Lepton Flavour Violation

has to occur, $[m_\nu]$ says so

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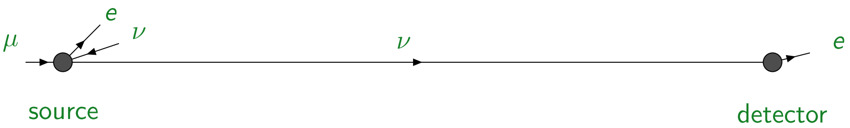
1. what is LFV ?
2. but not see LFV : bounds...
3. what to do/learn from non-detaction ?
   — parametrise exptal bounds
   — explore what data tells
   — constrain models
4. (Non-Standard ν Interactions)

What is Lepton Flavour Change=LFV ?

- three lepton flavours in the Standard Model : $\boldsymbol{e}, \boldsymbol{\mu}, \tau$

(flavour $\equiv$ charged mass eigenstate, so CLFV tautological)

- LFV $\equiv$ charged lepton flavour change, at a point $=$ ν osc. not count.



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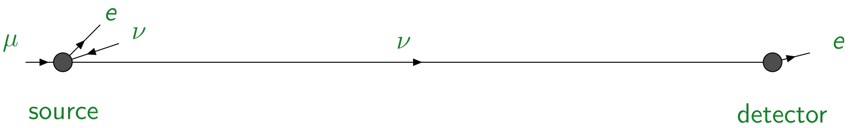
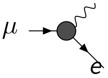


- ex of LFV : $\mu \to e\gamma$



- Lepton Flavour Change occurs with m_ν, U

because all lepton flavours are unconserved$\Leftrightarrow$ no symmetries

if m_ν renormalisable Dirac :

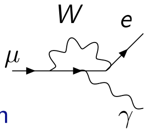
$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48} \quad \text{GIM suppressed(like quarks)}$$

(NB : caln diverges for non-renormalisable m_ν)

$\Rightarrow$ if see LFV, lepton flavour sector different from quarks !

suppose m_ν NOT renormalisable $\Leftrightarrow$ New Physics heavy

(Calculating $\mu \to e\gamma$



The dipole coefficient is a challenge to calculate :
a loop, but not calculate at zero-external-momentum because, eg

$$\bar{e}\,\sigma_{\alpha\beta}F^{\alpha\beta}\mu \quad \propto \quad \pm 2q_\alpha\varepsilon_\beta\bar{u}_e\sigma^{\alpha\beta}u_\mu$$

(and same diagram gives charge renormalisation)

In a model, can follow, eg, Peskin+ Schroeder (caln of $g-2$ in QED) :
do the loop integral with finite external momentum, express it on a simple basis, eg

$$\{\varepsilon_\beta\gamma^\beta, \varepsilon_\beta(P_\mu + p_e)^\beta, \varepsilon_\beta(P_\mu - p_e)^\beta\}$$

(first is charge renorm, last vanishes by Ward, middle becomes dipole via Gordon decomp).

)

But we don't see LFV : only exptal bounds

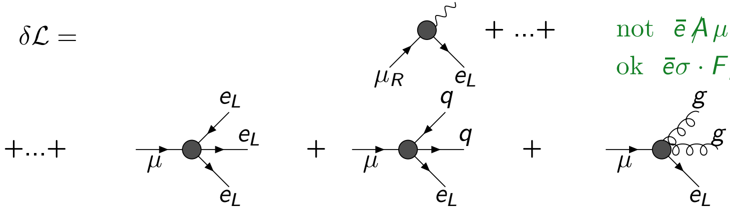
some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma\gamma$	$< 7.2 \times 10^{-11}$	
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2021, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	$10^{-(16 \to ?)}$ (Mu2e,COMET
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb
$\tau \to e\{\pi, \rho, \phi ...\}$	$\lesssim$ few $\times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm\{e, \mu\}^\mp$	$< 4.7, 2.5 \times 10^{-3}$	
$h \to \mu^\pm e^\mp$	$< 6.1 \times 10^{-5}$	
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$	

BR $\equiv$ Branching Ratio : (rate for process)/(total decay rate)

$\mu A \to eA \equiv \mu$ in $1s$ state of nucleus A converts to e

... add to $\mathcal{L}$: three- and four-point LFV contact interactions. Called "operators", should respect relevant gauge symmetries (QED*QCD at low E, SM above m_W), and can be classified by dimension.

$\delta\mathcal{L} =$



not $\bar{e} \not{A} \mu$
ok $\bar{e}\sigma \cdot F\mu$

+ h.c.

• below m_W, # legs is a better operator selection criteria than operator dimension, because SM masses can appear upstairs and downstairs. Eg,

$$\text{dim5} : \frac{m_\mu}{\Lambda_{NP}^2}\bar{e}\sigma \cdot F\mu \quad , \quad \text{dim7} : \frac{1}{m_Q \Lambda_{NP}^2}\bar{e}\mu GG$$

• chiral leptons motivated by 1) matching to SM at m_W, and 2) mass hierarchies : lighter leptons relativistic in decays of heavier leptons, so mass-suppressed interference between chiral operators

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Called "operators", should respect relevant gauge symmetries (QED*QCD at low E, SM above m_W), and can be classified by dimension.

building operators

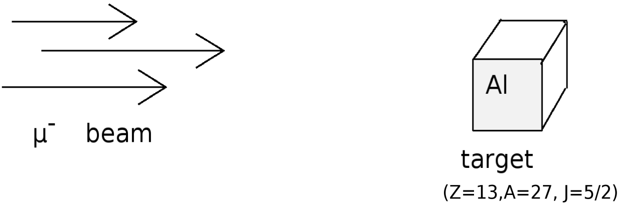
eg : 1008.4884,2005.00059,...

$$\delta\mathcal{L} \;=\; \sum_{n=1}^{3} \frac{1}{v^n} \sum_{X,\zeta} C_X^\zeta \mathcal{O}_X^\zeta + h.c.$$

$v \approx m_t$, $2\sqrt{2}G_F = 1/v^2$
$\{\mathcal{O}_X^\zeta\}$ = QED*QCD invar operators with 3 or 4 legs
X = Lorentz structure, ζ = flavour labels.
$\{C_X^\zeta\}$ dimless, $\equiv$ "Wilson coefficients".
calculable in models, constrained by exptal LFV bds

Now ask : what can we learn from those bounds ?

(What is $\mu A \to eA \equiv \mu \to e$ conversion ?



- μ^- captured by *Al* nucleus, tumbles down to 1*s*. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM : muon capture $\mu + p \to \nu + n$

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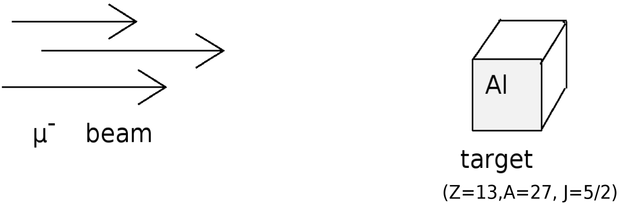

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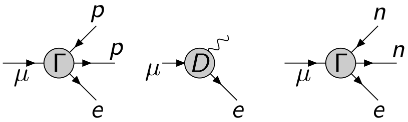

$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V, A, T\}$$

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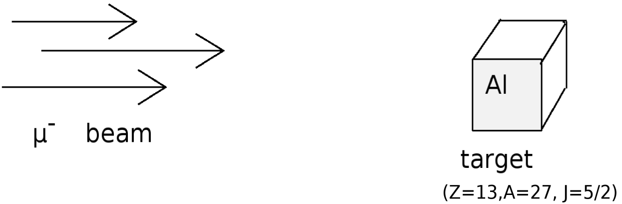

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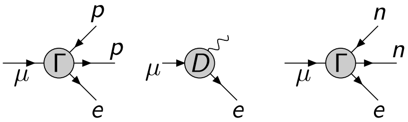

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$\approx$ WIMP scattering on nuclei
1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)
2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (Σ nucleons $\propto$ spin of only unpaired N)

Why is $\mu A \to eA$ conversion interesting ?

1. maybe where $\mu \to e$ could be discovered ?
   - experimental sensitivity to BR will improve : $10^{-12} \to 10^{-16}$ (expts under construction) $\to 10^{-18}$ (planned expts)
     (future sensitivity to $BR(\mu \to e\bar{e}e) \sim 10^{-16}$)
   - $\Gamma_{SI}(\mu A \to eA) \propto A^2$, whereas $\Gamma(\mu A \to \nu A')$ is not coherently enhanced. So

$$BR_{SI}(\mu A \to eA) \equiv \frac{\Gamma_{SI}(\mu A \to eA)}{\Gamma(\mu A \to \nu A')} \propto A^2 |\sum C|^2$$

   $\Rightarrow$ sensitivity $C \sim \sqrt{BR}/A$?

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2. ...its coherent $\Leftrightarrow$ many SI operators interfere

$$BR_{SI}(\mu A \to e_R A) = \frac{32 G_F^2 m_\mu^5}{\Gamma_{capt}} \Big| \widetilde{C}^{pp}_{V,R} V_A^{(p)} + \widetilde{C}'^{pp}_{S,L} S_A^{(p)} + \widetilde{C}^{nn}_{V,R} V_A^{(n)} + \widetilde{C}'^{nn}_{S,L} S_A^{(n)} + \frac{C_{D,L}}{4} D \Big|^2$$

if observed, different nuclear targets (w/wo spin) can potentially distinguish which coefficients,*but more theory calculations required*
(nuclear physics, χPT corrections to lepton-nucleon interactions...)

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Complementarity of $\mu \leftrightarrow e$, $\tau \leftrightarrow \ell$ and colliders

- bds on $\mu \to e\gamma, \mu \to e\bar{e}e$ and $\mu A \to eA$ much more restrictive than τs :

$$10^{-7} \sim \frac{\Gamma(\tau\to\ell+...)}{\Gamma(\tau\to\ell\nu\bar{\nu})} \approx \sqrt{\frac{\Gamma(\mu\to e+...)}{\Gamma(\mu\to e\nu\bar{\nu})}} \to 10^{-8}$$

(recall $\Gamma(\tau \to \ell\nu\bar{\nu}) \simeq \Gamma_\tau/6$ because hadronic τ decays)
because $\tau_\mu \sim 10^7 \tau_\tau$, so can make muon beams : PSI plans $\sim 10^{10}\mu$/sec (allows $BR(\mu \to e\bar{e}e) \sim 10^{-16}$), vs Belle-II aims to pair-produce $\sim 100\tau$/sec.
$\Rightarrow$? Can one search for $\ell \to \tau$ with intense ℓ beams on target ? Cirigliano etal 2102.06176

- more bds in $\tau \leftrightarrow \ell$ sector :better than $\mu \leftrightarrow e$ for discriminating models ?
In ParticleDataBook, $BR \lesssim 10^{-6} \to 10^{-8}$ for $\tau \to \ell + \{\pi, \eta, \rho, \phi, \omega, K, l_i \bar{l}_j, ..\}$.
And isospin, parity/lorentz properties of mesons restrict the operators that can contribute :

$$BR(\tau \to e\pi) \propto |C_A^{e\mu uu} - C_A^{e\mu dd}|^2 + \#|C_P^{e\mu uu} - C_P^{e\mu dd}|^2 + \text{interference}$$
$$BR(\tau \to e\eta) \propto |C_A^{e\mu uu} + C_A^{e\mu dd} + \# C_A^{e\mu ss}|^2 + \#\text{pseudoscalar}$$
$$BR(\tau \to e\rho) \propto |C_V^{e\mu uu} - C_V^{e\mu dd}|^2$$

to compare to

$$BR(\mu Ti \to eTi) \propto |C_S^{e\mu uu} + C_S^{e\mu dd} + .2(C_V^{e\mu uu} + C_V^{e\mu dd}) + .05 C_S^{e\mu ss} + .01 C_S^{e\mu cc} + .002 C_S^{e\mu bb}|^2$$

how to calculate $\tau \to \mu\pi_0$?

If add to Lagrangian :

$$\delta\mathcal{L} = \frac{1}{v^2}(\overline{\mu}\gamma^\mu P_L \tau)\Big(C_R^{uu}(\overline{u}\gamma^\mu P_R u) + C_L^{uu}(\overline{u}\gamma^\mu P_L u) + \{u \leftrightarrow d\}\Big)$$

obtain matrix elements like

$$\mathcal{M}(\tau \to \mu\pi_0) = \frac{C_L^{uu}}{v^2}(\overline{u}_\mu \gamma^\alpha P_L u_\tau)\langle 0|\overline{u}\gamma^\mu P_L u|\pi(P)\rangle + \{L \leftrightarrow R, u \leftrightarrow d\}$$

A pedagogical intro to χPT is 1804.05664, by A Pich, where :

$$\langle 0|\overline{d}\gamma^\beta\gamma_5 u|\pi^+(P)\rangle = iP^\beta\sqrt{2}f_\pi \quad \Rightarrow \quad \Gamma(\tau \to \pi\nu) = \frac{G_F^2 f_\pi^2 m_\tau^3}{8\pi}$$

$$\frac{1}{2}\langle 0|(\overline{u}\gamma^\beta\gamma_5 u - \overline{d}\gamma^\beta\gamma_5 d)|\pi^0(P)\rangle = iP^\beta f_\pi$$

with $f_\pi = 92.2$ MeV. (f_π depends on $\sqrt{2}$ conventions, can be 130 MeV). Then

$$\delta\mathcal{L} \supset \frac{1}{v^2}(\overline{\mu}\gamma^\beta P_L\tau)\Big(\frac{C_R^{uu} - C_L^{uu}}{2}(\overline{u}\gamma_\beta\gamma_5 u) + \frac{C_R^{dd} - C_L^{dd}}{2}(\overline{d}\gamma_\beta\gamma_5 d) + \ldots$$

$$\supset \frac{1}{v^2}(\overline{\mu}\gamma_\beta P_L\tau)\Big(\frac{C_R^{uu} - C_L^{uu} - C_R^{dd} + C_L^{dd}}{4}[(\overline{u}\gamma_\beta\gamma_5 u) - (\overline{d}\gamma_\beta\gamma_5 d)] + \ldots$$

so can calculate BR, or $\frac{\Gamma(\tau\to\pi_0\mu)}{\Gamma(\tau\to\pi^-\nu)}$

Complementarity, ctd : and colliders ?

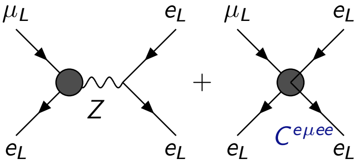
- sufficiently energetic colliders can produce new heavy particles 

- colliders probe LFV processes with heavy legs : h, t, Z

$\tau \leftrightarrow \ell$: LHC competitive with low energy ;

$\mu \leftrightarrow e$: better sensitivity at low energy, *but could be cancellations.* eg

$$\delta\mathcal{L} = \frac{g}{c_W} C_Z^{e\mu} \bar{e} \not{Z} P_L \mu \to \frac{\Gamma(Z\to e^\pm \mu^\mp)}{\Gamma(Z\to\mu\bar{\mu})} \simeq \frac{|C_Z^{e\mu}|^2}{|g_L^e|^2} \leq \frac{7.5\times 10^{-7}}{3.4\times 10^{-2}} \Rightarrow |C_Z^{e\mu}| \lesssim 10^{-3}$$



$$BR(\mu \to 3e) \sim |g_L^e C^{e\mu} + C^{e\mu ee}|^2 \leq 10^{-12}$$

$$\Rightarrow |g_L^e C_Z^{e\mu} + C^{e\mu ee}| \leq 10^{-6}$$

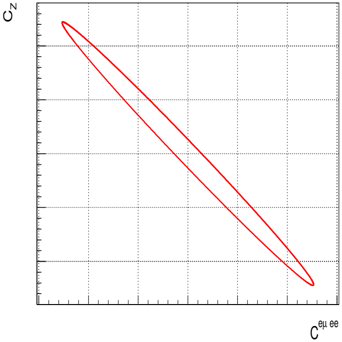
but hesitate to use "naturalness" notions for EFT concellations (coefficients are functions of model parameters, and cancellations depend on operator basis choice, eg could use gradient operator at Z vertex)

sensitivity $\equiv$ how small a coefficient could one see ?
$\Leftrightarrow$ "setting bounds one operator at a time"

$$|g_L^e C_Z^{e\mu} + C^{e\mu ee}| \leq 10^{-6} \Leftrightarrow |g_L^e C_Z^{e\mu}|, |C^{e\mu ee}| \leq 10^{-6}$$

constraints
$\equiv$ values of a coefficients excluded by the data

$|g_L^e C_Z^{e\mu} + C^{e\mu ee}| \leq 10^{-6}$
$|C_Z^{e\mu}| \leq 10^{-3}$
$\Leftrightarrow |C_Z^{e\mu}|, |C^{e\mu ee}| \lesssim 10^{-3}$



Are those bounds restrictive ? What does $BR(\mu \to e\bar{e}e) < 10^{-12}$ mean ?

$m_\mu = .105$ GeV
$v = 174$ GeV

BRs of μ, τ normalised to *weak* decay, mediated by
$\delta\mathcal{L} = 2\sqrt{2}G_F(\bar{e}\gamma^\alpha P_L\nu)(\bar{\nu}\gamma^\alpha P_L\mu), \quad 2\sqrt{2}G_F = 1/v^2$

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4}$$

so if $\delta\mathcal{L}_{LFV} = (\bar{e}\gamma^\alpha P_L\mu)(\bar{e}\gamma^\alpha P_L e)/\Lambda_{LFV}^2 \quad \Leftrightarrow C = v^2/\Lambda_{LFV}^2$

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$$\Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3\Lambda_{LFV}^4} \Rightarrow BR \lesssim \left\{ \begin{array}{l} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{array} \right.$$

vs $\Lambda_{LNV} \sim \mathbf{v^2}/m_\nu \sim 3 \times 10^{\mathbf{11}}$ TeV

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Compare to $\frac{(g-2)_\mu}{2} \equiv a \simeq \alpha_{em}/\pi$ (electromagnetic *amplitude*) :
torque $\vec{\tau} = \vec{\mu} \times \vec{B}$; $\vec{\mu} = g\frac{e}{2m}\vec{S}$

$$\begin{aligned} \Delta a &\equiv a^{SM} - a^{exp} \simeq 3 \times 10^{-9} \\ &\sim \frac{m_\mu^2}{16\pi^2 \Lambda_{NP}^2} \end{aligned}$$

$\Rightarrow \Lambda_{NP} \sim m_t$.

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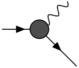
Compare to decays of $\Upsilon(2s)$ electromagnetically decaying $\bar{b}b$ state

$$BR(\Upsilon \to \tau\bar{\mu}) = \frac{\Gamma(\Upsilon \to \tau\bar{\mu})}{\Gamma(\Upsilon \to \mu\bar{\mu})} \sim \left| \frac{1/\Lambda_{LFV}^2}{e^2 Q_b/m_\Upsilon^2} \right|^2 \leq \frac{6 \times 10^{-6}}{2.5 \times 10^{-2}}$$

which excludes $\Lambda_{LFV} \lesssim 100$ GeV.

Two dipole operators contribute to $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_R^D \overline{\mu_R} \sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_L^D \overline{\mu_L} \sigma^{\alpha\beta} e_R F_{\alpha\beta} \right)$$

$$BR(\mu \to e\gamma) = 384\pi^2 (|C_R^D|^2 + |C_L^D|^2) < 4.2 \times 10^{-13}$$

$$\Rightarrow |C_X^D| \lesssim 10^{-8}$$

MEG expt, PSI

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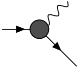

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$$BR(\mu \to e\gamma) = 384\pi^2 (|C_R^D|^2 + |C_L^D|^2) < 4.2 \times 10^{-13}$$

$$\Rightarrow |C_X^D| \lesssim 10^{-8}$$

MEG expt, PSI

How big does one expect C to be ?

			$n = 1$	$n = 2$
$C \frac{m_\mu}{v^2} \sim \frac{e m_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$ probes	$\Lambda \lesssim$	100 TeV	10 TeV
$C \frac{m_\mu}{v^2} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$ probes	$\Lambda \lesssim$	3000 TeV	300 TeV

2-loop sensitivity to New Particles that are beyond the reach of the LHC...

But QED loops are $\mathcal{O}(\alpha/4\pi)$...negligeable in discovery mode for LFV ?

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$$\delta\mathcal{L}(m_W) = 2\sqrt{2}G_F\ C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$$

1 : forget loops Match to nucleons $N \in \{n, p\}$ as

$\widetilde{C}_T^{NN} = \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4} C_T^{uu}$

nuclear matrix elements : EngelRTO, KlosMGS

$$\Rightarrow \boldsymbol{BR \approx BR_{SD} \approx \frac{1}{2}|C_T|^2}$$

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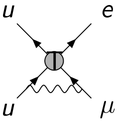
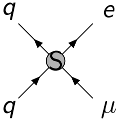
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$ 

$+ \ldots \quad \Rightarrow$ 

$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y \mu)$

$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7} C_T^{uu}(m_W)$

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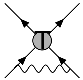
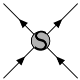
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y\mu)$ 

$+ \ldots \Rightarrow$ 

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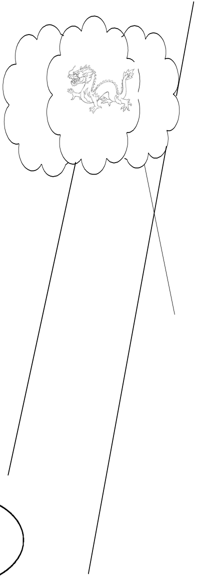
Then match to nucleons : $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle\Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 8Z^2 BR_{SD}}$$

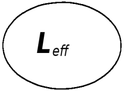
loops can change Lorentz structure/external legs $\Rightarrow$ different operator whose coefficient better constrained

Recall Outline :

1. SM leptons, m_ν and LFV
2. but not see LFV : bounds...
3. what to do/learn from non-detaction ?
  — parametrise exptal bounds
  — explore what data tells
  — **constrain models**
4. Non-Standard ν Interactions



Assume New LFV physics heavy : $M_{NP} \gg m_{SM} \sim \Lambda_{expt}$
Wilsonian interpretation of renormalisation says
that translations in scale mean
add/remove loops of dynamical (= SM) particles



data↔models : how to include loops ?

1. you have intuition to find the right model
   ⇒ calculate $\mathcal{M}$ = matrix element for observable !
   but : SM calns conceptually+ technically difficult
   = expand in loops and many couplings,
   many different mass scales in integrals+ need to resum QCD...
   also, have to repeat as many times as your taste in models changes

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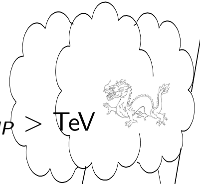
   many different mass scales in integrals+ need to resum QCD...
   also, have to repeat as many times as your taste in models changes
2. only dynamics below M_{NP} is SM : use EFT
   (already met EFT as parametrisation-via-contact-interactions of heavy New Physics)
   EFT simplifies loop calculations by expanding in scale ratios
   eg replace NP by contact interactions
   (and allows to get right=Wilsonian logs in dim reg ! See Georgi, "EFT", ARNPS, 1993)
   - match model to EFT at M_{NP} by equating matrix elements
     $\mathcal{M}_{\rm model} = \mathcal{M}(C_I)$
   - SM loop corrections to $\mathcal{M}$ obtained via Renormalisation Group running of operator coefficients (coefficients "run" like all coupling constants)
     Once operator RGEs are calculated, its done !

EFT to translate in scale ;top-down or bottom-up

match model to $\{C_I\}$ at M_{NP} (equate $\mathcal{M}$s)

run $\{C_I\}$ to Λ_{expt} with RGEs

$\mathcal{M}(C_I) = \mathcal{M}(\text{model})$ $\quad M_{NP} > \text{TeV}$



$\{Z, W, \gamma, g, h, t, f\}$

$\mathcal{M}(C_{above}) = \mathcal{M}(C_{below})$ $\quad m_W \sim m_h \sim m_t$

$\{\gamma, g, e\mu, \tau, u, d, c, s, b\}$

$$\text{RGEs} : \mu \frac{\partial}{\partial \mu} \vec{C} = \vec{C}\mathbf{\Gamma}$$

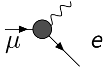
$$\Rightarrow \vec{C}(m_\tau) \sim \vec{C}(m_W) \exp\{-\mathbf{\Gamma} \log\}$$

GeV $\sim m_c, m_b, m_\tau$

$\{\gamma, e, \mu, p, n, (\pi)\}$ $\quad$ data

Many models, data fixed : lets take $\mu \to e$ data to M_{NP}

Counting constraints : $\boldsymbol{\mu \to e\gamma + \mu \to e\bar{e}e}$



Two dipole operators contribute to $\mu \to e\gamma$:

$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta} e_R F_{\alpha\beta} \right)$$

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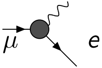

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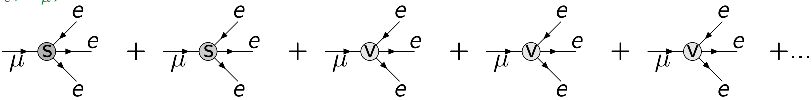
2dipoles + 6 4-f-ops contribute to $\mu \to e\bar{e}e$, (most interference between ops $\propto m_e^2/m_\mu^2$)



$(\bar{e}P_L\mu)(\bar{e}P_Le)$ $(\bar{e}P_R\mu)(\bar{e}P_Re)$ $(\bar{e}\gamma P_L\mu)(\bar{e}\gamma P_Le)$ $(\bar{e}\gamma P_R\mu)(\bar{e}\gamma P_Re)$ $(\bar{e}\gamma P_L\mu)(\bar{e}\gamma P_Re)$

$$BR(\mu \to e\bar{e}e) = \frac{|C_{S,LL}|^2 + |C_{S,RR}|^2}{8} + 2|C_{V,RR}|^2 + 2|C_{V,LL}|^2 + |C_{V,LR}|^2 + |C_{V,RL}|^2$$

$$\leq 10^{-12} \quad \Rightarrow \quad \boldsymbol{|C_X| \lesssim 10^{-6}\sqrt{BR/10^{-12}}}$$

see nothing in $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\Rightarrow$ all 8 Cs small

see something $\Rightarrow$ distinguish operator via angular distributions ?

adding up constraints from $\mu A \to eA$, $\mu \to e\gamma$ + $\mu \to e\bar{e}e$

1. constrain 2 dipoles +6 4ℓ coeffs with $\mu \to e\gamma$ + $\mu \to e\bar{e}e$
2. $\mu\mathrm{A} \to e\mathrm{A}$, Spin Indep, now : two targets {Au, Ti }
two constraints/target (outgoing e_L, e_R)
SI future : 3 independent targets ?
3. Spin-Dependent, now : (?) 2 counstraints ? (Ti ?)
SD future : 4 constraints ?

$$\Rightarrow \left\{ \begin{array}{cc} \text{now} & 12 \to 14 \\ \text{future} & \lesssim 20 \end{array} \right\} \text{constraints} \left\{ \begin{array}{cc} \ll & \#\text{operators} \\ \lesssim & \#\text{parameters in many models} \end{array} \right.$$

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• In $\mu \to e$ sector, expt probes ∼12-dim subspace of ∼ 100- dim space of operators. (= Many "flat directions" ; this is feature of data, not problem of EFT).
Have exptal subspace @ M_{NP}, just need to project models onto this subspace ...*lots to do !*

• "basis vectors" of exptal subspace have projection on most operators ⇔ most operators mix via loops into observables. So expts have *sensitivity* to most operators

Can still calculate sensitivities...

sensitivity : "one at a time bound" = below, parameter is to small to see in expt. (But larger possible, if cancelled by another contribution.)

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu - e$ conv.
$\lvert C_{D,X}\rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX}\rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY}\rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX}\rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX}\rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY}\rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX}\rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX}\rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY}\rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX}\rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY}\rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX}\rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

Table – Current sensitivities of $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. to the coefficients, at m_W, of QCD×QED-invariant 2- and 4-lepton operators. $X, Y \in \{L, R\}, X \neq Y$.

Summary

Lepton Flavour Violation is BSM that exists. Not yet detected, but experimental sensitivities for $\mu \leftrightarrow e$ should improve by orders of magnitude in coming years ($\rightarrow BR \sim 10^{-16} \rightarrow 10^{-18}$).

Current experimental constraints on $\mu \leftrightarrow e$ are few, but restrictive $\Rightarrow$ sensitivity to loop-induced LFV at scales beyond the LHC. But $\tau \leftrightarrow \ell$ maybe better for discriminating models ?

If assume LFV induced by heavy New Physics, can parametrise LFV interactions with non-renormalisable operators, and systematically include SM loop effects via RGEs for operator coefficients. Find that, in $\mu \leftrightarrow e$ sector

- (many) more operators than constraints
- but constraints probe independent directions in operator space
- It would be interesting to explore the ability of $\mu \leftrightarrow e$ observables to discriminate models...

But we already found some BSM in neutrino oscillations, maybe we could find some more = NSI !

NSI

BSM to find in ν oscillations

(not neccessarily BSM where to learn about mass mechanism)

Non-Standard Interactions

Wolfenstein, Valle, GuzzoMasieroPetcov

NSI : $\delta\mathcal{L} = -2\sqrt{2}G_F\varepsilon_f^{\rho\sigma}(\overline{\nu}_\rho\gamma_\alpha P_L\nu_\sigma)(\overline{f}\gamma^\alpha f)$, $f \in \{e, d, u\}$ ε matrix

QED×QCD invariant.

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QED×QCD invariant. At finite density

$$\langle \mathrm{medium}|\overline{\hat{f}}\gamma_\alpha \hat{f}(x)|\mathrm{medium}\rangle \to \delta_{\alpha 0} n_f \ ,$$

contributes to forward scattering amplitude ⇔ "effective $\Delta m^2/E \sim \sqrt{2}G_F n_e$" to oscillation Hamiltonian

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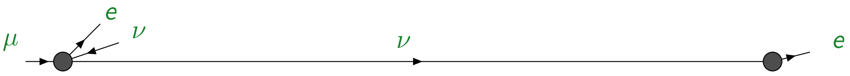

urce, CC,messy

clean, quantum, NC
probe (little known) ν propagator

detector, CC, messy

interest : ν oscillations= quantum mechanics on macroscopic scales = *very* sensitive window to probe poorly known ν propagator (⇔ m_ν + NC ν interactions)

Current constraints on NSI from oscillation data + COHERENT

EstebanGonzalezGarciaMaltoniEtal

Add NSI to low-E $\mathcal{L}_\nu$ (add no new CC), suppose $\varepsilon_f^{\alpha\beta} = \varepsilon^{\alpha\beta}\varepsilon_f$.

$-.008 < \varepsilon_u^{ee} < .62$ $-.01 < \varepsilon_d^{ee} < .56$ $-.01 < \varepsilon_e^{ee} < 2.0$	$-.06 < \varepsilon_u^{e\mu} < .05$ $-.06 < \varepsilon_d^{e\mu} < .05$ $-.18 < \varepsilon_e^{e\mu} < .15$	$-.25 < \varepsilon_u^{e\tau} < .11$ $-.21 < \varepsilon_d^{e\tau} < .11$ $-.86 < \varepsilon_e^{e\tau} < .35$
	$-.11 < \varepsilon_u^{\mu\mu} < .40$ $-.10 < \varepsilon_d^{\mu\mu} < .36$ $-.36 < \varepsilon_e^{\mu\mu} < 1.3$	$-.012 < \varepsilon_u^{\mu\tau} < .009$ $-.011 < \varepsilon_d^{\mu\tau} < .009$ $-.035 < \varepsilon_e^{\mu\tau} < .35$
		$-.11 < \varepsilon_u^{\tau\tau} < .40$ $-.10 < \varepsilon_d^{\tau\tau} < .36$ $-.35 < \varepsilon_e^{\tau\tau} < 1.40$

$\approx$ *constraints* = bigger is incompatible with data.

Comments...

- ?oscillations (maybe) have separate sensitivity to NSI on u and d because the sun is made of protons ?
- Oscillations only sensitive to $\varepsilon^{\alpha\alpha} - \varepsilon^{\beta\beta}$, but COHERENT lifts degeneracy (NC scattering, sensitive to $\varepsilon^{\sigma\rho}$)
- ranges *neglect* other solutions where SM parameters disconnected from bestfit values (LMA-Dark solution) !
- $\varepsilon_e^{\alpha\alpha} \sim 1$ allowed because flips sign of SM $(\bar{\nu}\gamma P_L \nu)(\bar{f}\gamma P_L f)$ (oscillations sensitive to signs, but only of flavour differences...)
- not matched onto SMEFT, so not accounting for potential contribution to flav-diagonal "SM" inputs by CC or charged-lepton components of the SMEFT operator.
- Energy scales : $q^2 \to 0$ in matter effect, 30-70 MeV at COHERENT.

But Standard Model neutrinos are in a doublet $\ell_\rho = \begin{pmatrix} \nu_\rho \\ e_\rho \end{pmatrix}$...LFV ?

New Physics must respect SM gauge symmetries : given bounds on (charged) Lepton Flavour Violation, can NSI be detectably large ?

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- ex : SU(2) invariant dimension 6 operators that induce $\nu_\tau \to \nu_\mu$ NSI on e

$$\varepsilon^{\tau\mu}_{(3)\ell\ell}(\overline{\ell}_\tau \gamma_\alpha \tau^a \ell_\mu)(\overline{\ell}_e \gamma^\alpha \tau^a \ell_e)\ ,\ \varepsilon^{\tau\mu}_{\ell\ell}(\overline{\ell}_\tau \gamma_\alpha \ell_\mu)(\overline{\ell}_e \gamma^\alpha \ell_e)\ ,\ \varepsilon^{\tau\mu}_{ee}(\overline{\ell}_\tau \gamma^\alpha \ell_\mu)(\overline{e}_e \gamma_\mu e_e)$$

NSI $\propto \varepsilon^{\tau\mu}_{(3)\ell\ell} + \varepsilon^{\tau\mu}_{\ell\ell}$, $\varepsilon^{\tau\mu}_{ee}$

$\widetilde{BR}(\tau \to 3l) \simeq |\varepsilon^{\tau\mu}_{(3)\ell\ell} - \varepsilon^{\tau\mu}_{\ell\ell}|^2 + |\varepsilon^{\tau\mu}_{ee}|^2 \lesssim 10^{-7}$ ...

$\Rightarrow$ LFV constraints, applied at tree level, exclude several (combinations of) dim 6 operators from inducing observable NSI.

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$\Rightarrow$ LFV constraints, applied at tree level, exclude several (combinations of) dim 6 operators from inducing observable NSI.

• To avoid LFV constraints, build NSI at dim 8 $f \in \{e, u, d, q_1, \ell_e\}$:

$$\frac{C_f^{\rho\sigma}}{\Lambda^4}(\overline{\ell}_\rho H)\gamma_\alpha(H^\dagger \ell_\sigma)(\overline{f}\gamma^\alpha f) \stackrel{H\to v}{\longrightarrow} \frac{C_f^{\rho\sigma} v^2}{\Lambda^4}(\overline{\nu}_\rho \gamma_\alpha \nu_\sigma)(\overline{f}\gamma^\alpha f)\ ,\ \varepsilon_f^{\rho\sigma} = \frac{C_f^{\rho\sigma} v^4}{\Lambda^4}$$

$\varepsilon_f^{\rho\sigma} \gtrsim 10^{-2} \Leftrightarrow \Lambda \lesssim .3 \to 1$ TeV $\Rightarrow$ *is there a model ?*

Is there a model ?

1. $10^{-2} \lesssim \varepsilon \lesssim 1$ suggests feebly-coupled mediator, $m \ll m_W$?

   - $\sim$10 MeV Z', flav.diag. coupling $g' \sim 10^{-4}$ to $\ell_\mu, \ell_\tau, q_{L,1}, u_R, d_R$.
   - light Z' feebly coupled to quarks and $\nu_{sterile}$, small $m\nu_s\nu_{SM}$. Farzan

   PospelovPradler

   avoid some ν scattering bounds if $m^2_{mediator} \ll \langle q^2 \rangle$
   avoid inducing LFV by chosing couplings...

2.

3. heavy New Physics, $m_{mediator} \gtrsim m_W$ recipe :GavelaHernandezOtaWinter
   tune NP masses/cplgs so tree LFV coefficients vanish(dim 6 and 8) :
   eg on e at dimension 6, need

$$\varepsilon^{\tau\mu}_{(3)\ell\ell} - \varepsilon^{\tau\mu}_{\ell\ell} = \varepsilon^{\tau\mu}_{ee} = 0$$

   ex : scalar + vector leptoquark with tuned masses/couplings.
   or scalar bilepton S, with $L=2$, $Q_{em}=1$, $S\ell_i^\alpha \epsilon^{ij} \ell_j^\beta$, induces only $2e2\nu$

   $\star$can do EFT = results that apply to many models

Summary

Lepton Flavour Violation is BSM that exists. Not yet detected, but experimental sensitivities for $\mu \leftrightarrow e$ should improve by orders of magnitude in coming years ($\rightarrow BR \sim 10^{-16} \rightarrow 10^{-18}$).

Current experimental constraints on $\mu \leftrightarrow e$ are few, but restrictive $\Rightarrow$ sensitivity to loop-induced LFV at scales beyond the LHC. But $\tau \leftrightarrow \ell$ maybe better for discriminating models ?

If assume LFV induced by heavy New Physics, can parametrise LFV interactions with non-renormalisable operators, and systematically include SM loop effects via RGEs for operator coefficients. Find that, in $\mu \leftrightarrow e$ sector

- (many) more operators than constraints
- but constraints probe independent directions in operator space
- It would be interesting to explore the ability of $\mu \leftrightarrow e$ observables to discriminate models...

But we already found some BSM in neutrino oscillations, maybe we could find some more = NSI !

Some Phenomenological LFV/EFT Reviews :

$\mu \leftrightarrow e$: Kuno and Okada, Rev.Mod.Phys. 73(2001)151, hep-ph/9909265
BelleII : The BelleII Physics Book, 1808.10567
EFT : Georgi, Ann.Rev.Nucl.Part.Phys. (1993)
Les Houches school, July 2017, Neubert, Manohar, Silvestrini (quarks)

I use chiral Dirac spinors, 4 degrees of freedom(dof)

$$\psi = \left(\begin{array}{c} \psi_L \\ \psi_R \end{array} \right) \quad , \; \{\gamma^\alpha\} = \left\{ \left[\begin{array}{cc} 0 & I \\ I & 0 \end{array} \right], \left[\begin{array}{cc} 0 & \sigma_i \\ -\sigma_i & 0 \end{array} \right] \right\}$$

$$\{\sigma_i\} = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right], \left[\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right], \left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right]$$

$$\psi_L = \; P_L \, \psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \quad , \; \psi_R = \; P_R \, \psi$$

chirality *not* observable ($\to$ helicity $= \pm \hat{s} \cdot \hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

notation : $\overline{(\psi_R)} = (P_R \psi)^\dagger \gamma_0 = \psi^\dagger P_R \gamma_0 = \psi^\dagger \gamma_0 P_L = (\overline{\psi})_L$

$(\psi^c)_L = P_L(-i\gamma_0\gamma_2\gamma_0\psi^*) = -i\gamma_0\gamma_2\gamma_0\psi^*_R$

operator basis for $\mu A \to eA$, $\mu \to e\bar{e}e$, $\mu \to e\gamma$ at Λ_{expt}

Kuno Okada

μ interaction with nucleon $N \in \{n, p\}$ parametrised by 20 4-f operators :

$S,\ V$	$\bar{e}P_X\mu\overline{N}N$	$\bar{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N$	$X \in \{L, R\}$
$A,\ T$	$\bar{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N$	$\bar{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N$	
$P,\ Der$	$\bar{e}P_X\mu\overline{N}\gamma_5 N$	$\bar{e}\gamma^\alpha P_X\mu(\overline{N}i\overleftrightarrow{\partial}_\alpha\gamma_5 N)$	

Matching in χPT gives Derivative. But absorb in matching into $G_O^{N,q}$= quark matrix elements in nucleons. and 2 dipoles

$$D \qquad \bar{e}\sigma^{\alpha\beta}P_X\mu F_{\alpha\beta}$$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$. For $\mu \to e\bar{e}e$

V	$(\bar{e}\gamma^\alpha P_Y\mu)(\bar{e}\gamma_\alpha P_Y e)$	$(\bar{e}\gamma^\alpha P_Y\mu)(\bar{e}\gamma_\alpha P_X e)$
S	$(\bar{e}P_Y\mu)(\bar{e}P_Y e)$	

28 operators

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

$\mu \leftrightarrow e$ operators (at scale $m_W \leftrightarrow m_\tau$) ; otherwise flav.diag.

$$m_\mu(\bar{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta}$$ *dim* 5 Kuno-Okada

$$\begin{array}{rl}
(\bar{e}\gamma^\alpha P_Y\mu)(\bar{e}\gamma_\alpha P_Y e) & (\bar{e}\gamma^\alpha P_Y\mu)(\bar{e}\gamma_\alpha P_X e) \\
(\bar{e}P_Y\mu)(\bar{e}P_Y e) & (\bar{e}P_Y\mu)(\bar{\mu}P_Y\mu) \\
(\bar{e}\gamma^\alpha P_Y\mu)(\bar{\mu}\gamma_\alpha P_Y\mu) & (\bar{e}\gamma^\alpha P_Y\mu)(\bar{\mu}\gamma_\alpha P_X\mu) \\
(\bar{e}\gamma^\alpha P_Y\mu)(\bar{f}\gamma_\alpha P_Y f) & (\bar{e}\gamma^\alpha P_Y\mu)(\bar{f}\gamma_\alpha P_X f) \\
(\bar{e}P_Y\mu)(\bar{f}P_Y f) & (\bar{e}P_Y\mu)(\bar{f}P_X f) \\
(\bar{e}\sigma P_Y\mu)(\bar{f}\sigma P_Y f) &
\end{array}$$ *dim* 6

$$\begin{array}{ll}
\frac{1}{m_t}(\bar{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} & \frac{1}{m_t}(\bar{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \\
\frac{1}{m_t}(\bar{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} & \frac{1}{m_t}(\bar{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}
\end{array}$$ *dim* 7

...zzz...

$f \in \{\tau, u, d, c, s, b\}$, $P_X \neq P_Y = (1 \pm \gamma_5)/2$

82 operators. + 80 more if allow quark flavour-changing.$\times$3 to account for $\mu \leftrightarrow e$, $\tau \leftrightarrow e$, and $\tau \leftrightarrow \mu$. + 48 $\Delta L_i = 2$.

To calculate SI $\mu\mathrm{A} \to e\mathrm{A}$ (at LO in χPT...)

differs from WIMP scattering in that μ and nucleus charged

1. suppose start with $\mu \leftrightarrow e$ operators involving gluons, γ, u, d, s, c, b
2. match quark/gluon operators onto nucleon ($N \in \{n, p\}$) operators :

$$\bar{q}(x)\Gamma_O q(x) \to G_O^{N,q} \bar{N}(x)\Gamma_O N(x)$$

Gs in Appendix

eg, $\langle N|\bar{q}(x)q(x)|N\rangle = G_O^{N,q}\langle N|\bar{N}(x)N(x)|N\rangle = G_O^{N,q}\overline{u_N}(P_f)u_N(P_i)e^{-i(P_f-P_i)x}$

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3. *imagine* to build the atom as a bound state of nucleus and muon in 1s state

in P+S

$$|\mu A(\vec{P}_i = 0)\rangle = \sqrt{\frac{2(M_A + m_\mu)}{4M_A m_\mu}} \sum_s \int \frac{d^3k}{(2\pi)^3} \widetilde{\psi}_\mu(\vec{k})\, |A(-\vec{k})\rangle \otimes |\mu(\vec{k}, s)\rangle$$

then build nucleus as bd state of nucleons (app. B of 1203.3542),gives :

SD overlap int : guess from SD DM targets

$$\langle e, A|\widetilde{C}_O O|\mu A\rangle \propto \widetilde{C}_O(\bar{u}_e \Gamma_O u_\mu) \int d^3x \psi_\mu^{1s} |f_N(x)|^2 \psi_e (\bar{N}\Gamma_O N)$$

where "overlap integral" over nucleus, of muon wavefn ($\widetilde{\psi}_\mu^{1s}$), nucleon density ($|f_N(x)|^2$), e wavefn ($\psi_e \sim e^{-iqx}$) and operator performed in KitanoKoikeOkada.

To calculate SI $\mu A \to e A$ (at LO in χPT...)

differs from WIMP scattering in that μ and nucleus charged

1. suppose start with $\mu \leftrightarrow e$ operators involving gluons, γ, u, d, s, c, b
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3. look up rate in KitanoKoikeOkada, PRD (2002), eqn 14, check your operator normalisation against KKO eqn 1, read numerical value of overlap integrals from table I, and divide by capture rate in table VIII of KKO.

Shortcut to calculate $\mu A \to eA$

shortcut for current bounds (Gold and Titanium) :write

$$BR_{SI}(\mu A \to eA) = B_A \left[|\hat{v}_A \cdot \vec{C}_L|^2 + |\hat{v}_A \cdot \vec{C}_R|^2 \right]$$

where

$$\vec{C}_L = (\widetilde{C}_{D,R}, \widetilde{C}^{pp}_{S,R}, \widetilde{C}^{pp}_{V,L}, \widetilde{C}^{nn}_{S,R}, \widetilde{C}^{nn}_{V,L})$$

$$B_A \equiv \frac{32 G_F^2 m_\mu^5 |\vec{v}_A|^2}{\Gamma_{cap}(A)} = \begin{cases} 250 & Ti \\ 300 & Au \\ 142 & Al \end{cases}$$

and normalised overlap integrals of KKO are lined up in target vectors

$$\hat{v}_{Ti} = (0.250, 0.426, 0.458, 0.503, 0.541)$$

$$\hat{v}_{Au} = (0.222, 0.289, 0.458, 0.432, 0.686)$$

(Spin Dep : ?likely in noise of SI signal—RGEs of QED mix T,A$\to$ S, V. To calculate, need nuclear caln, see discussion inCiriglianoDKuno, DKunoSaporta)

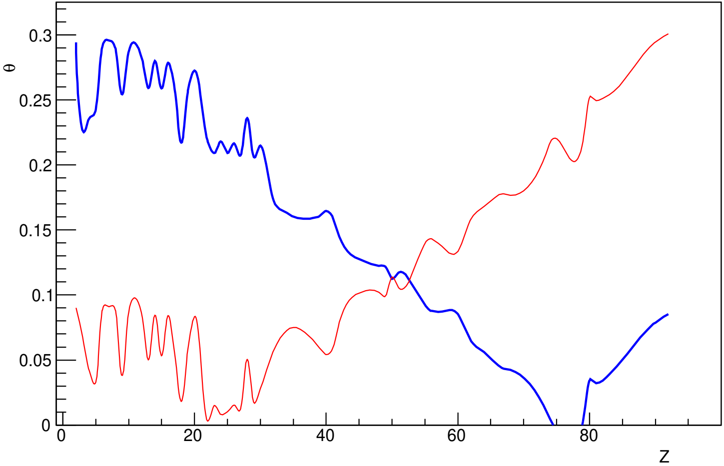
Gs

$G_V^{p,u} = G_V^{n,d} = 2$	$G_V^{p,d} = G_V^{n,u} = 1$	$G_V^{p,s} = G_V^{n,s} = 0$
$G_A^{p,u} = G_A^{n,d} = 0.84$ $G_A^{p,u} = G_A^{n,d} = 0.863$	$G_A^{p,d} = G_A^{n,u} = -0.43$ $G_A^{p,d} = G_A^{n,u} = -0.345$	$G_A^{p,s} = G_A^{n,s} = -.085$ $G_A^{p,s} = G_A^{n,s} = -.0240$
$G_S^{p,u} = 5.9$ $(G_S^{p,u} = 9.0)$ $G_S^{n,u} = 5.0$ $(G_S^{n,u} = 8.1)$ $G_S^{N,c} = \frac{2m_N}{17m_c}$	$G_S^{p,d} = 5.0$ $(G_S^{p,d} = 8.2)$ $G_S^{n,d} = 6.0$ $(G_S^{n,d} = 9.0)$ $(G_S^{N,b} = \frac{2m_N}{17m_b}$	$G_S^{p,s} = 0.42$ $G_S^{n,s} = 0.42$ $(G_S^{n,s} = 0.42)$
$G_P^{p,u} = 144 = G_P^{n,d}$	$G_P^{p,d} = -150 = G_P^{n,u}$	$G_P^{p,s} = -4.9 = G_P^{n,s}$
$G_T^{p,u} = G_T^{n,d} = 0.77(7)$	$G_T^{p,d} = G_T^{n,u} = -0.23(3)$	$G_T^{p,s} = G_T^{n,s} = .008(9)$
$G_S^{N,gg} = -8\pi m_N/(9\alpha_s v)$		

Table – Matching coefficients between nucleon and operators and gluon or light-quark(flavour-diagonal) operators. References in DKunoSaporta. Scalar Gs in parentheses are EFT caln, otherwise from lattice. In all cases, the $\overline{\text{MS}}$ quark masses at $\mu = 2$ GeV are taken as $m_u = 2.2$ MeV, $m_d = 4.7$ MeV, and $m_s = 96$ MeV. The nucleon masses are $m_p = 938$ MeV and $m_n = 939.6$ MeV.

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$

$BR(\mu Au \to eAu) < 7 \times 10^{-13} \quad (Au: Z = 79)$



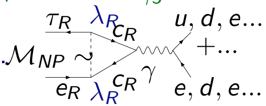
$$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)}), \text{ and } BR \propto |\vec{v}_A \cdot \vec{C}|^2$$

$$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta \text{ ...plot } \theta \text{ on vertical axis}$$

To calculate loops in EFT

regularise with dim.reg., renormalise with $\overline{MS}$, + Fierz and γ_5 in 4-d because only ever looking for $1/\epsilon$ poles.

- Suppose renormalisable model, mediates LFV.
  eg New leptoquark of mass Λ_{NP}.



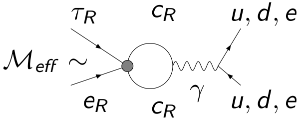
- Allows to calculate $\mathcal{M}_{NP}(\tau \to e\rho)$ (also $\tau \to 3e$), which is finite

$$\langle e(p_e, s_e), \rho(p_\rho)|\boldsymbol{S}|\tau(\vec{P} = 0, S_\tau)\rangle = i\tilde{\delta}^4(P - p_e - p_\rho)\mathcal{M}_{NP}$$

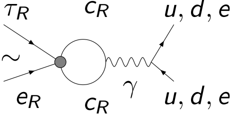
- But $\Lambda_{NP} \gg m_\tau$?
  1 : just calculate *relevant* dynamics = SM.
  2 : $\mathcal{O}(m_\tau^2/\Lambda^2)$ part of $\mathcal{M}_{NP}$ accurate enough ?

At tree level, in $\mathcal{L}$, replace LQ $\to \underbrace{\frac{\lambda_R \lambda_R}{2\Lambda_{NP}^2}}_{\text{coeff. } C_{LQ}} \underbrace{(\overline{e}\gamma P_R \tau)(\overline{c}\gamma P_R c)}_{\text{operator } O_{LQ}}$

- And calculate $\mathcal{M}_{eff}$



To calculate, ctd

$\mathcal{M}_{eff} \sim$ 

- *Yeeks !* $\mathcal{M}_{eff}$ diverges...
  ...regularise+add counterterms (for all operators generated)
  ($\Rightarrow \delta\mathcal{L} : C_{LQ}\mathcal{O}_{LQ} \rightarrow \mu^{2\epsilon}\sum_N C_{LQ}Z_{LQ,N}Z_\psi^{n/2}\mathcal{O}_N$)

- With renormalised $\mathcal{L}$, obtain finite $\mathcal{M}_{eff,ren}$ ...except, still logμ in $\mathcal{M}_{eff,ren}$.

- cancel μ-dependence of time-ordered-product-of-fields in usual way :
  require coupling constants (= operator coefficients) to be μ-dependent
  $\Rightarrow$ RGEs : $\mu\frac{d}{d\mu}\vec{C} = \vec{C}\left(\mu\frac{d}{d\mu}[Z]\right)Z^{-1} \equiv \vec{C}\cdot[\Gamma]$

- *Eureeka !* $\mathcal{M}_{eff,ren}$ with running coefficients $C(\mu)$, is finite + μ-indep.
  But :had to renormalise operators—result *cannot* depend on associated μ,
  or on scheme (no operators in renormalisable models),*so only can calculate*
  scheme-indep, μ-indep quantities !
  $\Rightarrow$ *at one loop, coeff of* $(1/\epsilon + \log)$ is scheme-indep
  $\Rightarrow$ allows to obtain *all* $(\frac{\log}{16\pi^2})^n$ terms ! (see Barr-Zee comments in a few slides)

But what is that scheme-indep log-term in $\mathcal{M}_{eff,ren}$?

- to calculate $C(\mu)$:

  1. match Greens fns in model, to tree level Greens fns in EFT... gives coeff $C(\Lambda_{NP}) \sim \lambda^2/\Lambda^2$

  2. scale evolution of $\vec{C}$ from RGEs (soln is "scale-ordered" exponential) ; run coeffs from $\Lambda_{NP} \to m_\tau$

- evaluate operator matrix element at m_τ : (more difficult for 4q)

  $\langle e,\rho|(\overline{e}\gamma P_R\tau)(\overline{q}\gamma q)|\tau\rangle \propto \overline{u}_e\gamma P_R u_\tau\langle\rho|(\overline{q}\gamma q)|0\rangle$

  $\langle e,\bar{e},e|(\overline{e}\gamma P_R\tau)[(\overline{e}\gamma e)]|\tau\rangle \propto (\overline{u}_e\gamma P_R u_\tau)(\overline{u}_e\gamma P_R u_e)$

  (troubles with quarks below m_τ where QCD becomes strong...)

- then $\mathcal{M}_{eff,ren} \simeq C_j(m_\tau)\langle f|\mathcal{O}_j|i\rangle(m_\tau)$

("Generalised Neutrino Interactions") ...

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$2\nu 2f$ four-fermion interactions, ν light, can be sterile $\simeq \nu_R$, $f \in \{e, d, u\}$

$$(\overline{\nu}_\rho \gamma P_L \nu_\sigma)(\overline{f} \gamma P_X f)\ ,\quad (\overline{\nu}_\rho P_L \nu_\sigma)(\overline{f} P_X f)\ ,\quad (\overline{\nu}_\rho \sigma P_L \nu_\sigma)(\overline{f} \sigma P_L f)$$

interest : COHERENT measured Coherent Elastic ν-Nucleus Scattering [CEνNS : $\sigma(\nu A \to \nu A)$, $q^2 \sim 50$ MeV so $\mathcal{M}(\nu A \to \nu A) \propto A\mathcal{M}(\nu n \to)$]
CEνNS *not* forward scattering, sensitive to more operators, in different combo from (incoherent) high-E σ.

NSI : coherent, some flavour combos interfere with SM
scalar GNI : coherent, not interfere SM (outgoing ν_R)
axial/pseudoscalar/tensor : f current $\to$ nucleon spin, incoherent on unpolaised target...

Lets stick to NSI...

Bounds on NSI from Neutral Current ν scattering (high energy)

BerezhianiRossi
DavidsonPenaGaraySantamariaRius

chiral ε ($g_L^f \neq g_R^f$ in SM), weaker bd to fit on slide

$-.4 < \varepsilon^{ee}_{u,L,R} < .7$ $-.6 < \varepsilon^{ee}_{d,L,R} < .5$ $-1, < \varepsilon^{ee}_{e} < .5$	$-.5 < \varepsilon^{e\mu}_{u,L,R} < .5$ $-.5 < \varepsilon^{e\mu}_{d,L,R} < .5$ $-.18 < \varepsilon^{e\mu}_{e} < .15$	$-.5 < \varepsilon^{e\tau}_{u,L,R} < .5$ $-.5 < \varepsilon^{e\tau}_{d,L,R} < .5$ $-.7 < \varepsilon^{e\tau}_{e} < .7$
	$-.008 < \varepsilon^{\mu\mu}_{u,L,R} < .003$ $-.008 < \varepsilon^{\mu\mu}_{d,L,R} < .015$ $-.03 < \varepsilon^{\mu\mu}_{e,L,R} < .03$	$-.05 < \varepsilon^{\mu\tau}_{u,L,R} < .05$ $-.05 < \varepsilon^{\mu\tau}_{d,L,R} < .05$ $-.1 < \varepsilon^{\mu\tau}_{e,L,R} < .1$
		$< \varepsilon^{\tau\tau}_{u,L,R} <$ $< \varepsilon^{\tau\tau}_{d,L,R} <$ $-.6, -.4 < \varepsilon^{\tau\tau}_{e,L,R} < .4, .6$

LSND : $\nu_e e \to \nu e$
CHARM : $\nu_e q \to \nu q$
CHARMII : $\nu_\mu e \to \nu e$
NuTeV : $\nu_\mu q \to \nu q$

LEP-1 : $Z \to \nu\nu\gamma$