TENSOR NETWORK STATES FOR QUANTUM FIELD THEORY

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(Garching)

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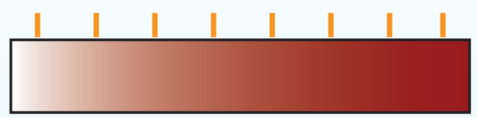
TENSOR NETWORKS

WHAT ARE TNS?

- TNS = Tensor Network States

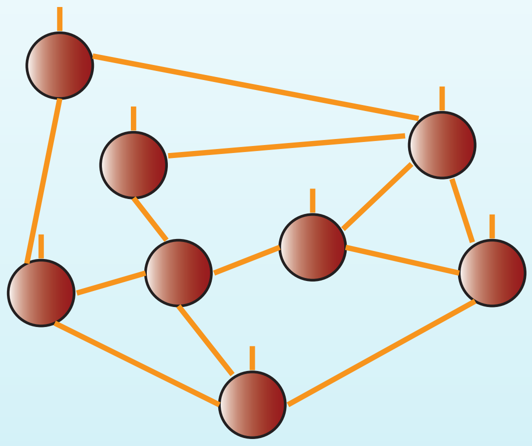
A general state of the N-body Hilbert space has exponentially many coefficients

$$|\Psi\rangle = \sum_{i_j} \boxed{c_{i_1 \dots i_N}} |i_1 \dots i_N\rangle$$



N-legged tensor

d^N



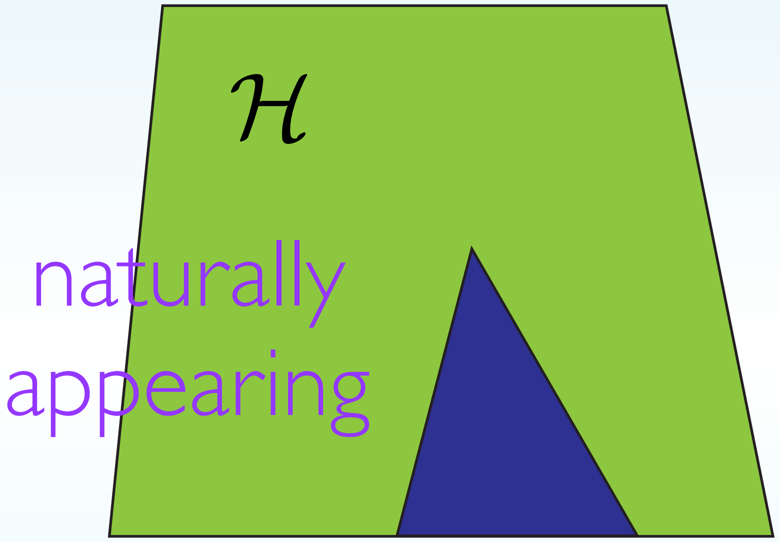
$\mathrm{poly}(N)$

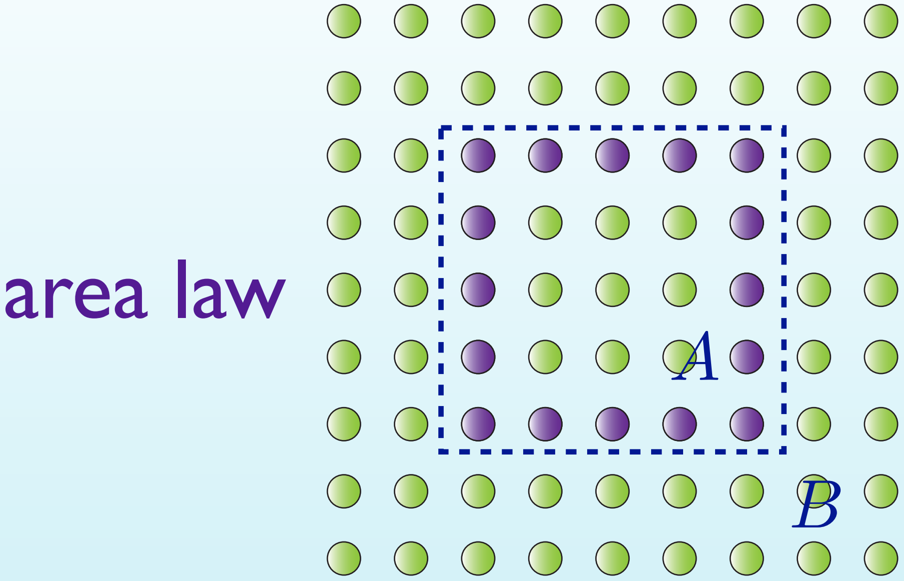
A TNS has only a polynomial number of parameters

WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product





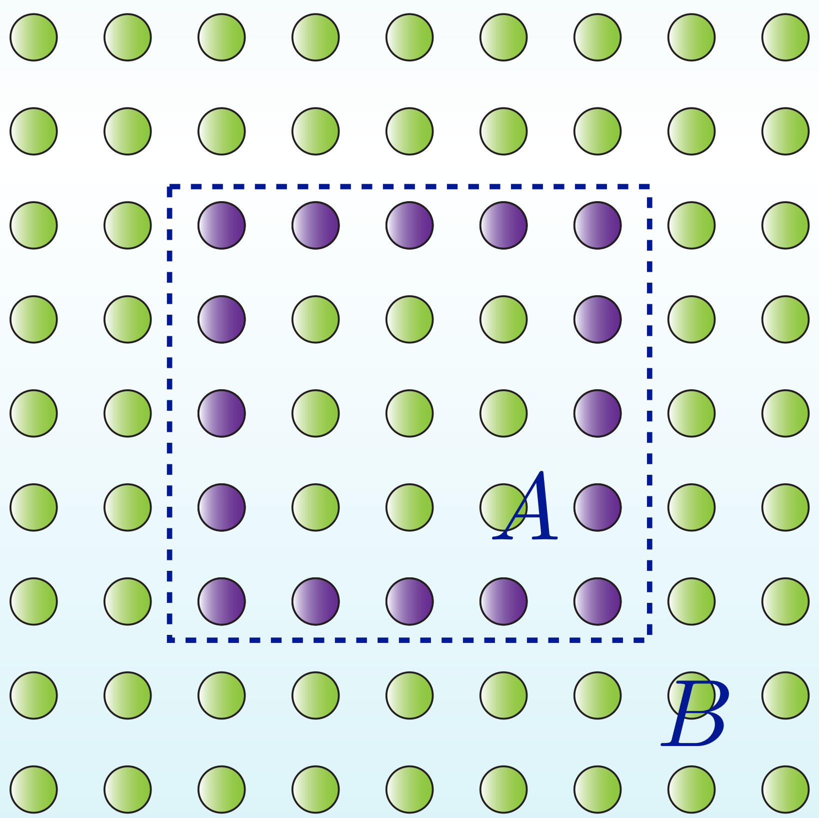
We look for states with *little* entanglement

TNS = entanglement based ansatz

WHY SHOULD TNS BE USEFUL?

Which properties characterize ground states of relevant Hamiltonians?

Area law



local gapped 1D Hamiltonians

have ground states

with area law of entanglement

$$S_{A\,\max} \propto |\delta A|$$ Hastings 2007

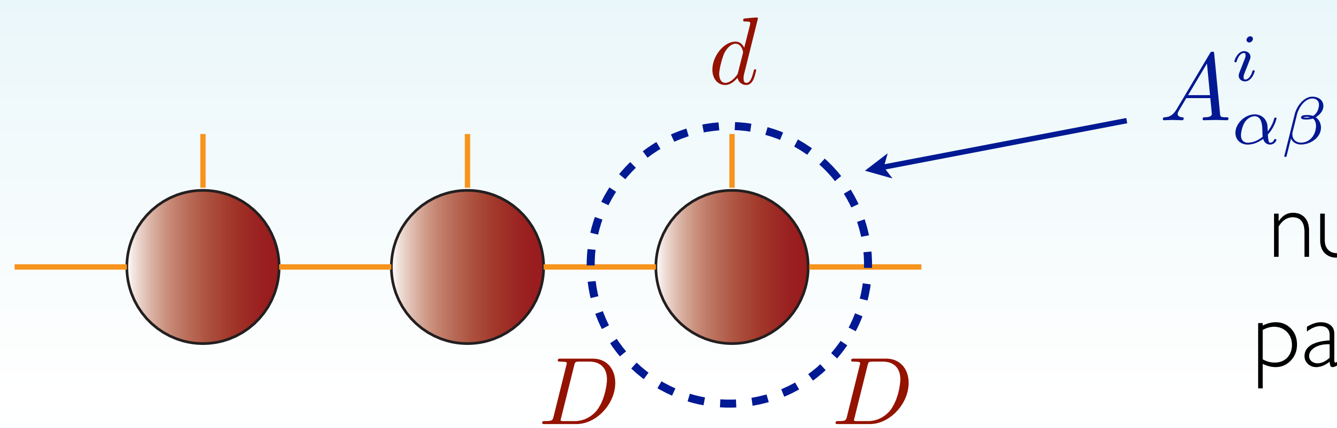
in 1D critical systems,
logarithmic corrections

$$S_{A\,\max} \propto |\delta A| \log A$$ Calabrese, Cardy 2004

satisfied at finite temperature Wolf, Verstraete, Hastings, Cirac, PRL 2008

MPS

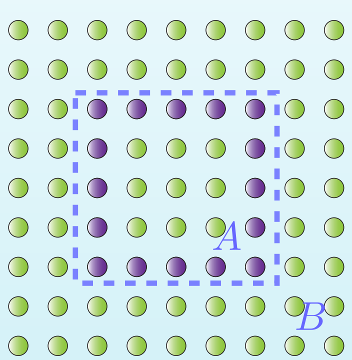
Matrix Product States



number of parameters

NdD^2

$$|\Psi\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}(A_1^{i_1} A_2^{i_2} \ldots A_N^{i_N}) |i_1 \ldots i_N\rangle$$



Area law by construction

Bounded entanglement $\quad S(L/2) \leq \log D$

MPS

Matrix Product States

SOME PROPERTIES

good approximation of ground states

area law (ground state)
critical states: D~poly(N)

Verstraete, Cirac, PRB 2006
Hastings, J. Stat. Phys 2007

efficient calculation of expectation values

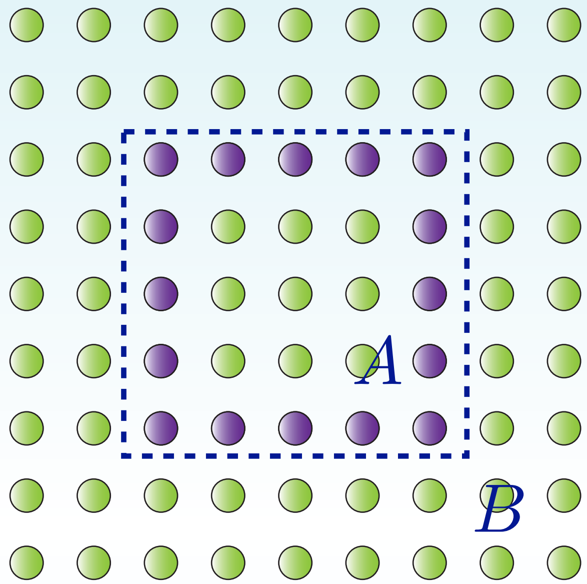
exponentially decaying correlations

can be prepared efficiently

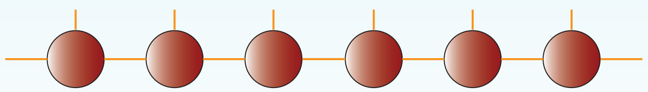
natural generalisation to higher dimensions: PEPS

TNS = entanglement based ansatz

area law

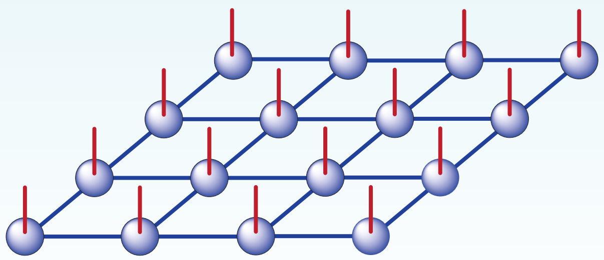


MPS



Schollwöck Ann.Phys.2011

PEPS



Verstraete et al. Adv. Phys. 2008

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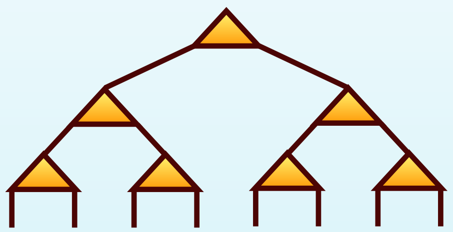
other TNS

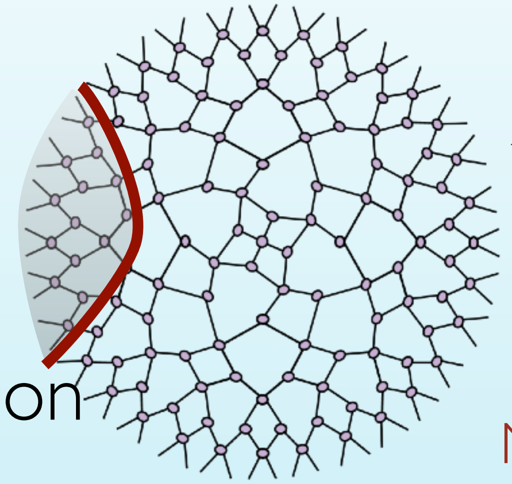
TTN

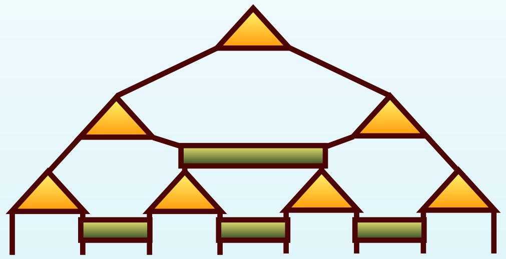


Shi et al PRA 2006

Vidal PRL 2007

MERA



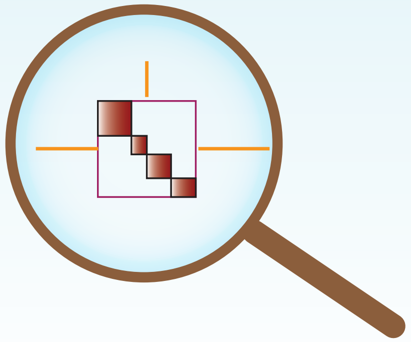


suggested connection
to AdS/CFT

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

USING TNS FOR QMB

a formal approach



classifying tensors

constructing states

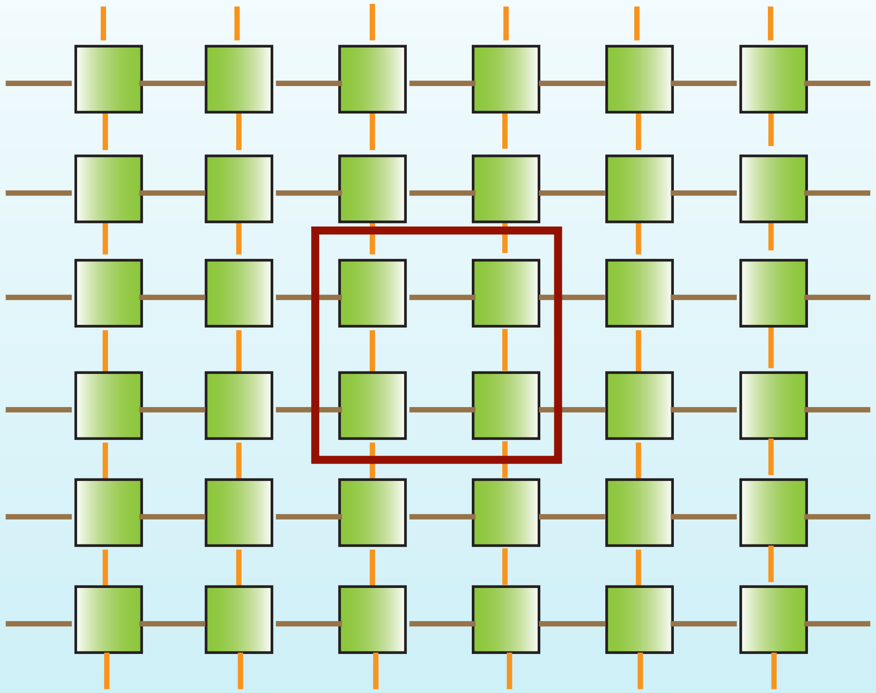
great descriptive power: phases,
topological chiral states, anyons...

Chen et al PRB 2011
Schuch et al PRB 2011
Wahl et al PRL 2013; Yang et al PRL 2015
Haegeman et al, Nat. Comm. 2015

no sign problem

numerical algorithms

tensor networks describe
partition functions (observables)

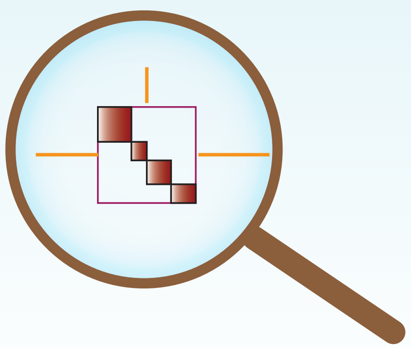


need to contract a TN
TRG approaches

Nishino, JPSJ 1995
Levin & Wen PRL 2008
Xie et al PRL2009; Zhao et al PRB 2010

USING TNS FOR QMB

a formal approach



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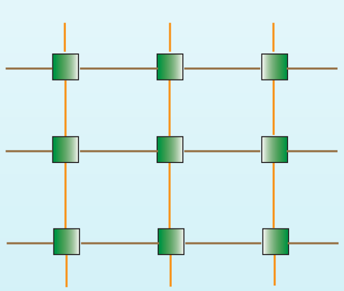

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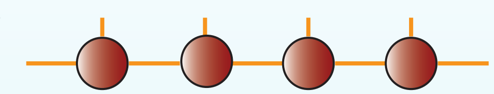


TNS as ansatz for the state

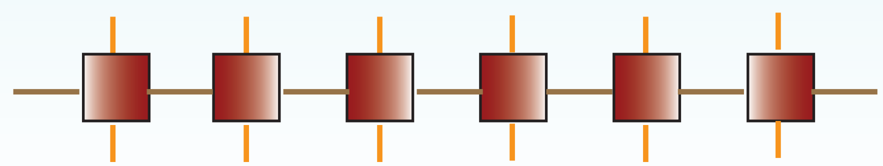
efficient algorithms for GS, low excited states, thermal, dynamics

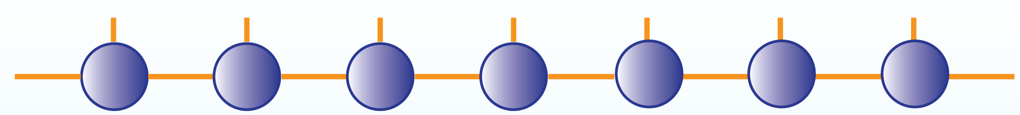
White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

NUMERICAL ALGORITHMS

variational minimization of energy

local
Hamiltonian

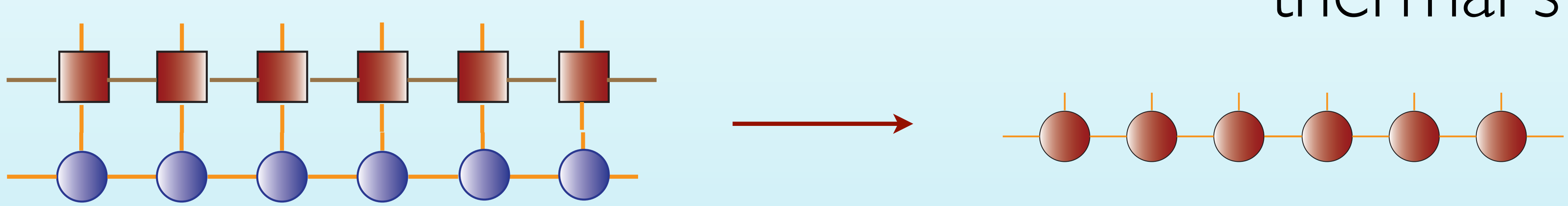
$H =$ 

$|E_0\rangle \simeq$ 

ground state
excitations

apply local operators → simulate time evolution

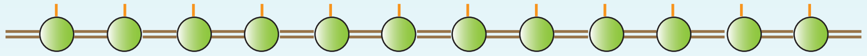
imaginary time → ground state
thermal state



various evolution algorithms

evolving the (pure state) ansatz

Vidal, PRL 2003, PRL 2007
White, Feiguin, PRL 2004
Daley et al., 2004
Haegeman et al., 2011

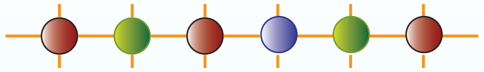


entanglement can grow fast!

Osborne, PRL 2006
Schuch et al., NJP 2008

evolving operators: Heisenberg picture

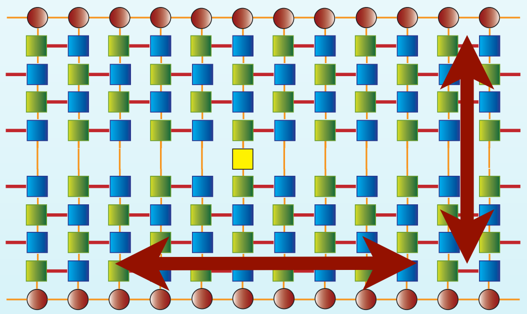
Hartmann et al, PRL 2009



also for mixed states
operator space entanglement

Prosen Pizorn, PRL 2008

observables as TN to contract



different *entanglement* quantities

MCB, Hastings, Verstraete, Cirac, PRL 2009
Müller-Hermes et al., NJP 2012
Hastings, Mahajan 2014

others: density of states

Yang, Iblisdir, Cirac, MCB, 1909.01398

USING TNS FOR QFT

http://heptnseminar.org

Tensor Networks in HEP Seminar

gqfi.aei.mpg.de/node/25

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gqfi.aei.mpg.de/node/25

Upcoming Seminars

Frank Verstraete

When: January, 2020
Title: TBA
Abstract: TBA

Past Seminars

1. Ignacio Cirac (Opening seminar of the series)

When: November 8, 2019 @ 15.00 (GMT +1 hour, Berlin time)
Title: *Tensor Networks and Lattice Gauge Theories*
Abstract: Certain Quantum Many-body states can be efficiently described in terms of tensor networks. Those include Matrix Product States (MPS), Projected Entangled-Pair Etates (PEPS), or the Multi-scale Entanglement Renormalization Ansatz. They play an important role in quantum computing, error correction, or the description of topological order in condensed matter physics, and are widely used in computational physics. In the last years, it has also been realized their suitability to describe Lattice Gauge Theories, at least in the context of MPS in low dimensions. In this talk I will review some of the basic ideas about tensor networks and their applications to lattice gauge theories, and explain current efforts to extend them to higher dimensions using PEPS.
YouTube link: https://www.youtube.com/watch?v=hdb82b1kazw&feature=youtu.be

2. Bartlomiej Czech

When: December 6, 2019 @ 15.30 (GMT +1 hour, Berlin time)
Title: *What does the Chern-Simons formulation of AdS3 gravity tell us about complexity?*
Abstract: I will explain how to realize the wavefunction of a CFT2 ground state as a network of Wilson lines in the Chern-Simons

USING TNS FOR LGT

an ongoing LGT-TNS roadmap...

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

Schwinger model U(1) in 1D

precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
S. Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

finite density

S. Kuehn et al, PRL118 (2017) 071601

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

2+1 dimensions

work in progress

full LQCD in 3+1 dimensions

MCB, K. Cichy 1910.00257
(2019 ROPP)

OTHER MODELS

THIRRING MODEL

MCB, K. Cichy, Y.-J. Kao, C.-J. D. Lin, Y.-P. Lin, D. T.-L. Tan, arXiv:1908.04536

THIRRING MODEL

probing two phases in lattice Thirring model

- one is critical, $c=1$, non-vanishing chiral condensate
- one is gapped, with exponentially decaying correlations
- transition compatible with BKT

outlook

- non-perturbative study of scaling (e.g. $\bar{\psi}\psi$)
- excitations (cont. limit $g>\pi/2$)

arXiv:1908.04536

REAL TIME THERMAL FIELD THEORY

with M. Heller, K. Jansen, J. Knaute, V. Svensson, arXiv:1912.08836

THERMAL QUENCHES IN ISING MODELS

Thermal response determined by retarded 2-point function

linear response

source

$$\delta\langle\mathcal{O}(t,p)\rangle = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega\, \boxed{G_R(\omega,p)}\, \delta J(-\omega,-p)\, e^{-i\,\omega\,t}$$

$$G_R(\Delta t,\Delta x) = i\mathrm{tr}\left(Z_\beta^{-1}\, e^{-\beta H}\, [\mathcal{O}(\Delta t,\Delta x),\mathcal{O}(0,0)]\right)$$

structure of singularities determines time response

→ holographic QFTs: single poles with imaginary part (transients) BH quasi normal modes

→ CFT in 1+1 dim, scalar operator

$$\omega = \pm p - i\,2\pi\,T\,(\Delta + 2\,n)$$

scaling dimension

THERMAL QUENCHES IN ISING MODELS

Thermal states well described by TNS Hastings 2006; Molnar et al. 2015

real time evolution: thermal response functions

Barthel 2013

→ use to identify characteristic frequency modes

→ connect lattice and field model

Compute 2-point correlators at different times

analyse real-time retarded 2-point function

→ zero or finite momentum can be reconstructed

→ numerical extraction of frequencies: Prony

arXiv:1912.08836

THERMAL QUENCHES IN ISING MODELS

Ising spin chain

$$H = -J \left(\sum_{j=1}^{L-1} \sigma_z^j \sigma_z^{j+1} + h \sum_{j=1}^{L} \sigma_x^j + g \sum_{j=1}^{L} \sigma_z^j \right)$$

$g = 0$ free fermions

$h = 1$ critical

near criticality Ising field theory

$$H = \int_{-\infty}^{\infty} dx \left\{ \frac{1}{2\pi} \left[\frac{i}{2} \left(\psi(x)\partial_x \psi(x) - \bar{\psi}(x)\partial_x \bar{\psi}(x) \right) - i M_h \bar{\psi}(x)\psi(x) \right] + \bar{g}\sigma(x) \right\}$$

$$M_h = 2J|1-h|$$

$$\bar{g} = \frac{2}{\bar{s}} J^{15/8} g$$

$M_h = 0$ exactly solvable QFT

Fonseca, Zamolodchikov 2006

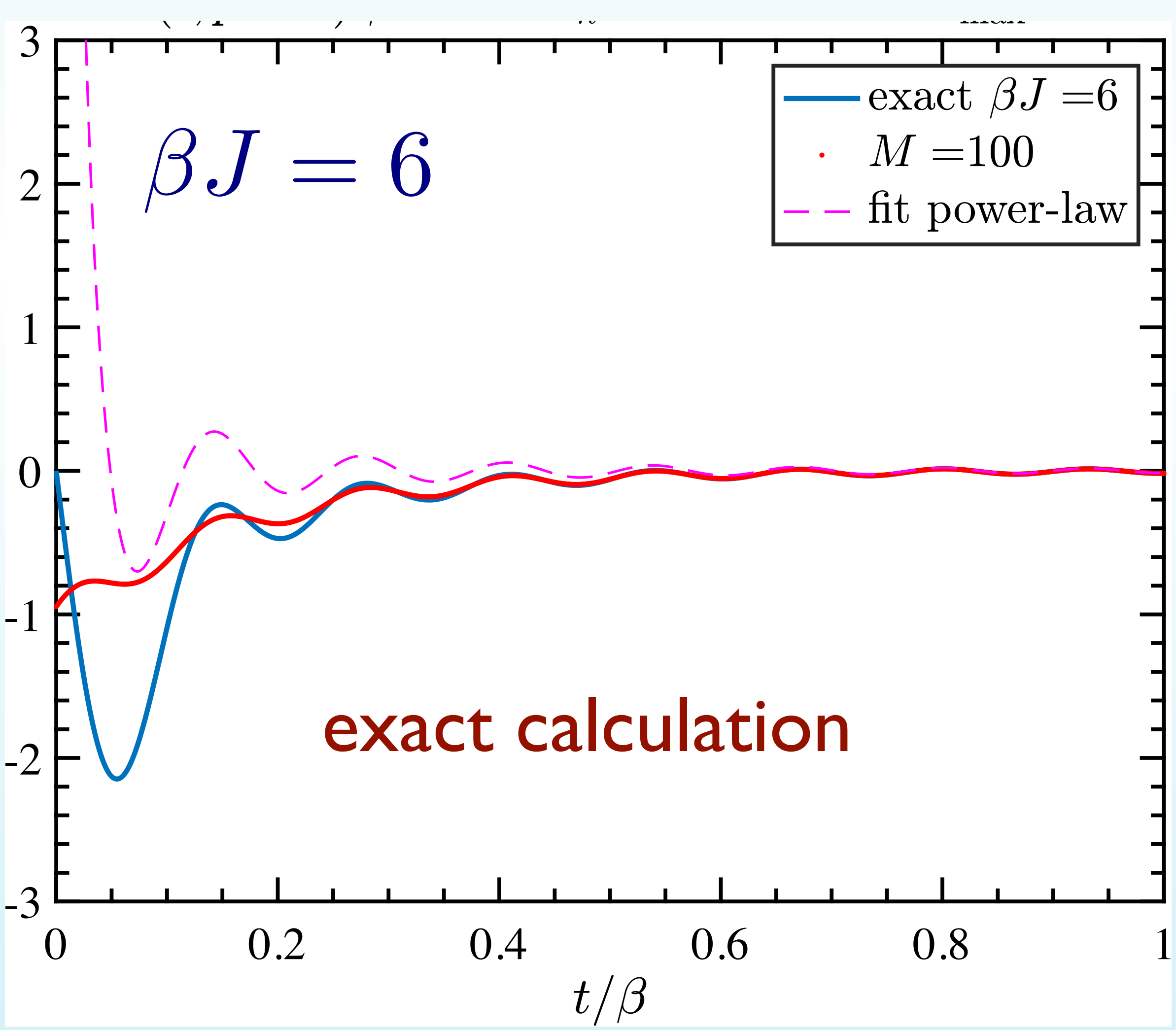
adimensional parameters $\quad (\beta M_h,\ \beta|\bar{g}|^{8/15},\ \beta J)$

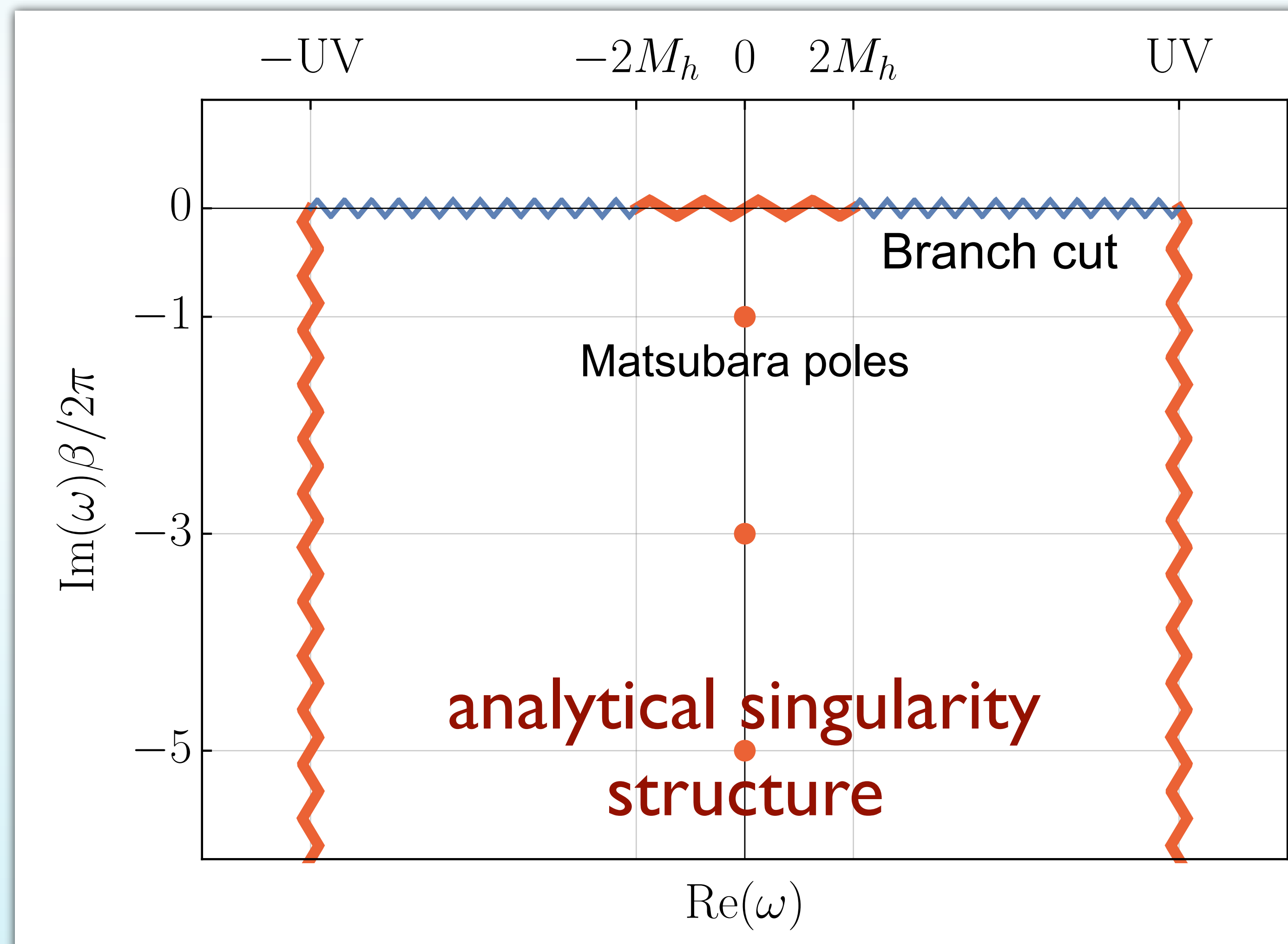
arXiv:1912.08836

THERMAL QUENCHES IN ISING MODELS

VALIDATING THE METHOD: FREE FERMIONIC CASE

transverse magnetization $\sigma_x \longrightarrow \bar{\psi}\psi \quad \Delta_{\bar{\psi}\psi} = 1$





recall CFT $\omega = \pm p - i\,2\pi\,T\,(\Delta + 2\,n)$

arXiv:1912.08836

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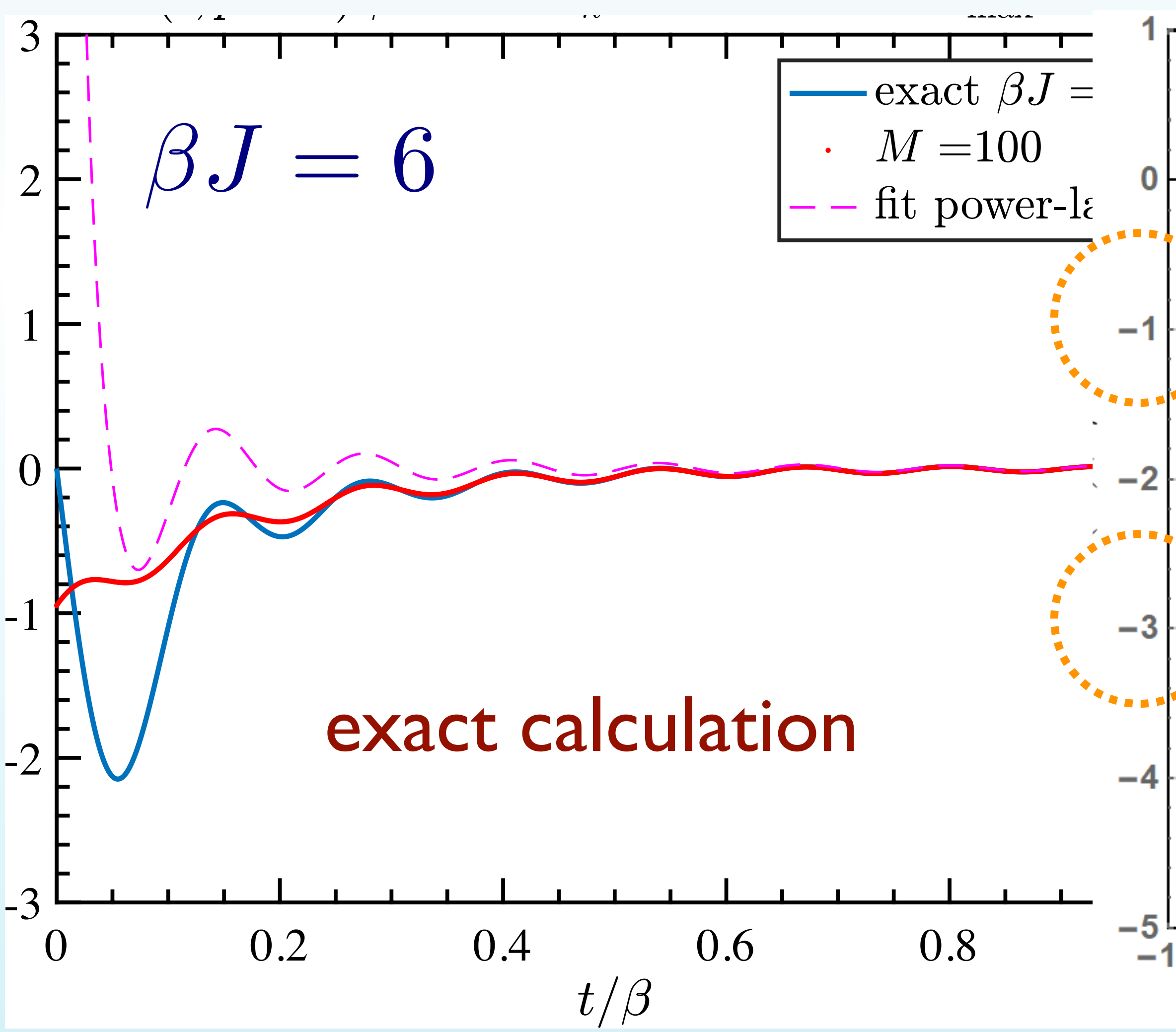
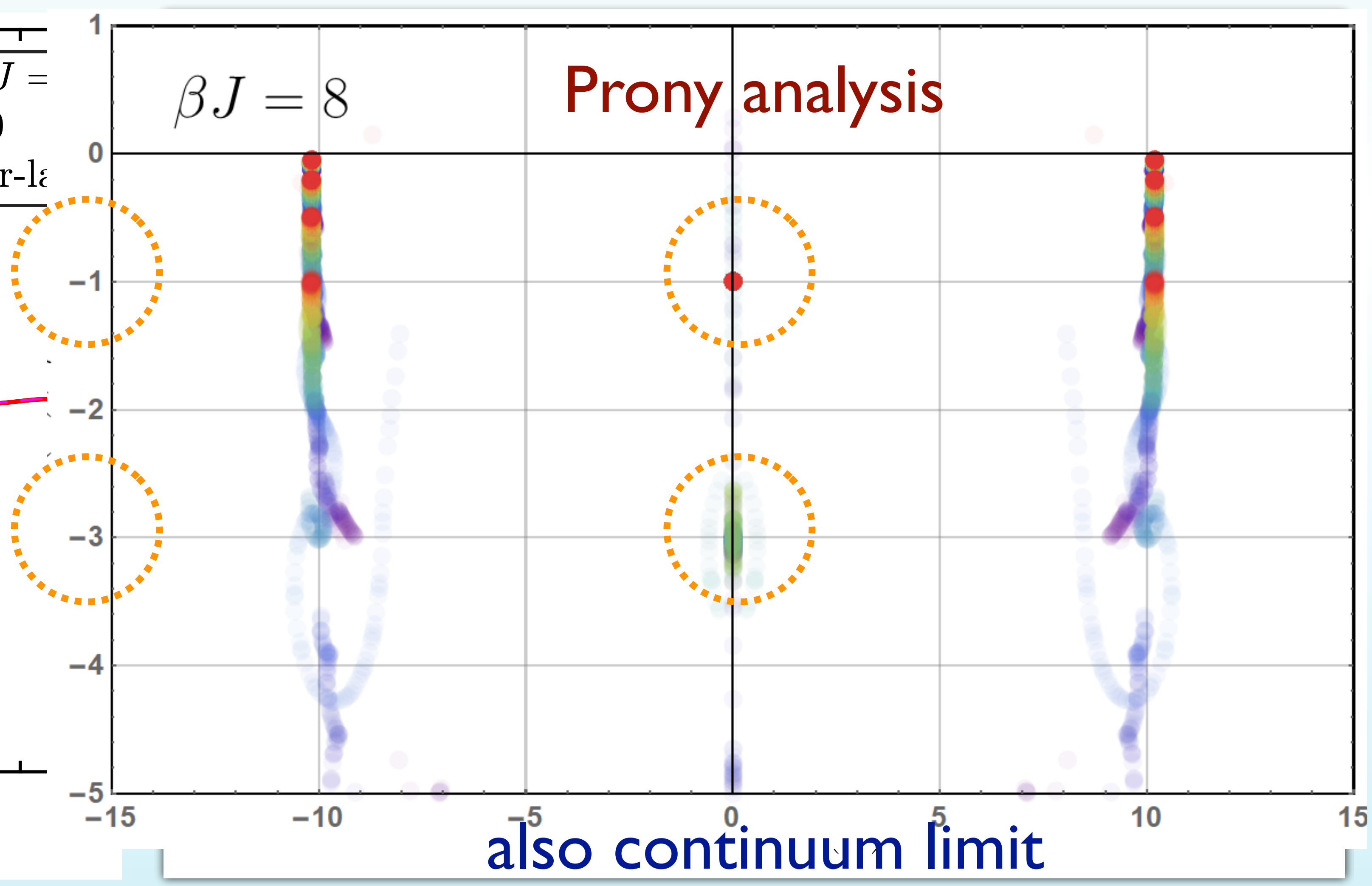




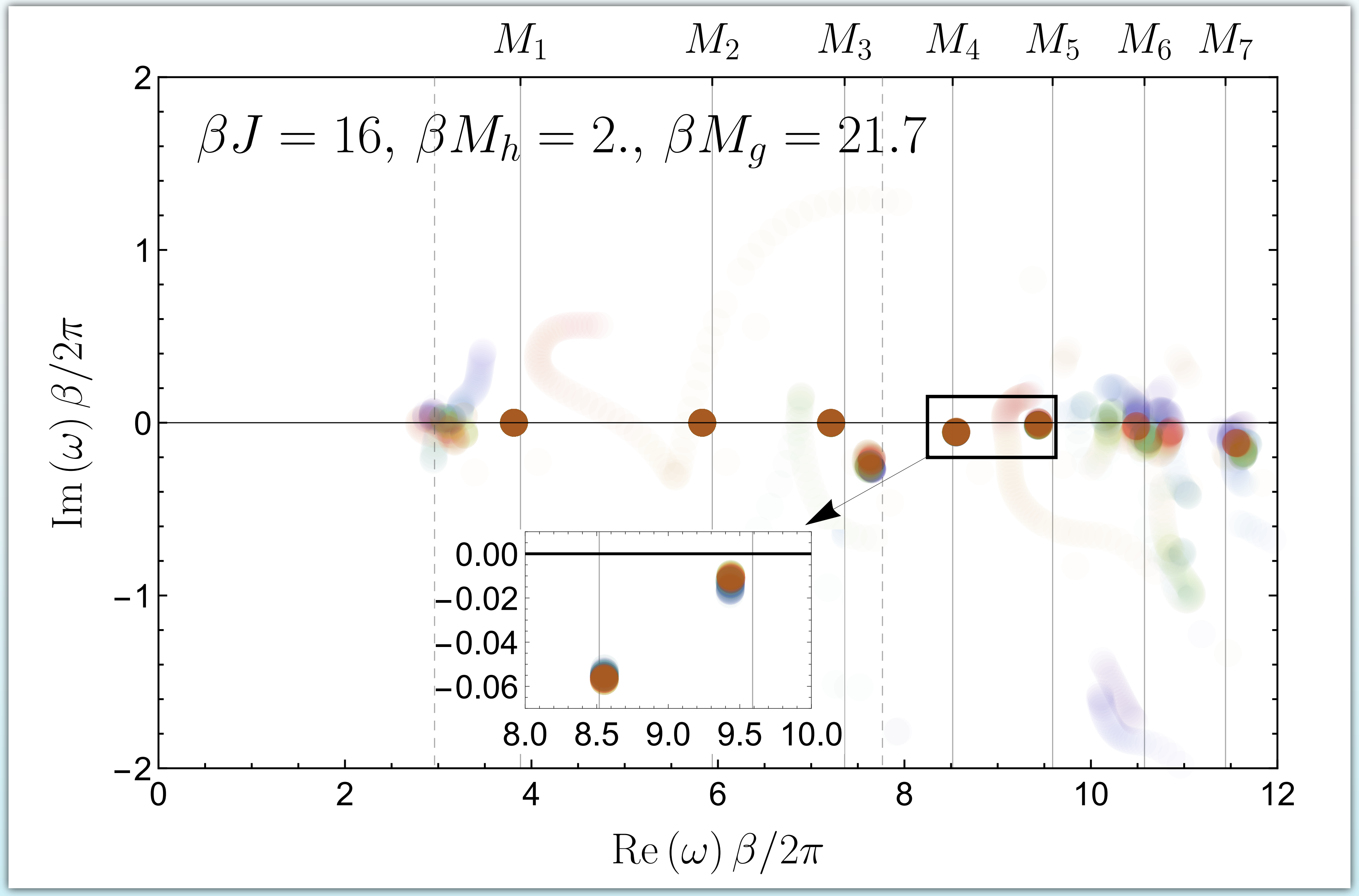
recall CFT $\omega = \pm p - i\, 2\pi\, T\, (\Delta + 2\, n)$

arXiv:1912.08836

THERMAL QUENCHES IN ISING MODELS

BEYOND EXACTLY SOLVABLE

transverse magnetization $g \neq 0$ $\longrightarrow$ bound states (mesons)



seen as single poles

also imaginary parts!

very good numerical agreement with theoretical calculations

arXiv:1912.08836

THANKS

http://heptnseminar.org

TNS = entanglement based ansatz

- versatile algorithms: spectrum, thermal equilibrium, (some) dynamics

applicability for QFT problems

- LGT: systematically probed in 1D, progress in 2D

  review: MCB, K. Cichy arXiv:1910.00257

- suitable for other problems
  - Thirring model

    MCB, K. Cichy, Y.-J. Kao, C.-J. D. Lin, Y.-P. Lin, D.T.-L. Tan, arXiv:1908.04536
  - thermal response in Ising field theory

    MCB, M. Heller, K. Jansen, J. Knaute, V. Svensson, arXiv:1912.08836

MPQ

MCQST