Brans-Dicke as an effective Dynamical Dark Energy model †

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October 17, 2019

† Based on *Brans-Dicke gravity with a cosmological constant smoothes out ΛCDM tensions* J.Solà, A. Gómez-Valent, J. de Cruz Pérez and C. Moreno-Pulido (arXiV: 1909.02554)

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The cosmological constant problems

- It is well established by observations that our universe is expanding in an accelerated way.
- The name "Dark Energy" has been used to designate the negative pressure fluid responsible of this acceleration. It seems to include almost 70% of the energy content of our universe.
- The Cosmological constant, Λ, appears in Einstein's equations. In principle, it takes into account the energy density of the vacuum. It is assumed to be part of the content of DE.

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

The cosmological constant problems

- Observations of its value in natural units at present time seem not to match predictions (by a lot):

$$\rho_{\Lambda}^{obs} \equiv \frac{\Lambda}{8\pi G_N} \sim 10^{-47}\ \mathrm{GeV}^4, \qquad \rho_{EW} \approx v^2 m_H^2 \sim 10^{8}\ \mathrm{GeV}^4$$

- This enormous difference is what we commonly call "the cosmological constant problem".
- There is also another issue when we compare its current value with the current content of cold dark matter of the universe

$$\rho_{DE}^{obs} \sim \rho_{CDM}^{obs},$$

this is called the "coincidence problem".

Dynamical vacuum energy

- Consider a FLRW metric

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2).$$

- In the ΛCDM (the current framework accepted in cosmology), Λ is constant and $\rho_\Lambda = p_\Lambda$.
- One way to alleviate the coincidence problem is to let a dynamical behaviour of the cosmological constant.
- The departure from ΛCDM is controlled by small parameters.
- An example of dynamical vacuum model is the running vacuum ‡ , where ρ_Λ evolves with the Hubble function,

$$\rho_\Lambda(H) = \frac{3}{8\pi G_N}(C_0 + \nu H^2),$$

where $\nu \sim 10^{-3} - 10^{-2}$ and $H \equiv \dot{a}/a$.

‡ see J. Solà, J. de Cruz Pérez & A. Gómez-Valent, MNRAS 478 (2018) 4357; EPL 121 (2018) 39001

Brans-Dicke action

Our attempt is to work in the context of modified gravity. We use the action

$$S_{BD}[\psi] \equiv \int dx^4 \sqrt{-g} \left[\frac{1}{16\pi} \left(R\psi - \frac{\omega_{BD}}{\psi} g^{\mu\nu} \partial_\nu \psi \partial_\mu \psi \right) - \rho_\Lambda \right] + \int dx^4 \sqrt{-g} \mathcal{L}_m(\phi_m, g_{\mu\nu}).$$

- Modification of the Einstein-Hilbert action.
- $[\psi] = [G_N]^{-1} = M^2$.
- Coined in the 60's by Brans & Dicke in order to incorporate the Mach's principle to General Relativity.
- It contains a scalar field ψ which encapsulates the variation of the Newton Gravitational constant, G_N.
- We find the Einstein-Hilbert action if $|\omega_{BD}| \to \infty$.

Background equations

The equations of motion for this action are

$$3H^2 + 3H\frac{\dot{\varphi}}{\varphi} - \frac{\omega_{BD}}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 = \frac{8\pi G_N}{\varphi}\sum_j \rho_j \qquad (1)$$

$$2\dot{H} + 3H^2 + \frac{\ddot{\varphi}}{\varphi} + 2H\frac{\dot{\varphi}}{\varphi} + \frac{\omega_{BD}}{2}\left(\frac{\dot{\varphi}}{\varphi}\right)^2 = -\frac{8\pi G_N}{\varphi}\sum_j p_j \qquad (2)$$

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{8\pi G_N}{2\omega_{BD}+3}\left(3\sum_j p_j - \sum_j \rho_j\right), \qquad (3)$$

where j runs over the different components (radiation, cold dark matter (CDM), vacuum energy…) and we have defined $\varphi \equiv G_N\psi$.

Density contrast perturbation

To do the numerical treatment, we need to find the linear perturbation equations that dictate the evolution of the growth of matter. At subhorizon scales, this equation is

$$\ddot{\delta}_m + 2H\dot{\delta}_m - \frac{4\pi G_N}{\bar{\varphi}}\bar{\rho}_m\delta_m\left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right) = 0. \tag{4}$$

where $\delta_m \equiv \delta\rho_m/\bar{\rho}_m$. In order to fix initial conditions, we could work at the matter dominated epoch. Then,

$$\delta_m(t) = t^{2/3}\left[A_1 + \epsilon_{BD}A_2 + \epsilon_{BD}\frac{A_3}{t^{5/3}} + \epsilon_{BD}\frac{A_4}{t} + \epsilon_{BD}A_5 ln(t)\right],$$

where $A_1, \ldots, A_5$ are constants and $\epsilon_{BD} \equiv 1/\omega_{BD}$. If $\epsilon_{BD} = 0$, we obtain the same as in ΛCDM.

Effective General Relativity

We can split the BD action in the following way,

$$S_{BD}[\psi] = S_{EH} + S_\psi + S_m,$$

where S_{EH} is the traditional EH action, S_m is the action of matter fields, and

$$S_\psi \equiv \int dx^4 \sqrt{-g} \frac{1}{16\pi G_N} \left[R(\psi G_N - 1) - \frac{G_N \omega_{BD}}{\psi} g^{\mu\nu} \partial_\nu \psi \partial_\mu \psi \right].$$

- We can write now the equations of motion for an effective GR theory with an extra ingredient, namely S_ψ.
- The point is to see the new energy density that appears associated to ψ, which can be a dynamical component of the Dark Energy.

Effective General Relativity

A new energy density and a pressure associated to ψ arises from the previous action,

$$\rho_\varphi \equiv -\frac{3H^2(\varphi-1)}{8\pi G_N} - \frac{3H\dot{\varphi}}{8\pi G_N} + \frac{\omega_{BD}}{16\pi G_N}\frac{\dot{\varphi}^2}{\varphi},$$

and

$$p_\varphi \equiv \frac{3}{8\pi G_N}H^2(\varphi-1) + \frac{\dot{H}}{4\pi G_N}(\varphi-1) + \frac{\ddot{\varphi}}{8\pi G_N} + \frac{H\dot{\varphi}}{4\pi G_N} + \frac{\omega_{BD}}{16\pi G_N}\frac{\dot{\varphi}^2}{\varphi},$$

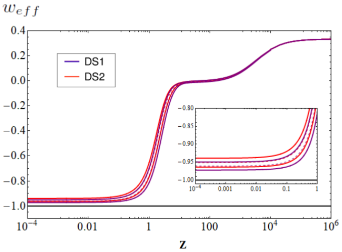
where, again, $\varphi = G_N\psi$.

- Due to the Bianchi identity, ρ_φ is covariantly conserved: $\dot{\rho}_\varphi + 3H\left(\rho_\varphi + p_\varphi\right) = 0$.
- Obviously ρ_φ is dynamical.

Effective General Relativity

The BD fluid is described by ρ_{φ} and ρ_{φ}, but is more useful to consider the EoS# of the full "Effective" DE:

$$\omega_{eff}(t) \equiv \frac{p_\Lambda + p_\varphi}{\rho_\Lambda + \rho_\varphi} = -1 + \frac{2\dot{H}(\varphi - 1) + \ddot{\varphi} - H\dot{\varphi} + \omega_{BD}\frac{\dot{\varphi}^2}{\varphi}}{8\pi G_N \rho_\Lambda - 3H^2(\varphi - 1) - 3H\dot{\varphi} + \frac{\omega_{BD}}{2}\frac{\dot{\varphi}^2}{\varphi}}.$$



#see J. Solà, Int. J. Mod. Phys. D27 (2018) 1847029; J. de Cruz Pérez and J. Solà, Mod. Phys. Lett. A33 (2018) 1850228

Results

DS1 with Bispectrum (SnIa+H(z)+BAO+LSS+RSDs+WL+CMB+R19)

Parameter	ΛCDM	BD
H_0 (km/s/Mpc)	$68.65^{+0.38}_{-0.40}$	$71.03^{+0.91}_{-0.86}$
ω_m^0	0.2955 ± 0.0048	0.2742 ± 0.0077
ω_b^0	0.0476 ± 0.0004	0.0453 ± 0.0010
τ	$0.063^{+0.010}_{-0.012}$	$0.081^{+0.015}_{-0.018}$
n_s	$0.9700^{+0.0038}_{-0.0040}$	$0.9891^{+0.0070}_{-0.0082}$
$\sigma_8(0)$	$0.804^{+0.007}_{-0.009}$	0.801 ± 0.010
ϵ_{BD}	0	$-0.00277^{+0.00170}_{-0.00154}$
φ_{ini}	1	$0.924^{+0.021}_{-0.023}$
$\varphi(0)$	1	$0.904^{+0.028}_{-0.029}$
$\omega_{eff}(0)$	-1	$-0.961^{+0.012}_{-0.011}$
$\dot{G}(0)/G(0)(10^{-13}/yr)$	0	$3.149^{+1.741}_{-1.924}$
$\triangle DIC(\triangle AIC)$	-	8.34(7.72)

Results

DS2 with Bispectrum (SnIa+H(z)+BAO+LSS+RSDs+WL+CMB+R19)

Parameter	ΛCDM	BD
H_0 (km/s/Mpc)	$68.69^{+0.38}_{-0.39}$	$72.00^{+1.00}_{-1.10}$
ω_m^0	0.2950 ± 0.0047	0.2665 ± 0.00847
ω_b^0	0.0476 ± 0.0004	0.0443 ± 0.0012
τ	$0.063^{+0.010}_{-0.011}$	$0.084^{\pm}0.018$
n_s	0.9704 ± 0.0038	$0.9945^{+0.0081}_{-0.0086}$
$\sigma_8(0)$	$0.804^{+0.007}_{-0.008}$	$0.803^{+0.011}_{-0.010}$
ϵ_{BD}	0	$-0.00315^{+0.00168}_{-0.00175}$
φ_{ini}	1	$0.901^{+0.026}_{-0.025}$
$\varphi(0)$	1	0.879 ± 0.032
$\omega_{eff}(0)$	-1	$-0.951^{+0.012}_{-0.013}$
$\dot{G}(0)/G(0)(10^{-13}/yr)$	0	$3.625^{+1.994}_{-1.954}$
$\triangle DIC(\triangle AIC)$	-	9.89(9.94)

Conclusions & remaining work

- The BD theory assumes a change in time of the effective Gravitational constant which, despite small, can affect the structure formation.
- This model seems able to reduce the H_0 tension, without producing additional tension in other quantities such as $\sigma_8(0)$.
- The effective model can mimic a ΛCDM framework with an effective dynamical vacuum energy density.
- When confronted with cosmological data, seems to be a possible good candidate for a new model of gravity.
- A more complete analysis from the dynamical system point of view will be done in the future, as well to test the model with the new likelihood of Planck 2018.

This is the end

Thank you for your attention!

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