Quantum walks: background geometry and gauge invariance

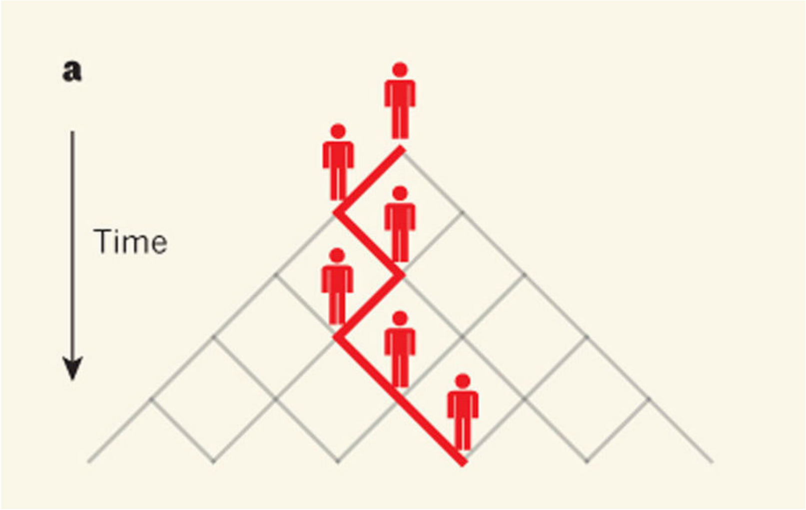
Iván Márquez Martín

20/12/2019

Index

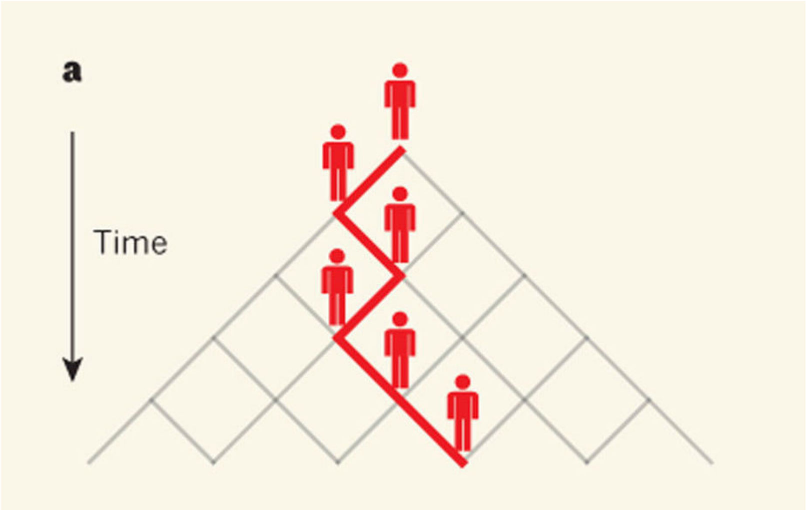
I. Introduction to QW

In the CRW



Same prob. to move left or right

I. Introduction to QW



Hilbert space $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_c$

$\Psi \in \mathcal{H}$

- $\mathcal{H}_p$ position sites $|i\rangle$ canonical basis $i \in \mathbb{Z}$
- $\mathcal{H}_c$ coin state $|c\rangle$ canonical basis of $\mathcal{H}_c$ $c \in \uparrow, \downarrow$

I. Introduction to QW

Unitary operator $U = S(\mathbb{I} \otimes C)$

C: coin operator

$$C = \begin{pmatrix} \cos\theta & \mathrm{i}\sin\theta \\ \mathrm{i}\sin\theta & \cos\theta \end{pmatrix}$$

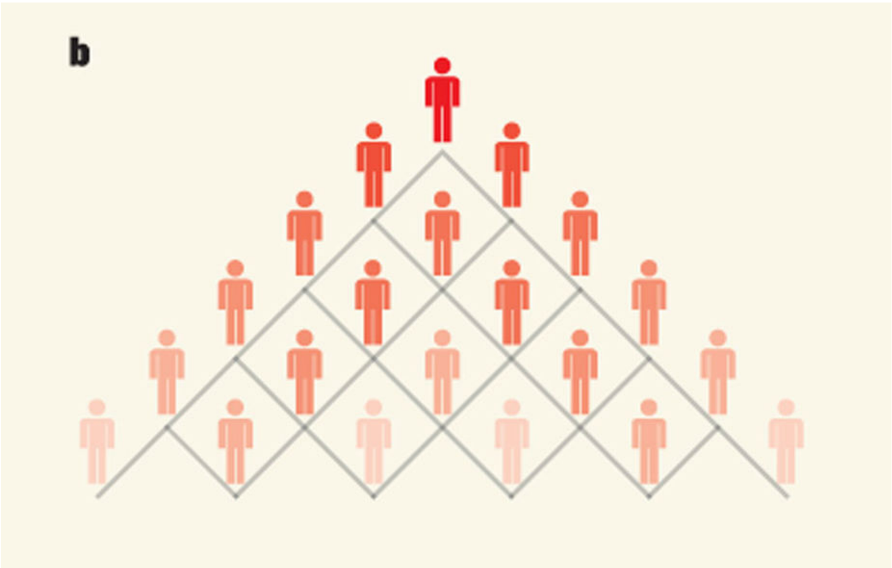
$$C : |\uparrow\rangle \to \cos\theta\,|\uparrow\rangle + \mathrm{i}\sin\theta\,|\downarrow\rangle$$
$$C : |\downarrow\rangle \to \mathrm{i}\sin\theta\,|\uparrow\rangle + \cos\theta\,|\downarrow\rangle$$

Superposition

S: shift operator

$$S = \sum_{i\in\mathbb{Z}} |i+1\rangle\langle i| \otimes |\uparrow\rangle\langle\uparrow| + |i-1\rangle\langle i| \otimes |\downarrow\rangle\langle\downarrow|$$

Time



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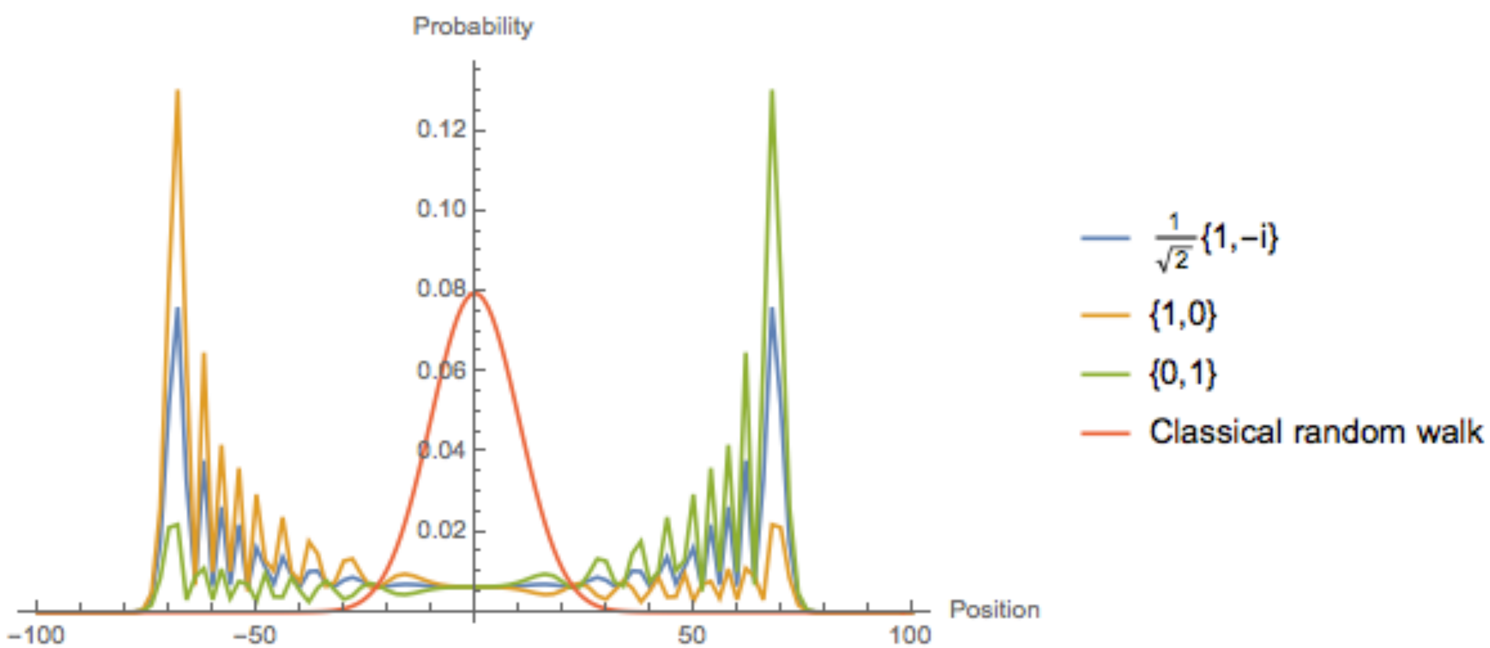
I. Introduction to QW

Probability density $P(i;t) = |\Psi_{t,i}|^2$



Spreading faster than CRW

Vniversitat de València — Facultad de Física

I. Introduction to QW

Quantum algorithm

Universal for quantum computation

Andrew M. Childs. "Universal computation by quantum walk". In: *Physical Review Letters* 102.18 (2009)

Neil B. Lovett, Sally Cooper, Matthew Everitt, et al. "Universal quantum computation using the discrete-time quantum walk". In: Physical Review A - Atomic, Molecular, and Optical Physics 81.4 (2010)

Vniversitat de València Facultad de Física

I. Introduction to QW

Quantum algorithm

Searching in graphs

Hypercube

$\mathcal{O}(\sqrt{N})$

Quantum random-walk search algorithm, Phys. Rev. A 67, 052307 (2003)

2D

$\mathcal{O}(\sqrt{N \log N})$

Avatar Tulsi. “Faster quantum-walk algorithm for the two-dimensional spatial search”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 78.1 (2008)

G. Abal, R. Donangelo, F. L. Marquezino, et al. “Spatial search on a honeycomb network”. Mathematical Structures in Computer Science 20.6 (2010), pp. 999–1009

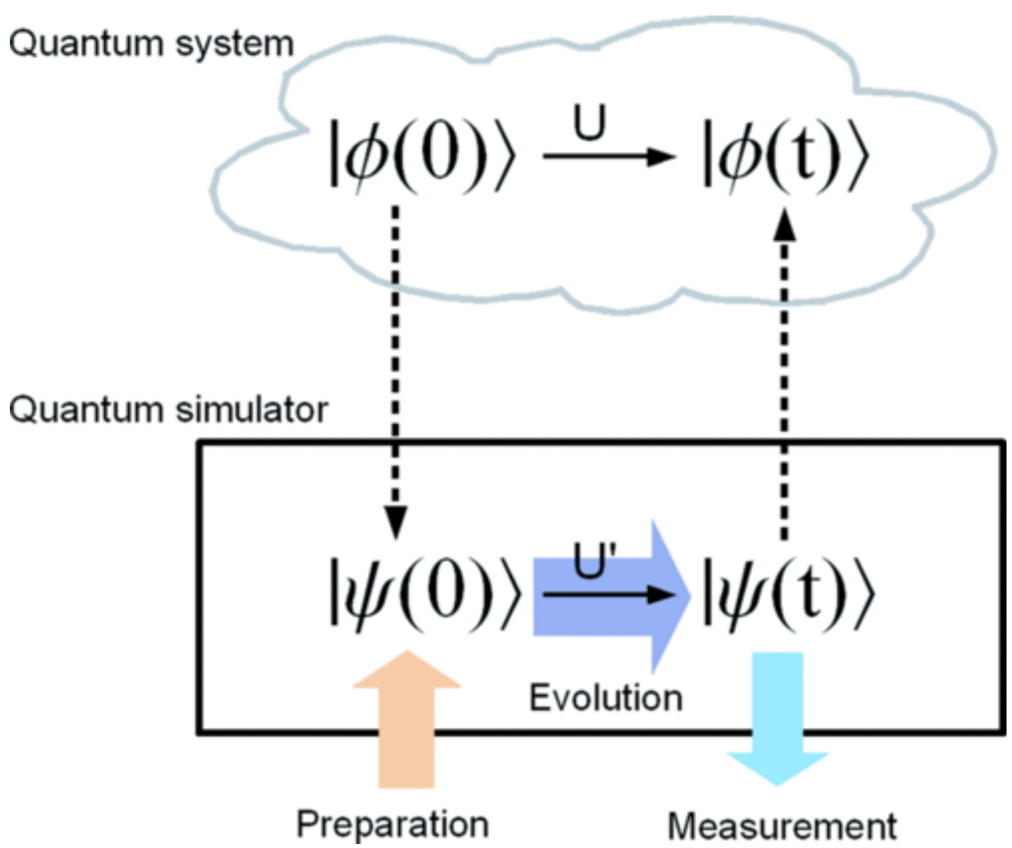
G. Abal, R. Donangelo, M. Forets, et al. “Spatial quantum search in a triangular network”. Mathematical Structures in Computer Science 22.3 (2012), pp. 521–531. issn: 09601295.

Element distinctness problem

Andris Ambainis. “Quantum Walk Algorithm for Element Distinctness”. Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science. FOCS ’04. Washington, DC, USA: IEEE Computer Society, 2004

I. Introduction to QW

Quantum simulation



I. M. Georgescu, S. Ashhab, and Franco Nori
Rev. Mod. Phys. **86**, 153

Index

I. Introduction to QWs
II. QWs as high energy physics simulators
  - **From QWs to the Dirac equation**
  - Brane theories dynamics
  - Electromagnetic gauge invariance in 2D
III. QWs in hexagonal and triangular lattices
  - Continuous limit: Dirac dynamics in free and curved space-time
IV. Conclusions

II. From QWs to Dirac equation

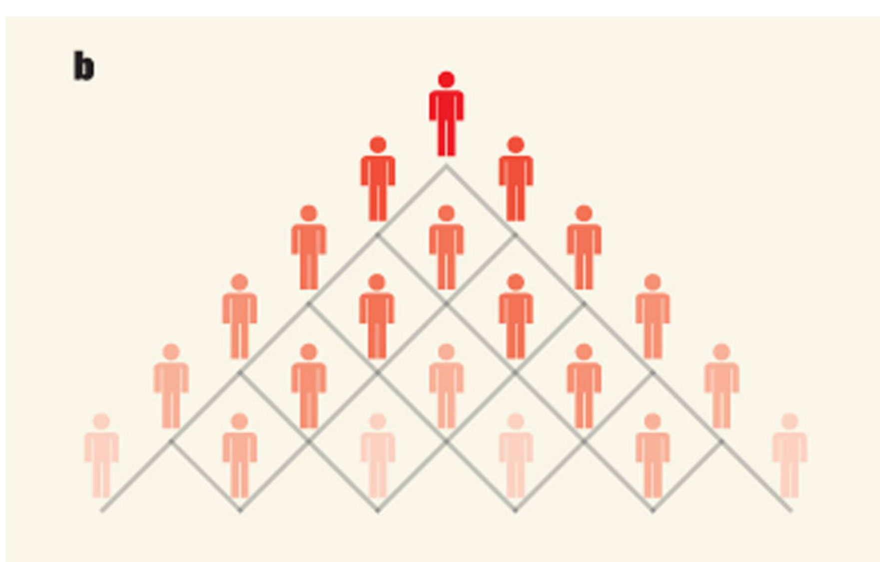
Unitary operator $U = S(\mathbb{I} \otimes C)$

C: coin operator $C = \begin{pmatrix} \cos\theta & \mathrm{i}\sin\theta \\ \mathrm{i}\sin\theta & \cos\theta \end{pmatrix}$

S: shift operator

$$S = \sum_{i \in \mathbb{Z}} |i+\epsilon\rangle \langle i| \otimes |\uparrow\rangle \langle\uparrow| + |i-\epsilon\rangle \langle i| \otimes |\downarrow\rangle \langle\downarrow|$$

Time ϵ



The finite difference eqs:

$$\begin{aligned} \Psi^{\uparrow}_{t+1,i} &= \cos\theta \Psi^{\uparrow}_{t,i+\epsilon} + \mathrm{i}\sin\theta \Psi^{\downarrow}_{t,i+\epsilon} \\ \Psi^{\downarrow}_{t+1,i} &= \mathrm{i}\sin\theta \Psi^{\uparrow}_{t,i-\epsilon} + \cos\theta \Psi^{\downarrow}_{t,i-\epsilon} \end{aligned}$$

$$\theta = \epsilon m$$

Continuous limit

$$\epsilon \longrightarrow 0$$

$$\begin{aligned} \partial_t \Psi^{\uparrow} &= -\partial_i \Psi^{\uparrow} - im\Psi^{\downarrow} \\ \partial_t \Psi^{\downarrow} &= \partial_i \Psi^{\downarrow} - im\Psi^{\uparrow} \end{aligned}$$

Index

I. Introduction to QWs

II. QWs as high energy physics simulators
- From QWs to the Dirac equation
- **Brane theories dynamics**
- Electromagnetic gauge invariance in 2D

III. QWs in hexagonal and triangular lattices
- Continuous limit: Dirac dynamics in free and curved space-time

IV. Conclusions

II. Brane theories dynamics

We analyze the properties of a quantum walk that are inspired by the idea of a brane-world model proposed by Rubakov

1 **QW with a site-dependent coin** → Localization

↕ **Continuous limit**

2 **Brane-world** → Extra spatial dimensions (3+N+1)

II. Brane theories dynamics

Background

Space-time 3+N+1 dimensions but the particles with low energy are confined in the N dimensions by a **potential well**.

$$\mathcal{L}_{\psi} = i\bar{\Psi}\Gamma^{A}\partial_{A}\Psi + h\varphi\bar{\Psi}\Psi \longrightarrow \boxed{i\Gamma^{A}\partial_{A}\Psi + h\varphi\Psi = 0}$$

$g_{AB} = (1, -1, -1, -1, -1) \qquad A = 0, 1, 2, 3, 4$

Effective position-dependent mass

$$\varphi(x^4) = \frac{m}{\sqrt{\lambda}} \tanh(\frac{mx^4}{\sqrt{2}})$$

Phenomenology expected

Low energy, particles lives on the ordinary dimensions

High energy, particles escape to the extra one

II. Brane theories dynamics

Consider a QW over discrete two-dimensional space and discrete time which is driven by an inhomogeneous coin acting on $\mathcal{H}_{spin}$

$$\begin{bmatrix} \psi^{\uparrow}_{j+1,p,q} \\ \psi^{\downarrow}_{j+1,p,q} \end{bmatrix} = S_y Q^{+}(\theta_q) S_x Q^{-}(\theta_q) \begin{bmatrix} \psi^{\uparrow}_{j,p,q} \\ \psi^{\downarrow}_{j,p,q} \end{bmatrix}$$

$$Q^{\pm}(\theta_q) = \begin{pmatrix} \cos\theta_q^{\pm} & i\sin\theta_q^{\pm} \\ i\sin\theta_q^{\pm} & \cos\theta_q^{\pm} \end{pmatrix}$$

$$\theta_q^{\pm} = \pm\frac{\pi}{4} - \epsilon\bar{\theta}_q$$

Considering the angle $\bar{\theta}_q$

$$\bar{\theta}_q = h\frac{m}{\sqrt{\lambda}}\tanh(\frac{mq}{\sqrt{2}})$$

Effective mass

$$\varphi(x^4) = \frac{m}{\sqrt{\lambda}}\tanh(\frac{mx^4}{\sqrt{2}})$$

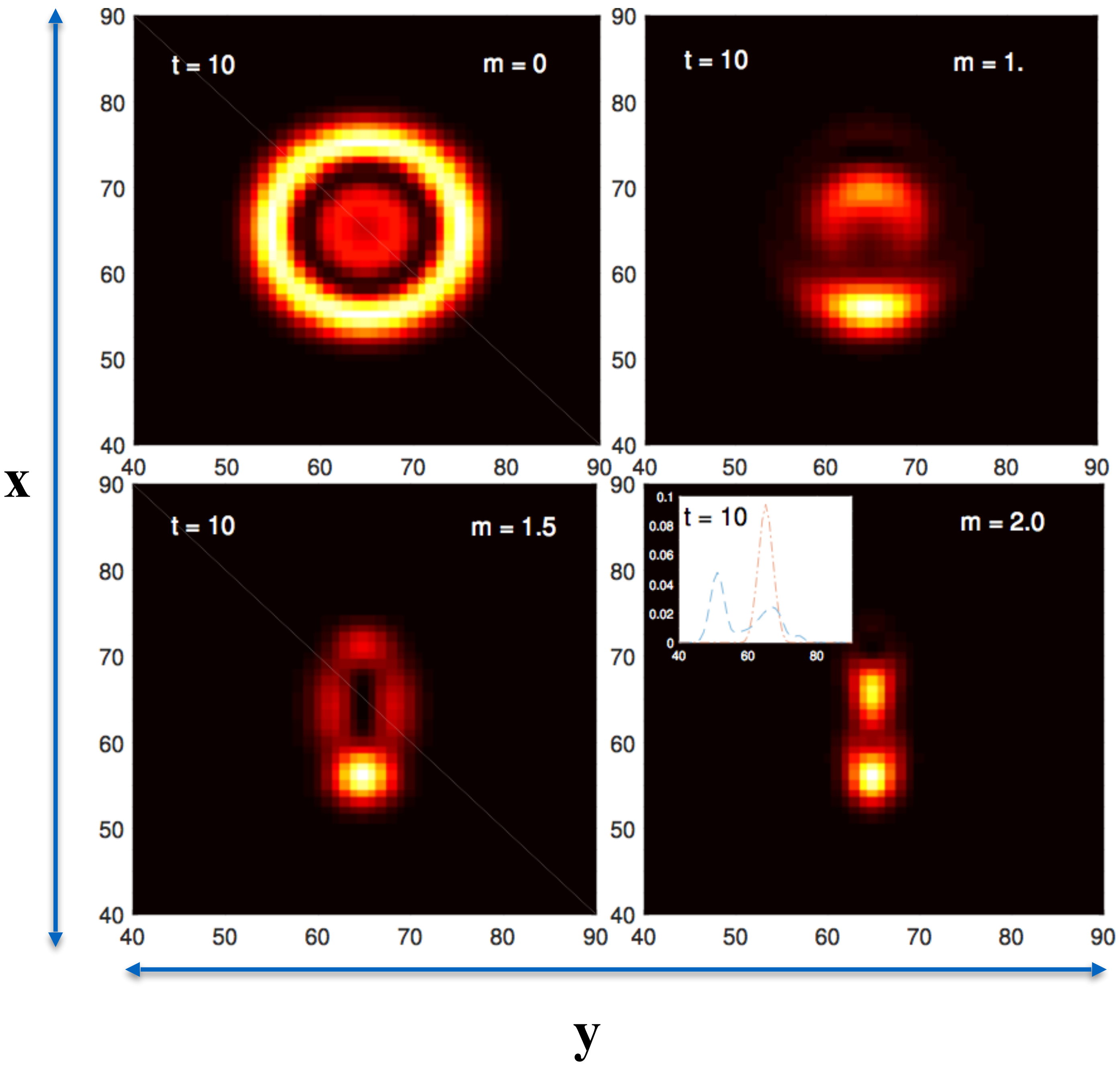
Continuous limit

$$i\Gamma^A\partial_A\Psi + h\varphi\Psi = 0$$

II. Brane theories dynamics

Probability distribution

$\lambda = 60, h = 70, \epsilon = 0.04$



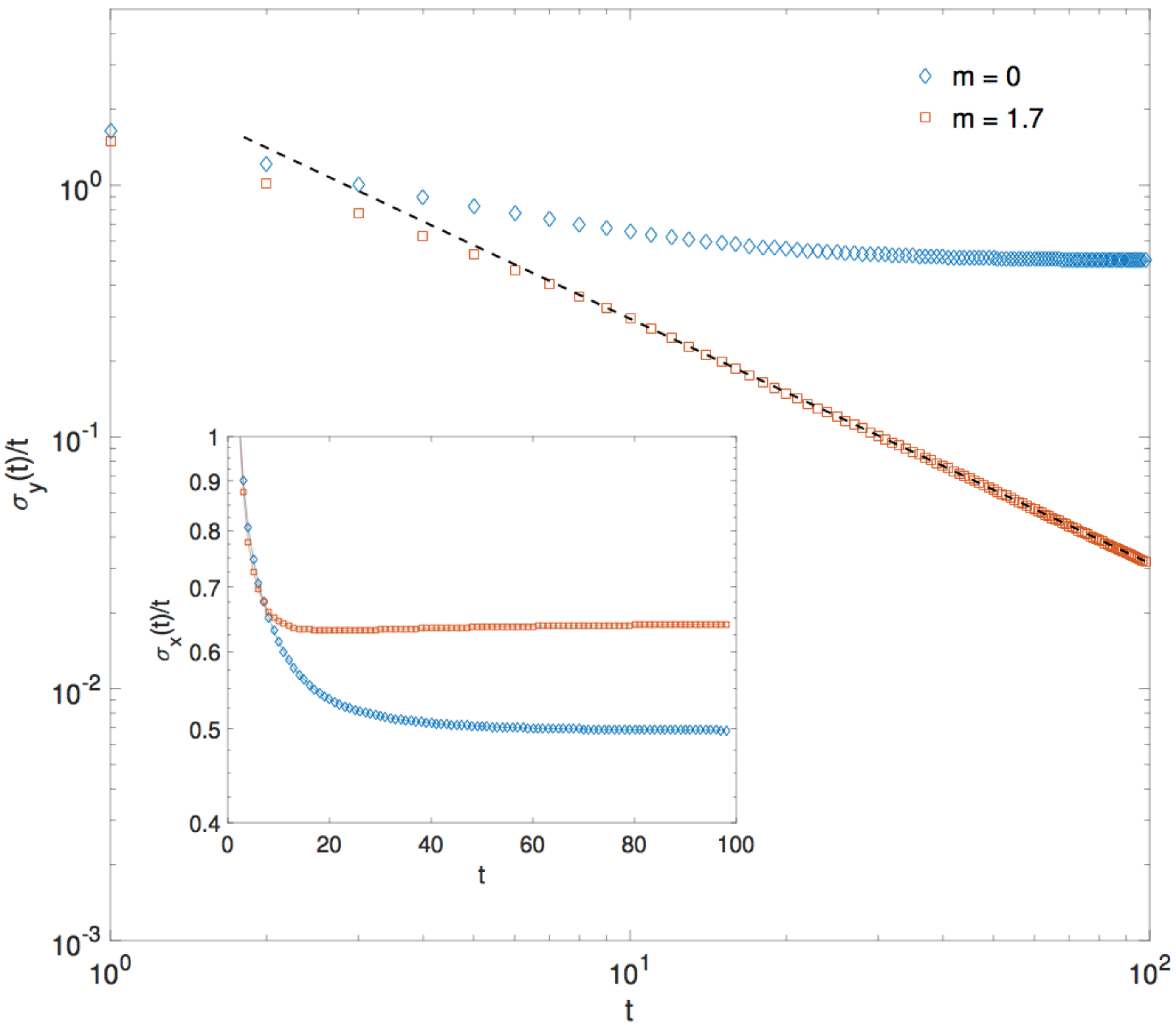
Initial condition $\Psi(0, x_p, y_q) = \sqrt{n(x_p, y_q)} \otimes (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})^\top$

As the effective mass increases, the probability becomes **localized** in the y direction

Roughly speaking, the y direction plays **the role of extra dimension**

II. Brane theories dynamics

Standard deviation divided by time-step



$\lambda = 60, h = 70, \epsilon = 0.02$

Initial condition $\Psi(0, x_p, y_q) = \sqrt{n(x_p, y_q)} \otimes (0, 1)^\top$

For m = 0 both quotients tend to a constant

As m increases, there is an exponential decay of $\sigma_y(t)/t$

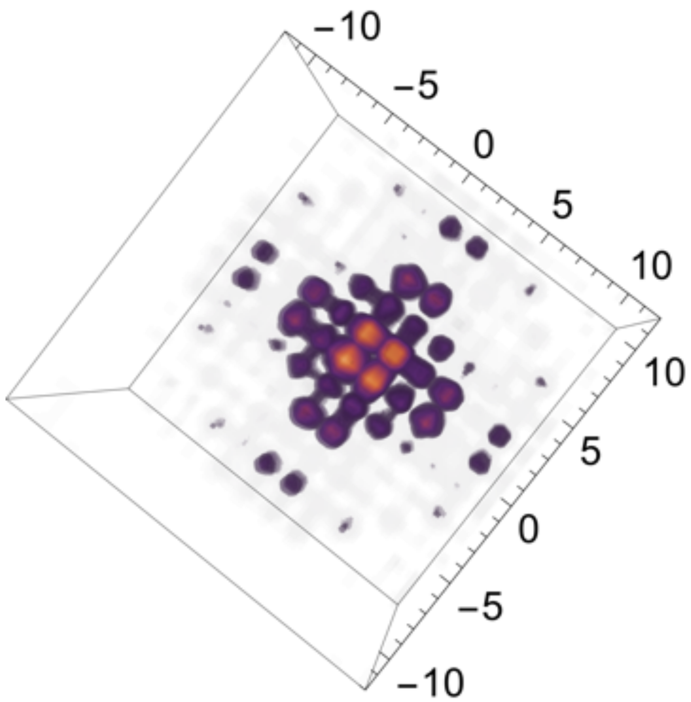
↓

Localization

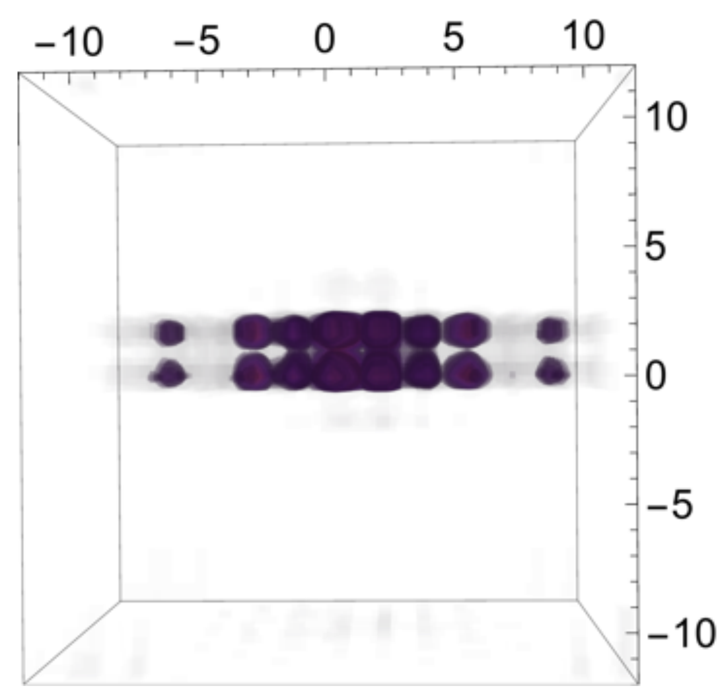
II. Brane theories dynamics

3D QW

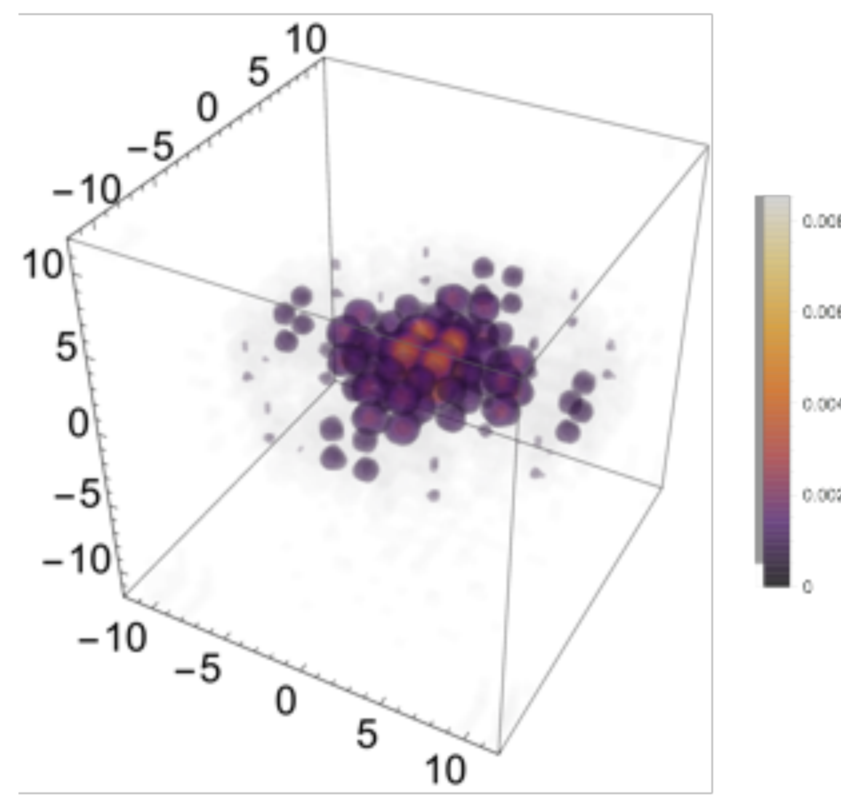
$j = 20, \lambda = 90, h = 4, m = 11$



x-y side



x-z side



The potential embeds a 3D QW in a 2D spacetime lattice

II. Brane theories dynamics

Summary

It can be established a parallelism between a high-energy QFT and a QW model

This QW shows localization from a coin which changes in space in a regular manner

Index

II. Electromagnetic gauge invariance in 2D QW

Gauge invariance plays a fundamental role in theoretical physics

From classical electrodynamics

$$\mathscr{L} = \frac{1}{2} m \dot{\overrightarrow{x}}^2 + q \dot{\overrightarrow{x}} \cdot \overrightarrow{A} - q\phi \qquad \overrightarrow{E} = -\nabla\phi - \partial_t \overrightarrow{A} \qquad \overrightarrow{B} = \nabla \times \overrightarrow{A}$$

Gauge transformation $\phi \to \phi - \partial_t \alpha \qquad \overrightarrow{A} \to \overrightarrow{A} + \overrightarrow{\nabla}\alpha$

$$\mathscr{L}(t) \to \mathscr{L}'(t) = \mathscr{L} + q\frac{d}{dt}\alpha$$

$\int \mathscr{L} dt$ **Invariant!**

II. Electromagnetic gauge invariance in 2D QW

Gauge invariance in quantum mechanics

Local gauge invariance $\psi(\vec{x},t) \rightarrow \psi'(\vec{x},t) = e^{i\alpha(\vec{x},t)}\psi(\vec{x},t)$

$$i\hbar\partial_t\psi'(\vec{x},t) = -\frac{\hbar^2}{2m}\nabla^2\psi'(\vec{x},t) + U(\vec{x})\psi'(\vec{x},t)$$

$$i\hbar\partial_t\psi(\vec{x},t) = -\frac{\hbar^2}{2m}\left(\nabla + iq\vec{A}(\vec{x},t)\right)^2\psi(\vec{x},t) + \left(U(\vec{x}) + \hbar\phi(\vec{x},t)\right)$$

Gauge fields appears in order to preserve invariance

$$\phi(\vec{x},t) \rightarrow \phi(\vec{x},t) - \partial_t\alpha(\vec{x},t) \qquad \vec{A}(\vec{x},t) \rightarrow \vec{A} + \frac{1}{q}\nabla\alpha(\vec{x},t)$$

Gauge invariance is one of the main principles to develop successful theories such a the standard model

II. Electromagnetic gauge invariance in 2D QW

What about QWs?

G. Di Molfetta, F. Debbasch, and M. Brachet, Quantum walks in artificial electric and gravitational fields, Physica A **397**, 157 (2014).

Pablo Arnault and Fabrice Debbasch. “Quantum walks and discrete gauge theories”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 93.5 (2016), pp. 1–6.

Miquel Montero. “Invariance in Quantum Walks”. In: *Research Advances in Quantum Dynamics*.

The way of implementing gauge invariance is not unique

II. Electromagnetic gauge invariance in 2D QW

Let us define a 1D QW

$$|\psi_{j+1}\rangle = U_{j+1}|\psi_j\rangle \qquad C(\theta) = e^{i\sigma^1\frac{\theta}{2}} = \begin{bmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$U = SC \qquad S(\alpha_j, \xi_j) = \begin{bmatrix} e^{-i\mathcal{K}}e^{i(\xi_j-\alpha_j)} & 0 \\ 0 & e^{-i(\xi_j+\alpha_j)}e^{i\mathcal{K}} \end{bmatrix}$$

$$= Te^{i(\beta_-)_j}\Lambda_R + e^{-i(\beta_+)_j}T^\dagger\Lambda_L$$

$$\beta_- = \xi - \alpha$$
$$\beta_+ = \xi + \alpha$$

The order matters!

Continuous limit

$$\alpha_{j,p} = \epsilon_A q A^0_{j,p}$$
$$\xi_{j,p} = \epsilon_A q A^1_{j,p}$$

Gauge fields

II. Electromagnetic gauge invariance in 2D QW

U(1) lattice gauge invariance

$$\psi_{j,p} \rightarrow \psi'_{j,p} = e^{iq\chi_{j,p}} \psi_{j,p}$$

Gauge fields transforms as:

$$(A'_\mu)_{j,p} = (A_\mu)_{j,p} - (d_\mu \chi)_{j,p}$$

$$d_0 = \frac{1}{\epsilon_A} \Delta_0 \Sigma_1 \qquad (\Sigma_\mu Q)_{p_\mu} = Q_{p_\mu + 1} + Q_{p_\mu}$$

$$d_1 = \frac{1}{\epsilon_A} \Delta_1 \Sigma_0 \qquad (\Delta_\mu Q)_{p_\mu} = Q_{p_\mu + 1} - Q_{p_\mu}$$

Time and space is treated equally on the contrary to other models

Discrete derivatives are similar as those used in standard LGT, except by the use of $\Sigma' s$

II. Electromagnetic gauge invariance in 2D QW

U(1) lattice gauge invariance in 2D $U^{2D}_{j+1} = U^{(2)}_{j+1} U^{(1)}_j$

$$\psi_{j,p} \rightarrow \psi'_{j,p} = e^{iq\chi_{j,p}} \psi_{j,p}$$

Gauge fields transforms as:

$$(A'_0)_{j,p,q} = \begin{cases} (A_0)_{j,p,q} - (d_0^1 \chi)_{j,p,q} & \text{for } j \text{ even} \\ (A_0)_{j,p,q} - (d_0^2 \chi)_{j,p,q} & \text{for } j \text{ odd} \end{cases}$$

$$(A'_i)_{j,p,q} = (A_i)_{j,p,q} - (d_i \chi)_{j,p,q}$$

$$d_0^i = \frac{1}{\epsilon_A} \Delta_0 \Sigma_i$$

$$d_i = \frac{1}{\epsilon_A} \Delta_i \Sigma_0$$

Again we treat space and time on the same footing

Both directions of the lattices are treated equally

II. Electromagnetic gauge invariance in 2D QW

The way of implementing gauge invariance is not unique!

Pablo Arnault and Fabrice Debbasch. “Quantum walks and discrete gauge theories”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 93.5 (2016), pp. 1–6.

They introduce the gauge fields on the coin, without sub-step

$$d_0 = (L - \Sigma_2\Sigma_1)/\epsilon_A, \quad d_1 = \Delta_1/\epsilon_A, \quad d_2 = \Delta_2\Sigma_1/\epsilon_A$$

$$\begin{aligned}
(LQ)_{j,p,q} &= Q_{j+1,p,q} \\
(\Sigma_1 Q)_{j,p,q} &= \left(Q_{j,p+1,q} + Q_{j,p-1,q}\right)/2 \\
(\Sigma_2 Q)_{j,p,q} &= \left(Q_{j,p,q+1} + Q_{j,p,q-1}\right)/2 \\
(\Delta_1 Q)_{j,p,q} &= \left(Q_{j,p+1,q} - Q_{j,p-1,q}\right)/2 \\
(\Delta_2 Q)_{j,p,q} &= \left(Q_{j,p,q+1} - Q_{j,p,q-1}\right)/2
\end{aligned}$$

Our discrete derivatives are simpler

However, we have two time derivatives

II. Electromagnetic gauge invariance in 2D QW

Continuity equation and conserved current

The continuity equation can be defined as:

$$[\Delta_0^{\text{sym.}} J^0](t,x,y) = \frac{1}{2}\langle\psi(t)|\left(U^\dagger(t)\Lambda_{x,y}U(t) - U(t)\Lambda_{x,y}U^\dagger(t)\right)|\psi(t)\rangle$$

↓

$$\Delta_\mu^{\text{sym.}} J^\mu = 0$$

$$[\Delta_0^{\text{sym.}} f]\,(t,x,y) \equiv \frac{1}{2}\left(f(t+\epsilon,x,y) - f(t-\epsilon,x,y)\right)$$

$$[\Delta_1^{\text{sym.}} f]\,(t,x,y) \equiv \frac{1}{2}\left(f(t,x+\epsilon,y) - f(t,x-\epsilon,y)\right)$$

$$[\Delta_2^{\text{sym.}} f]\,(t,x,y) \equiv \frac{1}{2}\left(f(t,x,y+\epsilon) - f(t,x,y-\epsilon)\right)$$

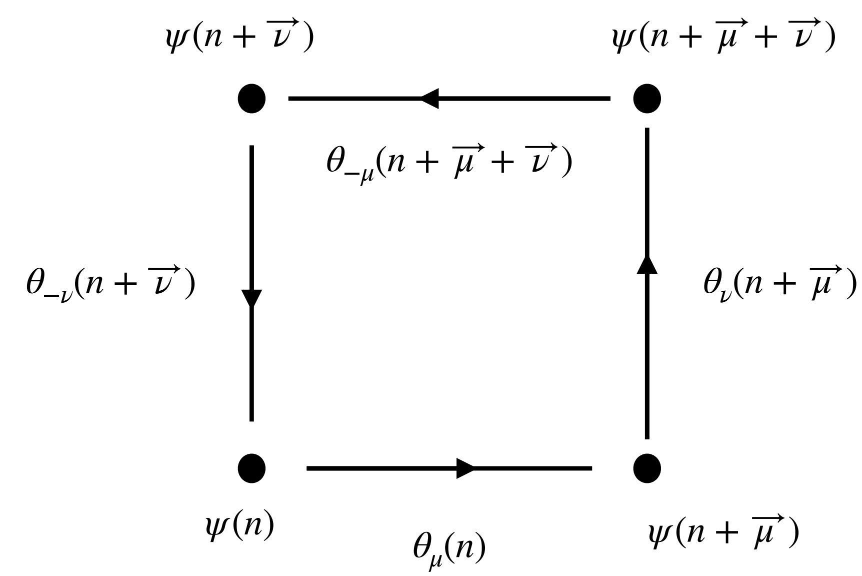
All three coordinates are treated equally

Truncation error $O(\epsilon^3)$

II. Electromagnetic gauge invariance in 2D QW

Close analogies to LGT

Discrete derivatives are similar as those used in standard LGT, except by the use of $\Sigma' s$



Unified framework to understand U(1) gauge invariance, in DTQW with coin spaces of arbitrary dimensions

C. Cedzich, T. Geib, A. H. Werner, et al. "Quantum walks in external gauge fields". In: *Journal of Mathematical Physics* 60.1 (2019)

II. Electromagnetic gauge invariance in 2D QW

QWs allow us to study in a deeper way physical models, i.e. relativistic particles

from discrete to continuous

This approach could potentially be used to understand wider theories such as Quantum field theory, Quantum Gravity…

A quantum cellular automaton for one-dimensional QED. P.Arrighi, C.Bény, T.Farrely arXiv:1903.07007

What happens if we change the geometry of the lattice?

Index

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  - From QWs to the Dirac equation
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  - Electromagnetic gauge invariance in 2D
III. QWs in hexagonal and triangular lattices
  - Continuous limit: Dirac dynamics in free and curved space-time
IV. Conclusions

III. QWs in hexagonal and triangular lattices

Previous work

Topological phases

Takuya Kitagawa, Mark S. Rudner, Erez Berg, et al. “Exploring topological phases with quantum walks”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 82.3 (2010)

Search algorithms

G. Abal, R. Donangelo, F. L. Marquezino, et al. “Spatial search on a honeycomb network”. Mathematical Structures in Computer Science 20.6 (2010), pp. 999–1009

G. Abal, R. Donangelo, M. Forets, et al. “Spatial quantum search in a triangular network”. Mathematical Structures in Computer Science 22.3 (2012), pp. 521–531. issn: 09601295.

Localization processes

Changyuan Lyu, Luyan Yu, and Shengjun Wu. “Localization in quantum walks on a honeycomb network”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 92.5 (2015)

III. QWs in hexagonal and triangular lattices

Possible physical implementation of QWs on graphene

Ioannis G. Karafyllidis. “Quantum walks on graphene nanoribbons using quantum gates as coins”. In: *Journal of Computational Science* 11 (2015)

Hamza Bougroura, Habib Aissaoui, Nicholas Chancellor, et al. “Quantum- walk transport properties on graphene structures”. In: *Physical Review A* 94.6 (2016)

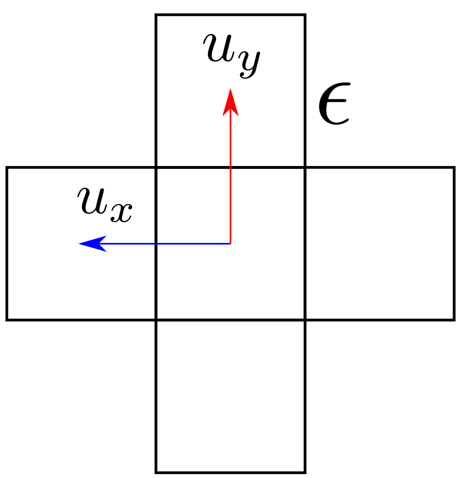
Can we also the Dirac equation with other background lattices?

P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. The Dirac equation as a quantum walk over the honeycomb and triangular lattices. Phys. Rev. A 97, 062111

P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. From curved spacetime to spacetime-dependent local unitaries over the honeycomb and triangular Quantum Walks. *Scientific Reports* **volume 9**, Article number: 10904 (2019)

III. Dirac dynamics in free and curved space-time

2D QW in the grid



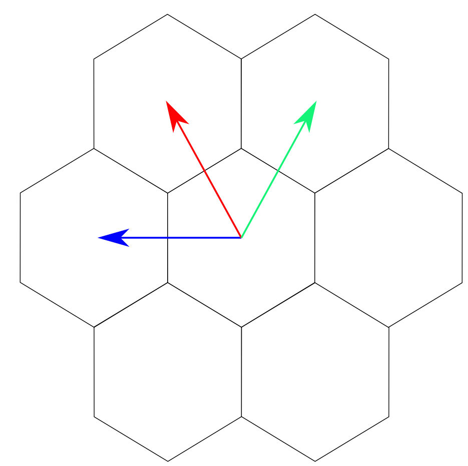
Continuous limit

$$\epsilon \to 0$$

Dirac Hamiltonian 2D

$$H_D = p_x\sigma_x + p_y\sigma_y + m\sigma_z$$

2D QW in a hexagonal lattice



$$u_i = \cos{(i\frac{2\pi}{3})}u_x + \sin{(i\frac{2\pi}{3})}u_y$$

$$i = 0, 1, 2$$

Continuous limit

$$\epsilon \to 0$$

Equivalent Dirac Hamiltonian 2D

$$H'_D = H_D$$

$$H'_D = \sum_i \pi_i\tau_i + m\sigma_z$$

Momentum along *i* direction

$$\pi_i = u_i^\nu p_\nu$$

New unitary matrices

$$\nu = 1, 2$$

III. Dirac dynamics in free and curved space-time

In order to find the τ_i matrices

i) $$\sum_i \pi_i \tau_i = \sigma_x p_x + \sigma_y p_y$$

ii) Eigenvalues $\{1, -1\}$

$$\tau_i = U_i^\dagger \sigma_z U_i$$

→

Unique solution, up to a sign

$$\tau_0 = \frac{2}{3}\sigma_x + \xi\sigma_z$$

$$\tau_1 = -\frac{1}{3}\sigma_x + \frac{\sqrt{3}}{3}\sigma_y + \xi\sigma_z$$

$$\tau_2 = -\frac{1}{3}\sigma_x - \frac{\sqrt{3}}{3}\sigma_y + \xi\sigma_z$$

$$\xi = \pm\frac{\sqrt{5}}{3}$$

III. Dirac dynamics in free and curved space-time

Let's define a QW over this hexagonal grid such a in the continuous limit recovers H_D

Evolution of a state $$|\psi(t+\epsilon)\rangle = e^{-i\epsilon H_D}\,|\psi(t)\rangle$$

$$H_D = \sum_i \pi_i \tau_i + m\sigma_z$$

Using Lie-Trotter product formula

$$e^{-\mathrm{i}\epsilon\left(\sum_i \pi_i\tau_i + m\sigma_z\right)} \approx \prod_{i=0}^{2} e^{-\mathrm{i}\epsilon m\sigma_z} e^{-\mathrm{i}\epsilon\pi_i\tau_i}$$

removing second order epsilon terms

III. Dirac dynamics in free and curved space-time

Making use of the condition that the eigenvalues are $\{1, -1\}$

$$e^{-\mathrm{i}\epsilon\tau_i\pi_i} = e^{-\mathrm{i}\epsilon U_i^\dagger \sigma_z U_i \pi_i} = U_i^\dagger e^{-\mathrm{i}\epsilon\sigma_z\pi_i} U_i = U_i^\dagger T_{i,\epsilon} U_i$$

Translation operator along u_i

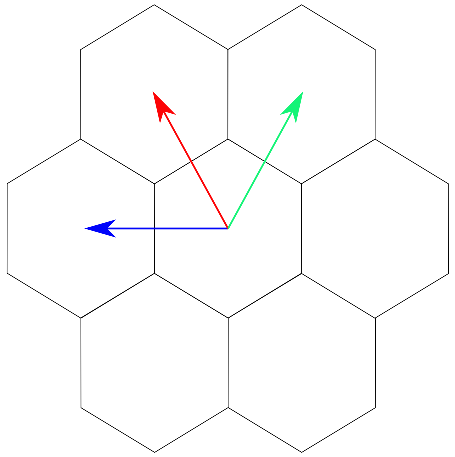
III. Dirac dynamics in free and curved space-time

QW over the hexagonal lattice

$$|\widetilde{\psi}(t+\varepsilon)\rangle = (WT_{2,\varepsilon}WT_{1,\varepsilon}WT_{0,\varepsilon})\,|\widetilde{\psi}(t)\rangle$$

$$W = U_0 S U_0^{\dagger} M \qquad |\widetilde{\psi}(t)\rangle \equiv U_0\,|\psi(t)\rangle$$

$$S = e^{\mathrm{i}\frac{\pi}{3}}\mathcal{R}_{\sigma_z}(\frac{2\pi}{3})$$



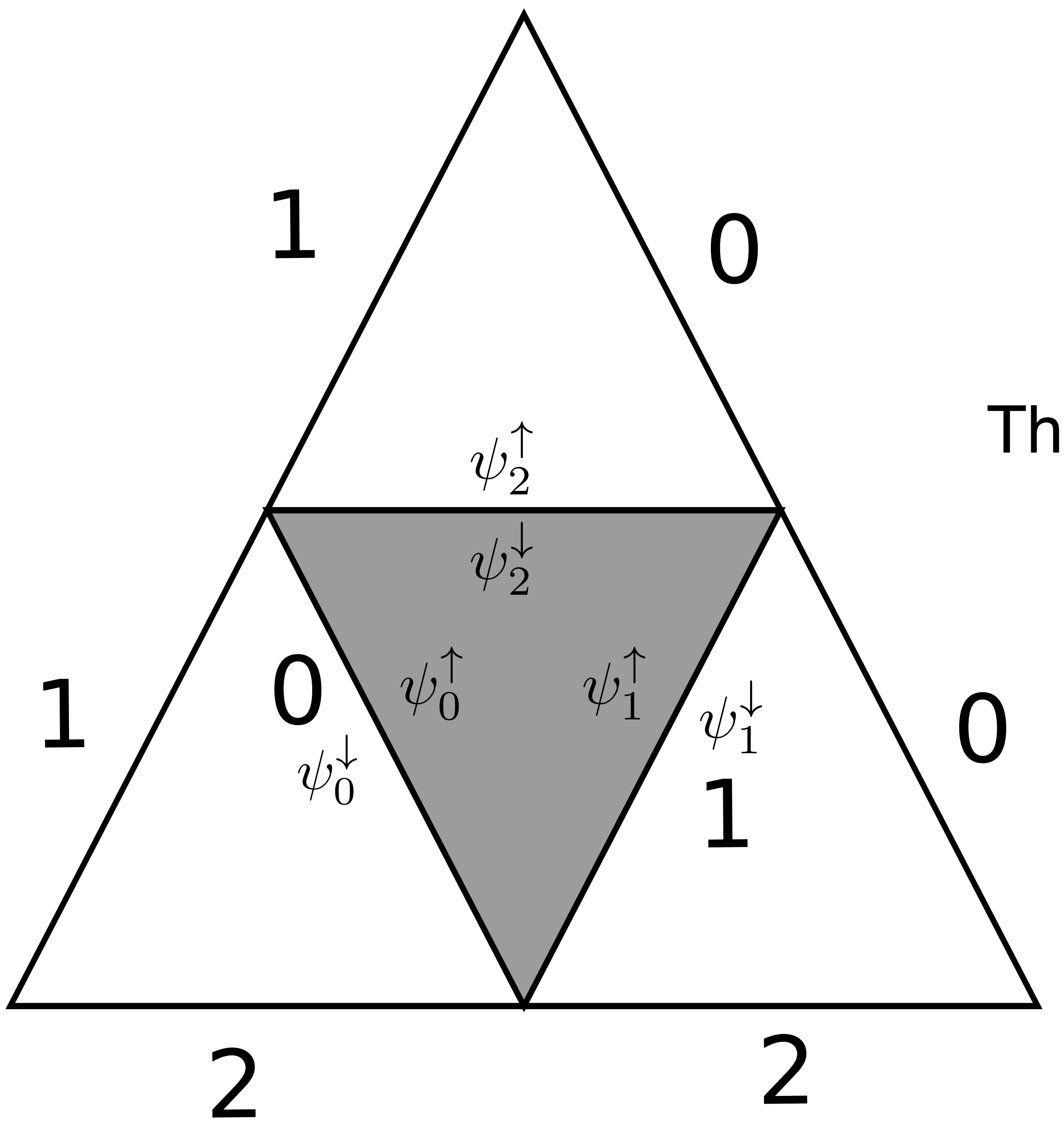
By construction, Dirac eq is recover when $\epsilon \to 0$

Having understood the honeycomb QW, it will be easier to tackle the triangular case

P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. The Dirac equation as a quantum walk over the honeycomb and triangular lattices. Phys. Rev. A 97, 062111

III. Dirac dynamics in free and curved space-time

Triangular QW

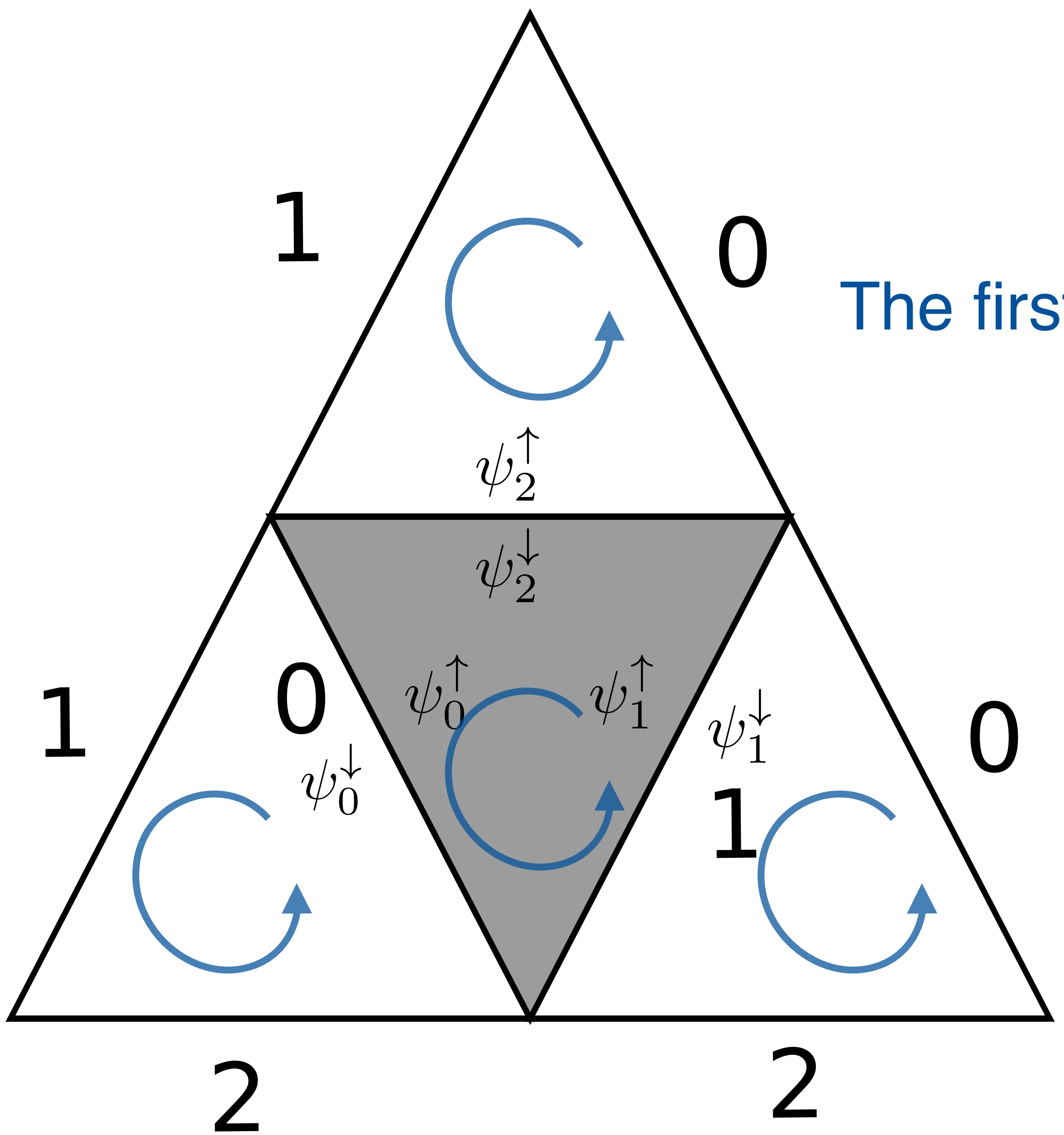


The walker lies on the edges

The dynamics: composition of two operators

III. Dirac dynamics in free and curved space-time

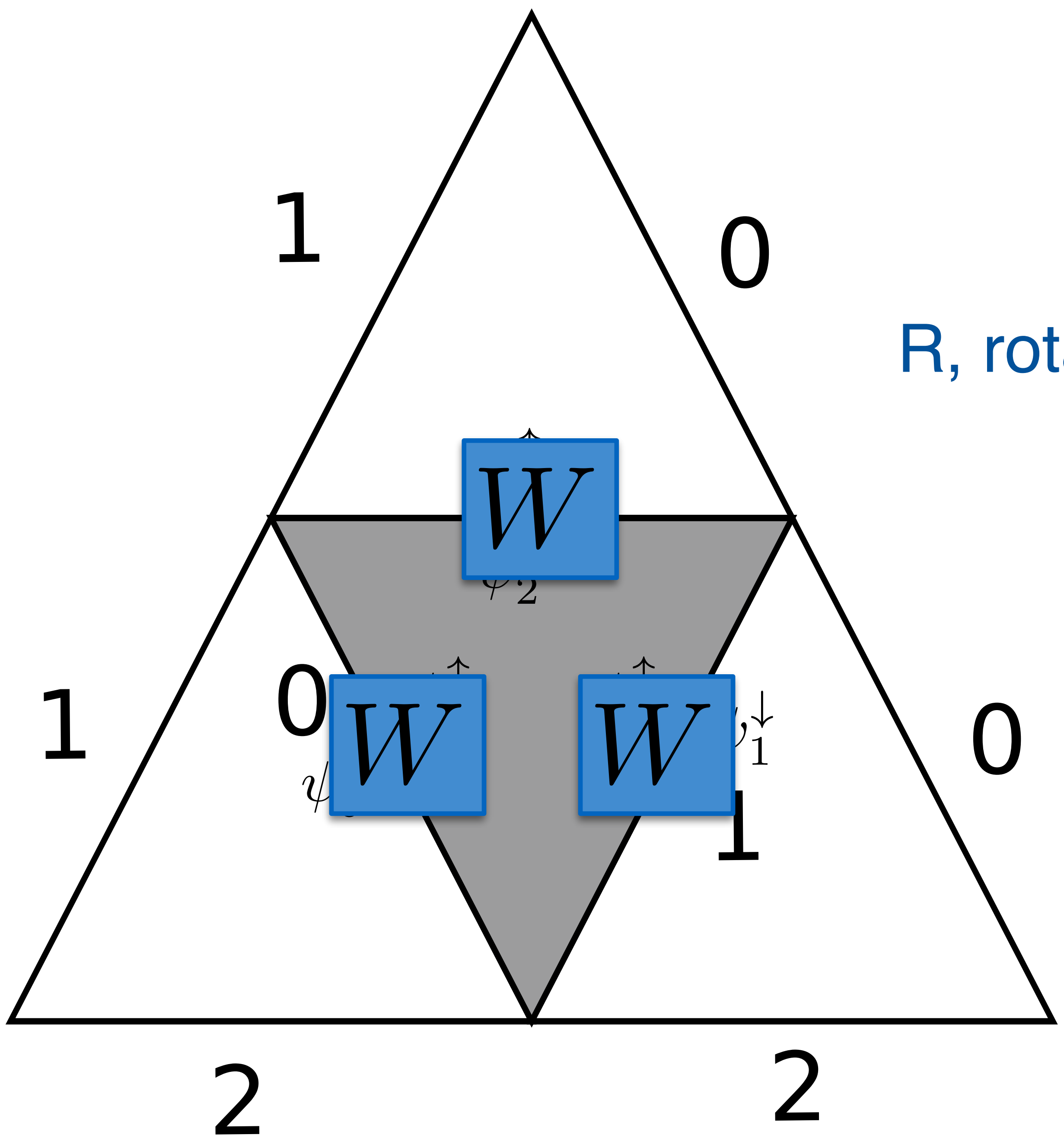
Triangular QW



The first operator, R, rotates every triangle anti-clockwise

III. Dirac dynamics in free and curved space-time

Triangular QW

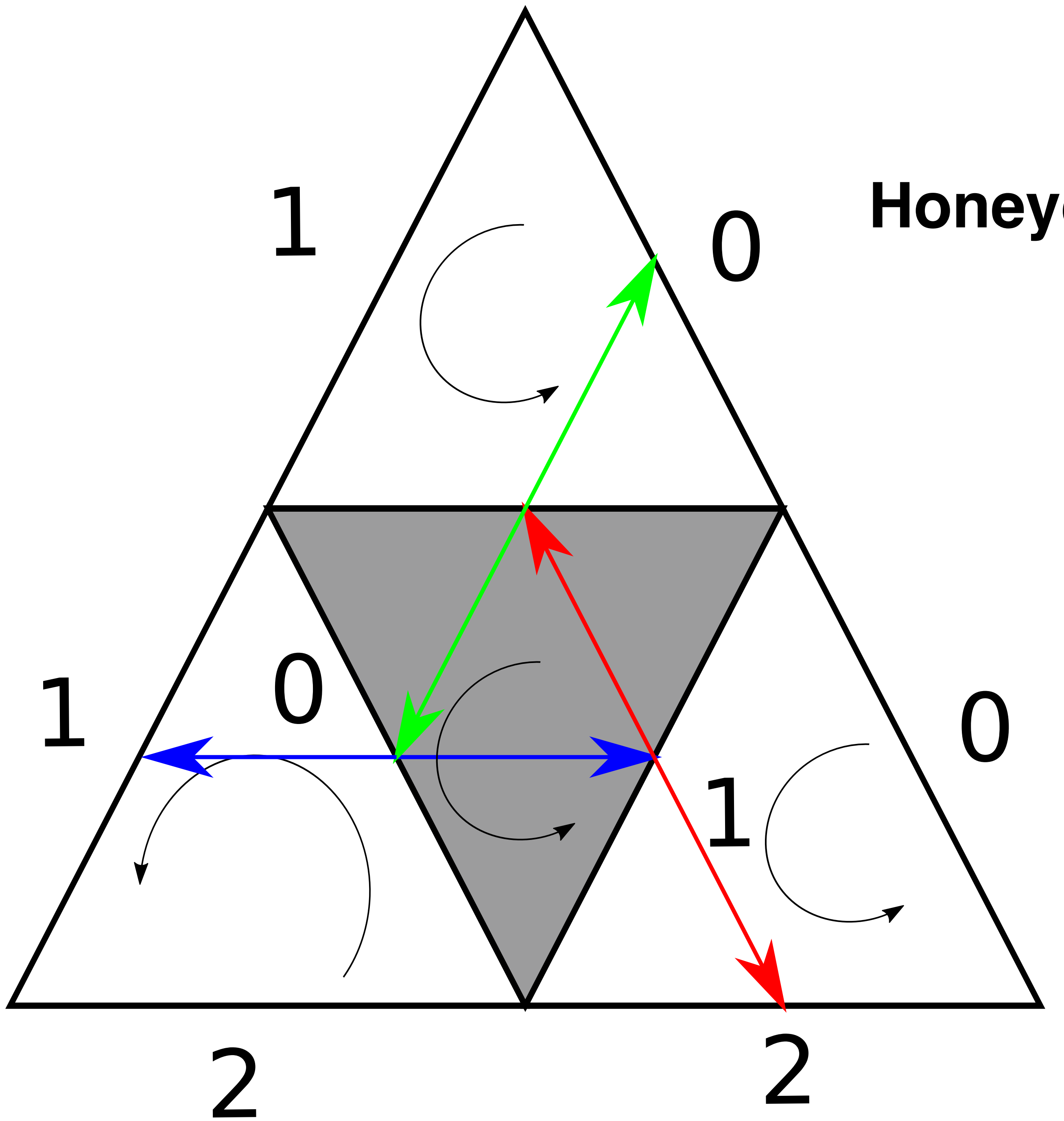


R, rotates every triangle anti-clockwise

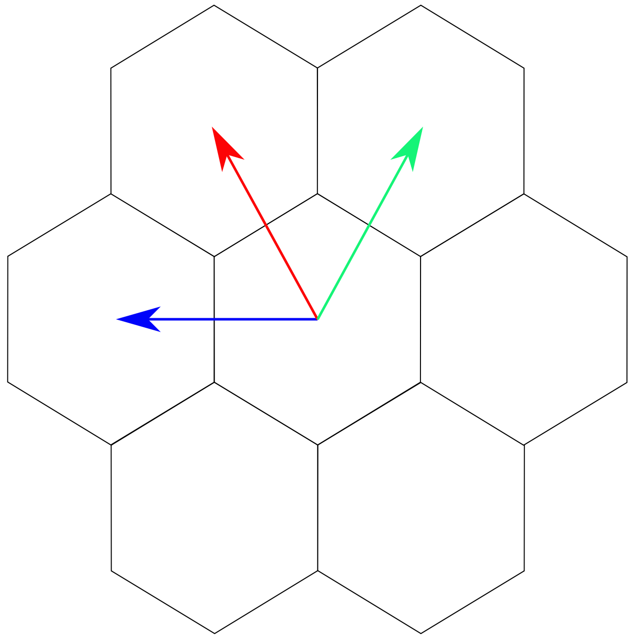
W, U(2) matrix

$$W = U_0 S U_0^\dagger M$$

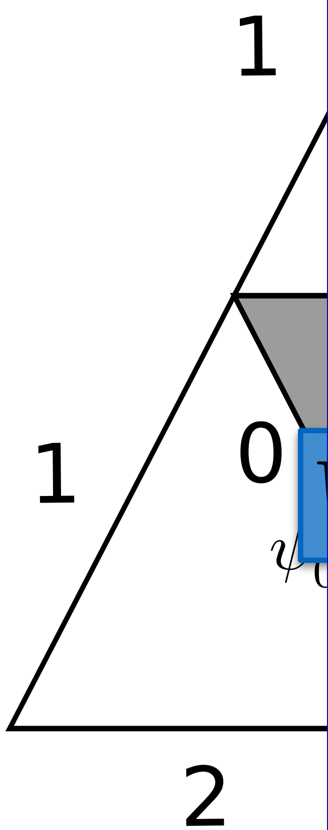
III. Dirac dynamics in free and curved space-time

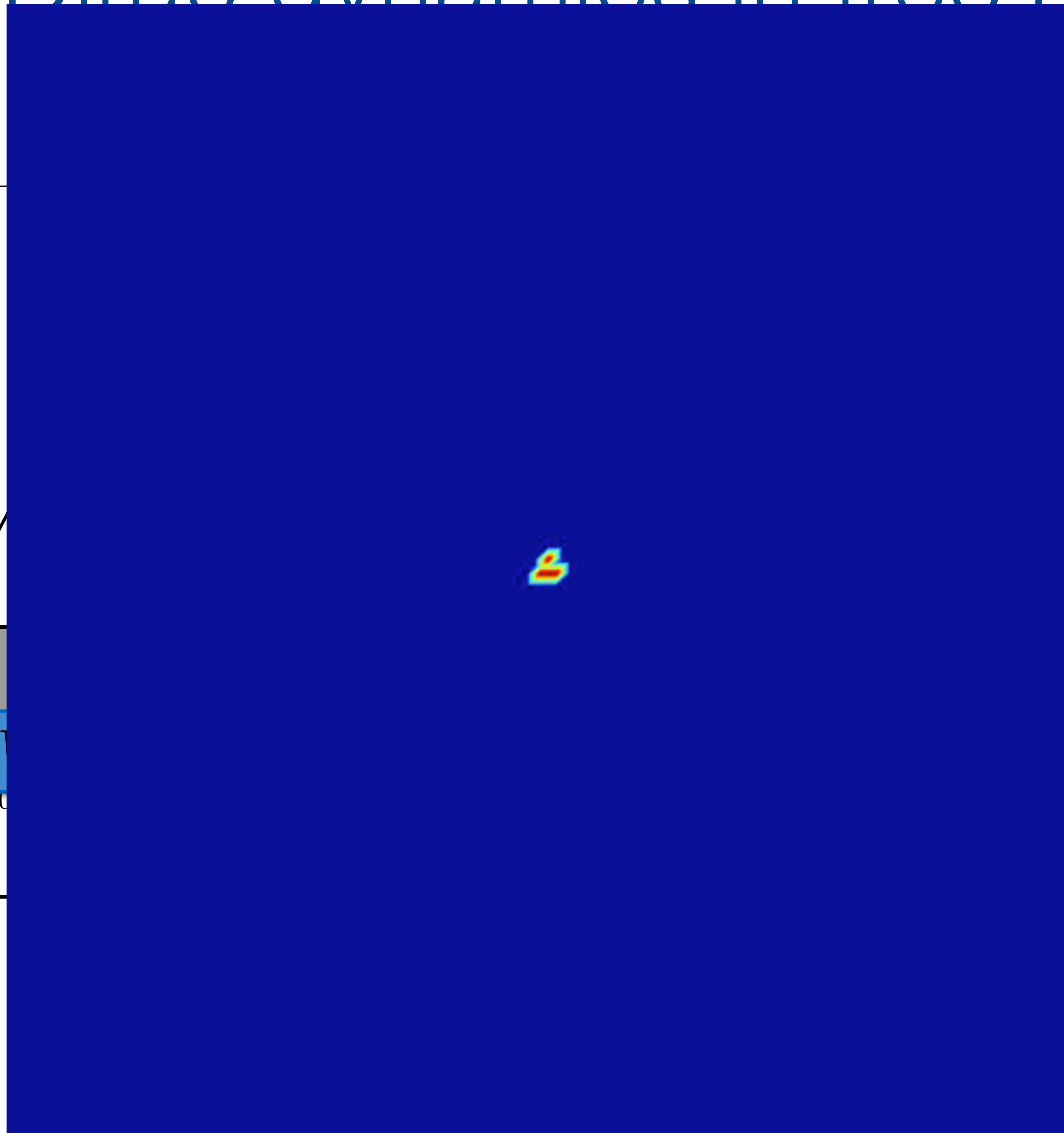


Honeycomb QW dynamics in a covert way in 3 time-steps



III. Dirac dynamics in free and





clockwise

III. Dirac dynamics in free and curved space-time

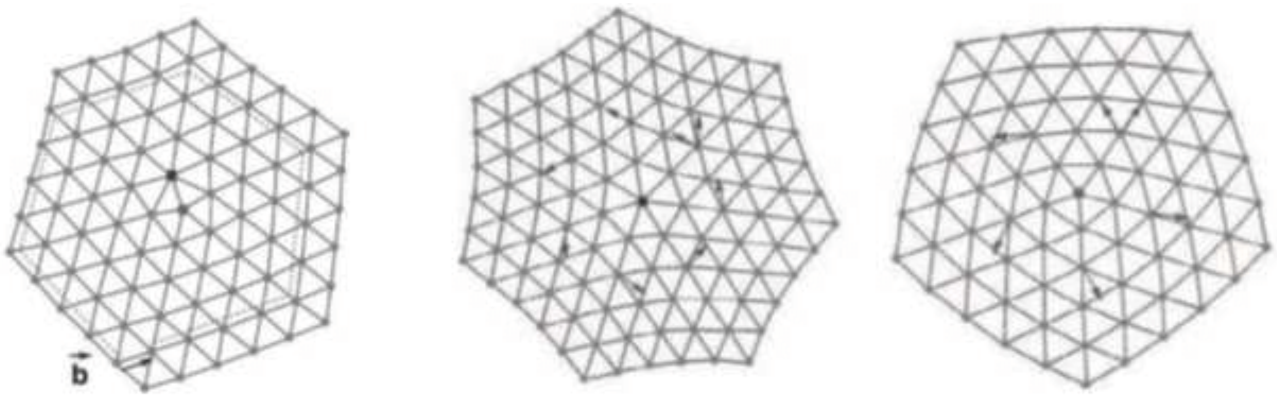
We recover again the Dirac eq in the continuous limit

$$\mathrm{i}\partial_t \psi = \left(p_x \sigma_x + p_y \sigma_y + m\sigma_z\right)\psi$$

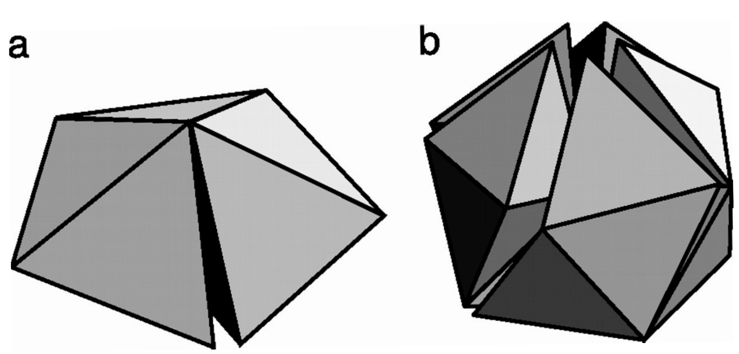
P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. The Dirac equation as a quantum walk over the honeycomb and triangular lattices. Phys. Rev. A 97, 062111

III. Dirac dynamics in free and curved space-time

Defects/Deformation



3D



http://www.pnas.org/content/103/28/10612.full

III. Dirac dynamics in free and curved space-time

Understanding propagation in discretized curved spacetime

Dirac equation in 2+1 curved space-time

Tetrad field $g_{\mu\nu}(x) = e_\mu{}^a(x) e_\nu{}^b(x) \eta_{ab}$

$$i\partial_t \chi + \frac{i}{2}\{B^s, \partial_s\}\chi - \frac{m}{e^t{}_0}\beta\chi = 0,$$

$$\chi = g^{1/4}(e^t{}_0)^{1/2}\psi \qquad B^s = \alpha^a \frac{e^s{}_a}{e^t{}_0} \qquad \beta \equiv \gamma^0 \qquad \alpha^a \equiv \beta\gamma^a$$

III. Dirac dynamics in free and curved space-time

Previous works

Giuseppe Di Molfetta, Marc Brachet, and Fabrice Debbasch. Quantum walks in artificial electric and gravitational fields. *Physica A: Statistical Mechanics and its Applications*, 397:157–168, 2014.

P. Arrighi and F. Facchini. Quantum walking in curved space-time: (3+1) dimensions, and beyond. *Quantum Information and Computation*, 17(9-10):0810–0824, 2017

Pablo Arnault and Fabrice Debbasch. "Quantum walks and gravitational waves". In: *Annals of Physics* 383 (2017),

Simpler scheme to simulate curvature

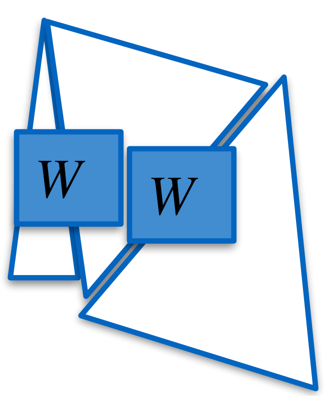
P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. From curved spacetime to spacetime-dependent local unitaries over the honeycomb and triangular Quantum Walks. *Scientific Reports* **volume 9**, Article number: 10904 (2019)

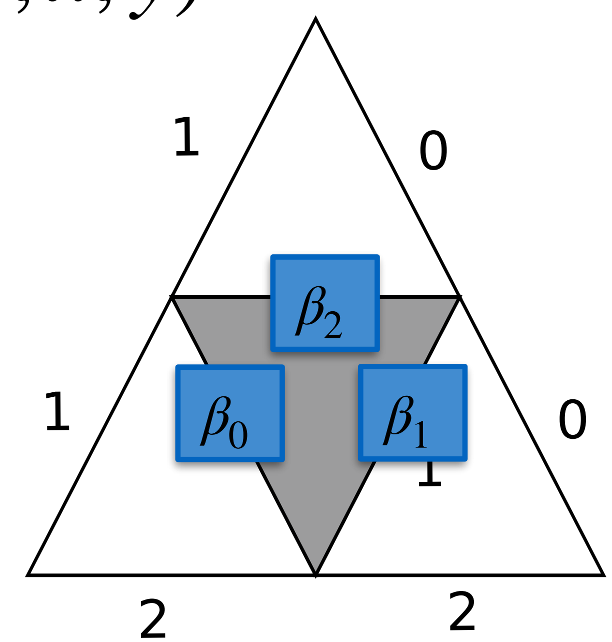
III. Dirac dynamics in free and curved space-time

Starting from the free QW, we will impose a local deformation to obtain the curved Dirac QW

Duality

Instead of deforming the lattice, we absorb the deformation in a set of matrices $\beta^i(t, x, y)$





$$\Lambda_k^j(t,x,y)u_i^k\tau^i = u_i^j\beta^i(t,x,y)$$

III. Dirac dynamics in free and curved space-time

Instead of deforming the lattice, we absorb the deformation in a set of matrices $\beta^i(t,x,y)$

i) $$\Lambda_k^j(t,x,y)u_i^k\tau^i = u_i^j\beta^i(t,x,y)$$

ii) $$\beta^i(t,x,y) = U_i^\dagger(t,x,y)\sigma^z U_i(t,x,y)$$

In square lattices this duality does not work!

$$\Lambda_k^j\sigma^k = \beta^j \longrightarrow \boxed{\sum_k (\Lambda_k^j)^2 = 1}$$

III. Dirac dynamics in free and curved space-time

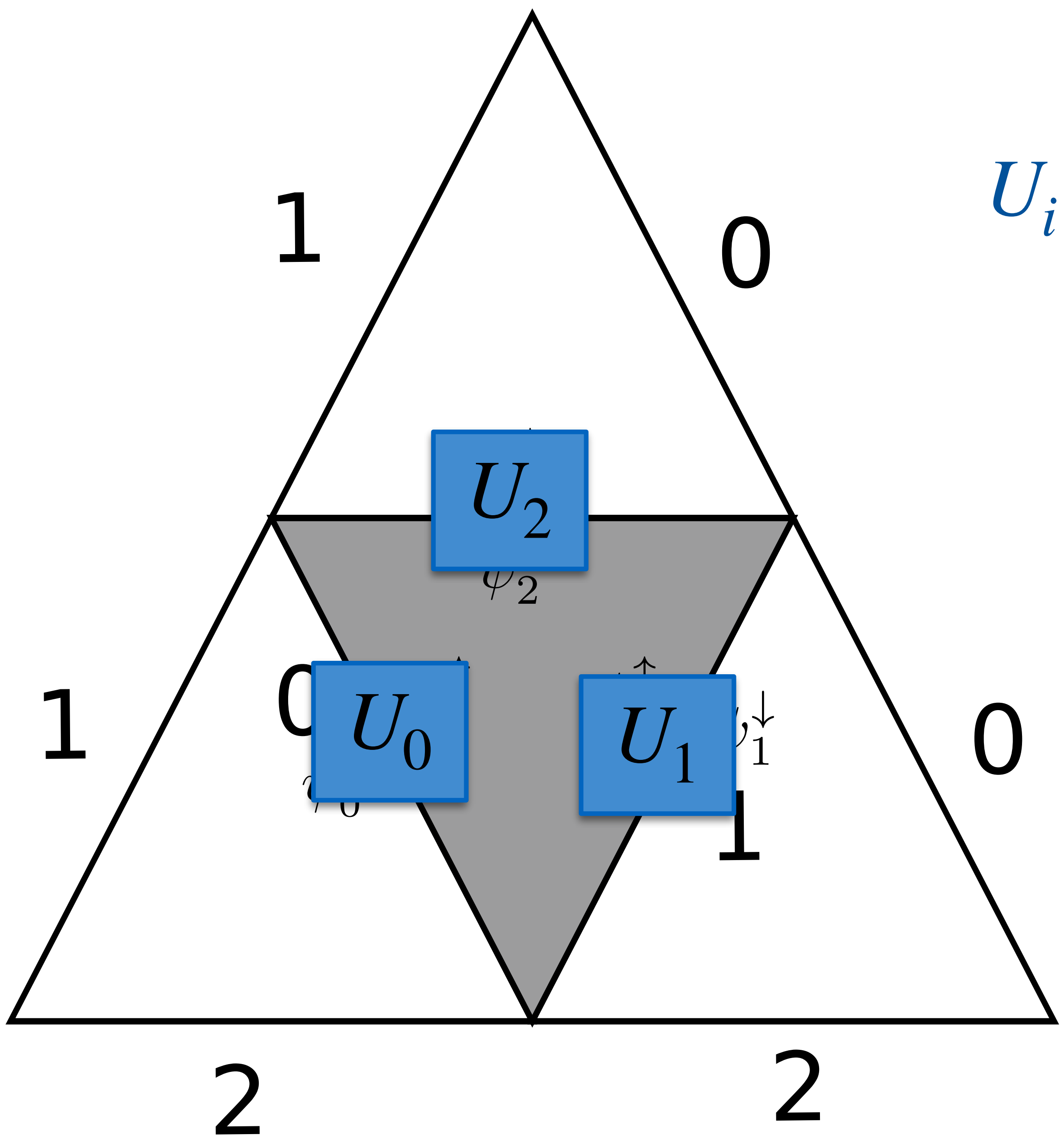
Honeycomb curved QW

$$\psi(t+\epsilon) = e^{-i\widetilde{m}\varepsilon\sigma^z} \prod_i U^\dagger{}_i T_i U_i e^{-i\epsilon\gamma_i} \psi(t)$$

$$\beta_i = U_i^\dagger \sigma_z U_i = \begin{pmatrix} \cos\theta_i & e^{-i\phi_i}\sin\theta_i \\ e^{i\phi_i}\sin\theta_i & -\cos\theta_i \end{pmatrix} \qquad U_i = \begin{pmatrix} e^{\frac{i\phi_i}{2}}\cos\frac{\theta_i}{2} & e^{-\frac{i\phi_i}{2}}\sin\frac{\theta_i}{2} \\ -e^{\frac{i\phi_i}{2}}\sin\frac{\theta_i}{2} & e^{-\frac{i\phi_i}{2}}\cos\frac{\theta_i}{2}, \end{pmatrix}.$$

III. Dirac dynamics in free and curved space-time

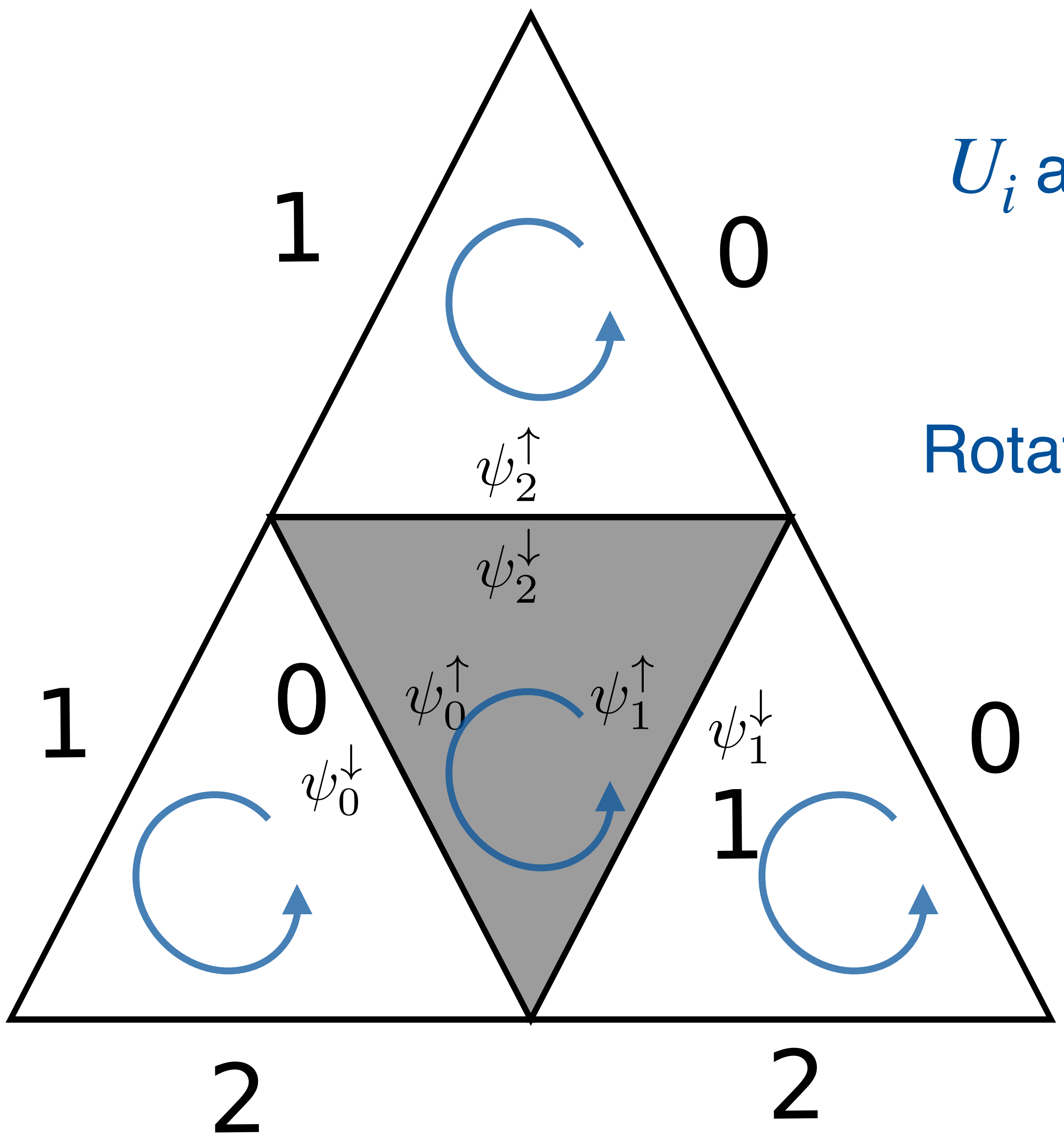
Triangular QW



U_i are applied (now are space-time dependent)

III. Dirac dynamics in free and curved space-time

Triangular QW

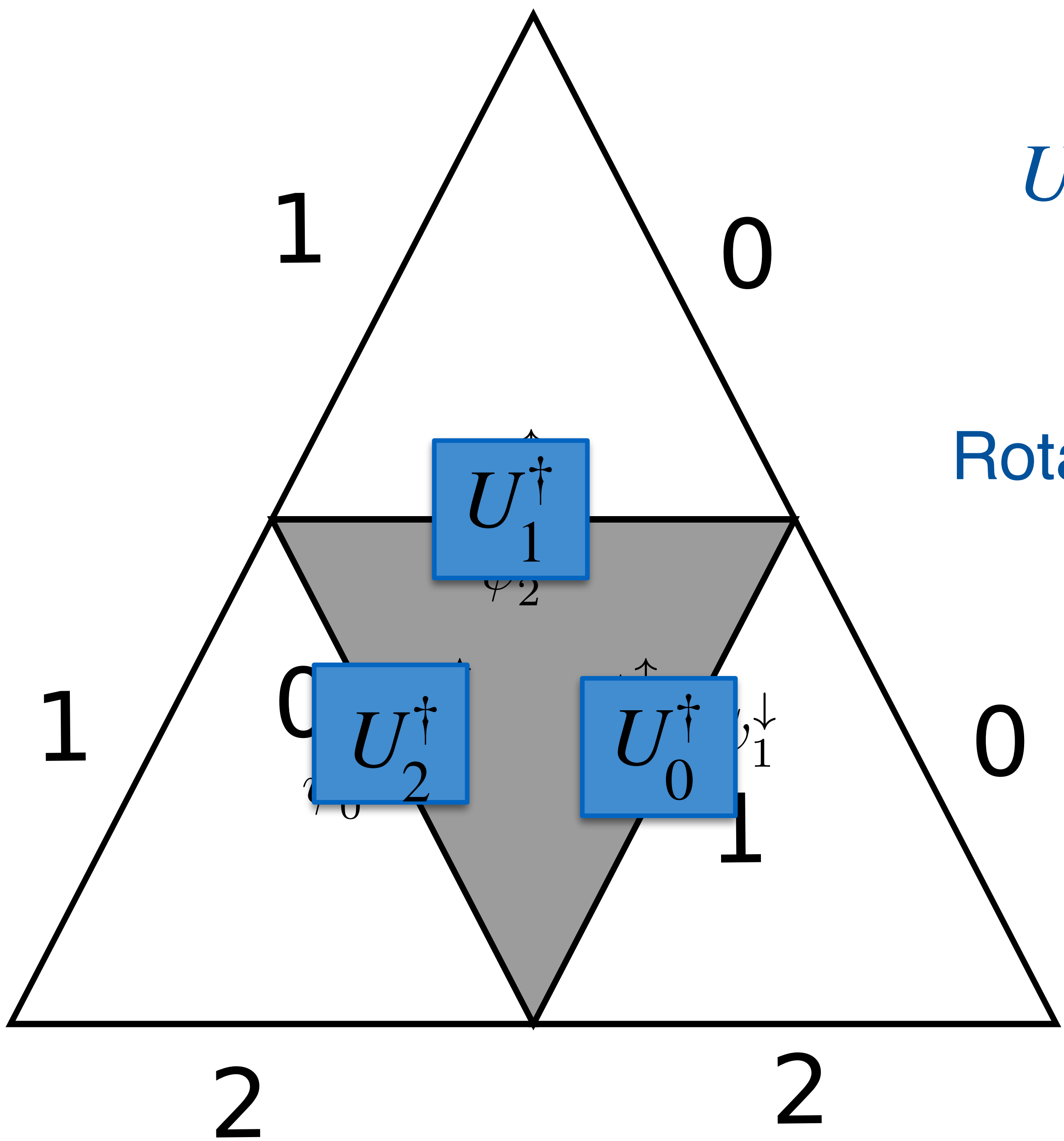


U_i are applied (now are space-time dependent)

Rotation at every triangle anti-clockwise

III. Dirac dynamics in free and curved space-time

Triangular QW



U_i are applied (now are space-time dependent)

Rotation at every triangle anti-clockwise

$U_i^\dagger$ are applied

III. Dirac dynamics in free and curved space-time

Triangular QW

The evolution can be written as:

$$\psi(t+\varepsilon/3, v, k) = U_i^\dagger(t,v,k)\left[P^\uparrow U_i(t,v,k-1)e^{-i\epsilon\gamma_i}\psi(t,v,k-1) \oplus P^\downarrow U_i(t,e(v,k),k-1)e^{-i\epsilon\gamma_i}\psi(t,e(v,k),k-1)\right]$$

After 3 time-step, in the continuum we recover the curved Dirac equation

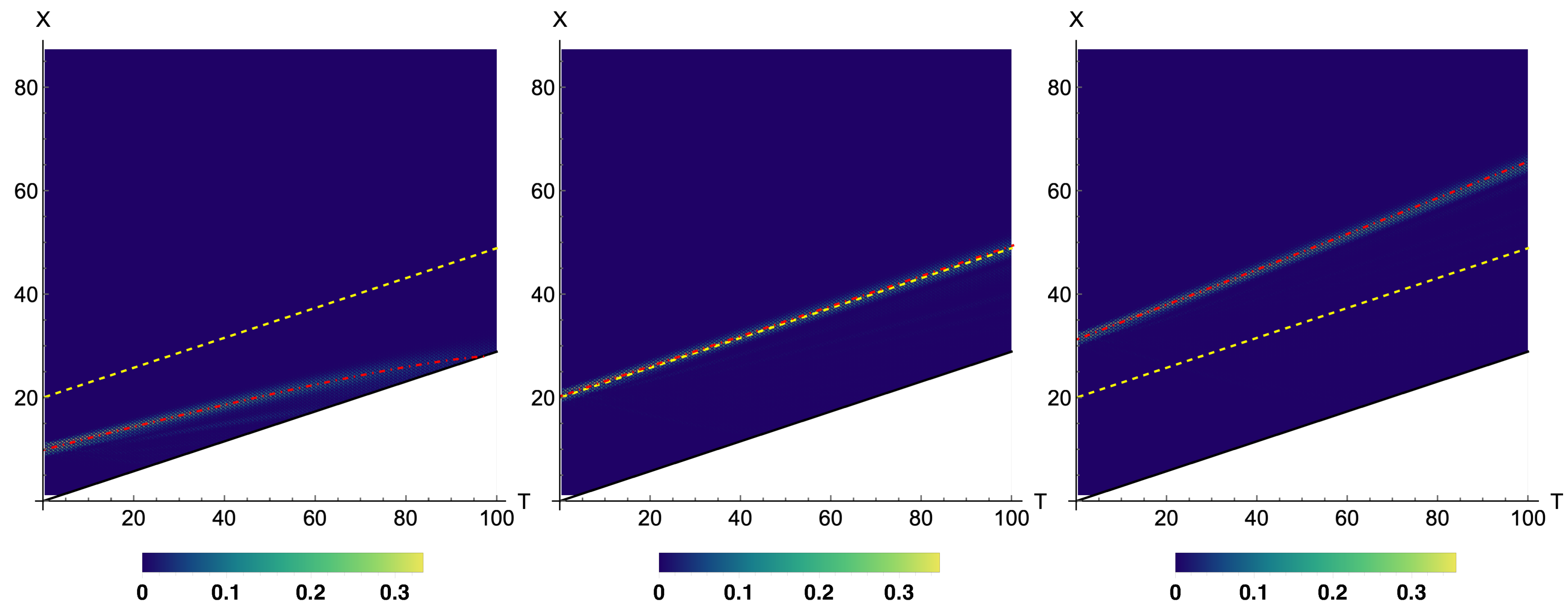
$$\partial_t\psi = (U_0^\dagger\sigma^z U_0 - \frac{1}{2}U_1^\dagger\sigma^z U_1 - \frac{1}{2}U_2^\dagger\sigma^z U_2)\partial_x\psi + \frac{\sqrt{3}}{2}(U_1^\dagger\sigma^z U_1 - U_2^\dagger\sigma^z U_2)\partial_y\psi +$$
$$\partial_x(U_0^\dagger\sigma^z U_0 - \frac{1}{2}U_1^\dagger\sigma^z U_1 - \frac{1}{2}U_2^\dagger\sigma^z U_2)\psi + \frac{\sqrt{3}}{2}\partial_y(U_1^\dagger\sigma^z U_1 - U_2^\dagger\sigma^z U_2)\psi - i\widetilde{m}\sigma^z$$

$$B^x \equiv (\beta^0 - \frac{1}{2}\beta^1 - \frac{1}{2}\beta^2) \qquad B^y \equiv \frac{\sqrt{3}}{2}(\beta^1 - \beta^2)$$

III. Dirac dynamics in free and curved space-time

Black hole simulation

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{d\rho^2}{1 - \frac{r_s}{r}} - r^2 d\theta^2$$



Probability density of a QW in the plane (t,x), compared with the classical geodesic

III. Dirac dynamics in free and curved space-time

Motivations

Understanding the fermion propagation under discrete space-time

Graphene applications

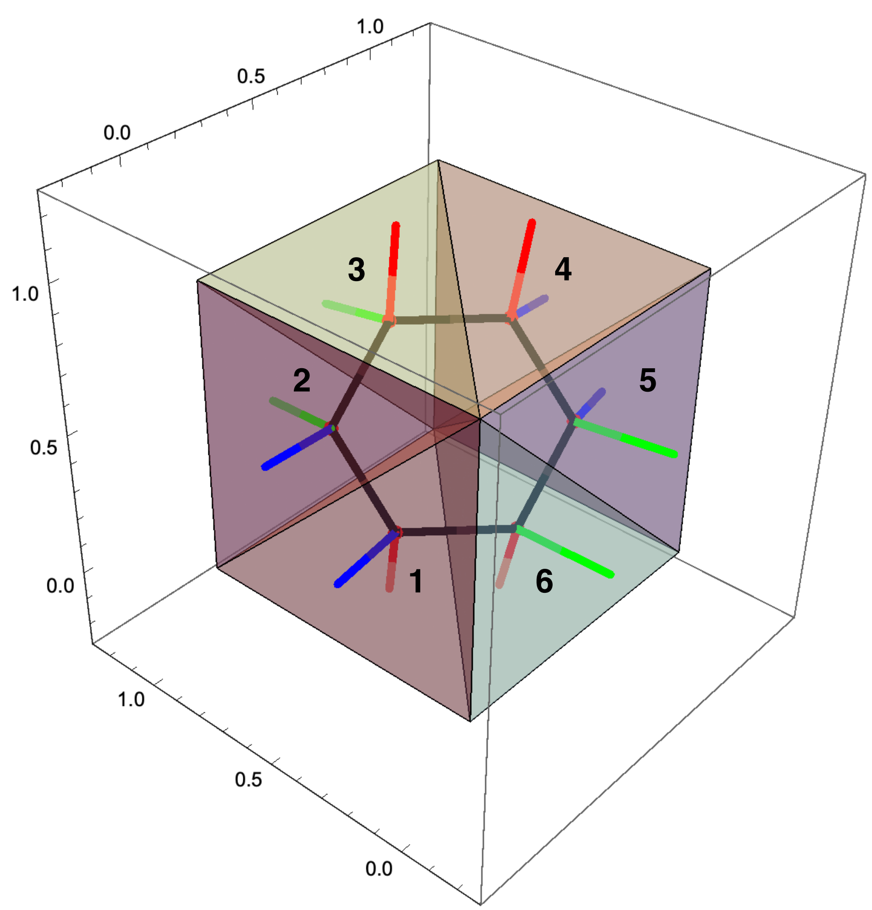
Antonio Gallerati. "Graphene properties from curved space Dirac equation". In: *European Physical Journal Plus* 134.5 (2019)

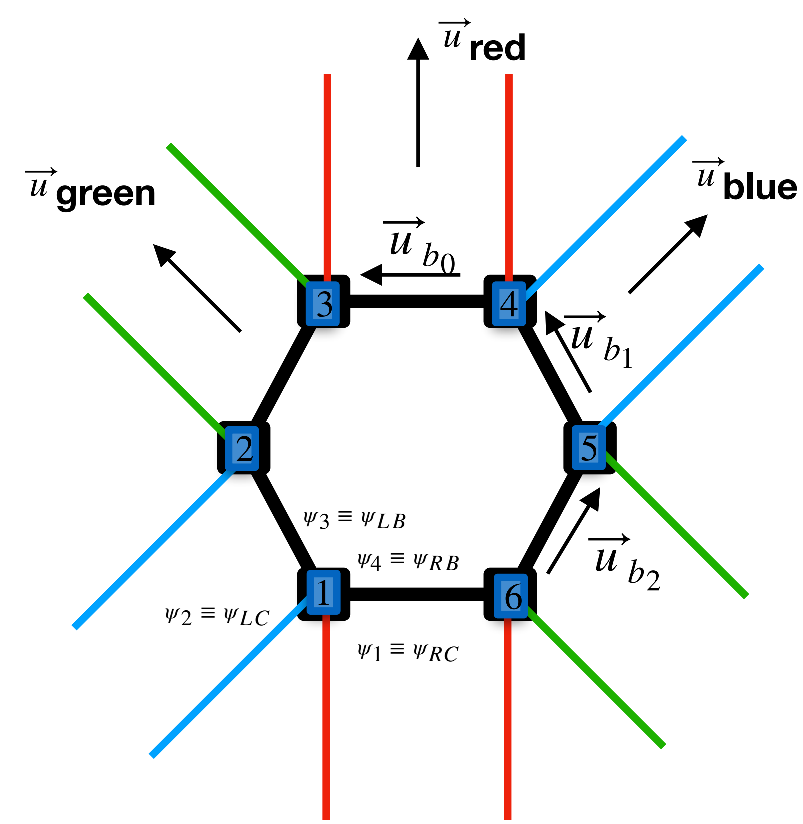
Kyriakos Flouris, Sauro Succi, and Hans J. Herrmann. "Quantized Alternate Current on Curved Graphene". In: *Condensed Matter* 4.2 (2019)

Posible applications in quantum algorithms

III. Dirac dynamics in free and curved space-time

Quantum simulation in 3D QW triangulated?





Conclusions

We establish connections between high energy physics and QWs

What about other brane models? What kind of dynamics can QWs describe?

QWs ⟷ LGT

QWs over non-rectangular lattices can be used as a QS schemes

Advantages in 3D? (Duality)

Thank you!