Exotic QCD states LHCb observation of $X(6900)$

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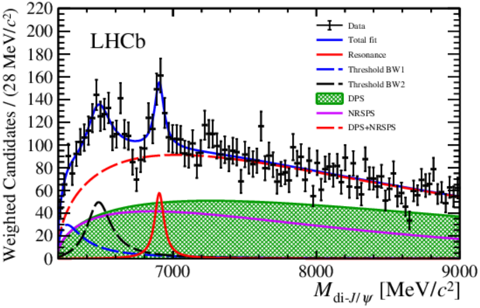
History: experiments

- Several claims for unusual hadrons
- Z baryons in the 60s, with strangeness $S = +1$, based on adventurous analyses of KN scattering without spin measurements
- Baryonium in the 70s. Peaks in the $\bar{p}$ induced reactions. Not confirmed at LEAR and elsewhere. But enhancements in baryon-antibaryon distributions from quarkonium or heavy flavored mesons
- Scalar mesons. If supernumerary, $qq\bar{q}\bar{q}$ contribute. Still open.
- Light pentaquark Peak in nK^+ of $\gamma d \to K^+K^-pn$ at LEPS. First confirmed in other data sets, with even partners. Later, no confirmation in high-statistics experiments.

History: experiments

- $X(3872)$ at Belle and other XYZ states in several experiments. It really revitalized the field.
- $X(3872)$ confirmed in several experiments, but its real structure not yet settled. Probably versatile. Cf. G.F. Chew: the Δ looks as a πN resonance when you study πA scattering, and qqq when you do hadron spectroscopy.
- LHCb pentaquarks peaks in $J/\psi p$ from $\Lambda_b \to J/\psi p K^-$. But the exact number of peaks and possible QN not fully stabilized.
- $d^*(2380)$ dibaryon
- Many inconclusive searches.
  - H of Jaffe (more than 20 exp.)
  - P of Gignoux et al. and Lipkin (Fermilab, HERA)
  - etc.

- Recently, evidence for a peak in J/ψ-J/ψ, again at LHCb, interpreted as a $cc\bar{c}\bar{c}$ resonance
- Motivated many studies



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History: phenomenology

- 70s Baryonium qq-$\bar{q}\bar{q}$ preferentially coupled to baryon-antibaryon
  - Rediscovered for cq-$\bar{c}\bar{q}$ (some of XYZ)
  - Rediscovered for cc-$\bar{c}\bar{c}$ (Chao, Riska et al.)
  - Small width due to angular-momentum barrier
  - Or to $6 - \bar{6}$ color structure (“mock baryonium”, Chan H.M. et al)
- 70s Scalar mesons with hundreds of contributions
  - $qq\bar{q}\bar{q}$ in S-wave challenge $q\bar{q}$ states in P-wave
  - At least the lighter ones, favored by the chromomagnetic interaction (Jaffe)
- 80s Meson-meson molecules (Törnqvist, Manohar, Ericson, . . .)
  - First on the basis of one-pion-exchange
  - Generalized to one-boson-exchange (e.g., Hozaka et al.)
  - Reformulated with effective theories (e.g., Oset et al.)

History

- 80s sqq. Explicit quark model calculations
  - Many bag model estimates, but . . .
  - Potential models
  - Standard 4-body, 5-body, 6-body problems
  - Calculations either carefully done
  - Or with unjustified approximations
  - Interesting analogies and differences with atomic physics,
  - For instance $qq\bar{q}\bar{q}$ vs. $(+,+,-,-)$ molecules
- 90s sqq. More elaborate approaches: sum rules, lattice

Remarks about diquarks

- QQq 3-body or two-steps?
  - First solve for QQ
  - Next solve for $QQ - q$
- For HO with usual Jacobi variables $\boldsymbol{x}$ and $\boldsymbol{y}$
- Exact

$$H = \frac{\boldsymbol{p}_x^2}{M} + \frac{\boldsymbol{p}_y^2}{\mu} + K\,(\frac{3}{4}\,\boldsymbol{x}^2 + \boldsymbol{y}^2)$$

- Diquark

$$H' = \frac{\boldsymbol{p}_x^2}{M} + \frac{\boldsymbol{p}_y^2}{\mu} + K\,(\frac{1}{2}\,\boldsymbol{x}^2 + \boldsymbol{y}^2)$$

- $\sim 25\%$ decreases of the $M^{-1/2}$ term in the energy
- Similarly, for a $\bar{3}3$ tetraquark with HO, exact

$$H = \frac{\boldsymbol{p}_x^2}{M} + \frac{\boldsymbol{p}_y^2}{m} + \frac{\boldsymbol{p}_z^2}{\mu} + K\,(\frac{3}{4}\,(\boldsymbol{x}^2 + \boldsymbol{y}^2) + \boldsymbol{z}^2)$$

- Diquark $3/4 \to 1/2$! artificially $\searrow$ energy

Chromomagnetism and scalar mesons

- Very popular after De Rújula-Georgi-Glashow's paper
- Late 70s (Jaffe): S-wave $qq\bar{q}\bar{q}$ compete with P-wave $q\bar{q}$
- The exotic $qq\bar{q}\bar{q}$ (such as $I = 2$) are pushed up by the chromomagnetic operator

$$\tilde{H}_{\mathrm{CM}} = -C \sum_{i<j} \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$$

- Many other applications (see below for H and P)
- For instance $X(3872)$ (Hogaasen et al., Stancu, S.L. Zhu ...) Diagonalizing

$$\tilde{H}_{\mathrm{CM}} = -\sum_{i<j} C_{ij}\, \tilde{\lambda}_i.\tilde{\lambda}_j\, \boldsymbol{\sigma}_i.\boldsymbol{\sigma}_j$$

(C_{ij} borrowed from ordinary hadrons) gives some of the important properties of $X(3872)$

Chromoelectricity and tetraquarks

- Less advertised than CM. Nevertheless rather interesting achievements
- Lipkin, Stanley and Robson and others discussed the 1/2 rule:
  - $q\bar{q}$ in mesons $V(r)$
  - qqq in baryons $1/2 \sum V(r_{ij})$
  - i.e., pairwise and color-octet exchange
  - Simultaneous meson and baryon phenomenology
- Generalized for multiquarks as

$$-\frac{3}{16} \sum_{1 \le i < j \le N} \tilde{\lambda}_i . \tilde{\lambda}_j \, V(r_{ij})$$

providing a starting point for multiquark calculations.

Chromoelectricity-2

- 1978 Gavela and the Orsay group (Oliver et al.) solved

$$\sum_{i=1}^{4} \frac{\boldsymbol{p}_i^2}{2\,m} - \frac{3\,K}{16} \sum_{1\leq i<j\leq N} \tilde{\lambda}_i.\tilde{\lambda}_j\, r_{ij}^2$$

  for $qq\bar{q}\bar{q}$ states with $L > 0$ baryonium
  Interesting selection rules for decay into mesons, first guessed by Chan H.M., Jaffe, . . .
- Namely $(qq) \underset{L}{-} (\bar{q}\bar{q})$
- In 1985 (A. Martin): rigorous proof that excited baryons have such a structure, namely $(qq) \underset{L}{-} q$
- Spontaneous generation of quark-diquark structure without postulating diquark beforehand.

Chromoelectricity-3

- 1981 Ader et al. resumed the work by Gavela et al., restricting to $L = 0$ but introducing different masses and a more general potential, namely for $QQ\bar{q}\bar{q}$

$$H(QQ\bar{q}\bar{q}) = \sum_{i=1}^{2} \frac{\mathbf{p}_i^2}{2\,M} + \sum_{i=3}^{4} \frac{\mathbf{p}_i^2}{2\,m} - \frac{3\,K}{16} \sum_{1 \le i < j \le N} \tilde{\lambda}_i . \tilde{\lambda}_j \, v(r_{ij})$$

- Discovered that $QQ\bar{q}\bar{q}$ becomes stable if M/m is large enough
- New mechanism of binding!
- Confirmed by Heller et al. (Los Alamos), Zouzou et al. (Grenoble), Rosina et al. (Slovenia), Valcarce et al. (Spain), Brink et al., etc.,
- But often advertently or inadvertently omitted from the refs.

Chromoelectricity-4

- Why $(QQ\bar{q}\bar{q})$ becomes stable? Favorable symmetry breaking

$$\sum_{i=1}^{2} \frac{\boldsymbol{p}_i^2}{2\,M} + \sum_{i=3}^{4} \frac{\boldsymbol{p}_i^2}{2\,m} - \frac{3\,K}{16} \sum_{1 \le i < j \le N} \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij}) = H_{\text{even}} + H_{\text{odd}}$$

$$H_{\text{even}} = \left(\frac{1}{4\,M} + \frac{1}{4\,m}\right)\left(\sum_i \boldsymbol{p}_1^2\right) + V$$

$$H_{\text{odd}} = \left(\frac{1}{4\,M} - \frac{1}{4\,m}\right)(\boldsymbol{p}_1^2 + \boldsymbol{p}_2^2 - \boldsymbol{p}_3^2 - \boldsymbol{p}_4^2)$$

- Hence $E(H) \le E(H_{\text{even}})$ but H and H_{even} have the same threshold! Stability improved if M/m increases at fixed $M^{-1} + m^{-1}$
- Same mechanism explains why H_2 is more stable than Ps_2 in atomic physics

Combining CM and CE

- $QQ\bar{u}\bar{d}$ with $I=0$ benefits from the CE attraction between QQ, that is not in the threshold
- $\bar{u}\bar{d}$ with $S=I=0$ benefits from a favorable CM interaction, not in the threshold
- Model calculations indicate the possibility of binding for $cc\bar{u}\bar{d}$ (Rosina et al., Barnea et al., ...), and for sure, binding for heavier quarks, typically
  - 180 MeV for $bb\bar{u}\bar{d}$
  - 10-20 MeV for $cc\bar{u}\bar{d}$
- Important role of $6\bar{6} \leftrightarrow \bar{3}3$ for weak binding

Fully-heavy tetraquarks

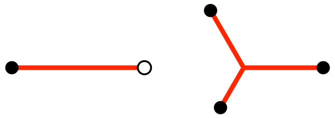
- The color additive model $\sum \tilde{\lambda}_i.\tilde{\lambda}_j\, v(r_{ij})$ does not bind $QQ\bar{Q}\bar{Q}$
- Unlike some claims (erroneous removal of the center-of-mass, diquark approximation, etc.)
- Resonances? Likely, but the calculation requires dedicated techniques
- More precisely: an unconverged variational calculation with a bound-state type of wave function is not a good approach of resonances!
- Appropriate methods are available: complex scaling (J-M. Combes, ...), variational complex scaling (Okopinska, ...), real scaling (Hiyama, ...) but requires some work!
- Work in progress

Improvements for tetraquarks

- Relativistic kinematics

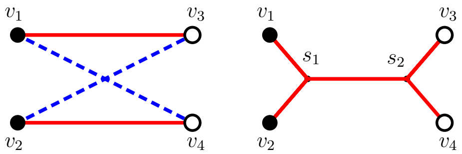
$$\frac{\boldsymbol{p}^2}{2\,m} \to \sqrt{\boldsymbol{p}^2 + m^2} - m$$

  decreases the mass of mesons and baryons, but the net effect of tetraquarks under study (lower mass, but less binding with respect to the threshold)
- Non-pairwise interaction? String potential
- Linear part $\sum \tilde{\lambda}_i.\tilde{\lambda}_j\, r_{ij} \to$



- Not too much change for baryons

String dynamics for tetraquarks

- For tetraquarks: minimum of flip-flop and connected string
- 

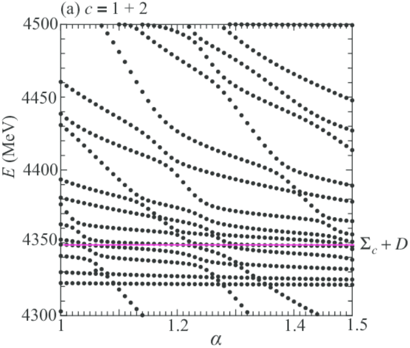
- More attractive than the pairwise model
- Dominated by flip-flop
- Provided there is no antisymmetrization constraint.
- So $bb'\bar{b}\bar{b}'$ is stable if $b' \neq b$ but *nearly the same mass*.

Pentaquarks

- The LHCb pentaquarks attracted much attention
- Anticipated in “molecular approach” and in pure CM models
- Experimentally, search for other production mechanisms and decay final states
- Also search for partners with other QN and/or strangeness
- In constituent models, some contributions
    - Stabilization method
    - Possibility of bound states

Pentaquarks: stabilization

- Hiyama et al. $\Psi^{(i)}(\boldsymbol{x}, \ldots)$ from a standard variational calculation in a large basis, $i = 1, 2, \ldots$ radial excitations with same QN
- Rescale as $\Psi_\alpha^{(i)} \propto \Psi^{(i)}(\alpha\, \boldsymbol{x}, \ldots)$ and look at $\langle \Psi_\alpha^{(i)} | H | \Psi_\alpha^{(i)} \rangle$
- Plateaux near resonances, and distance between avoided crossings (or density of states) $\to$ width



Pentaquarks: bound states?

- Systematics of $\bar{c}cqqq$ in constituent models (R., Valcarce, Vijande)
- Reveals the possibility of stable bound states (if internal $c\bar{c}$ annihilation is neglected)
- Or narrow (e.g., dissociation with D-wave meson-baryon)
- Not always translated into s or b analogs, as it is a coherent effect of CM and CE
- Look e.g., at $\eta_c\, p\, \pi$, i.e., beyond J/ψ trigger

Hexaquarks

- One of the most studied systems
- $H = uuddss$ unbound in simple constituent models
- Many studies as baryon-baryon systems with Yukawa forces (Oka et al., Riska, etc.)
- Many studies also related to $d^*(2380)$
- In potential models, no obvious bound hexaquark containing heavy quarks
- Recently *bbbccc* claimed to be stable in lattice calculation (Mathur et al.)
- Not confirmed in a 6-body quark-model calculation (R., Valcarce, Vijande)

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Outlook

- Striking spectrum of resonances in experiments
- $X(3872)$ and other XYZ, hidden-charm pentaquarks, di-J/ψ,
- Stimulated new investigations within several approaches
- Also pushed quark-model practitioners beyond the day-to-day routine of mesons and baryons
- Subtleties of N body problem with $N > 2$ often ignored
- Challenging piece of few-body physics (hopefully) related to QCD
- Very few bound states! In most cases,
  - constraints from the Pauli statistics
  - the threshold benefits more from CM and other effects
- The search would require new triggers

THE END

Backup slides

Tetraquarks. Rigorous results

Symmetry breaking in strength factors

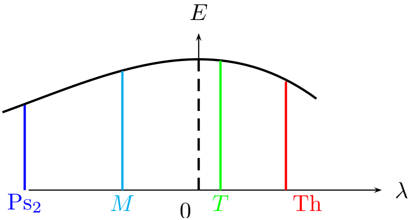
- Comparison of $6\bar{6}$ vs. $\bar{3}3$ color states

$$H = \sum_{i}^{4} \frac{\boldsymbol{p}_i^2}{2\,M} + \sum_{i<j} g_{ij}\, V(r_{ij})$$

- With $\sum_{i<j} g_{ij} = 2$ (V is attractive)
- Special case $g_{12} = g_{34} = 1/3 - \lambda$ other $g_{ij} = 1/3 + \lambda/2$
- "Mock" $M = 6\bar{6}$ $\lambda = 7/12$
- "True" $T = \bar{3}3$ $\lambda = -1/6$
- Ps_2 $\lambda = 4/3$
- Threshold made of two mesons $\lambda = -2/3$

Symmetry breaking in tetraquark and Ps2

$$H = \sum_i^4 \frac{\boldsymbol{p}_i^2}{2\,M} - \frac{3}{16} \sum_{i<j} \langle \tilde{\lambda}_i . \tilde{\lambda}_j \rangle\, V(r_{ij}) \quad \text{vs.} \quad \sum_i^4 \frac{\boldsymbol{p}_i^2}{2\,M} + \sum_{i<j} \frac{q_i\, q_j}{r_{ij}}$$



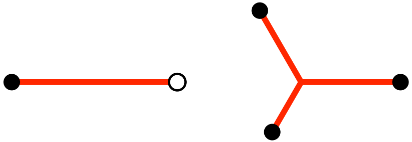
- *Tetraquarks penalized by the non-Abelian algebra!!!*
- Almost a proof of the stability of PS_2 without calculus

Pairwise or multibody interaction?

Steiner tree: baryons-1

- For baryons, the linear confinement is described by a Y-shape interaction (Artru, Merkuriev, Dosch, Kuti et al., Kogut et al., etc.)

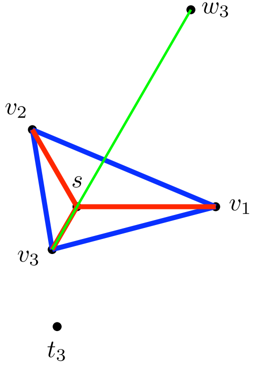
$$v = \sigma\, r_{12}\ , \qquad V_Y = \sigma\, \min_J \sum_{i=1}^{3} r_{iJ}\ .$$

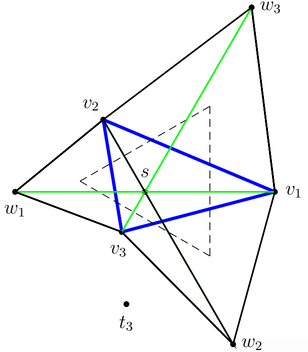


- No dramatic change for baryon spectroscopy, as compared to the $1/2$ rule.
- Except for solving the 3-body problem (Taxil et al., Semay et al., etc.)

Steiner tree: baryons-2

- This baryon potential is the solution of the famous Fermat-Torricelli problem of the minimal path linking three points, with an interesting symmetry restoration, intimately related to a theorem by Napoleon.





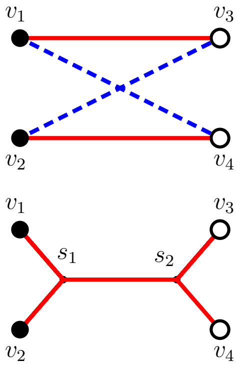
Steiner tree: tetraquarks-1

$$U = \min\{V_{\text{flip-flop}}, V_{\text{Steiner}}\}$$

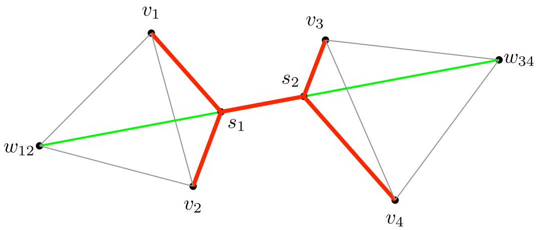
$$V_{\text{flip-flop}} = \min\{d_{13} + d_{24}, d_{14} + d_{23}\}\ ,$$

$$V_{\text{Steiner}} = \min_{s_1, s_2}\left(\|v_1 s_1\| + \|v_2 s_1\| + \|s_1 s_2\| + \|s_2 v_3\| + \|s_2 v_4\| \right),$$

U dominated by the flip-flop term,



Steiner tree: tetraquarks-2



In the planar case, very simple construction of the connected term of the potential (this speeds up the computation).

$$V_4 = \sigma \left\| w_{12} w_{34} \right\| ,$$

maximal distance between the two Melznak points.

Steiner tree: tetraquarks-3

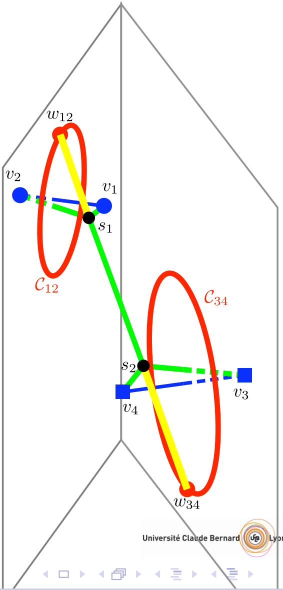
$$V_4 = \sigma\,\|w_{12}w_{34}\|\ ,$$

maximal distance between the two Melznak circles.

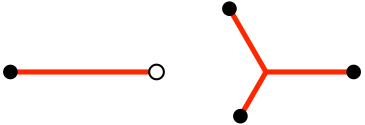
$$V_4 \leq \sigma\left\{\frac{\sqrt{3}}{2}\,[\|\boldsymbol{x}\| + \|\boldsymbol{y}\|] + \|\boldsymbol{z}\|\right\}\ ,$$

which is exactly solvable. The Jacobi var.

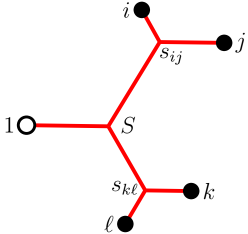
$$\boldsymbol{x} = v_1v_2,$$
$$\boldsymbol{y} = v_3v_4,$$
$$\boldsymbol{z} = (v_1+v_2)/2 - (v_3+v_4)/2\ ,$$



Steiner tree: pentaquark

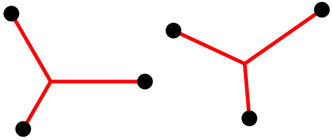


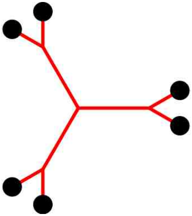
- $U = \min\{$flip-flop, Steiner$\}$,
- Flip-flop
- Connected Steiner tree



  - $(\bar{q}qqqq)$, as well as $(\overline{Q}qqqq)$, $(\bar{q}qqqQ)$ for $M \gg m$, and probably many other configurations bound vs. spontaneous dissociation. (hyperscalar approx. with flip-flop alone sufficient to prove binding)
  - But short-range forces and antisymmetrisation constraints not yet included.
  - $(\bar{c}uuds)$ should survive, as spin effects might help.

Steiner-tree: hexaquark

- Same scenario: flip-flop and connected diagrams,
- The latter, more interesting, but less important for the dynamics,
- Binding is obtained in most cases, where antisymmetrization is neglected.
- 



Steiner-tree: baryon-antibaryon

- Again: flip-flop and connected diagrams,
- Binding obtained in some cases.
-