1)$$

Real to imaginary magnetic field $h \to i\theta \Rightarrow \cosh\frac{h}{2} \to \cos\frac{\theta}{2} \Rightarrow z' \geq 1 \to z \leq 1$ equivalent to a rescaling of z

Ising 1D: $y(z)|_\mu = Y(e^{-\frac{\mu}{2}}z)$, rescaling of z equivalent to a change in the coupling

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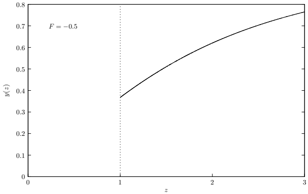

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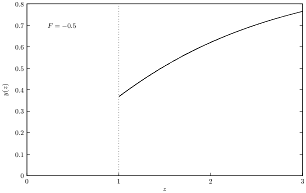

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Reconstructing the θ Dependence: 1D Ising Model

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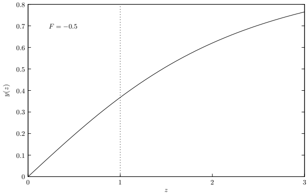

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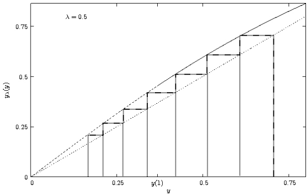
Reconstructing the θ Dependence: Scaling Transformation

Analytic continuation substituted with a more controllable extrapolation

Define $y_\lambda(z) = y\left(e^{\frac{\lambda}{2}} z\right)$, $\lambda > 0$ (recall $y(z) = \frac{\langle Q \rangle_h}{\tanh \frac{h}{2}}$, $z = \cosh \frac{h}{2}$)

- Measure $y(z), y_\lambda(z)$ at $z > 1$ through numerical simulations
- Extrapolate $y_\lambda(y)$ towards $y, y_\lambda = 0$
  ($y(0) = y_\lambda(0) = 0$ unless the order parameter diverges at $z = 0$, i.e. $\theta = \pi$)
- Reconstruct $y(z)$ at $z < 1$



Given $y_i = y(z_i)$, define $y_{i+1} = y(z_{i+1})$ through $y_\lambda(y_{i+1}) = y_i$. By definition $y_\lambda(y_{i+1}) = y(e^{\frac{\lambda}{2}} z_{i+1})$
$\Rightarrow z_{i+1} = e^{-\frac{\lambda}{2}} z_i$ (requires monotonicity)

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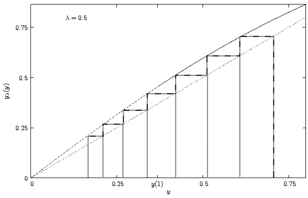
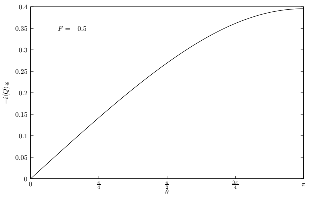


$\Rightarrow$



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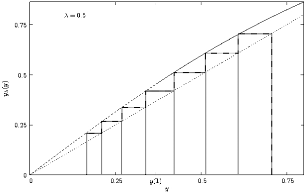

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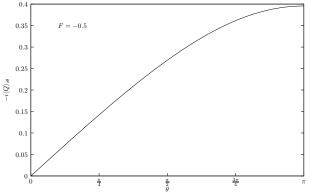


Successfully applied to CP^N models in 2D [Azcoiti *et al.* (2003),(2004),(2007)] and to Ising model in 2D and 3D [Azcoiti *et al.* (2011)]

Outline

$O(3)$ Nonlinear Sigma-Model

2D $O(3)$ Nonlinear Sigma-Model with θ term, $S_\theta = S - i\theta Q$

$$S = \frac{1}{2g^2} \int d^2x \, \partial_\mu \vec{s}(x) \cdot \partial_\mu \vec{s}(x)$$

Topological charge Q

$$Q = \frac{1}{8\pi} \int d^2x \, \epsilon_{\mu\nu} \, \vec{s} \cdot (\partial_\mu \vec{s} \wedge \partial_\nu \vec{s})$$

Relation with 1D quantum spin chains: [Haldane (1983), Affleck (1991)]

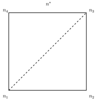
- Large spin S, long-range physics $\sim O(3)$ NLσMθ, $g = \frac{1}{2S}$, $\theta = 2\pi S$
- Haldane conjecture:
  integer spin $\sim$ mass gap ($\theta = 0$), half-integer spin $\sim$ gapless ($\theta = \pi$)
- Second-order phase transition expected at $\theta = \pi$, restoration of parity

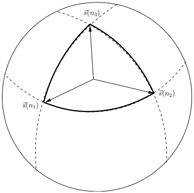
Studied with different techniques [Bietenholz *et al.* (1995)][Allés and Papa (2008)] and using CP^1 [Azcoiti *et al.* (2007)] which should be equivalent to $O(3)$

Discretise the theory on the lattice

$$S = \beta \sum_{n} \sum_{\mu=1,2} \left(1 - \vec{s}(n) \cdot \vec{s}(n+\hat{\mu})\right) , \qquad \beta = \frac{1}{g^2}$$

Geometric topological charge [Berg and Lüscher (1981)]





$$Q = \sum_{n^*} q(n^*)$$

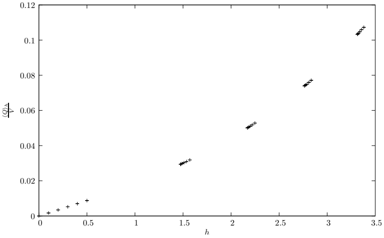
$$q(n^*) = \frac{1}{4\pi} \left\{ A(n_1, n_2, n_3) + A(n_1, n_3, n_4) \right\}$$

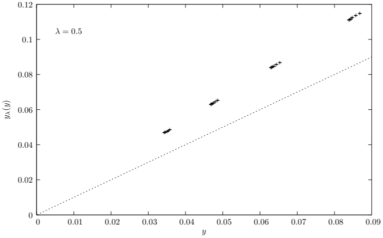
$A(n_1, n_2, n_3)$: signed area of a spherical triangle

$Q \in \mathbb{N}$ if periodic boundary conditions are imposed

Numerical simulations with action $S_{-ih} = S - hQ$ at imaginary $\theta = -ih$

Preliminary Results I





$\beta = 1.0,\ V = 100 \times 100$

20000 measurements (low statistics)

$$\langle Q\rangle_h = \frac{\int \mathcal{D}\vec{s}\, e^{-S[\vec{s}]+hQ[\vec{s}]}\ Q[\vec{s}]}{\int \mathcal{D}\vec{s}\, e^{-S[\vec{s}]+hQ[\vec{s}]}}$$

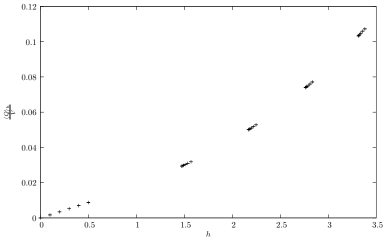
$$y(z) = \frac{\langle Q\rangle_h/V}{\tanh\frac{h}{2}},\ z = \cosh\frac{h}{2}$$

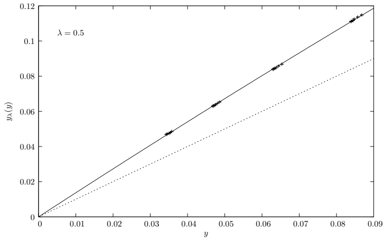
$$y_\lambda(z) = y\left(e^{\frac{\lambda}{2}} z\right) \text{ with } \lambda = 0.5$$

Quadratic fit to extrapolate $y_\lambda(y)$ towards $y = 0$, and so reconstruct $y(z)$ at $z < 1$

Point at $h = 0$, $y(1) = 2\chi_t$, $\chi_t = \frac{\langle Q^2\rangle_0}{V}$ determined with larger statistics

Not included in the fit, used to check





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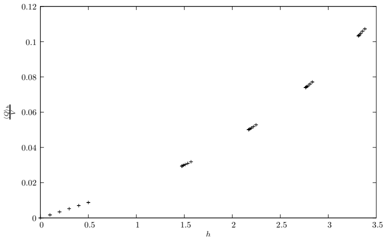

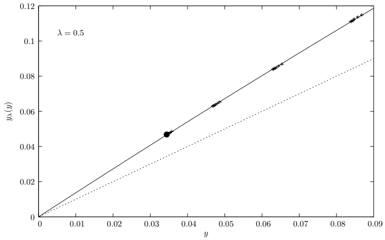

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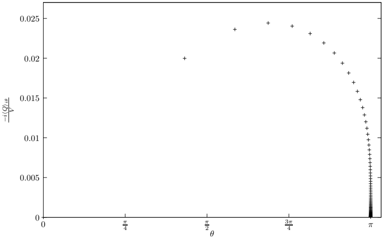

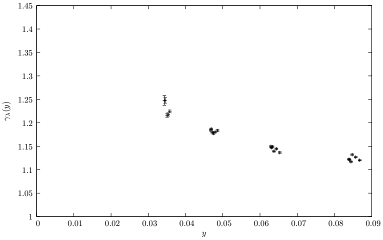

Order parameter

$$\mathcal{O}(\theta) = \frac{-i\langle Q\rangle_{i\theta}}{V} = \tan\frac{\theta}{2} y(z)\,,\ z = \cos\frac{\theta}{2}$$

At $\theta \sim \pi$, $z \sim (\pi - \theta)$, $\tan\frac{\theta}{2} \sim z^{-1}$

1. Reconstruct $\mathcal{O}(\theta)$ from $y_\lambda(y)$

Near $\theta = \pi$, $\mathcal{O}(\theta) \propto (\pi - \theta)^\epsilon$

$\epsilon = 0$: broken P, $\epsilon \in (0,1)$: 2^{nd} order, $\chi_t \to \infty$

$$\epsilon \simeq 0.3$$

2. Effective exponent $\gamma_\lambda(y) = \frac{2}{\lambda}\log\frac{y_\lambda(y)}{y}$

$\gamma_\lambda(0) - 1$: critical exponent at $\theta = \pi$

Near $z = 0$, $y(z) \sim z^{\gamma_\lambda}$, $\gamma_\lambda \equiv \gamma_\lambda(0)$

$\gamma_\lambda = 1$: broken P, $\gamma_\lambda \in (1,2)$: 2^{nd} order, $\chi_t \to \infty$

$$\gamma_\lambda(0) - 1 \simeq 0.4$$

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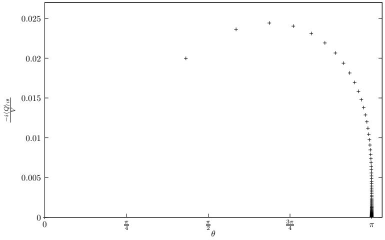

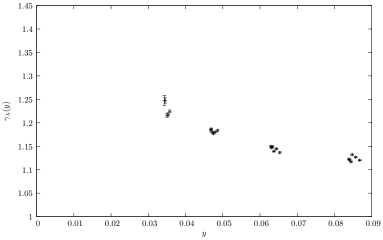

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At $\theta \sim \pi$, $z \sim (\pi - \theta)$, $\tan\frac{\theta}{2} \sim z^{-1}$

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Near $\theta = \pi$, $\mathcal{O}(\theta) \propto (\pi - \theta)^\epsilon$

$\epsilon = 0$: broken P, $\epsilon \in (0,1)$: 2^{nd} order, $\chi_t \to \infty$

$$\epsilon \simeq 0.3$$

2. Effective exponent $\gamma_\lambda(y) = \frac{2}{\lambda}\log\frac{y_\lambda(y)}{y}$

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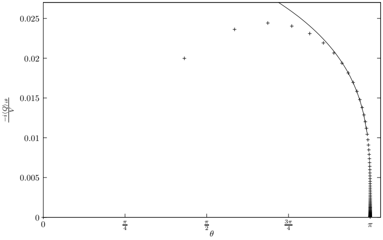

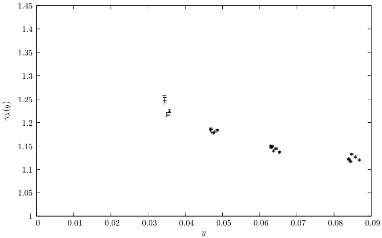

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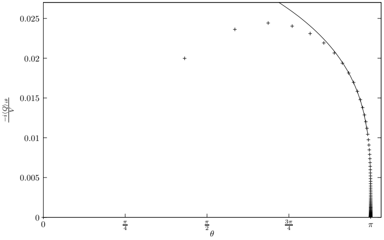

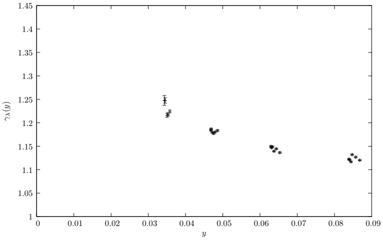

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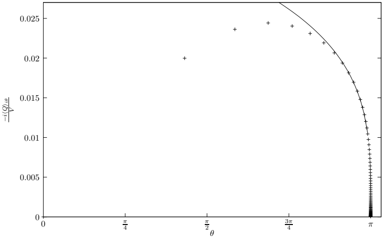

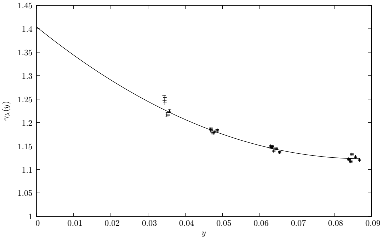

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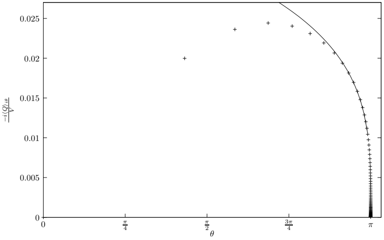

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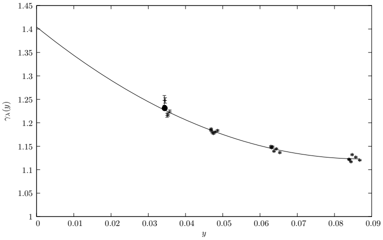

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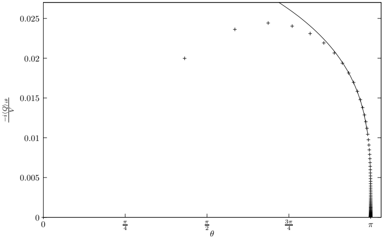

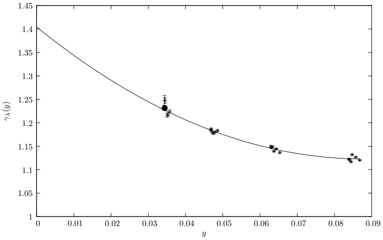

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Outline

Outlook

Results are preliminary, but the method seems to work well for the $O(3)$ nonlinear sigma model

In progress:

- Improve the statistics
- Study the dependence on β
- Test the relation between $O(3)$ and "auxiliary field" formulation of CP^1
- Gauge theories are the ultimate goal

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