Combining $e^+e^- (\tau)$ hadronic spectra and applications

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In collaboration with Michel Davier, Andreas Hoecker, Zhiqing Zhang
“Main application”: Hadronic Vacuum Polarization and Muon \((g - 2)_\mu\)

Dominant uncertainty for the theoretical prediction: from lowest-order HVP piece Cannot be calculated from perturbative QCD (low mass scale), but one can use experimental data on \(e^+e^- \rightarrow\) hadrons cross section

\[
\text{Born: } \sigma^{(0)}(s) = \sigma(s)\left(\frac{\alpha}{\alpha(s)}\right)^2
\]

\[
12\pi \text{ Im} \Pi_\gamma(s) = \frac{\sigma^0 \left[ e^+e^- \rightarrow \text{hadrons} \left(\gamma_{FSR}\right) \right]}{\sigma_{pt}} \equiv R(s)
\]

\[
\text{Im} \left[\begin{array}{c}
\text{hadrons} \\
|\text{hadrons}|^2
\end{array}\right] \propto \left|\begin{array}{c}
\text{hadrons} \\
\text{hadrons}
\end{array}\right|
\]

\[
a^\text{had}_{\mu} = \frac{\alpha^2}{3\pi^2} \int_0^{\infty} ds \frac{K(s)}{s} R(s)
\]

\[\rightarrow \text{Precise } \sigma(e^+e^- \rightarrow \text{hadrons}) \text{ measurements at low energy are very important}\]
Data on $e^+e^- \rightarrow \text{hadrons}$

Combination of all $e^+e^-$ data:
focus on the combination procedure
(HVPTools and fit based on analyticity & unitarity)

Results on $a_\mu$

Comparison with $\tau$ data

Indications of uncertainties on uncertainties and on correlations & their implications for combinations

Discussion and conclusions
HVP: Low-energy data on $e^+e^- \rightarrow$ hadrons

$\sqrt{s}$ scan + radiative corrections: CMD-2&3, SND, BES etc.

KLOE (08&10) + $\mu\mu$ (12) (ISR)

BABAR (09) (ISR + Add. rad.)

Need: $e^+e^- \rightarrow$ hadrons bare (no VP) cross section
→ in addition to the dominant $\pi\pi$ channel, need to account for KK, $\pi^0\gamma$, $\eta\gamma$
  + channels with higher multiplicities
→ need to combine measurements in each channel & sum channels
→ Do not use hadronic $\tau$ decays data for g-2 (less precise + theory uncertainties)
Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel (2017)

Improved procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning

Goal: combine experimental spectra with arbitrary point spacing / binning

Requirements:

- Properly propagate uncertainties and correlations
  - Between measurements (data points/bins) of a given experiment (covariance matrices and/or detailed split of uncertainties in sub-components)
  - Between experiments (common systematic uncertainties, e.g. VP) – based on detailed information provided in publications
  - Between different channels – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones:
    - BABAR luminosity (ISR or BhaBha), efficiencies (photon, Ks, Kl, modeling);
    - BABAR radiative corrections; $4\pi 2\pi^0 - \eta\omega$
    - CMD2 $\eta\gamma - \pi^0\gamma$; CMD2/3 luminosity; SND luminosity;
    - FSR; hadronic VP (old experiments)

- Minimize biases

- Optimize $g-2$ integral uncertainty (without overestimating the precision with which the uncertainties of the measurements are known)
Combination procedure implemented in HVPTools software

→ Define a (fine) final binning (to be filled and used for integrals etc.)
→ Linear/quadratic splines: interpolate between the points/bins of each experiment
  - for binned measurements: preserve integral inside each bin
  - closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination
    → (non-)negligible bias for (linear)quadratic interpolation
→ Fluctuate data points taking into account correlations and re-do the splines for each (pseudo-)experiment
  - each uncertainty fluctuated coherently for all the points/bins that it impacts
  - eigenvector decomposition for (statistical & systematic) covariance matrices
Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize $\chi^2$ and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty:
  - allows to account for correlations between different channels (for integrals and spectra)

→ Checked consistency between the different approaches
Treatment of the KLOE correlation matrices

→ Statistical and systematic correlation matrices among the 3 measurements
Treatment of the KLOE data – eigenvector decomposition

Statistical cov. mat.  
KLOE 08-10-12

Systematic cov. mat.  
KLOE 08-10-12

→ “counting” the number of independent components (50) used to build the covariance matrix

\[ C = S \cdot D \cdot S^T \]

\[ D = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

\[ S = (V_1 \ldots V_n) \]

→ Problem of negative eigenvalues for previous systematic covariance matrix solved (informed KLOE collaboration about the problem in summer 2016)
Treatment of the KLOE data – eigenvector decomposition

→ Each normalized eigenvector \((\sigma_i^* V_i)\) treated as an uncertainty fully correlated between the bins
→ All these uncertainties are independent between each-other

\[
C = \sum_{i=1}^{N_{bins}} \sigma_i^2 \cdot C(V_i)
\]

→ Checked exact matching with the original matrices + with all \(a_\mu\) integrals and uncertainties published by KLOE

→ See backup for matrices and eigenvectors of combined KLOE data
Combination procedure: weights of various measurements

For each final bin:
→ Minimize $\chi^2$ and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements (do not over-estimate the weight of experiments with large bins)
→ weights in fine bins evaluated using a common (large) binning for measurements + interpolation → compare the precisions on the same footing

→ Bins used by KLOE larger than the ones by BABAR in $\rho$-$\omega$ interference region (factor $\sim 3$)

→ Average dominated by BaBar and KLOE, BaBar covering full range
Combination procedure: compatibility between measurements

For each final bin:

→ $\chi^2/\text{ndof}$: test locally the level of agreement between input measurements, taking into account the correlations

→ Scale uncertainties in bins where $\chi^2/\text{ndof} > 1$ (PDG): locally conservative

→ Observed tension between BABAR and KLOE measurements

→ Tension between measurements: indication of underestimated uncertainties

→ Motivates conservative uncertainty treatment in evaluation of weights
Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel

<table>
<thead>
<tr>
<th>√s [GeV]</th>
<th>Cross section [nb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>0.8</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1.2</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1.4</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1.6</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>1.8</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>2</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>2.2</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>2.4</td>
<td>$10^{-1}$</td>
</tr>
</tbody>
</table>

TOF  
OLYA  
CMD  
CMD-2 06  
CMD-2 03  
KLOE 08  
KLOE 10  
KLOE 12  
BESIII  
SND  
DM1  
DM2  
CLEO  
CLEO  
BABAR  

Combined
Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel

Cross section [nb]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\sqrt{s} [GeV] & 0.3 & 0.35 & 0.4 & 0.45 & 0.5 & 0.55 & 0.6 & 0.65 & 0.7 \\
Cross section [nb] & 200 & 400 & 600 & 800 & 1000 & 1200 & 1400 & 1600 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\sqrt{s} [GeV] & 0.72 & 0.73 & 0.74 & 0.75 & 0.76 & 0.77 & 0.78 & 0.79 & 0.8 \\
Cross section [nb] & 700 & 800 & 900 & 1000 & 1100 & 1200 & 1300 & 1400 & 1500 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\sqrt{s} [GeV] & 0.82 & 0.84 & 0.86 & 0.88 & 0.9 & 0.92 & 0.94 & 0.96 \\
Cross section [nb] & 100 & 200 & 300 & 400 & 500 & 600 & 700 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\sqrt{s} [GeV] & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 \\
Cross section [nb] & 10 & 20 & 30 & 40 & 50 & 60 & 70 \\
\end{array}
\]
Combination for the $e^+e^- \rightarrow \pi^+\pi^-$ channel

Slope between various results

Local tension
\[ v_{1,X}^{-}(s) = \frac{m_{\pi}^2}{6|V_{ud}|^2} \frac{B_{X^-}}{B_{e}} \frac{1}{N_X} \frac{dN_X}{ds} \times \left( 1 - \frac{s}{m_{\tau}^2} \right)^2 \left( 1 + \frac{2s}{m_{\tau}^2} \right)^{-1} \frac{R_{IB}(s)}{S_{EW}} \]

\[ R_{IB}(s) = \frac{\text{FSR}(s) \beta_0^3(s)}{G_{EM}(s) \beta_3^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2 \]

→ Comparing corrections used by Davier et al. with the ones by F. Jegerlehner

Plots by Z. Zhang, based on private communication with F. Jegerlehner
Comparison with IB-corrected \( \tau \) data

→ for \( a_\mu, e^+e^- - \tau \) difference of 2.2 \( \sigma \) (Davier et al.)

→ the \( \rho - \gamma \) mixing correction proposed in arXiv:1101.2872 (FJ) seems to over-estimate the \( e^+e^- - \tau \) difference
Improving $a_\mu$ through fits for the $e^+e^- \rightarrow \pi^+\pi^-$ channel

→ Fit bare form-factor using 6 param. model based on analyticity and unitarity

$$|F_\pi^0|^2 = |R(s) \times J(s)|^2$$

$$R(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - im_\omega \Gamma_\omega}$$

(1611.09359, C. Hanhart et al.)

$$J(s) = e^{1 - \frac{\delta_1(s_0)}{s}} \left(1 - \frac{s}{s_0}\right)^{\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi} \int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t - s}}$$

Omnès integral

(hep-ph/0402285, F.J. Yndurain et al.)

$$\cot \delta_1(s) = \frac{\sqrt{s}}{2k^3} \left(m_\rho^2 - s\right) \left[\frac{2m_\pi^3}{m_\rho^2 \sqrt{s}} + B_0 + B_1 \omega(s)\right]$$

(1102.2183, F.J. Yndurain et al.)

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}$$

$\sqrt{s_0} = 1.05$ GeV

→ Conservative $\chi^2$ (diagonal matrix) & local rescaling of input uncertainties

→ Full uncertainty propagation using pseudo-experiments

 DHMZ - 1908.00921
Fit performed up to 1 GeV: comparison with data

B. Malaescu (CNRS) – Valencia GenT workshop – December 2019
Fit performed up to 1 GeV, Result used up to 0.6 GeV

→ Use fit only below 0.6 GeV for $a_\mu$
    integral:
    - where data is less precise and scarce
    - less impacted by potential uncertainties of inelastic effects

<table>
<thead>
<tr>
<th>$\sqrt{s}$ range [GeV]</th>
<th>$a_\mu^{\text{had}} [10^{-10}]$ Fit</th>
<th>$a_\mu^{\text{had}} [10^{-10}]$ Data Integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3 - 0.6</td>
<td>$109.80 \pm 0.37_{\text{exp}} \pm 0.36_{\text{para}^*}$</td>
<td>$109.6 \pm 1.0_{\text{exp}}$</td>
</tr>
</tbody>
</table>

→ The difference $0.2 \pm 0.9$
   (72% correlation accounted for)

→ The fit improves the precision by a factor $\sim 2$

(*) Parameter uncertainty corresponds to variations with/without the $B_\gamma$ term in the phase shift formula and $\sqrt{s_0}$ varied from 1.05 GeV to 1.3 GeV (absolute values summed linearly), checked to be statistically significant
Combined results: \( \text{Fit} [<0.6\text{GeV}] + \text{Data}[0.6-1.8\text{GeV}] \)

→ Full uncertainty propagation using the same pseudo-experiments as for the spline-based combination: 62% correlation among the two contributions

<table>
<thead>
<tr>
<th>( \sqrt{s} ) range [GeV]</th>
<th>( a_\mu^{\text{had}} [10^{-10}] ) All data</th>
<th>( a_\mu^{\text{had}} [10^{-10}] ) All but BABAR</th>
<th>( a_\mu^{\text{had}} [10^{-10}] ) All but KLOE</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold - 1.8</td>
<td>506.9 ± 1.9_{\text{total}}</td>
<td>505.0 ± 2.1_{\text{total}}</td>
<td>510.6 ± 2.2_{\text{total}}</td>
</tr>
</tbody>
</table>

→ The difference “All but BABAR” and “All but KLOE” = 5.6 to be compared with 1.9 uncertainty with “All data”
- The local error inflation is not sufficient to amplify the uncertainty
- Global tension (normalisation/shape) not previously accounted for
- Potential underestimated uncertainty in at least one of the measurements?
- Other measurements not precise enough to discriminate BABAR / KLOE

→ Given the fact we do not know which dataset is problematic, we decide to:
  - Add half of the discrepancy (2.8) as an additional uncertainty (correcting the local PDG inflation to avoid double counting)
  - Take (“All but BABAR” + “All but KLOE”) / 2 as central value
Direct comparison of the 3 KLOE measurements
Local comparison of the 3 KLOE measurements

→ Local $\chi^2$/ndof test of the local compatibility between KLOE 08 & 10 & 12, taking into account the correlations: some tensions observed

→ Does not probe general trends of the difference between the measurements (e.g. slopes in the ratio)
→ Compute ratio between pairs of KLOE measurements
→ Full propagation of uncertainties and correlations using pseudo-experiments (agreement with analytical linear uncertainty propagation)

→ Good agreement between KLOE 10 and KLOE 12
Ratios between measurements

B. Malaescu (CNRS)
Valencia GenT workshop
December 2019

1.1
1.08
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
0.9

0.9
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1
√s [GeV]

50
45
40
35
30
25
20
15
10
5
0
n bin

50
45
40
35
30
25
20
15
10
5
0
n bin

50
45
40
35
30
25
20
15
10
5
0
n bin

50
45
40
35
30
25
20
15
10
5
0
n bin

50
Quantitative comparisons of the KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit

Comparison with Unity:
\( \chi^2 [0.35;0.85] \text{ GeV}^2 : 79.0 / 50(\text{DOF}) \)
P-value= 0.0056

\( \chi^2 [0.35;0.58] \text{ GeV}^2 : 46.2 / 23(\text{DOF}) \)
P-value= 0.0028

\( \chi^2 [p0 + p1 \sqrt{s}] : 36.1 / 21(\text{DOF}) \)
P-value= 0.02

\ p0 : 0.745 ± 0.085 \\
\ p1 : 0.341 ± 0.117 \\

→ Significant shift & slope (~2.5-3σ) at low \( \sqrt{s} \), no significant shift at high \( \sqrt{s} \) 
Similar shift & slope for KLOE 12 / KLOE 08 (see backup)

→ Should motivate conservative treatment of uncertainties and correlations in combination
Uncertainties on uncertainties and on correlations
\( \chi^2 \) definitions and properties

\[
\chi^2 (d; t) = \sum_{i,j} (d_i - t_i) \cdot [C^{-1}(t)]_{ij} \cdot (d_j - t_j)
\]

\[
\chi^2 (d; t) = \min_{\beta_a} \left\{ \sum_{i,j} \left[ \sum_{a} \beta_a \cdot (\epsilon^\pm_a (\beta_a))_i \cdot t_i \right] \cdot [C_{su}^{-1}(t)]_{ij} \cdot \left[ \sum_{a} \beta_a \cdot (\epsilon^\pm_a (\beta_a))_j \cdot t_j \right] + \sum_{a} \beta_a^2 \right\},
\]

→ Two \( \chi^2 \) definitions, with systematic uncertainties included in covariance matrix or treated as fitted “nuisance parameters”

→ Equivalent for symmetric Gaussian uncertainties

(1312.3524 - ATLAS)

→ Both approaches assume the knowledge of the amplitude, shape (phase-space dependence) and correlations of systematic uncertainties
Example: published uncertainties on correlations

1406.0076 – ATLAS jet energy scale uncertainties

Nominal correlation scenario

Weaker - stronger correlation scenarios
Two different approaches for combining \((e^+e^-)\) data

**DHMZ:**
→ $\chi^2$ computed locally (in each fine bin), taking into account correlations between measurements (see previous slides)
→ used to determine the weights on the measurements in the combination and their level of agreement
→ uncertainties and correlations propagated using pseudo-experiments or $\pm 1\sigma$ shifts of each uncertainty component

**KNT:**
→ $\chi^2$ computed globally (for full mass range)

\[
\chi^2_{\text{KNT}} = \sum_{i=1}^{N_{\text{tot}}} \sum_{j=1}^{N_{\text{tot}}} (R_{i}^{(m)} - R_{m}^{i,j}) C_{I}^{-1}(i^{(m)}, j^{(n)}) (R_{j}^{(n)} - R_{n}^{j,i})
\]

\[
\chi^2_{\text{KLOE-KMT}} = \sum_{i=1}^{195} \sum_{j=1}^{195} (\sigma_{\pi\pi(\gamma)}^{0}(i) - \sigma_{\pi\pi(\gamma)}^{0}(m)) C_{I}^{-1}(i^{(m)}, j^{(n)}) (\sigma_{\pi\pi(\gamma)}^{0}(j) - \sigma_{\pi\pi(\gamma)}^{0}(n))
\]

→ relies on description of correlations on long ranges

→ *One of the main sources of differences for the uncertainty on $a_\mu$*
Evaluation of uncertainties and correlations ($e^+e^-$)

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{\pi\gamma}$</th>
<th>$\sigma^0_{\pi\pi}$</th>
<th>$F^*$</th>
<th>$\Delta^\pi\pi a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reconstruction Filter</td>
<td>negligible</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Background subtraction</td>
<td>Tab. [1]</td>
<td>0.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trackmass</td>
<td>0.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pion cluster ID</td>
<td>negligible</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>0.1%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Acceptance</td>
<td>Tab. [2]</td>
<td>0.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unfolding</td>
<td>Tab. [3]</td>
<td>negligible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{s}$ dependence of $H$</td>
<td>Tab. [4]</td>
<td>0.2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luminosity</td>
<td>0.3%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>$\sigma_{\pi\gamma}$</th>
<th>$\sigma^0_{\pi\pi}$</th>
<th>$F^*$</th>
<th>$\Delta^\pi\pi a_\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FSR resummation</td>
<td>0.3%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radiator function $H$</td>
<td>0.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vacuum Polarization</td>
<td>0.1%</td>
<td>0.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory systematics</td>
<td>0.6%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the correlation of the systematic uncertainty due to the acceptance, only half of the KLOE10 uncertainty is correlated with the KLOE08 uncertainty in order to ensure that the photon detection acceptance that enters into the KLOE10 uncertainty (that is not present in the KLOE08 analyses) is not correlated and only the correlation of the pion tracks is duly accounted for.

Is this statement ("same impact of photon and pions on the acceptance") valid on the full $\sqrt{s}$ range?

Systematics evaluated in ~wide mass ranges with sharp transitions

KLOE 10 (1006.5313)

<table>
<thead>
<tr>
<th>$M^2_{\pi\pi}$ range (GeV$^2$)</th>
<th>Systematic error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.35 \leq M^2_{\pi\pi} &lt; 0.39$</td>
<td>0.6</td>
</tr>
<tr>
<td>$0.39 \leq M^2_{\pi\pi} &lt; 0.43$</td>
<td>0.5</td>
</tr>
<tr>
<td>$0.43 \leq M^2_{\pi\pi} &lt; 0.45$</td>
<td>0.4</td>
</tr>
<tr>
<td>$0.45 \leq M^2_{\pi\pi} &lt; 0.49$</td>
<td>0.3</td>
</tr>
<tr>
<td>$0.49 \leq M^2_{\pi\pi} &lt; 0.51$</td>
<td>0.2</td>
</tr>
<tr>
<td>$0.51 \leq M^2_{\pi\pi} &lt; 0.64$</td>
<td>0.1</td>
</tr>
<tr>
<td>$0.64 \leq M^2_{\pi\pi} &lt; 0.95$</td>
<td>-</td>
</tr>
</tbody>
</table>

| Source                      | $\sigma_{\pi\gamma}$ | $\sigma^0_{\pi\pi}$ | $|F^*|^2$ | $\Delta a_{\pi\pi}^\mu$ ($0.1$ - $0.85$ GeV$^2$) |
|-----------------------------|-----------------------|----------------------|----------|--------------------------------------------------|
| Background Filter           | 0.5% ; 0.1%           | negligible           |          |                                                  |
| Background subtraction      | 3.4% ; 0.1%           | 0.5%                 |          |                                                  |
| $f_0 + \rho \pi$ bkg.      | 6.5% ; negl.           | 0.4%                 |          |                                                  |
| $\Omega$ cut               | 1.4% ; negl.           | 0.2%                 |          |                                                  |
| Trackmass cut              | 3.0% ; 0.2%           | 0.5%                 |          |                                                  |
| $\pi$-e PID                | 0.3% ; negl.           | 0.2%                 |          | negligible                                       |
| Trigger                     | 0.3% ; 0.2%           | 0.2%                 |          |                                                  |
| Acceptance                  | 1.9% ; 0.3%           | 0.5%                 |          |                                                  |
| Unfolding                  | negl. ; 2.0%          | 1.0%                 |          |                                                  |
| Tracking                    | 0.3%                  |                      |          |                                                  |
| Software Trigger (L3)      | 0.1%                  |                      |          |                                                  |
| Luminosity                  | 0.3%                  |                      |          |                                                  |
| Experimental syst.         | 1.0%                  |                      |          |                                                  |
| FSR treatment               | -                     | 7% ; negl.            | 0.8%     |                                                  |
| Radiator function $H$      | -                     | 0.5%                 |          |                                                  |
| Vacuum Polarization         | - Ref. [34]           | - 0.1%               | 0.1%     |                                                  |
| Theory syst.               | -                     | 0.9%                 |          |                                                  |
### Evaluation of uncertainties and correlations ($e^+e^-$)

<table>
<thead>
<tr>
<th>Sources</th>
<th>0.3-0.4</th>
<th>0.4-0.5</th>
<th>0.5-0.6</th>
<th>0.6-0.9</th>
<th>0.9-1.2</th>
<th>1.2-1.4</th>
<th>1.4-2.0</th>
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</table>

→ **Systematics evaluated** in ~wide mass ranges with sharp transitions (statistics limitations when going to narrow ranges)

BABAR (1205.2228)
Uncertainties on uncertainties and correlations ($e^+e^-$)

→ Shapes of systematic uncertainties evaluated in ~wide mass ranges with sharp transitions

→ One standard deviation is statistically not well defined for systematic uncertainties

→ Systematic uncertainties like acceptance, tracking efficiency, background etc. not necessarily fully correlated between low and high mass

→ Are all systematic uncertainty components fully independent between each-other? (e.g. tracking and trigger)

→ Yield uncertainties on uncertainties and on correlations

→ Tensions between measurements (BABAR/KLOE; 3 KLOE results etc.): indications of underestimated uncertainties
Combining the 3 KLOE measurements

Local combination (DHMZ)

Information propagated between mass regions, through shifts of systematics - relying on correlations, amplitudes and shapes of systematics (KLOE-KT)
Combining the 3 KLOE measurements - $a_\mu^{\pi\pi}$ contribution

KLOE08 $a_\mu[0.6;0.9]: 368.3 \pm 3.2 [10^{-10}]$
KLOE10 $a_\mu[0.6;0.9]: 365.6 \pm 3.3$
KLOE12 $a_\mu[0.6;0.9]: 366.8 \pm 2.5$

→ Correlation matrix:

<table>
<thead>
<tr>
<th></th>
<th>08</th>
<th>10</th>
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<tr>
<td>12</td>
<td>0.35</td>
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</table>

→ Amount of independent information provided by each measurement

→ KLOE-08-10-12(DHMZ) - $a_\mu[0.6;0.9]: 366.5 \pm 2.8$ (Without $\chi^2$ rescaling: $\pm 2.2$)
→ Conservative treatment of uncertainties and correlations (not perfectly known) in weight determination

→ KLOE-08-10-12(KLOE-KT) - $a_\mu[0.6;0.9]$GeV: $366.9 \pm 2.2$ (Includes $\chi^2$ rescaling)

→ Assuming perfect knowledge of the correlations to minimize average uncertainty
Uncertainties on uncertainties & correlations: status and outlook

→ Numerous indications of uncertainties on uncertainties and on correlations, with a direct impact on combinations

Recommendation:

1) Short term (with current experimental inputs)
→ use combination approaches that do not exploit assumptions on long-range correlations that are experimentally not under control

2) Long term
→ provide measurements with information on uncertainties on uncertainties and on their correlations
Combination of measurements for various channels and total HVP contribution
Combination for the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ channel
Combination for the $e^+e^- \rightarrow K^+K^-$ channel

$\rightarrow$ Tension between measurements

$\rightarrow a_\mu[\rightarrow 1.8\text{GeV}]: 23.08 \pm 0.20 \text{ (stat.)} \pm 0.40 \text{ (syst.) } [10^{-10}] \text{ (enhancement x 2.2)}$
\[ e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-, \quad e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0 \]

\[ \rightarrow \text{Essentially normalization differences w.r.t. } \tau \text{ data: cross-checks very desirable} \]
Combination for the $e^+e^- \rightarrow KK\pi$ and $KK2\pi$ channels

B. Malaescu (CNRS) – Valencia GenT workshop – December 2019
Contributions from the 1.8 – 3.7 GeV region

Contribution evaluated from pQCD (4 loops) + $O(\alpha_s^2)$ quark mass corrections

Uncertainties: $\alpha_s$, truncation of perturbative series, CIPT/FOPT, $m_q$

1.8-2.0 GeV: $7.65\pm0.31$ (data excl.); $8.30\pm0.09$ (QCD); added syst. $0.65 \times 10^{-10}$

2.0-3.7 GeV: $25.82\pm0.61$ (data); $25.15 \pm 0.19$ (QCD); agreement within $1\sigma$
Contributions from the charm resonance region

\[ e^+e^- \rightarrow \text{Hadrons} \]

Graph showing cross section vs. \( \sqrt{s} \) in GeV. The graph includes data from PLUTO, CB, BES, and a combined dataset.
### Situation in arXiv:1908.00921

- 32 exclusive channels are integrated up to 1.8 GeV
- Only $0.016 \pm 0.016\%$ in missing (estimated) channels for $a_\mu$

### Table

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma^\text{had,LO}_{\mu} [10^{-10}]$</th>
<th>$\Delta \sigma^\text{had}(m_Z^2) [10^{-4}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0 \gamma$</td>
<td>$4.29 \pm 0.06 \pm 0.04 \pm 0.07$</td>
<td>$0.35 \pm 0.00 \pm 0.00 \pm 0.01$</td>
</tr>
<tr>
<td>$\eta \gamma$</td>
<td>$0.65 \pm 0.02 \pm 0.01 \pm 0.01$</td>
<td>$0.08 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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<tr>
<td>$\pi^+ \pi^-$</td>
<td>$507.80 \pm 0.83 \pm 3.19 \pm 0.60$</td>
<td>$34.49 \pm 0.06 \pm 0.20 \pm 0.04$</td>
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<td>$\pi^+ \pi^- \pi^0$</td>
<td>$46.20 \pm 0.40 \pm 1.10 \pm 0.86$</td>
<td>$4.60 \pm 0.04 \pm 0.11 \pm 0.08$</td>
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<tr>
<td>$3\pi^+ 2\pi^-$</td>
<td>$13.68 \pm 0.03 \pm 0.27 \pm 0.14$</td>
<td>$3.58 \pm 0.01 \pm 0.07 \pm 0.03$</td>
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<td>$\pi^+ \pi^- 2\pi^0$</td>
<td>$18.03 \pm 0.06 \pm 0.48 \pm 0.26$</td>
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<td>$2\pi^+ 2\pi^- \pi^0 (\eta \text{ excl.})$</td>
<td>$0.69 \pm 0.04 \pm 0.06 \pm 0.03$</td>
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<td>$\pi^+ \pi^- 3\pi^0 (\eta \text{ excl.})$</td>
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<td>$\pi^+ \pi^- 4\pi^0 (\eta \text{ excl., isospin})$</td>
<td>$0.08 \pm 0.01 \pm 0.08 \pm 0.00$</td>
<td>$0.03 \pm 0.00 \pm 0.03 \pm 0.00$</td>
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<td>$\eta \pi^+ \pi^-$</td>
<td>$1.19 \pm 0.02 \pm 0.04 \pm 0.02$</td>
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<td>$\eta \omega$</td>
<td>$0.35 \pm 0.01 \pm 0.02 \pm 0.01$</td>
<td>$0.11 \pm 0.00 \pm 0.01 \pm 0.00$</td>
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<tr>
<td>$\eta \pi^+ \pi^- 3\pi^0 (\text{non-}\omega, \phi)$</td>
<td>$0.34 \pm 0.03 \pm 0.03 \pm 0.04$</td>
<td>$0.12 \pm 0.01 \pm 0.01 \pm 0.01$</td>
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<tr>
<td>$\eta 2\pi^+ 2\pi^-$</td>
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<td>$\omega \pi^0$</td>
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<td>$0.02 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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<td>$\omega \pi^0 (\omega \rightarrow \pi^0 \gamma)$</td>
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<td>$\omega (\text{non-}3\pi^-, \pi^+, \pi\gamma)$</td>
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<td>$0.00 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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<tr>
<td>$K^+ K^-$</td>
<td>$23.08 \pm 0.20 \pm 0.33 \pm 0.21$</td>
<td>$3.35 \pm 0.03 \pm 0.05 \pm 0.03$</td>
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<td>$K_S K_L$</td>
<td>$12.82 \pm 0.00 \pm 0.18 \pm 0.15$</td>
<td>$1.74 \pm 0.01 \pm 0.03 \pm 0.02$</td>
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<td>$\phi (\text{non-}K K, 3\pi, \pi\gamma, \eta\gamma)$</td>
<td>$0.05 \pm 0.00 \pm 0.00 \pm 0.00$</td>
<td>$0.01 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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<tr>
<td>$K K \pi$</td>
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<td>$K K \omega$</td>
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<tr>
<td>$\eta \phi$</td>
<td>$0.33 \pm 0.01 \pm 0.01 \pm 0.00$</td>
<td>$0.11 \pm 0.00 \pm 0.00 \pm 0.00$</td>
</tr>
<tr>
<td>$\eta K K$ (non-\phi)</td>
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<td>$0.00 \pm 0.00 \pm 0.00 \pm 0.00$</td>
</tr>
<tr>
<td>$\omega 3\pi (\omega \rightarrow \pi^0 \gamma)$</td>
<td>$0.06 \pm 0.01 \pm 0.01 \pm 0.01$</td>
<td>$0.02 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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<tr>
<td>$7\pi (3\pi^+ 3\pi^- \pi^0 + \text{estimate})$</td>
<td>$0.02 \pm 0.00 \pm 0.01 \pm 0.00$</td>
<td>$0.01 \pm 0.00 \pm 0.00 \pm 0.00$</td>
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</table>

### $J/\psi$ (BW integral)
- $6.28 \pm 0.07$
- $7.09 \pm 0.08$

### $\psi(2S)$ (BW integral)
- $1.57 \pm 0.03$
- $2.50 \pm 0.04$

### R data [3.7 – 5.0] GeV
- $7.29 \pm 0.05 \pm 0.30 \pm 0.00$
- $15.79 \pm 0.12 \pm 0.66 \pm 0.00$

### $R_{QCD}$
- $[1.8 – 3.7 \text{ GeV}]_{uds}$ $33.45 \pm 0.28 \pm 0.65_{\text{dual}}$
- $[5.0 – 9.3 \text{ GeV}]_{uds,c}$ $8.68 \pm 0.04$
- $[9.3 – 12.0 \text{ GeV}]_{uds,c}$ $1.21 \pm 0.01$
- $[12.0 – 40.0 \text{ GeV}]_{uds,c}$ $1.64 \pm 0.00$
- $[> 40.0 \text{ GeV}]_{uds,c}$ $0.16 \pm 0.00$
- $[> 40.0 \text{ GeV}]_{c}$ $0.00 \pm 0.00$

### Sum
- $693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_{\psi} \pm 0.7_{QCD}$
- $275.43 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_{\psi} \pm 0.55_{QCD}$
Status of $a_\mu$

- Including latest results on $e^+e^- \rightarrow$ hadrons in the combination + latest QED calculation (Kinoshita et al.) yields

$$a_\mu^{SM}[e^+e^-] = (11\,659\,183.0 \pm 4.0 \pm 2.6 \pm 0.1) \times 10^{-10}$$

HVP  LBL  EW  (±4.8)

- E-821 updated result

$$a_\mu^{exp} = (11\,659\,209.1 \pm 6.3) \times 10^{-10}$$

- Deviation $$(26.1 \pm 7.9) \times 10^{-10}$$

$$(3.3 \sigma)$$
\( R_{e^+e^-} \rightarrow \text{Hadrons} \)

→ Full propagation of uncertainties and correlations

→ Performed non-trivial check: 
\( a_\mu \) from sum of individual channels and from \( R_{e^+e^-} \) integral < 1.8 GeV
Conclusion

→ Long standing discrepancy between data and SM on $a_\mu : 3.3\sigma$

→ The evaluation of the HVP contribution to $a_\mu^{\text{SM}}$ is a continuous effort, following the release of new experimental data: $693.9 \pm 4.0 \left[10^{-10}\right]$

→ Added uncertainty to account for BABAR-KLOE tension, not resolved by other existing data

→ Looking forward to the improved experimental result at Fermilab

→ For discussion at this workshop: other possible applications of $e^+e^-$ and $\tau$ data for e.g. New Physics searches
\[ \vec{\mu} = g \frac{e}{2m} \vec{s}, \quad a = \frac{(g - 2)}{2} \]

Dirac (1928) \quad g_e=2 \quad a_e=0

anomaly discovered:
Kusch-Foley (1948) \quad a_e = (1.19 \pm 0.05) \times 10^{-3}

and explained by \( O(\alpha) \) QED contribution:
Schwinger (1948) \quad a_e = \alpha/2\pi = 1.16 \times 10^{-3}

first triumph of QED

\[ \Rightarrow a_e \text{ sensitive to quantum fluctuations of fields} \]
More Quantum Fluctuations

\[ a = a_{QED} + a_{\text{had}} + a_{\text{weak}} + ? a_{\text{new physics?}} \]

typical contributions:

\textbf{QED} up to \( O(\alpha^5) \) (Kinoshita et al.)

\textbf{Hadrons} vacuum polarization

\textbf{Electroweak} light-by-light (models)

\( \Rightarrow \) \( a_\mu \) much more sensitive to high scales

B. Malaescu (CNRS) – Valencia GenT workshop – December 2019
HVP: Data on $e^+e^- \rightarrow$ hadrons


CMD-2 (2006)

SND (2006)

KLOE (ISR)

Cross section [nb]

Mass [GeV]

Cross section [nb]

Mass [GeV]

Cross section [nb]

Mass [GeV]

Cross section [nb]

$\left( M_{e^+e^-}^0 \right)^2$ [GeV$^2$]

$0.1$ $0.2$ $0.3$ $0.4$ $0.5$ $0.6$ $0.7$ $0.8$ $0.9$ $1$

Absolute $\mu^+\mu^-$ cross section agrees with NLO QED within 1.1%

BaBar diagonal errors (stat+syst)
Closure test of the combination method:
- replace all central values of the measured cross sections by predictions from of a Gounaris-Sakurai model (keeping uncertainties unchanged)
- perform combination and integration procedure
- compare integration result with expectation from integral of the model

→ Bias ~ one unit of $10^{-10}$ when using linear interpolation
→ Negligible bias for quadratic interpolation

→ Updated result:
$506.70 \pm 2.32 (\pm 1.01 \text{ (stat.)} \pm 2.08 \text{ (syst.)}) \left[10^{-10}\right]$ 
(after uncertainty enhancement by $\sim14\%$ caused by the tension between inputs, taken into account through a local rescaling)

Total uncertainty: $5.9 \text{ (2003)} \rightarrow 2.8 \text{ (2011)} \rightarrow 2.6 \text{ (2017)} \rightarrow 2.3 \text{ (2018)}$
Contribution $a_{\mu \pi \pi}$ [0.28; 1.8] GeV – spline-based (2018)

→ with KLOE-08-10-12 (KLOE-KT) used as input: $506.55 \pm 2.38 \times 10^{-10}$

(after uncertainty enhancement by 18\% caused by the tension between inputs, taken into account through a local rescaling)

→ Compensation between uncertainty reduction for KLOE-08-10-12 (KLOE-KT), inducing a change of weights in DHMZ combination, and tension enhancement
Fit parameters, uncertainties and correlations $e^+e^- \rightarrow \pi^+\pi^-$

<table>
<thead>
<tr>
<th>$\alpha_N$</th>
<th>$\kappa[10^{-4}]$</th>
<th>$B_0$</th>
<th>$B_1$</th>
<th>$m_\rho$ [MeV]</th>
<th>$m_\omega$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_N$</td>
<td>0.133 $\pm$ 0.020</td>
<td>0.52</td>
<td>$-0.45$</td>
<td>$-0.97$</td>
<td>0.90</td>
</tr>
<tr>
<td>$\kappa[10^{-4}]$</td>
<td>21.6 $\pm$ 0.5</td>
<td>$-0.33$</td>
<td>$-0.57$</td>
<td>0.64</td>
<td>$-0.08$</td>
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<tr>
<td>$B_0$</td>
<td>1.040 $\pm$ 0.003</td>
<td>0.40</td>
<td>$-0.40$</td>
<td>0.29</td>
<td></td>
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<tr>
<td>$B_1$</td>
<td>$-0.13 \pm 0.11$</td>
<td>$-0.96$</td>
<td>0.20</td>
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<td></td>
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<tr>
<td>$m_\rho$ [MeV]</td>
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<td>774.5 $\pm$ 0.8</td>
<td>$-0.17$</td>
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</tr>
<tr>
<td>$m_\omega$ [MeV]</td>
<td></td>
<td>782.0 $\pm$ 0.1</td>
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<td></td>
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</tbody>
</table>

$\kappa$ corresponds to a $\text{Br} (\omega \rightarrow \pi^+\pi^-)$ of $(2.09 \pm 0.09) \cdot 10^{-2}$, in agreement with the result extracted from the fit of arXiv:1810.00007, $(1.95 \pm 0.08) \cdot 10^{-2}$. Both values disagree with the PDG average $(1.51 \pm 0.12) \cdot 10^{-2}$, dominated by the result of arXiv:1611.09359 which uses fits to essentially the same data.

The fitted $\omega$ mass is found to be lower than the PDG average obtained from $3\pi$ decays by $(0.65 \pm 0.12 \pm 0.12_{PDG})$ MeV, in agreement with previous fits of the $\rho - \omega$ interference in the $2\pi$ spectrum (see e.g. arXiv:1205.2228 and arXiv:1810.00007).
<table>
<thead>
<tr>
<th>Channel</th>
<th>( g_{\text{had,LO}}^{[10^{-10}]} )</th>
<th>( \Delta g_{\text{had}}^{M_2^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^0 \gamma )</td>
<td>4.42 ± 0.08 ± 0.13 ± 0.12</td>
<td>0.36 ± 0.01 ± 0.01 ± 0.01</td>
</tr>
<tr>
<td>( \eta \gamma )</td>
<td>0.64 ± 0.02 ± 0.01 ± 0.01</td>
<td>0.08 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( \pi^+ \pi^- )</td>
<td>507.80 ± 2.22 ± 2.50 ± 0.56</td>
<td>34.43 ± 0.07 ± 0.17 ± 0.04</td>
</tr>
<tr>
<td>( \pi^+ \pi^- \pi^0 )</td>
<td>46.00 ± 0.42 ± 1.03 ± 0.98</td>
<td>4.58 ± 0.04 ± 0.11 ± 0.09</td>
</tr>
<tr>
<td>( 2\pi^+ \pi^- )</td>
<td>13.35 ± 0.10 ± 0.43 ± 0.29</td>
<td>3.49 ± 0.03 ± 0.12 ± 0.08</td>
</tr>
<tr>
<td>( \pi^+ \pi^- 2\pi^0 )</td>
<td>18.01 ± 0.14 ± 1.17 ± 0.40</td>
<td>4.43 ± 0.03 ± 0.29 ± 0.10</td>
</tr>
<tr>
<td>( 2\pi^+ 2\pi^- \pi^0 ) (( \eta ) excl.)</td>
<td>0.72 ± 0.04 ± 0.07 ± 0.03</td>
<td>0.22 ± 0.01 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>( \pi^+ \pi^- 3\pi^0 ) (( \eta ) excl., from isospin)</td>
<td>0.36 ± 0.01 ± 0.03 ± 0.01</td>
<td>0.11 ± 0.01 ± 0.01 ± 0.00</td>
</tr>
<tr>
<td>( 3\pi^+ 3\pi^- )</td>
<td>0.12 ± 0.01 ± 0.01 ± 0.00</td>
<td>0.04 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( 2\pi^+ 2\pi^- 2\pi^0 ) (( \eta ) excl.)</td>
<td>0.70 ± 0.05 ± 0.04 ± 0.09</td>
<td>0.25 ± 0.02 ± 0.02 ± 0.03</td>
</tr>
<tr>
<td>( \pi^+ \pi^- 4\pi^0 ) (( \eta ) excl., from isospin)</td>
<td>0.11 ± 0.01 ± 0.11 ± 0.00</td>
<td>0.04 ± 0.00 ± 0.04 ± 0.00</td>
</tr>
<tr>
<td>( \eta \pi^+ \pi^- )</td>
<td>1.15 ± 0.06 ± 0.08 ± 0.03</td>
<td>0.33 ± 0.02 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>( \eta \omega )</td>
<td>0.47 ± 0.04 ± 0.00 ± 0.05</td>
<td>0.15 ± 0.01 ± 0.00 ± 0.02</td>
</tr>
<tr>
<td>( \eta 2\pi^+ 2\pi^- )</td>
<td>0.02 ± 0.01 ± 0.00 ± 0.00</td>
<td>0.01 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( \eta \pi^+ \pi^- 2\pi^0 ) (estimated)</td>
<td>0.02 ± 0.01 ± 0.00 ± 0.00</td>
<td>0.01 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( \omega \pi^0 ) (( \omega \rightarrow \pi^0 \gamma ))</td>
<td>0.89 ± 0.02 ± 0.06 ± 0.02</td>
<td>0.18 ± 0.00 ± 0.02 ± 0.00</td>
</tr>
<tr>
<td>( \omega \pi^+ \pi^- \omega 2\pi^0 ) (( \omega \rightarrow \pi^0 \gamma ))</td>
<td>0.08 ± 0.00 ± 0.01 ± 0.00</td>
<td>0.03 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( \omega ) (non-3( \pi ), ( \pi \gamma ))</td>
<td>0.36 ± 0.00 ± 0.01 ± 0.00</td>
<td>0.03 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( K^+ K^- )</td>
<td>21.63 ± 0.27 ± 0.58 ± 0.36</td>
<td>3.13 ± 0.04 ± 0.08 ± 0.05</td>
</tr>
<tr>
<td>( K^0 \bar{K}^0 )</td>
<td>12.96 ± 0.18 ± 0.25 ± 0.24</td>
<td>1.75 ± 0.02 ± 0.03 ± 0.03</td>
</tr>
<tr>
<td>( \phi ) (non-( K \bar{K}, 3\pi, \pi \gamma ))</td>
<td>0.05 ± 0.00 ± 0.00 ± 0.00</td>
<td>0.01 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
<tr>
<td>( K \bar{K} \pi ) (partly from isospin)</td>
<td>2.39 ± 0.07 ± 0.12 ± 0.08</td>
<td>0.76 ± 0.02 ± 0.04 ± 0.02</td>
</tr>
<tr>
<td>( K \bar{K} 2\pi ) (partly from isospin)</td>
<td>1.35 ± 0.09 ± 0.38 ± 0.03</td>
<td>0.48 ± 0.03 ± 0.14 ± 0.01</td>
</tr>
<tr>
<td>( K \bar{K} 3\pi ) (partly from isospin)</td>
<td>-0.03 ± 0.01 ± 0.02 ± 0.00</td>
<td>-0.01 ± 0.00 ± 0.01 ± 0.00</td>
</tr>
<tr>
<td>( \phi \omega )</td>
<td>0.36 ± 0.02 ± 0.02 ± 0.01</td>
<td>0.13 ± 0.01 ± 0.01 ± 0.00</td>
</tr>
<tr>
<td>( \omega KK ) (( \omega \rightarrow \pi^0 \gamma ))</td>
<td>0.00 ± 0.00 ± 0.00 ± 0.00</td>
<td>0.00 ± 0.00 ± 0.00 ± 0.00</td>
</tr>
</tbody>
</table>

| \( J/\psi \) (Breit-Wigner integral) | 6.22 ± 0.16 | 7.03 ± 0.18 |
| \( \psi(2S) \) (Breit-Wigner integral) | 1.57 ± 0.03 | 2.50 ± 0.04 |

| \( R_{\text{data}} \) [3.7 – 5.0 GeV] | 7.29 ± 0.05 ± 0.30 ± 0.00 | 15.79 ± 0.12 ± 0.66 ± 0.00 |

| \( R_{\text{QCD}} \) | 33.45 ± 0.28 | 24.27 ± 0.19 |
| \( R_{\text{QCD}} \) [5.0 – 9.3 GeV] | 6.86 ± 0.04 | 34.89 ± 0.18 |
| \( R_{\text{QCD}} \) [9.3 – 12.0 GeV] | 1.21 ± 0.01 | 15.56 ± 0.04 |
| \( R_{\text{QCD}} \) [12.0 – 40.0 GeV] | 1.64 ± 0.01 | 77.94 ± 0.12 |
| \( R_{\text{QCD}} \) (> 40.0 GeV) | 0.16 ± 0.00 | 42.70 ± 0.06 |
| \( R_{\text{QCD}} \) (> 40.0 GeV) | 0.00 ± 0.00 | -0.72 ± 0.01 |

| Sum | 692.3 ± 1.4 ± 3.1 ± 2.4 ± 0.2 | 274.97 ± 0.17 ± 0.78 ± 0.37 ± 0.18 | 0.52 ± 0.00 |

B. Malaescu (CNRS) – Valencia GenT workshop – December 2019
Treatment of the KLOE data – eigenvector decomposition

→ Eigenvectors carry the general features of the correlations:
  - long-range for systematics
  - ~short-range for statistical uncertainties + correlations between KLOE 08 & 12
Treatment of the combined KLOE data

correlation matrix

eigenvalues of the covariance matrix

Uncertainty
Direct comparison of the 3 KLOE measurements

→ Quantitative comparison between the ratios and unity, taking into account correlations

<table>
<thead>
<tr>
<th>KLOE 10 / KLOE 08</th>
<th>KLOE 12 / KLOE 08</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2 [0.35;0.85] \text{ GeV}^2 : 79.0 / 50(\text{DOF})$</td>
<td>$\chi^2 [0.35;0.95] \text{ GeV}^2 : 73.7 / 60(\text{DOF})$</td>
</tr>
<tr>
<td>p-value= 0.0056</td>
<td>p-value= 0.11</td>
</tr>
<tr>
<td>$\chi^2 [0.35;0.58] \text{ GeV}^2 : 46.2 / 23(\text{DOF})$</td>
<td>$\chi^2 [0.35;0.58] \text{ GeV}^2 : 21.8 / 23(\text{DOF})$</td>
</tr>
<tr>
<td>p-value= 0.0028</td>
<td>p-value= 0.53</td>
</tr>
<tr>
<td>$\chi^2 [0.58;0.85] \text{ GeV}^2 : 29.7 / 27(\text{DOF})$</td>
<td>$\chi^2 [0.35;0.64] \text{ GeV}^2 : 27.5 / 29(\text{DOF})$</td>
</tr>
<tr>
<td>p-value= 0.33</td>
<td>p-value= 0.55</td>
</tr>
<tr>
<td>$\chi^2 [0.64;0.85] \text{ GeV}^2 : 20.7 / 21(\text{DOF})$</td>
<td>$\chi^2 [0.64;0.95] \text{ GeV}^2 : 39.4 / 31(\text{DOF})$</td>
</tr>
<tr>
<td>p-value= 0.47</td>
<td>p-value= 0.14</td>
</tr>
</tbody>
</table>
Direct comparison of the 3 KLOE measurements

→ Fitting the ratio taking into account correlations

→ Full propagation of uncertainties and correlations – 3 methods yielding consistent results: ±1σ shifts of each uncertainty, pseudo-experiments and fit uncertainties from Minuit

\[ \chi^2 [p0 + p1\sqrt{s}]: 20.7 / 27 (DOF) \]

p-value = 0.80

p0 : 0.876 ± 0.056

p1 : 0.159 ± 0.081

→ Significant shift and slope (∼2σ) at low √s, no significant shift at high √s

\[ \chi^2 [p0]: 38.4 / 30 (DOF) \]

p-value = 0.14

p0 : 1.009 ± 0.009
Direct comparison of the 3 KLOE measurements

$\chi^2[p_0]: 25.4 / 16$(DOF)
$p$-value = 0.06
$p_0: 0.979 \pm 0.008$

$\chi^2[p_0 + p_1 \sqrt{s}]: 29.5 / 26$(DOF)
$p$-value = 0.29
$p_0: 1.002 \pm 0.006$

$\chi^2[p_0 + p_1 \sqrt{s}]: 36.1 / 21$(DOF)
$p$-value = 0.02
$p_0: 0.745 \pm 0.085$
$p_1: 0.341 \pm 0.117$

→ Significant shift and slope ($\sim 2.5$-$3\sigma$) at low $\sqrt{s}$, no significant shift at high $\sqrt{s}$
Combining the 3 KLOE measurements