Pole vs running mass scheme: impact on different observables

Adrián Irles LAL – CNRS/IN2P3

IFIC-DESY workshop on top quark mass

15/10/2019

Outline

- Which scheme is preferable?
- The R-observable in the running mass scheme.
- The CMS observable TOP-18-004 (only for the +X jets, X>0)
  - Pole mass vs Running mass.

- Extra material:
  - R-observable at 13 TeV vs 8 TeV
  - Comparison of the TOP-18-004 CMS observable calculations with the tt+1Jet @NLO fixed order (Eur.Phys.J. C59 (2009) 625-646)

- All calculations are based on **Eur.Phys.J. C59 (2009) 625-646** (Dittmaier, Uwer, Weinzierl)

Requirements for a precise quark mass measurement

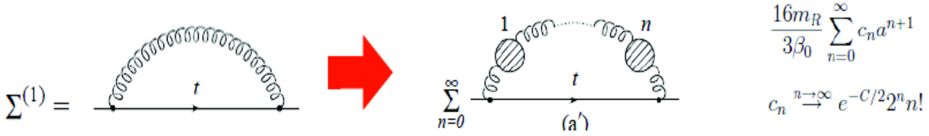
- Define an observable with **good sensitivity to the interesting parameter (i.e. mass, alpha_s, etc)**

$$\frac{\Delta O}{O} \leftrightarrow \frac{\Delta m_t}{m_t}$$

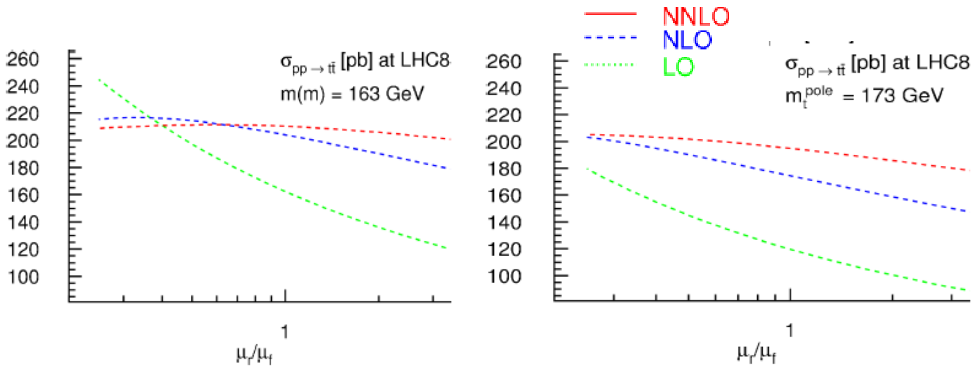
- The observable should have **small and understood theoretical uncertainties** (perturbative theory!!)
- Well defined **mass scheme** → **NLO calculations!**
- Measured observables have to be compared to calculations → parton level, particle level (if the calculation is possible).

Which mass scheme is better? (Pole vs $\overline{\text{MS}}$)

- The pole mass has an intrinsic ambiguity of the order of Λ_{QCD}
- For inclusive $\overline{t\bar{t}}$ cross sections, the running mass scheme ($\overline{m}(m)$) provides better convergence of the calculations → smaller uncertainties
- The threshold effects are badly described in e+e- when the pole mass scheme is used.
  - Specially designed mass schemes.
- In fact, the choice would depend on each observable/distribution.



[Bigi, Shifman, Uraltsev, Vainshtein 94 Beneke, Braun,94 Smith, Willenbrock 97]



Langenfeld, Moch, Uwer PRD 80, 054009 (2009)

Czakon, Fiedler, Mitov hep-ph/1303.6254

Switching between schemes

- All schemes are equivalent and, by definition, we can switch from one to another.

NLO approx

$$M_t^{\text{pole}} = m_t(\mu)\left(1+\hat{a}(\mu)\frac{4}{3}\left[1-\frac{3}{4}\ln\left(\frac{m_t^2}{\mu^2}\right)\right]\right)+O(\hat{a}^2) \quad (5)$$

with

$$\hat{a}(\mu) = \frac{\alpha_s^{(6)}(\mu)}{\pi} \quad (6)$$

- And this is also possible to be done with observables calculated at a fixed order

How? ***Eur.Phys.J. C77 (2017) no.11, 794 (2017-11-22)***
Fuster, A.I, Melini, Uwer, Vos

Switching between schemes

$$a(\mu) = \frac{\alpha_s^{(5)}(\mu)}{\pi}$$

$$\sigma = a(\mu)^3 \sigma^{(0)}(M_t^{\text{pole}}) + a(\mu)^4 \sigma^{(1)}(M_t^{\text{pole}}) + \ldots,$$

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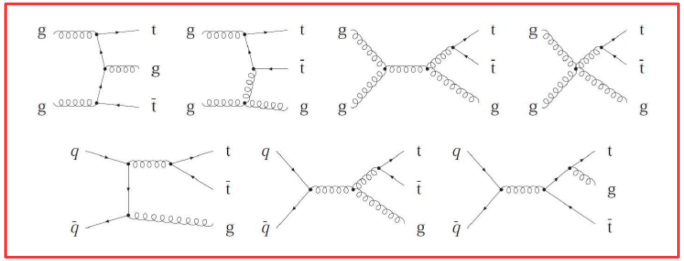
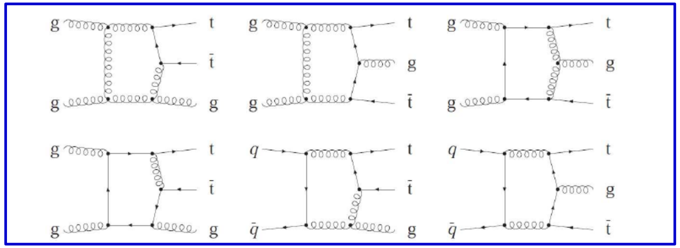
LO



Real correction (tt+2p) +

Virtual (loop) corrections



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$$\sigma = a(\mu)^3 \sigma^{(0)}(M_t^{\text{pole}}) + a(\mu)^4 \sigma^{(1)}(M_t^{\text{pole}}) + \ldots,$$

$$\sigma = a(\mu)^3 \sigma^{(0)} \left(m_t(m_t)(1 + \frac{4}{3} a(\mu) + \ldots) \right) + a(\mu)^4 \sigma^{(1)} \left(m_t(m_t)(1 + \frac{4}{3} a(\mu) + \ldots) \right) + \ldots$$

- The mass dependence can be written as follows.
  - Same precision in the perturbative expansion approach

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$$\sigma = a(\mu)^3\sigma^{(0)}\left(m_t(m_t)(1+\frac{4}{3}a(\mu)+\ldots)\right) + a(\mu)^4\sigma^{(1)}\left(m_t(m_t)(1+\frac{4}{3}a(\mu)+\ldots)\right) + \ldots$$

- Few steps further, the cross section as a function of the pole mass can be converted to the equivalent but as a function of the running mass.
  - Different for each mass scheme.
  - Valid in the perturbative expansion approach.

$$\sigma = a(\mu)^3\sigma^{(0)}(m_t(m_t)) + a(\mu)^4\left[\sigma^{(1)}(m_t(m_t)) + \frac{4}{3}m_t(m_t)\left.\frac{d\sigma^{(0)}(M_t^{\text{pole}})}{dM_t^{\text{pole}}}\right|_{M_t^{\text{pole}}=m_t(m_t)}\right] + O(a^5).$$

Eur.Phys.J. C77 (2017) no.11, 794 (2017-11-22)
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Switching between schemes

- This can be applied to integrated or differential cross sections as the R-observable or any other (i.e. see next section)

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}}} \frac{d\sigma_{t\bar{t}+1\text{-jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s),$$

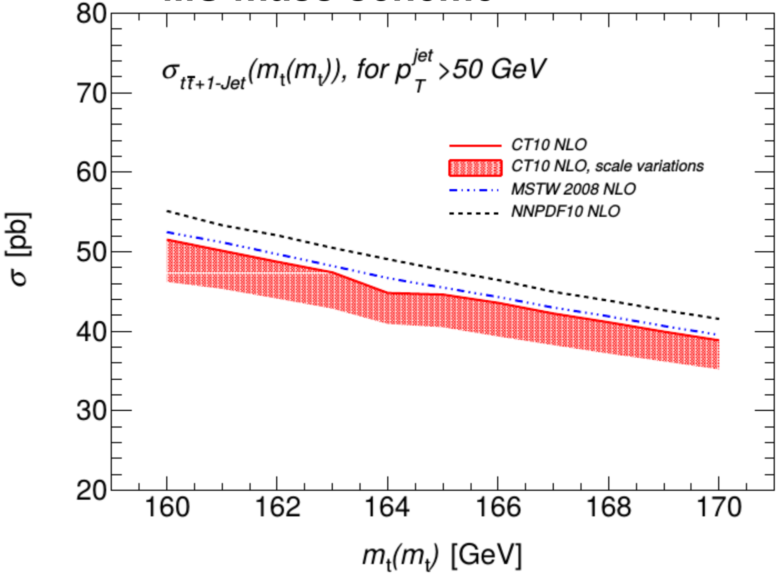
Inclusive tt+1jet @ NLO: pole vs running (7TeV)

pole mass scheme

	$\sigma_{t\bar{t}+1\text{-jet}}$ [pb] $p_T(jet) > 50\,\text{GeV}, \lvert\eta(jet)\rvert < 2.5$	
m_t^{pole} [GeV]	LO	NLO
160	66.727(5)	60.04(8)
165	57.615(4)	52.25(9)
170	$49.910(3)^{+30}_{-17}$	$45.45(6)^{+1}_{-6}$
172.5	$46.508(3)^{+28}_{-15}$	$42.37(6)^{+1}_{-6}$
175	45.372(3)	39.46(6)
180	37.800(2)	34.73(5)

Eur.Phys.J. C73 (2013) 2438 *(S. Alioli, P. Fernández, J. Fuster, A.I., S. Moch, P. Uwer, M. Vos)*

$\overline{\text{MS}}$ mass scheme



Eur.Phys.J. C77 (2017) no.11, 794 (2017-11-22)
Fuster, A.I, Melini, Uwer, Vos

- Slightly better convergence for the running mass, but basically due to kinematic effects (smaller mass values)

Theoretical sensitivity (reminder).

- **Eur.Phys.J. C73 (2013) 2438** *(S. Alioli, P. Fernández, J. Fuster, A.I., S. Moch, P. Uwer, M. Vos)*

$$\mathcal{S}(\rho_s) = \sum_{\Delta=\pm 5-10\,\mathrm{GeV}} \frac{|\mathcal{R}(170\,\mathrm{GeV},\rho_s) - \mathcal{R}(170\,\mathrm{GeV}+\Delta,\rho_s)|}{2|\Delta|\mathcal{R}(170\,\mathrm{GeV},\rho_s)} . \quad (5)$$

$$\left|\frac{\Delta\mathcal{R}}{\mathcal{R}}\right| \approx \left(m_t^{\mathrm{pole}}\mathcal{S}\right) \times \left|\frac{\Delta m_t^{\mathrm{pole}}}{m_t^{\mathrm{pole}}}\right| .$$

$$\frac{\Delta\mathcal{R}_\mu/\mathcal{R}(170\,\mathrm{GeV},\rho_s)}{\mathcal{S}(\rho_s)}$$

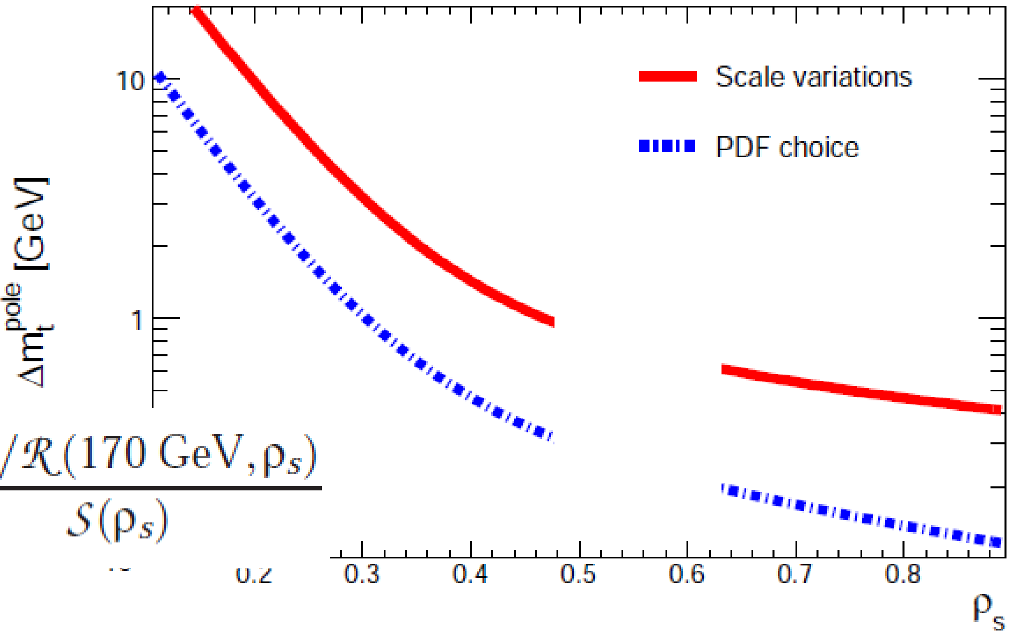
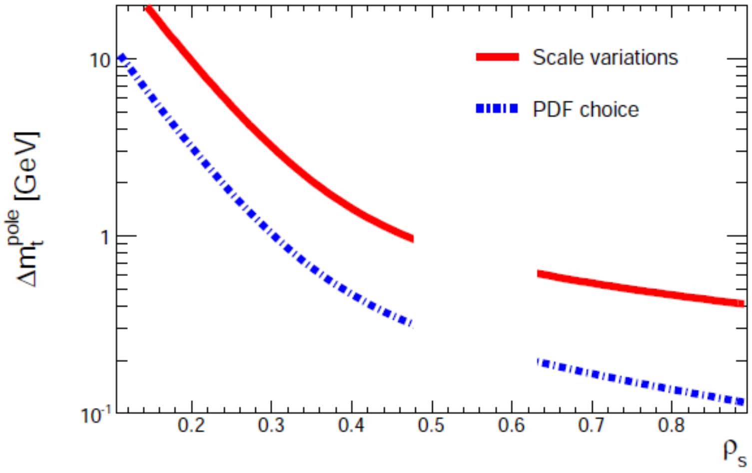


Fig. 6. Expected impact of scale (magenta line) and PDF (blue dashed line) uncertainties on the measured top-quark mass value. The region where $\mathcal{R}$ is essentially insensitive to the top-quark mass is not shown.

Differential tt+1jet @ NLO: pole vs running (7TeV)

pole mass scheme



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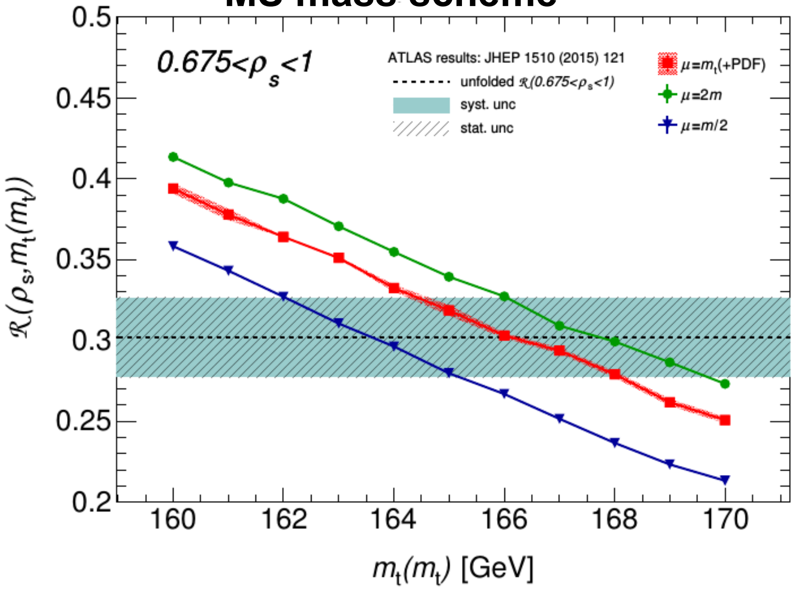


Eur.Phys.J. C77 (2017) no.11, 794 (2017-11-22)
Fuster, A.I, Melini, Uwer, Vos

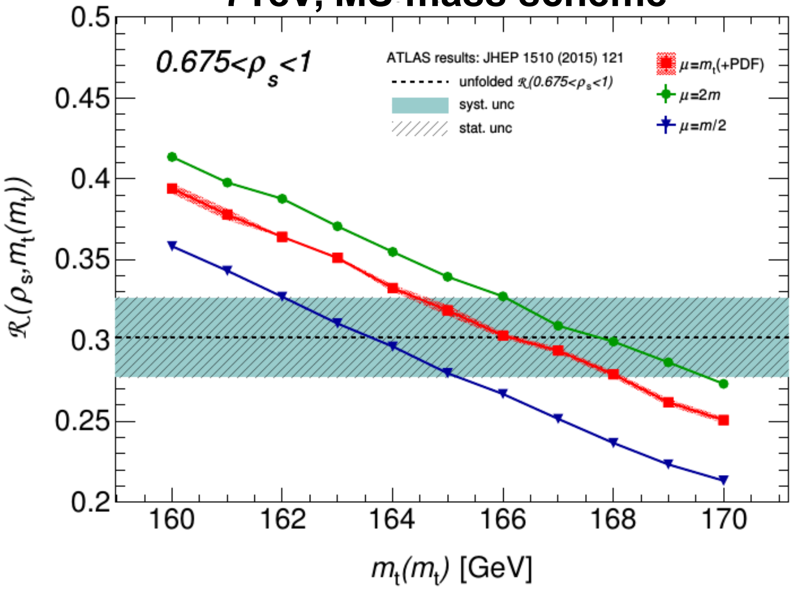
- The scale uncertainty for the running mass in the sensitive bin has an larger impact on the mass extraction. **~ 1-1.5 GeV**

Differential tt+1jet @ NLO: pole vs running

$$\sigma = a(\mu)^3\sigma^{(0)}(m_t(m_t)) + a(\mu)^4\left[\sigma^{(1)}(m_t(m_t)) + \frac{4}{3}m_t(m_t)\left.\frac{d\sigma^{(0)}(M_t^{\text{pole}})}{dM_t^{\text{pole}}}\right|_{M_t^{\text{pole}}=m_t(m_t)}\right] + O(a^5).$$

- Due to the large mass dependence near the threshold ! We introduce large corrections to the LO in this bin.

7TeV, $\overline{\text{MS}}$ mass scheme



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Top-quark mass determinations using R (ATLAS)

- 7 TeV
  - **Pole mass (ATLAS)** JHEP 10 (2015) 121,

$$M_t^{\text{pole}} = 173.7 \pm 1.5\,(\text{stat.}) \pm 1.4\,(\text{syst.})^{+1.0}_{-0.5}\,(\text{theory})\ \text{GeV}$$

  - **Running mass** Eur.Phys.J. C77 (2017) no.11, 794 (2017-11-22)

$$m_t(m_t) = 165.9 \pm 1.4\,(\text{stat.}) \pm 1.3\,(\text{syst.})^{+1.5}_{-0.6}\,(\text{theory})\ \text{GeV},$$

- 8 TeV

The value obtained for the pole-mass scheme is:

$$m_t^{\text{pole}} = 171.1 \pm 0.4\,(\text{stat}) \pm 0.9\,(\text{syst})\ ^{+0.7}_{-0.3}\,(\text{theo})\ \text{GeV}.$$

The extracted value in the running-mass scheme is:

$$m_t(m_t) = 162.9 \pm 0.5\,(\text{stat}) \pm 1.0\,(\text{syst})\ ^{+2.1}_{-1.2}\,(\text{theo})\ \text{GeV}.$$

CMS-TOP-18-004

CERN-EP-2019-028
2019/04/11

Measurement of $t\bar{t}$ normalised multi-differential cross sections in pp collisions at $\sqrt{s} = 13$ TeV, and simultaneous determination of the strong coupling strength, top quark pole mass, and parton distribution functions

The CMS Collaboration*

CMS-TOP-18-004

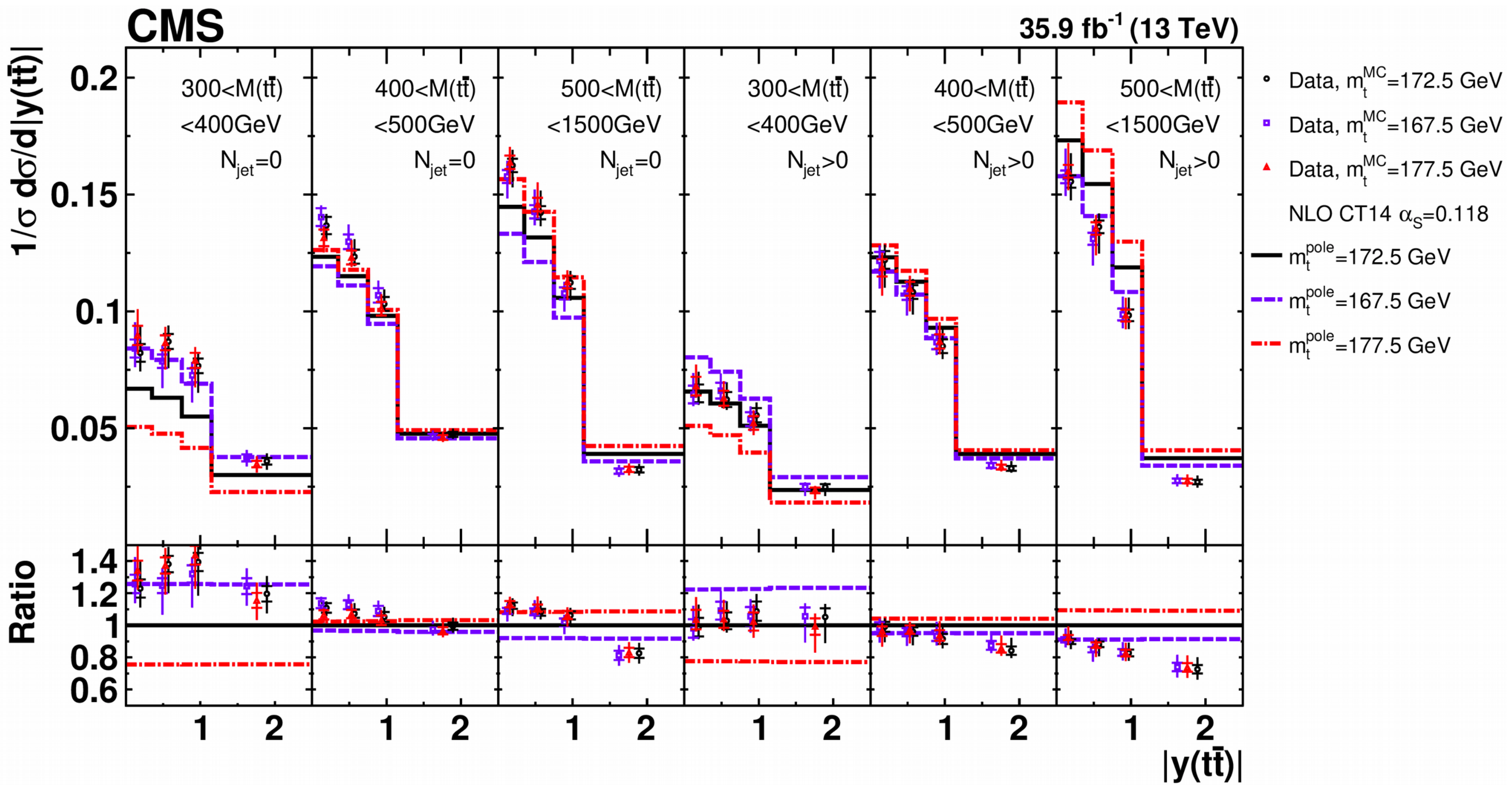
CERN-EP-2019-028
2019/04/11

Measurement of $t\bar{t}$ normalised multi-differential cross sections in pp collisions at $\sqrt{s} = 13$ TeV, and simultaneous determination of the strong coupling strength, top quark pole mass, and parton distribution functions

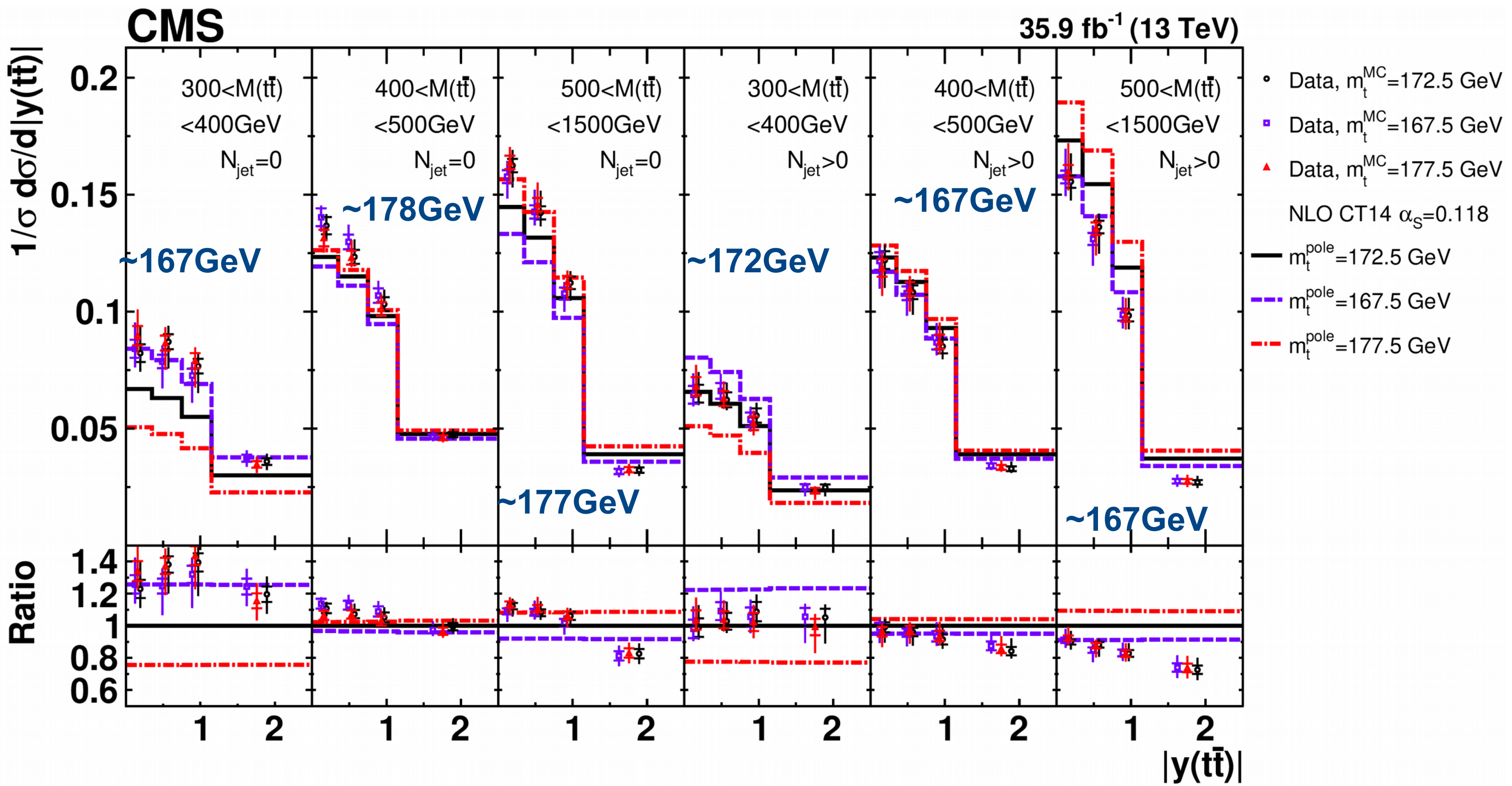
The CMS Collaboration*

- The observables are based on differential cross sections for ttbar +0 or >0 jets.
- Extra jets have pt>30 and |eta|<2.4, using anti-kt, R=0.4
- The measurement is done at particle level.
- They publish full unfolded data with statistical correlation and systematics, bin by bin.
- They also provide a C-factor to correct from parton to particle level.
- http://cms-results.web.cern.ch/cms-results/public-results/publications/TOP-18-004/

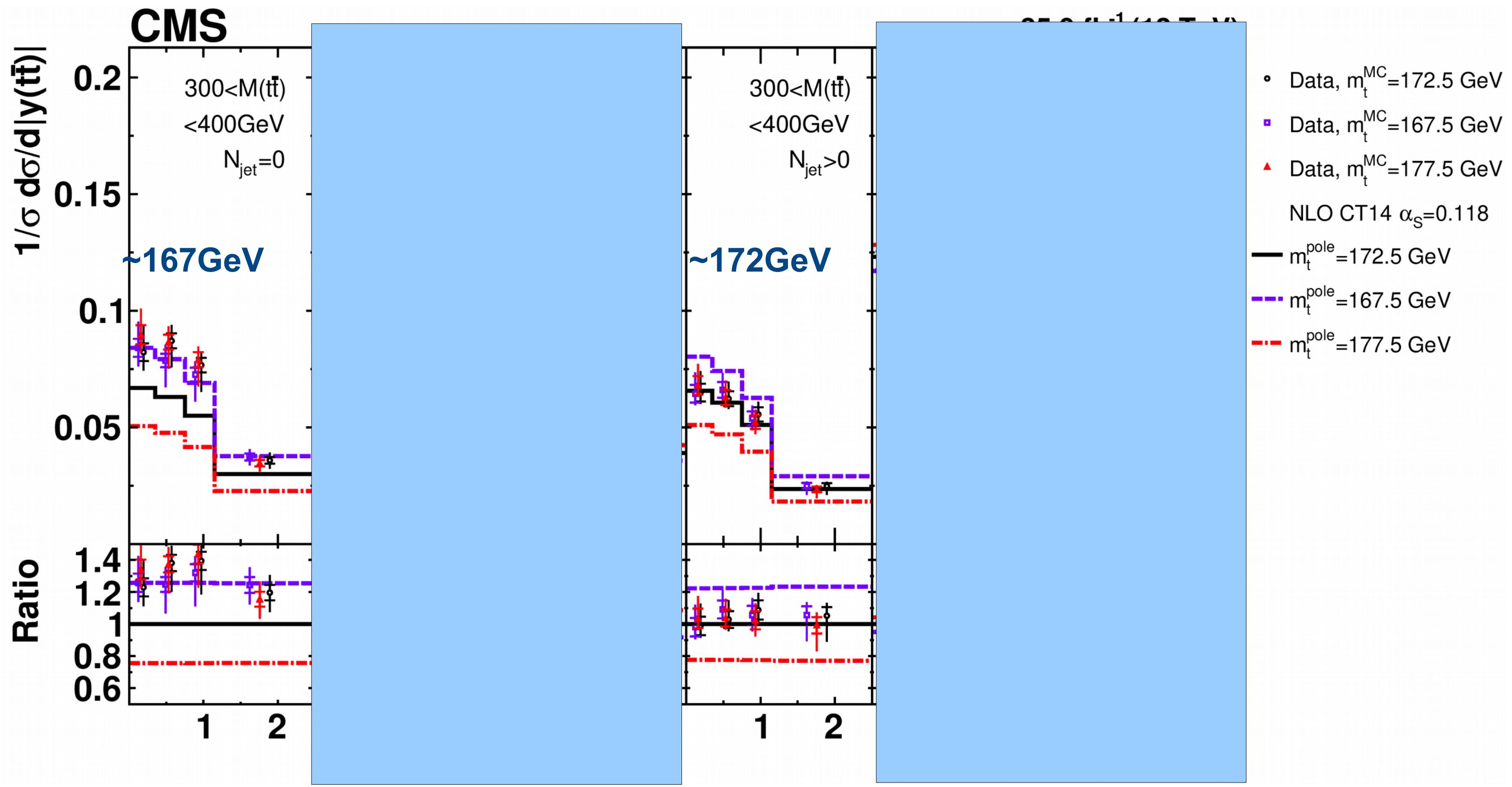
- They do a multidimensional fit, fitting PDFs, alpha_s and mass at the same time.
- The mass highest precision is obtained with the |mtt| dimension of the 3D distribution. The precision on the alpha_s extraction is dominated by the |y_tt| dimension



- Unfolded data (points) using different MC masses. Theory calculations are shown as histograms.



- Unfolded data (points) using different MC masses. Theory calculations are shown as histograms. **What are the theory uncertainties?**



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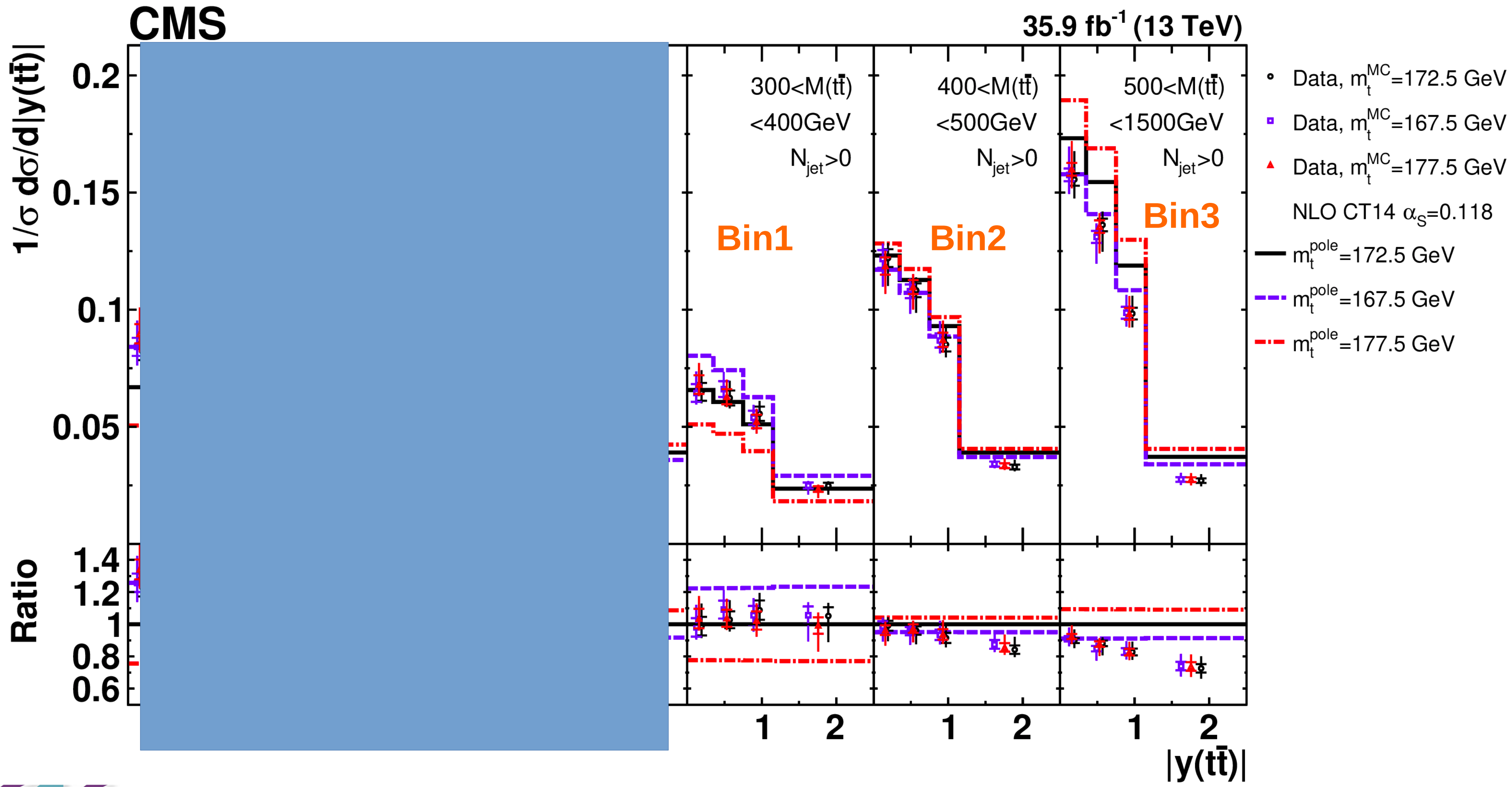

CMS
35.9 fb-1 (13 TeV)
1/σ dσ/d|y(tt̄)|
Ratio
300<M(tt̄) <400GeV Njet>0
400<M(tt̄) <500GeV Njet>0
500<M(tt̄) <1500GeV Njet>0
Bin1
Bin2
Bin3
|y(tt̄)|
Data, mtMC=172.5 GeV
Data, mtMC=167.5 GeV
Data, mtMC=177.5 GeV
NLO CT14 αS=0.118
mtpole=172.5 GeV
mtpole=167.5 GeV
mtpole=177.5 GeV

- We implement the calculation of the CMS observable (for Njets>0). With few differences (?) :
  - Our observable is defined for ttbar+1jet+X events :
  - The « 1jet » has pt>30GeV and |eta|<2.4
  - The « X » that follows has lower pT and whatever eta. For CMS, the comparison is done at particle level where the second jet has also pt>30 and |eta|<2.4
  - They do a “folding” from parton to particle level. Is their parton level equivalent to ours? I think so but… needs confirmation.

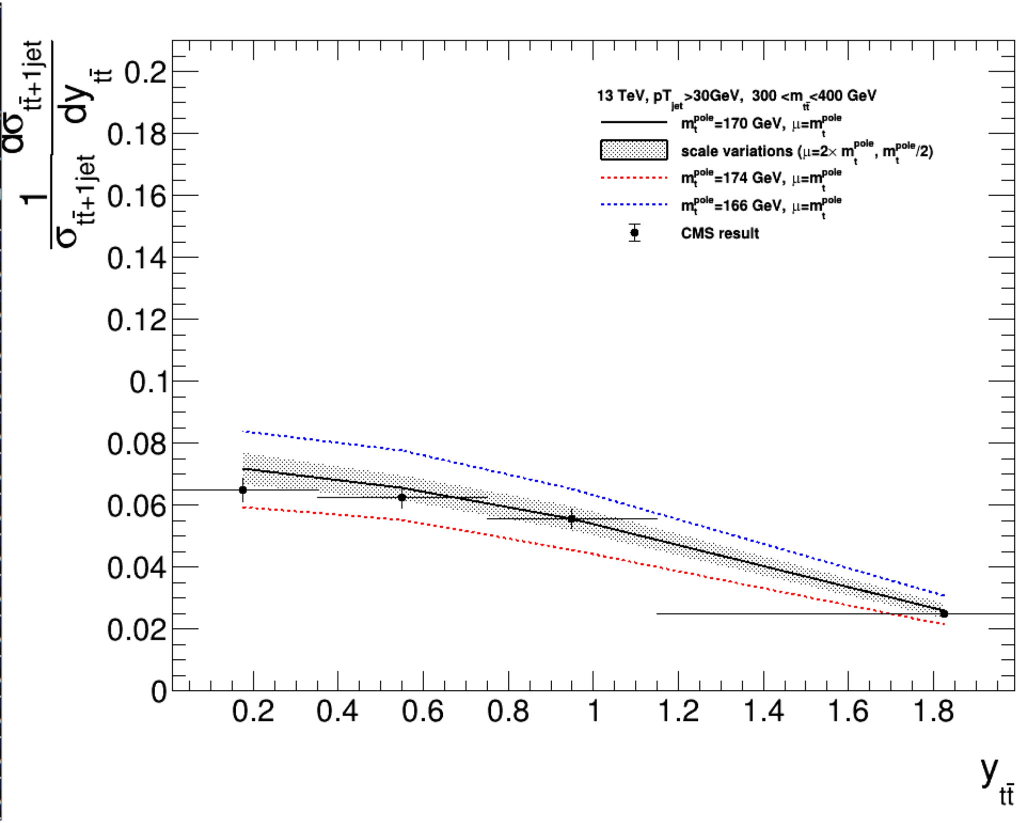
- I apply the C-factor given by CMS. (usually of the order of ~%)
  - Parton level to particle level

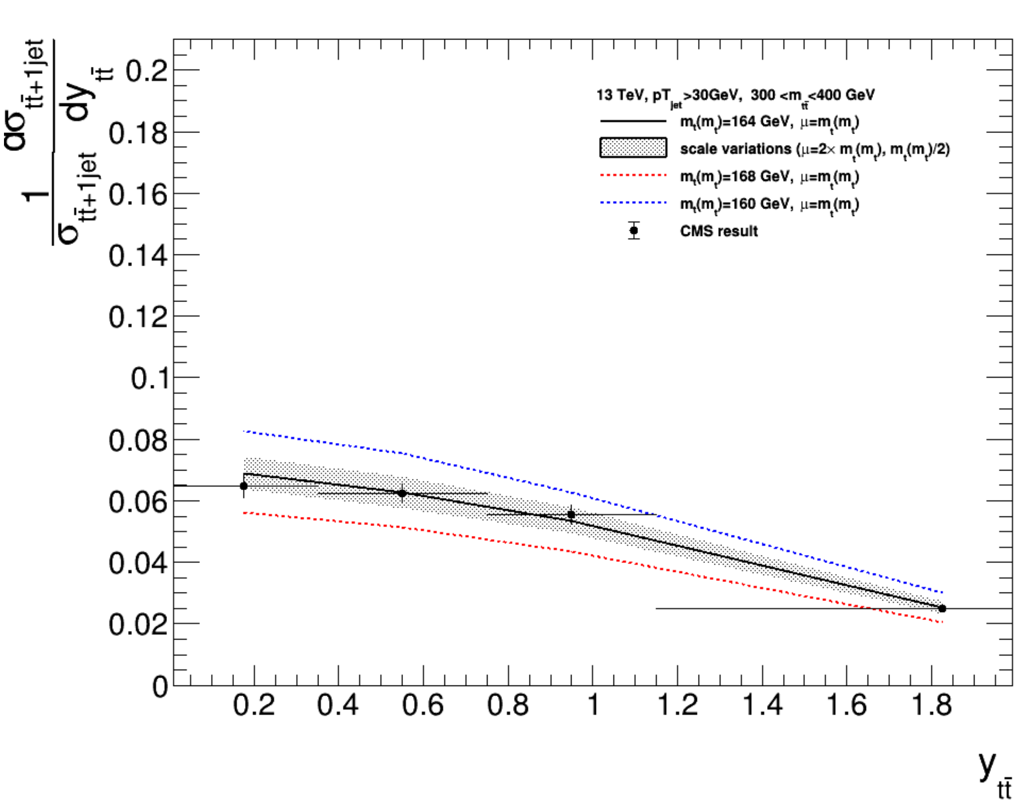
DISCLAIMER : Everything is at a very preliminary stage !

My masses points are a bit different than those shown by CMS.

Bin1

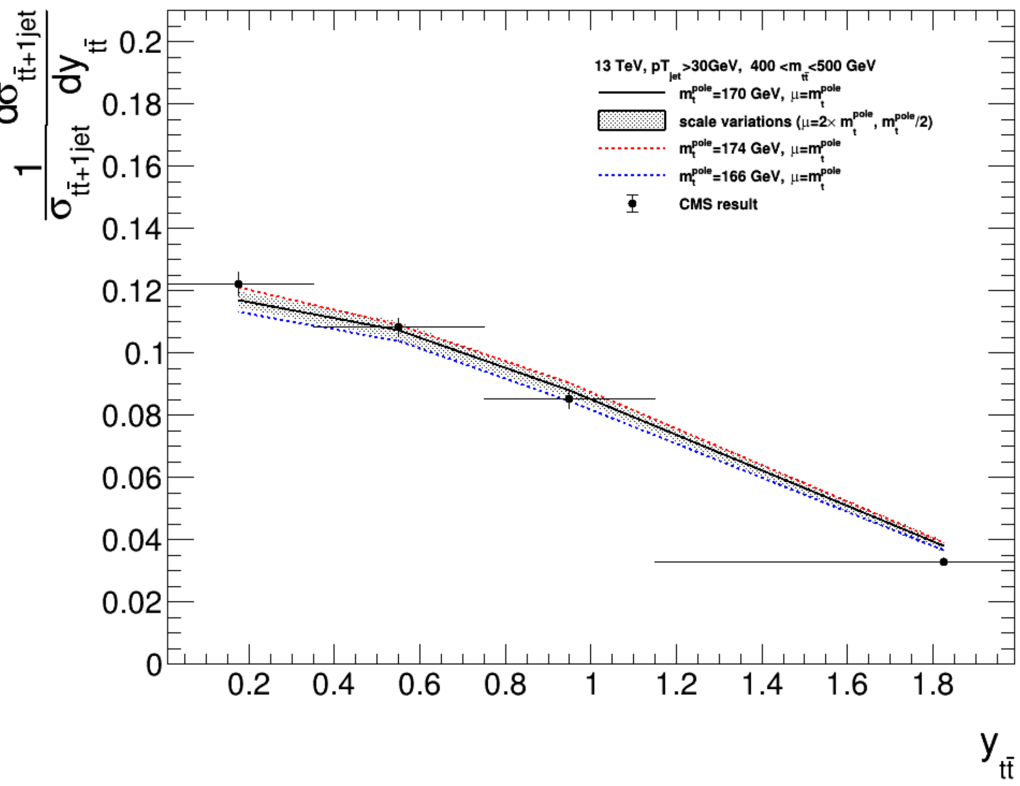
Pole vs running

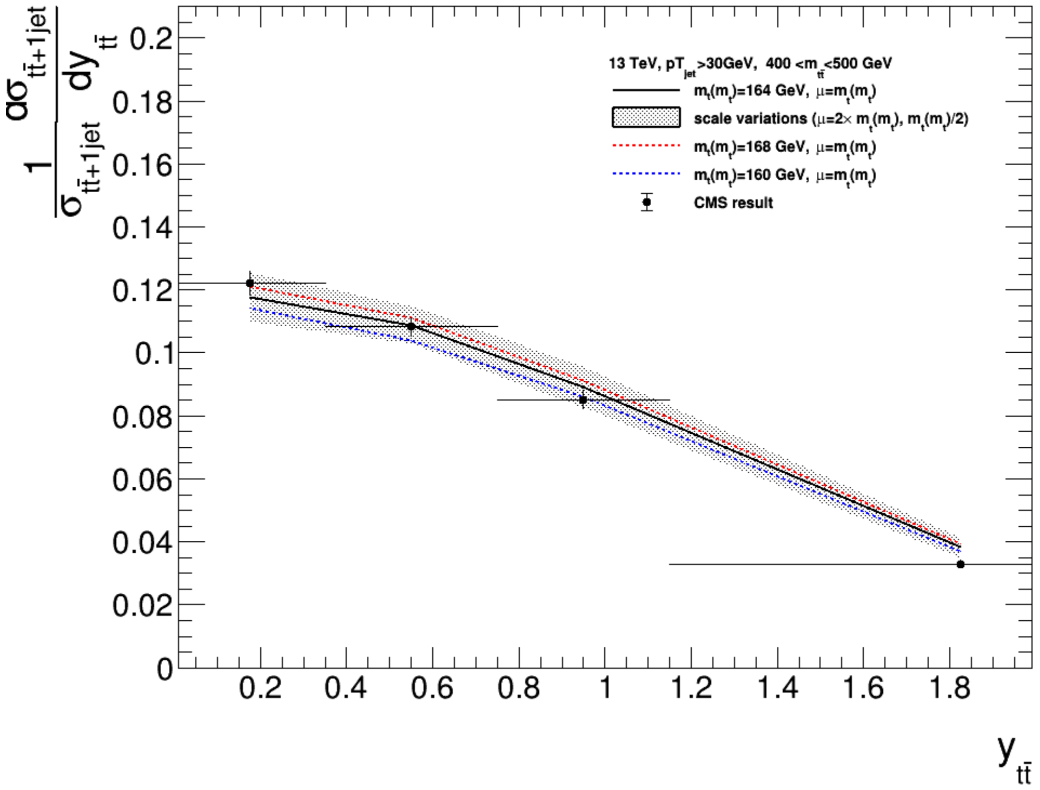




Bin2

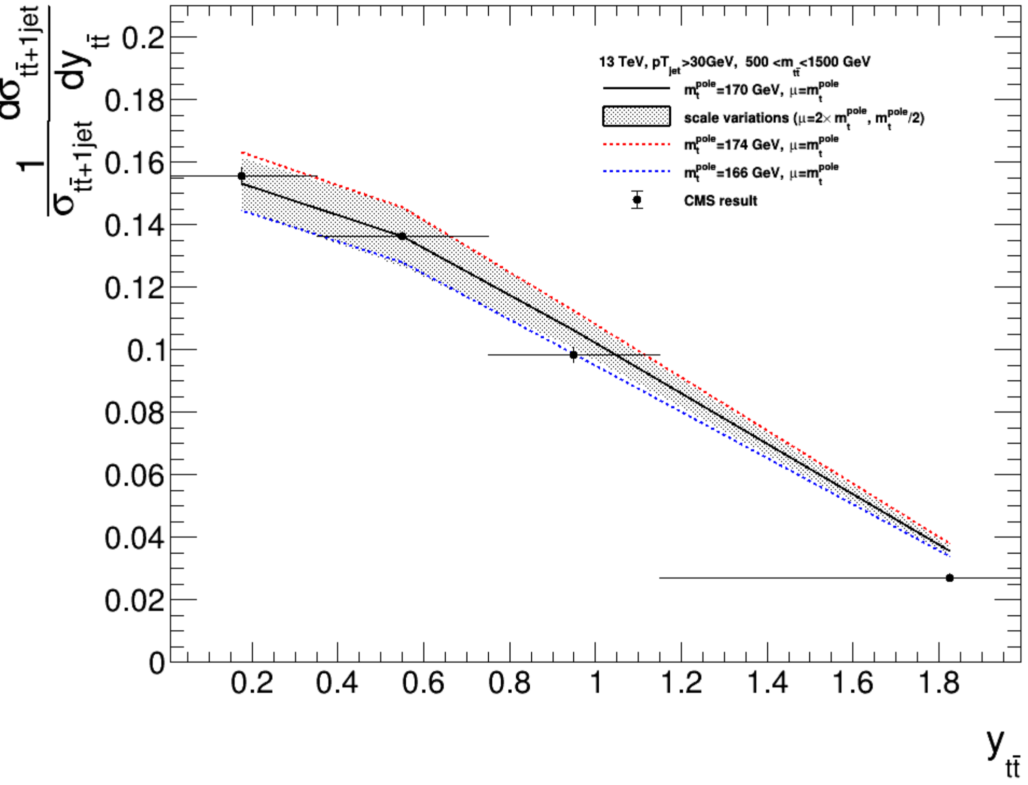
Pole vs running

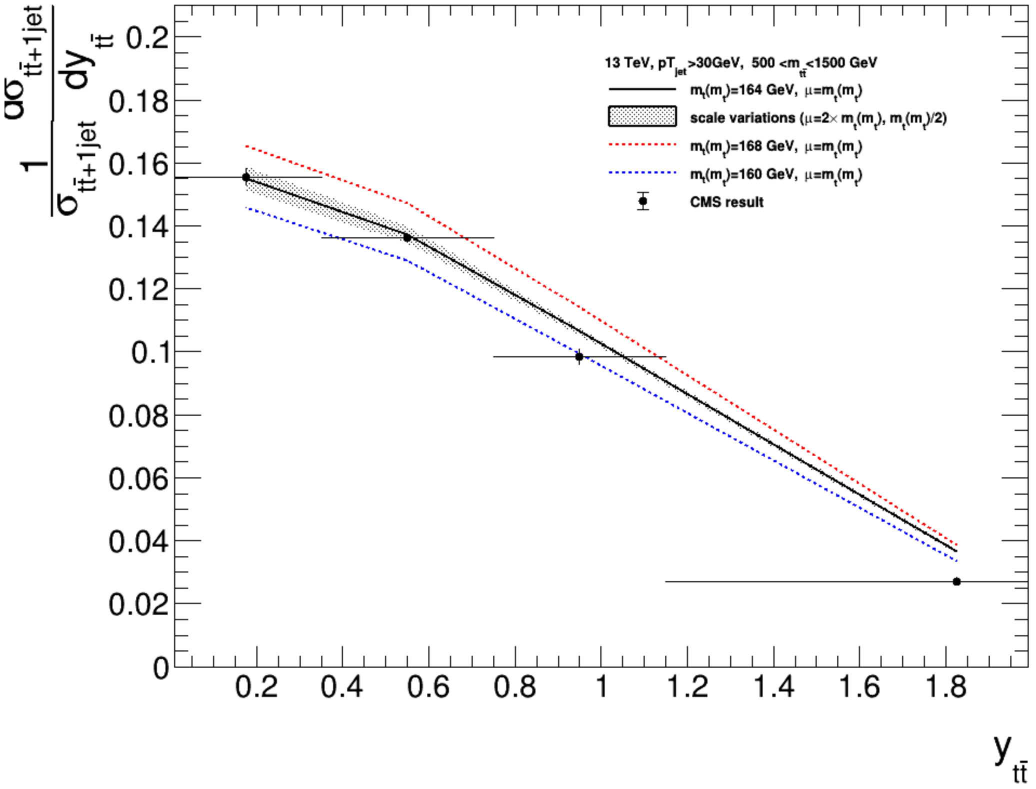




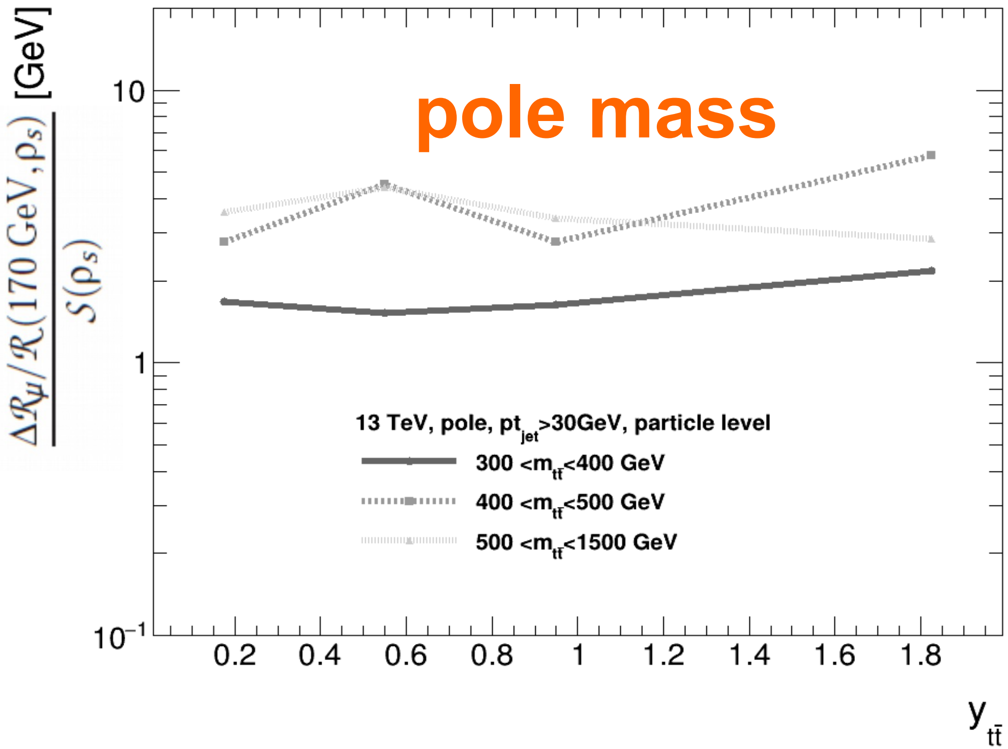
Bin3

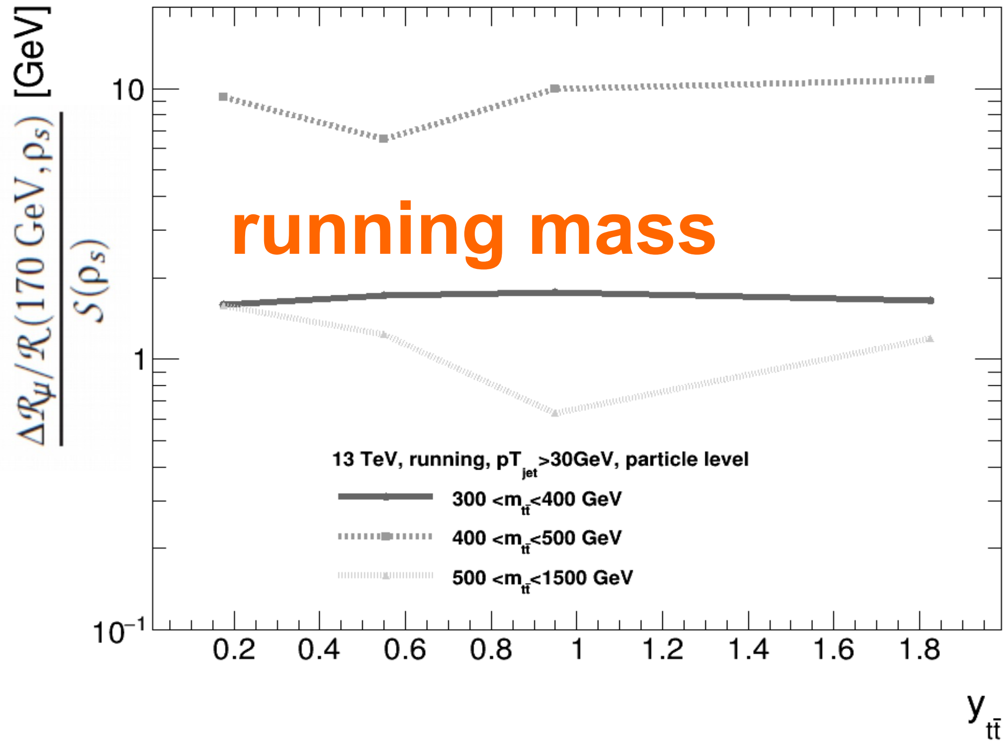
Pole vs running





Sensitivity: pole mass vs running scheme





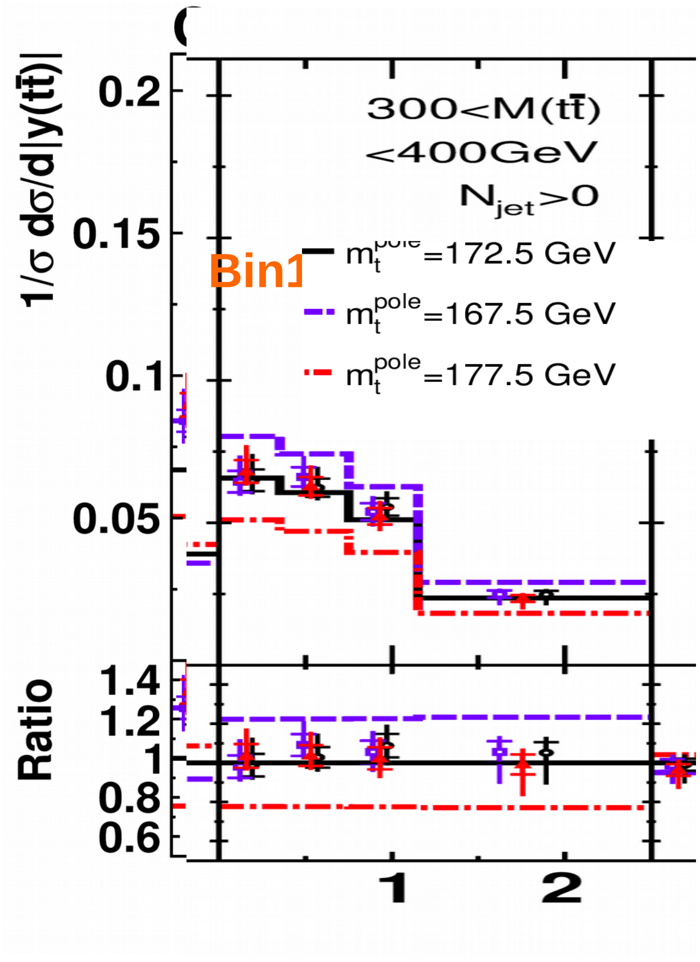
- For the running mass, we keep the sensitivity in the first bin (threshold!), and we add one for the highest invariant mass.

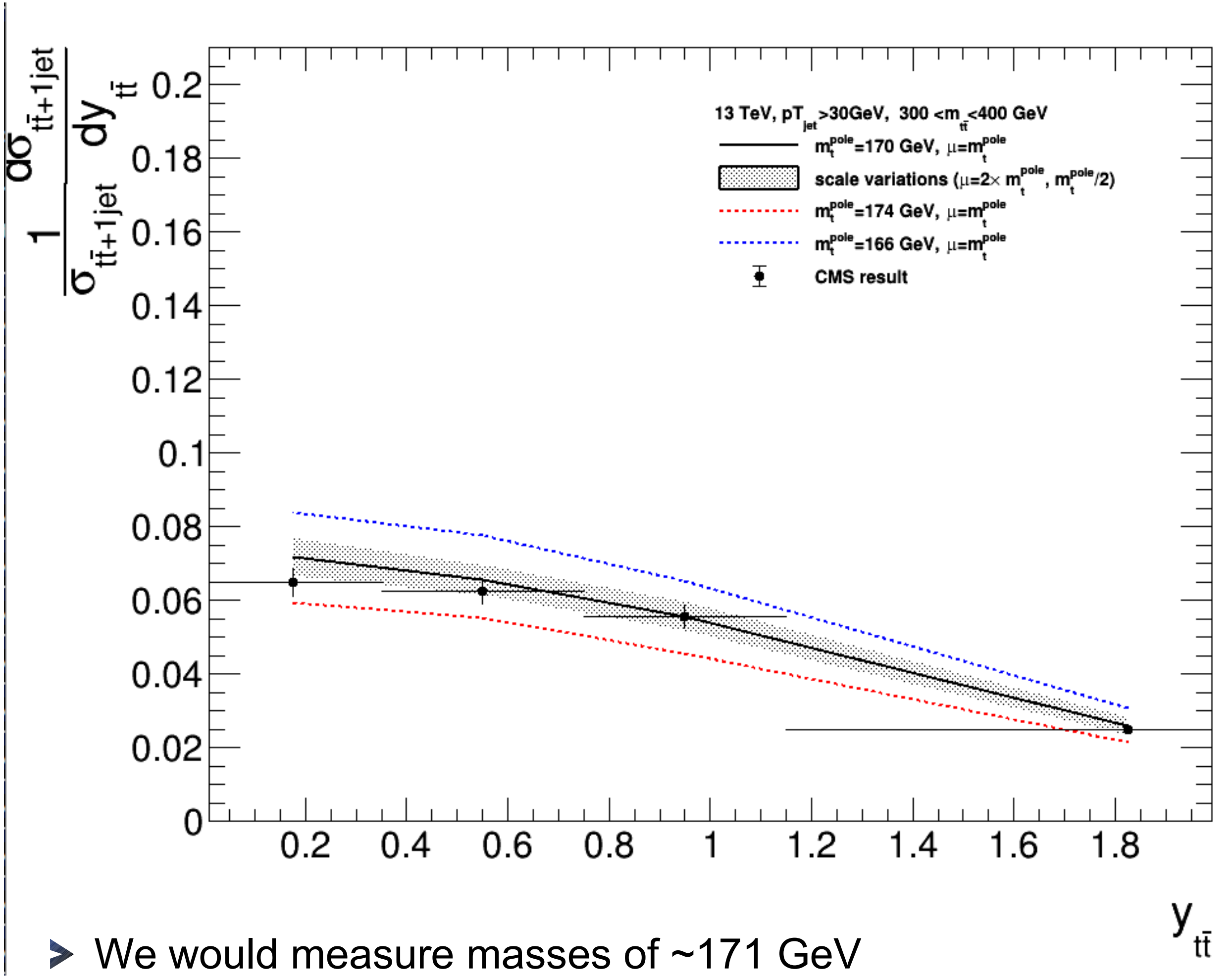
Some conclusions/thoughts

- The precision in the calculation of different observables depends on the mass scheme.
  - Every observable/calculation is different.
- For the CMS observable, the running mass scheme performs better than for the R-observable,
  - In comparison with the pole mass.
  - No other systematics are accounted!

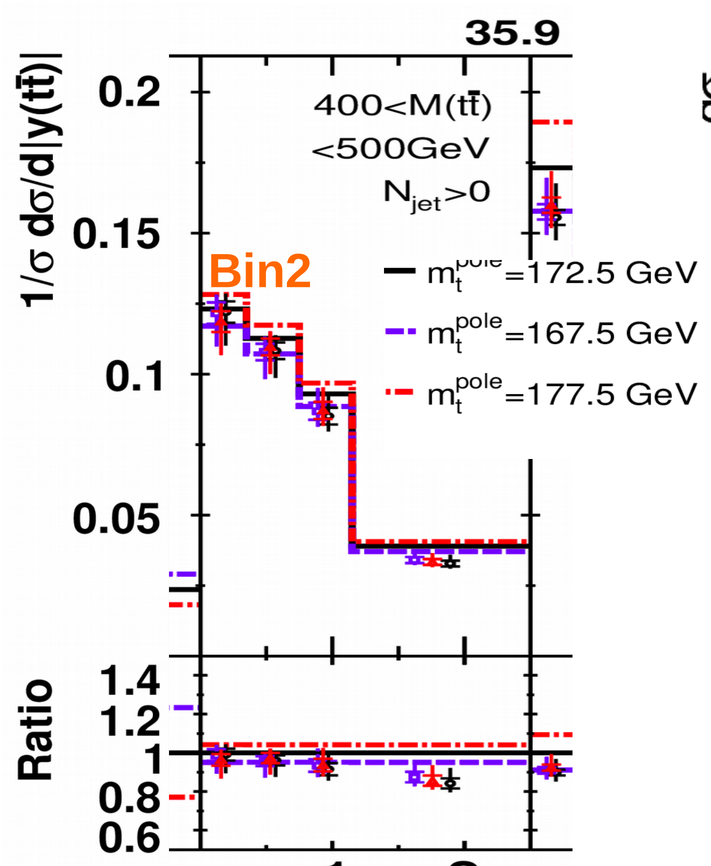
- Lots of potential!

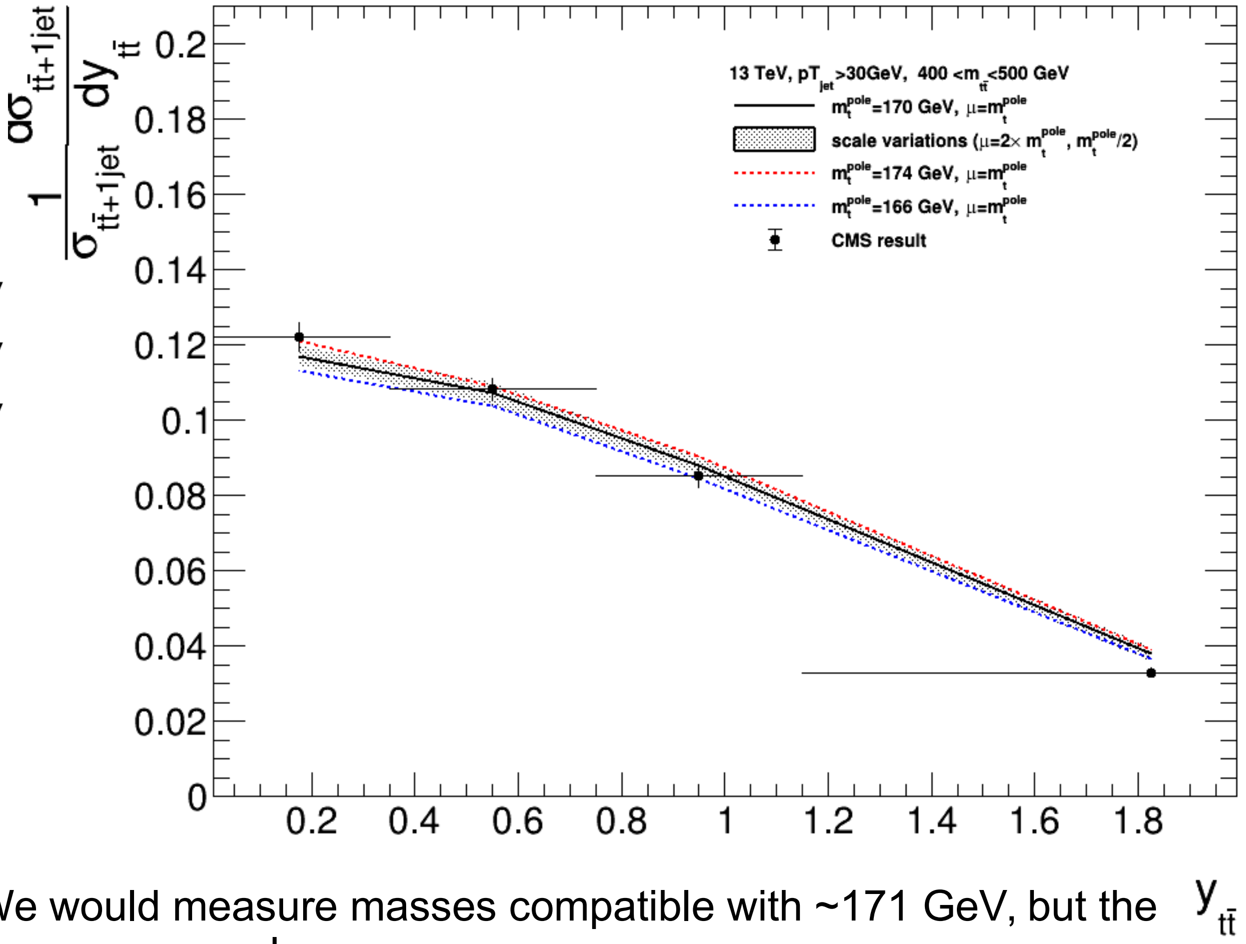
Back-up slides





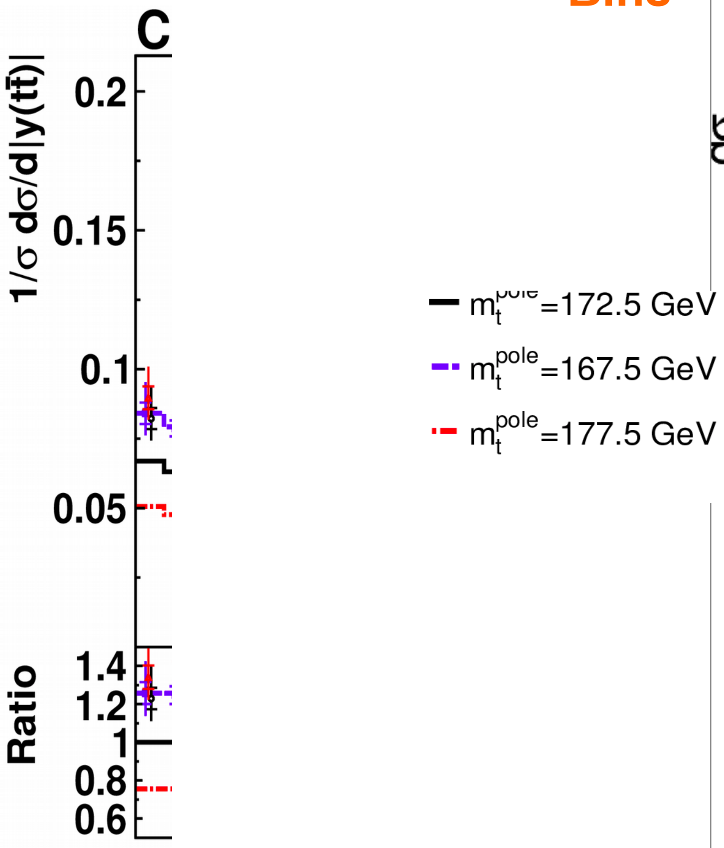
- We would measure masses of ~171 GeV
- CMS would measure masses of ~171 GeV

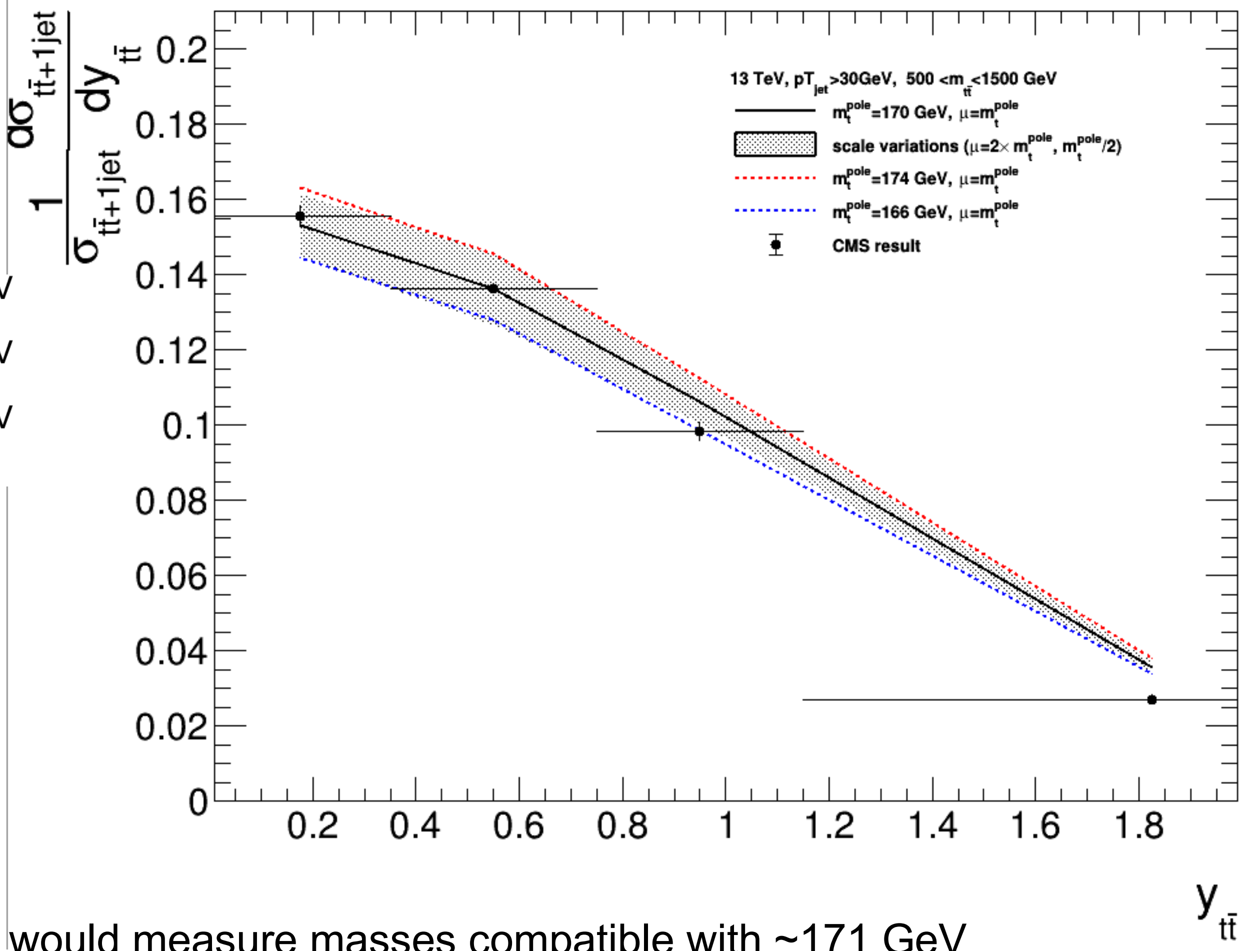




- We would measure masses compatible with ~171 GeV, but the theory unc. are large.
- CMS is more compatible with smaller masses than 167GeV

Bin3



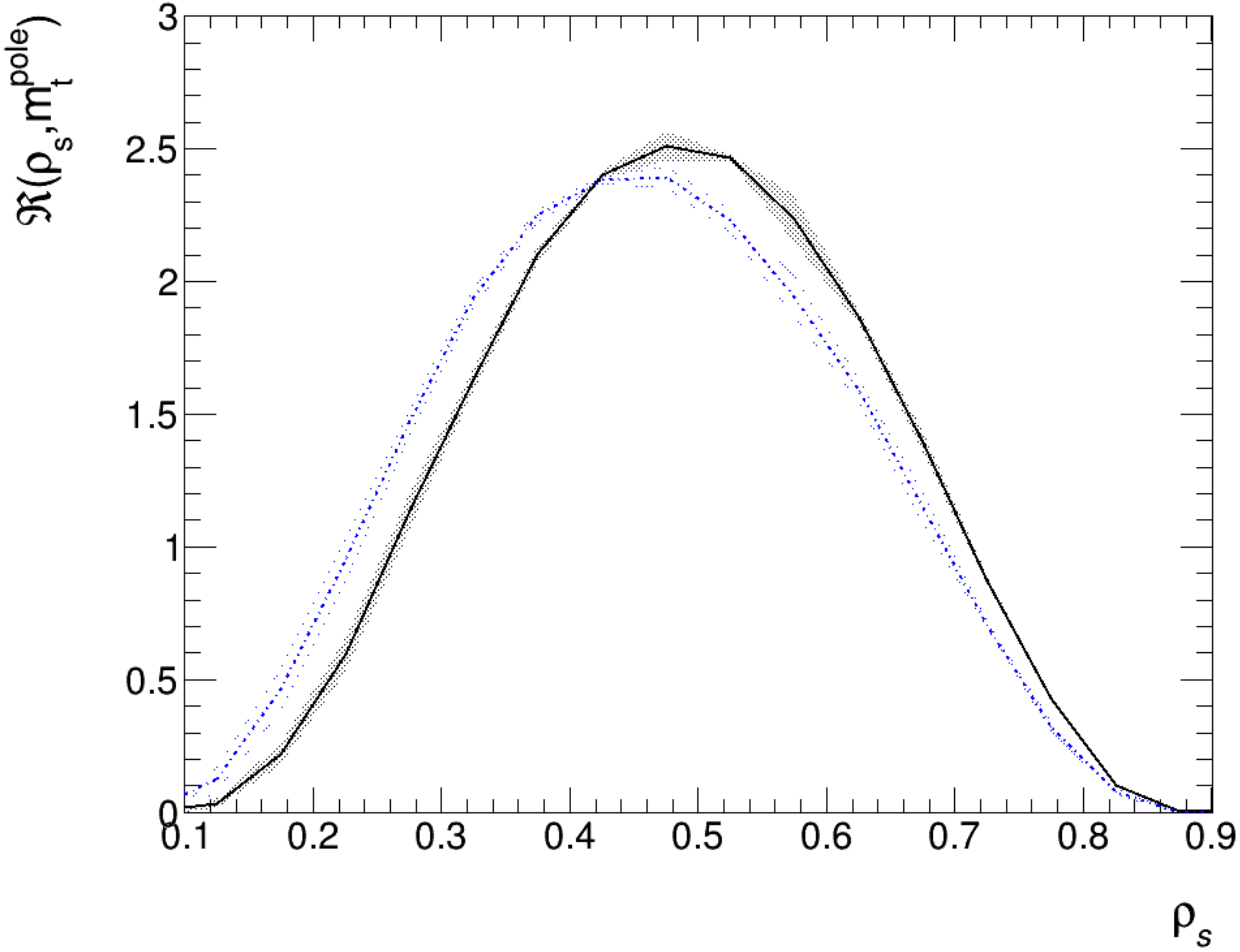


- We would measure masses compatible with ~171 GeV
- Again, CMS seems to predict lower masses...

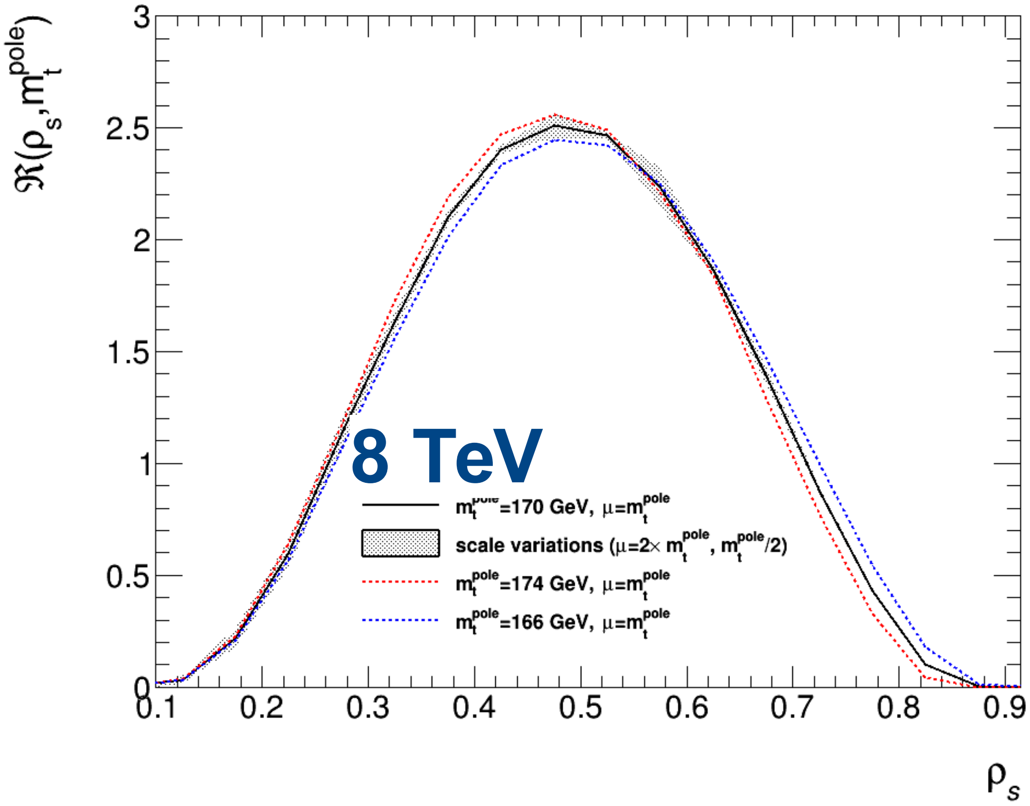
R distribution

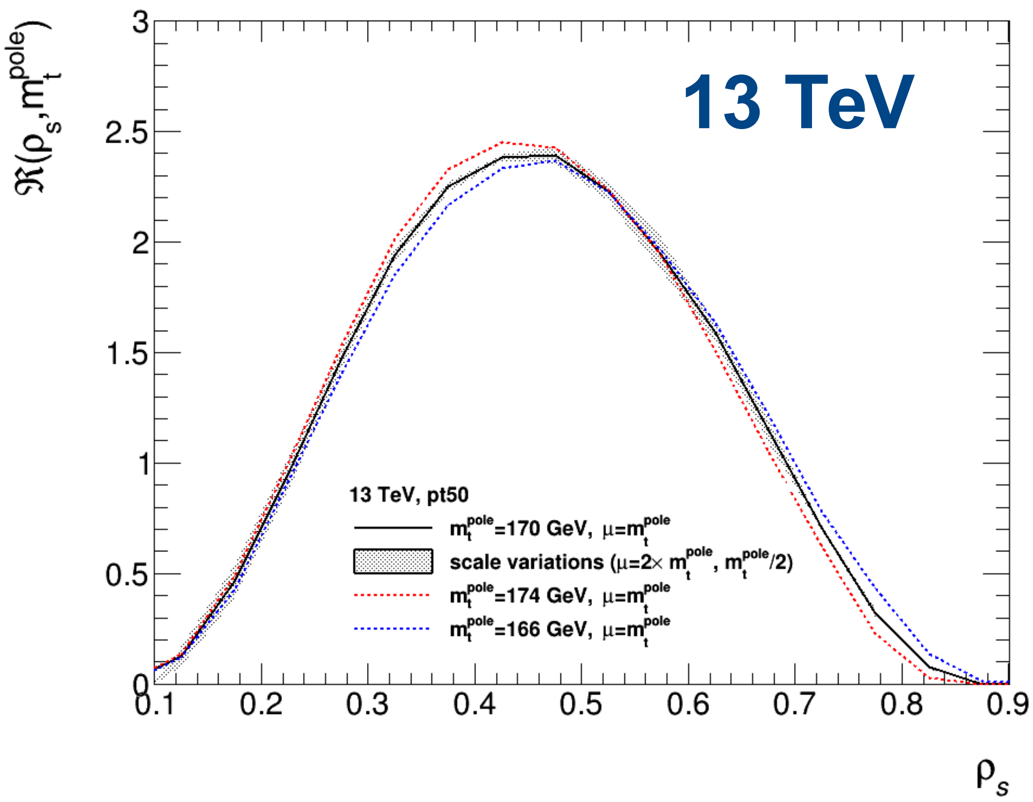
- **Blue 13 TeV,**
- **Black, 8TeV**

- both for pole mass and m=170 GeV
- Shaded areas = scale uncertainties.



R distribution, pole mass





Theoretical sensitivity. (7 vs 8 TeV, pole mass)

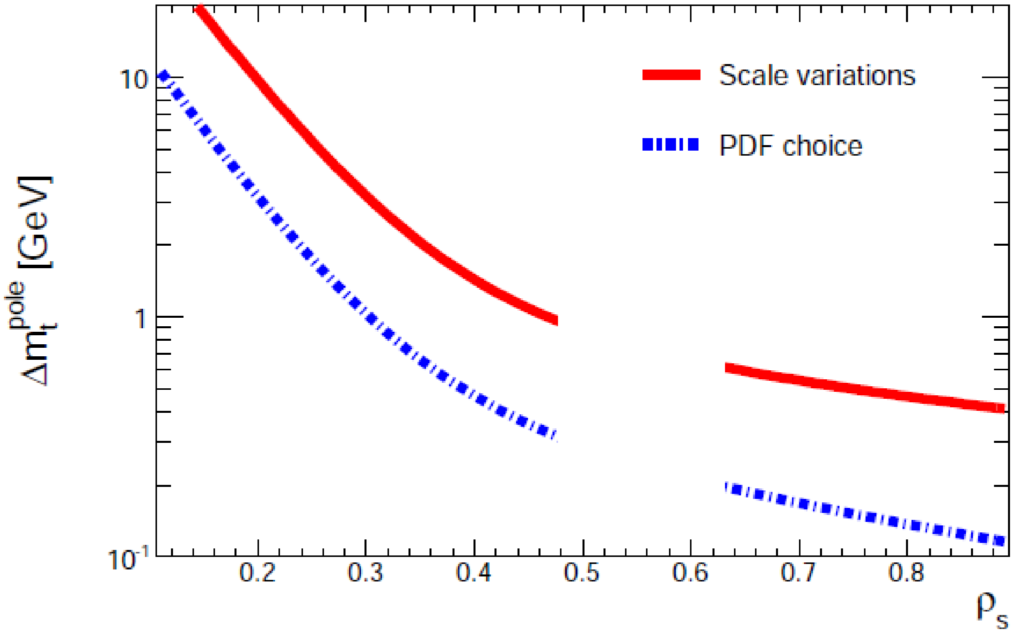
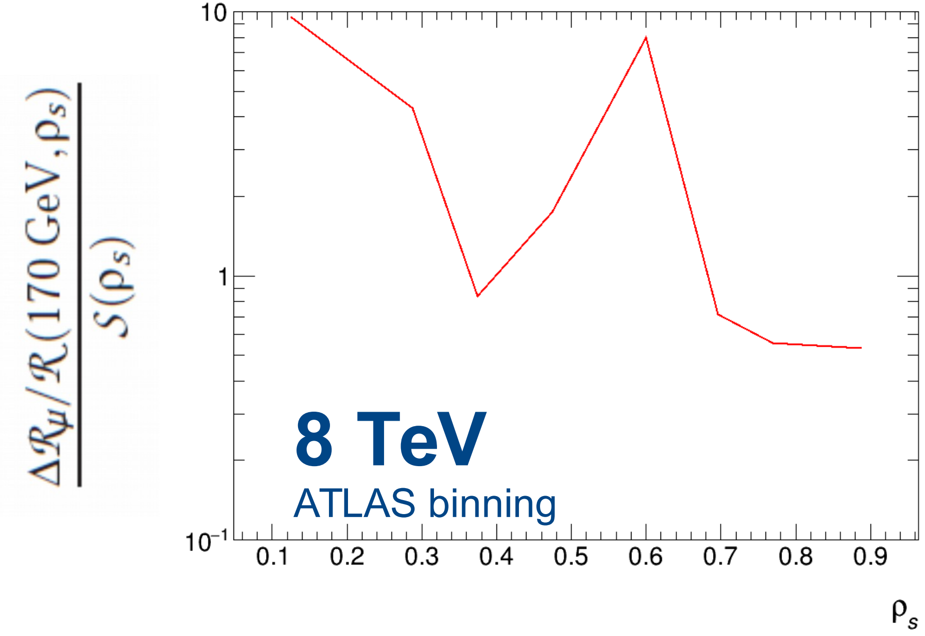
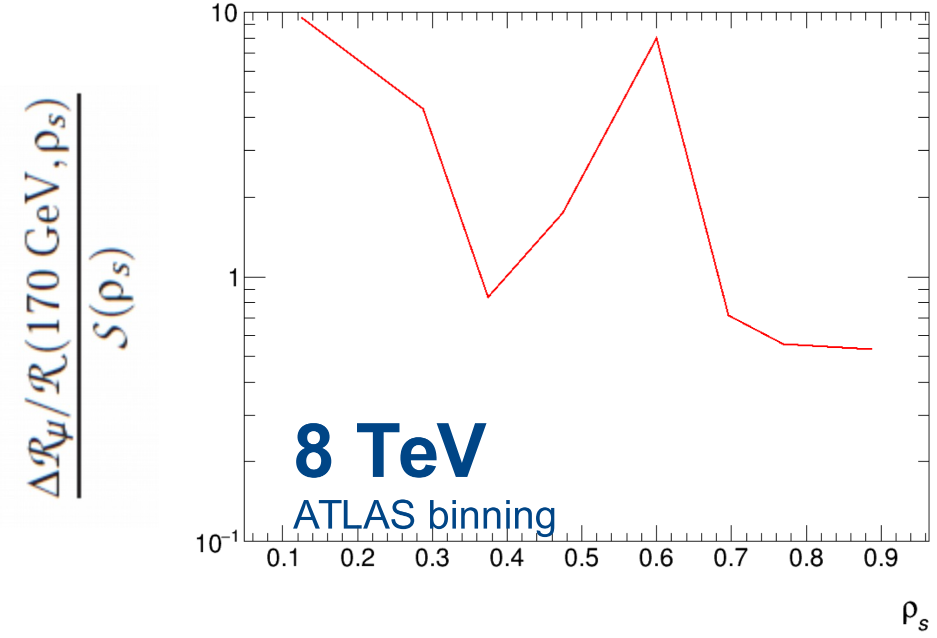
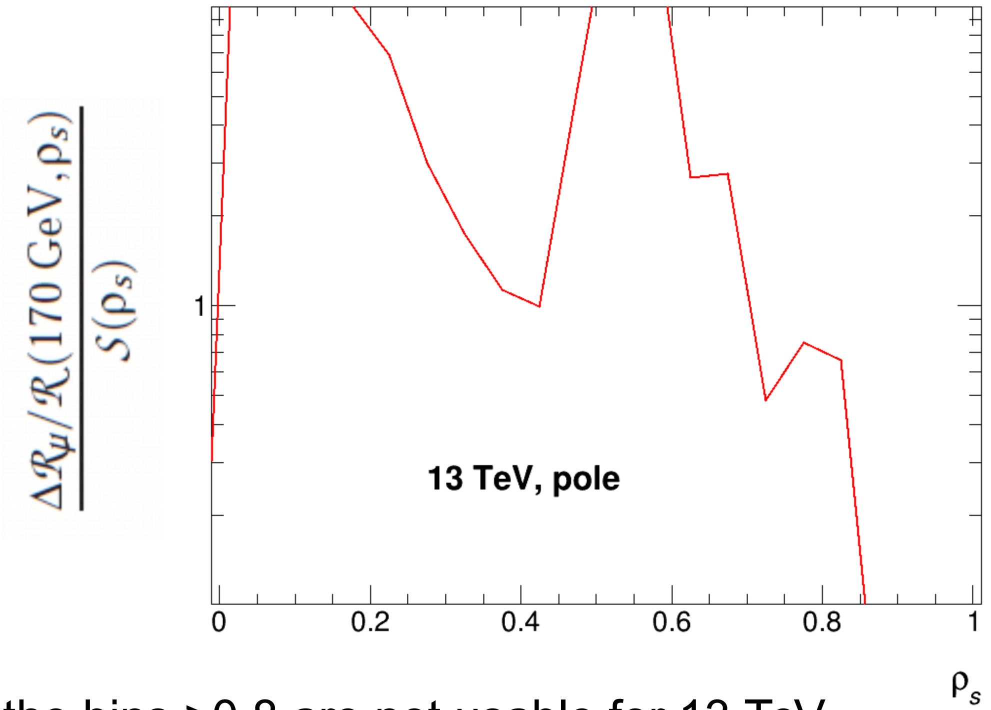


Fig. 6. Expected impact of scale (magenta line) and PDF (blue dashed line) uncertainties on the measured top-quark mass value. The region where $\mathcal{R}$ is essentially insensitive to the top-quark mass is not shown.



Theoretical sensitivity. (8 vs 13 TeV, pole mass)





- In principle, the sensitivities are similar but the bins >0.8 are not usable for 13 TeV (almost no cross section!)