New Consistent Truncations of M-theory

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Based on: GL, P. Ntokos, O. Varela – arXiv:1907.02087

GL, O. Varela – arXiv:1907.11027

Consistent Truncations: the idea

Supergravities tend to come with many fields:

- In $D = 4$ $\mathcal{N} = 8$, we have

$$\text{metric}, \quad 28 \text{ vectors}, \quad 70 \text{ scalars}, \; \dots$$

- For $D > 4$, we have *infinite* KK towers.

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How can we restrict ourselves to a smaller set *consistently*?

Lessons from pure gravity on $M_4 \times T^1$

Consider $\{g_{\mu\nu}(x), A_\mu(x), \phi(x)\} \hookrightarrow \hat{g}_{MN}(x,y)$ as

$$d\hat{s}_5^2 = e^{\frac{\phi}{\sqrt{3}}} ds_4^2 + e^{-\frac{2\phi}{\sqrt{3}}} (dy + A)^2$$

so that

$$\hat{R}_{MN} = 0 \quad \Longleftrightarrow \quad \begin{aligned} & G_{\mu\nu} = T_{\mu\nu}(F, \phi) \\ & \nabla^\mu \left(e^{-\sqrt{3}\phi} F_{\mu\nu} \right) = 0 \\ & \Box \phi = -\frac{\sqrt{3}}{4} e^{-\sqrt{3}\phi} F^2 \end{aligned}$$

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Is truncating the massive modes consistent? ✓
We are keeping the singlets of the U(1)-structure (non-singlets cannot be generated):

$$\hat{g}_{MN}(x,y) = \sum_{n\in\mathbb{Z}} \hat{g}_{MN}^{(n)}(x) e^{iny/L}$$

Some systematics

Gauntlett and Varela Conjecture [J.Gauntlett, O.Varela, '07]:

For any supersymmetric solution of $D = 10$ *or* $D = 11$ *sugra of type* $\mathrm{AdS}_d \times_w M$, there is a consistent Kaluza-Klein truncation on M to a minimal gauged supergravity in d dimensions.

Known examples:

- $D = 11$ Sugra on $\mathrm{AdS}_4 \times SE_7$
- $D = 11$ Sugra on SLAG-3 Flux-Geometry (warped M5-branes)
- Type IIB on general $\mathrm{AdS}_5 \times_w M_5$

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[Recent proof using Generalised Geometry methods [D.Cassani, G.Josse, M.Petrini, D.Waldram, '19]]

Outline

Two routes:

1. SU(3)-sector of $\mathcal{N} = 8$ SO(8)-gauged theory in $D = 4$.
2. Deformations of general $\mathcal{N} = 2$ – preserving solutions.

$\Rightarrow$ Intersection at minimal supergravity on CPW solution.

[R.Corrado, K.Pilch, N.P. Warner, '01]

$\mathcal{N} = 8$ sugra, the tensor hierarchy and $\Theta_M{}^\alpha$

The EoM's of the ungauged $\mathcal{N} = 8$ theory are invariant under global $E_{7(7)} \subset Sp(56,\mathbb{R})$ transformations.

For an $E_{7(7)}$-invariant Lagrangian, need ancillary higher p-forms in the *tensor hierarchy*:

$$A^M_{(1)} \,, \quad B_{(2)\,\alpha} \,, \quad C_{(3)\,\alpha}{}^M \,, \quad \ldots$$

Subgroups of $E_{7(7)}$ can be gauged, whose generators are picked by the embedding tensor $\Theta_M{}^\alpha$.

We focus on the electric SO(8) gauging, where only the 28 vectors that enter the Lagrangian have associated charges.

The SU(3) sector of SO(8)-sugra

Keeping singlets under SU(3) $\subset$ SO(8), find a U$(1)^2$-gauged, $\mathcal{N}=2$ sugra with a vector- and a hypermultiplet:

$$\begin{aligned}
\text{the metric} &: \quad ds_4^2 \, , \\
\text{2+4 scalars} &: \quad \varphi \, , \, \chi \, ; \, \phi \, , \, a \, , \, \zeta \, , \, \tilde{\zeta} \, , \\
\text{2+2 one-forms} &: \quad A^0 \, , \, A^1 \, , \, \tilde{A}_0 \, , \, \tilde{A}_1 \, , \\
\text{5 two-forms} &: \quad B^0 \, , \, B^2 \, , \, B^{ab} = B^{(ab)} \, , \\
\text{4 three-forms} &: \quad C^1 \, , \, C^{ab} = C^{(ab)}
\end{aligned}$$

with the scalars parametrising

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SU}(2,1)}{\mathrm{SU}(2)\times \mathrm{U}(1)} \subset \frac{\mathrm{E}_{7(7)}}{\mathrm{SU}(8)}$$

The SU(3) sector of SO(8)-sugra

- Only the metric, scalars and electric one-forms enter the Lagrangian.

- The other p-forms are determined by duality relations:

$$\tilde{H}_{\Lambda(2)} = \mathcal{I}_{\Lambda\Sigma} * H^{\Sigma}_{(2)} + \mathcal{R}_{\Lambda\Sigma} H^{\Sigma}_{(2)}$$

$$H_{(3)} = *Df(\text{scalars}) \,, \qquad H_{(4)} = h(\text{scalars})\,\text{vol}_4$$

- The scalar potential has 6 critical loci, including

$$\mathcal{N} = 8 \,,\ \text{SO}(8) \,, \qquad \mathcal{N} = 2 \,,\ \text{SU}(3) \times \text{U}(1)$$

Subsectors within SU(3)

Further truncations include:

- $\mathrm{SU}(3) \times \mathrm{U}(1)^2$: hypermultiplet out
- $\mathrm{SU}(3) \times \mathrm{U}(1)_{v,\,c,\,s}$ and $\mathrm{SU}(4)_{v,\,c,\,s}$: triality-inequivalent
- G_2 : $\mathcal{N} = 1$ sugra coupled to a chiral

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SU(4)$_s$ sector: minimal $\mathcal{N} = 2$ gauged sugra, with

$$A^1 = -A^0 \equiv \frac{1}{4}\bar{A}\,, \qquad\qquad ds_4^2 \equiv \frac{1}{4}d\bar{s}_4^2$$

obeying $\mathcal{L} = \bar{R}\,\overline{\mathrm{vol}}_4 - \frac{1}{2}\bar{F} \wedge \bar{*}\bar{F} + 6g^2\overline{\mathrm{vol}}_4$

Uplift to $D = 11$

Following [O.Varela, '15], we parametrise S^7 in terms of $\mathbb{R}^8$ embedding coordinates as

$$\mu^A \mu^B \delta_{AB} = 1$$

and split $A = (i,\ a)$ following $\mathbf{8}_v \to \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1}$.
The metric reads

$$d\hat{s}_{11}^2 = e^{-\varphi} X^{1/3} \Delta_1^{2/3} \Big[ds_4^2 + g^{-2} e^{\varphi} \Delta_1^{-1} \Big(D\mu_i D\mu^i + e^{2\varphi} X^{-1} Y (h^{-1})_{ab} D\mu^a D\mu^b \Big) + g^{-2} e^{3\varphi} X^{-1} Y^{-1} (Y - X) \Delta_1^{-2} \Big(Y J_{ij}^{(6)} \mu^i D\mu^j + h_{ab} \epsilon^{bc} \mu^a D\mu_c \Big)^2 \Big] \, .$$

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Restricting to smaller subsectors, in adapted coordinates,

$$d\hat{s}^2_{11}[\mathrm{SU}(4)_c] = e^{\frac{4}{3}\phi + \varphi}\, ds_4^2 + g^{-2}\left[e^{-\frac{2}{3}\phi} ds^2(\mathbb{CP}^3_+) + e^{\frac{4}{3}\phi - 2\varphi}(\boldsymbol{\eta}^{(7)}_+ + gA)^2\right]\ ,$$

$$d\hat{s}^2_{11}[\mathrm{SU}(4)_s] = \tfrac{1}{4}\, d\bar{s}_4^2 + g^{-2}\left[ds^2(\mathbb{CP}^3_-) + (\boldsymbol{\eta}^{(7)}_- + \tfrac{1}{4} g\bar{A})^2\right],$$

$$d\hat{s}^2_{11}[\mathrm{G}_2] = e^{-\varphi} X^{1/3} \Delta_1^{2/3} ds_4^2 + g^{-2} X^{1/3} \Delta_1^{-1/3}\left[e^{2\varphi} X^{-3} \Delta_1 d\beta^2 + \sin^2\!\beta\, ds^2(S^6)\right].$$

Uplift to $D = 11$

Similarly, the three form reads

$$\hat{A}_{(3)} = C^1 \mu_i \mu^i + C_{ab}\, \mu^a \mu^b - \tfrac{1}{12} g^{-1} \big[\big(B_a{}^a + 2\, A^1 \wedge \tilde{A}_1 \big)\, \delta_{ij} + 4 B^2\, J^{(6)}_{ij} \big] \wedge \mu^i D\mu^j$$

$$+ \tfrac{1}{2} g^{-1} \big[B_{ab} - A^0 \wedge \tilde{A}_0\, \delta_{ab} + B^0\, \epsilon_{ab} \big] \wedge \mu^a D\mu^b$$

$$+ \tfrac{1}{6}\, g^{-2}\, \tilde{A}_1 \wedge J^{(6)}_{ij} D\mu^i \wedge D\mu^j + \tfrac{1}{2}\, g^{-2} \tilde{A}_0 \wedge \epsilon_{ab} D\mu^a \wedge D\mu^b + A$$

with

$$A = -g^{-3}\, \Delta_1^{-1} \Big[\tfrac{1}{2}\, e^{4\varphi}\, \chi\, X^{-1} Y\, J^{(6)}_{ij} \mu^i D\mu^j \wedge \epsilon_{ab}\, D\mu^a \wedge D\mu^b$$

$$+ \tfrac{1}{2}\, \chi\, e^{2\varphi} \big(Y J^{(6)}_{ij} \mu^i D\mu^j + h_{ab} \epsilon^{bc} \mu^a D\mu_c \big) \wedge J^{(6)}_{kl} D\mu^k \wedge D\mu^l$$

$$- \tfrac{1}{4}\, e^{2\varphi}\, \big(V_1\, \mathrm{Re}\, \Omega^{(6)}_{ijk} + V_2\, \mathrm{Im}\, \Omega^{(6)}_{ijk} \big) \wedge \mu^i D\mu^j \wedge D\mu^k$$

$$+ \tfrac{1}{12}\, e^{2\phi}\, X\, \big(v_1\, \mathrm{Re}\, \Omega^{(6)}_{ijk} + v_2\, \mathrm{Im}\, \Omega^{(6)}_{ijk} \big) D\mu^i \wedge D\mu^j \wedge D\mu^k \Big]$$

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Uplift to $D = 11$

As before, $\hat{A}_{(3)}$ simplifies (drastically) in some subsectors. E.g.:

$$\hat{A}_{(3)}[\mathrm{G}_2] = C_1 \sin^2\beta + C_{88}\cos^2\beta + 4g^{-1}\sin\beta\cos\beta\, B_{77}\wedge d\beta$$
$$+ g^{-3}\chi\,\Delta_1^{-1}\sin^2\beta\Big[e^{2\varphi}X^{-1}\Delta_1\mathcal{J}\wedge d\beta$$
$$+ X^2\sin\beta\cos\beta\,\mathrm{Re}\,\Omega + e^{2\varphi}X\,\sin^2\beta\,\mathrm{Im}\,\Omega\Big]\,.$$

Upon using the duality relations, can give redundancy-free field strengths. E.g.:

$$\hat{F}_{(4)}[\mathrm{SU}(4)_s] = \tfrac{3}{8}\,g\,\overline{\mathrm{vol}}_4 - \tfrac{1}{4}\,g^{-2}\,\bar{*}\bar{F}\wedge \boldsymbol{J}_{-}^{(7)}\,.$$

A special case: Minimal supergravity on CPW

It is well known that [R.Corrado, K.Pilch, N.P. Warner, '01]

$$[\mathcal{N}=2\ ,\ \mathrm{SU}(3)\times\mathrm{U}(1)_c]-\text{point}\hookrightarrow \text{CPW solution in } D=11$$

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What is the minimal theory predicted by the GV conjecture?

The $D = 4$ theory admits a minimal truncation if we

1. Fix scalars to their vevs.
2. Identify $A^0 = -3A^1 \equiv \frac{1}{2}\bar{A}$ (also, $g_{\mu\nu} = \frac{1}{3\sqrt{3}}\bar{g}_{\mu\nu}$).

A special case: Minimal supergravity on CPW

Implementing this on the uplifting formulae provides a new family of consistent M-theory solutions:

$$d\hat{s}_{11}^2 = \frac{1}{3}\cdot 2^{-2/3}\,(1+2\sin^2\alpha)^{2/3}\Bigg[d\bar{s}_4^2 + g^{-2}\Big[\,2\,d\alpha^2 + \frac{6\cos^2\alpha}{1+2\sin^2\alpha}ds^2(\mathbb{CP}^2) + \frac{18\sin^2\alpha\cos^2\alpha}{1+8\sin^4\alpha}\,\boldsymbol{\eta}'^{\,2} + \frac{1+8\sin^4\alpha}{\left(1+2\sin^2\alpha\right)^2}\left(D\psi' - \frac{3\cos^2\alpha}{1+8\sin^4\alpha}\boldsymbol{\eta}'\right)^2\Big]\Bigg]\ ,$$

$$\hat{F}_{(4)} = \frac{g}{2\sqrt{3}}\overline{\mathrm{vol}}_4 + \frac{g^{-3}}{\sqrt{3}}\Bigg[-\frac{\cos^2\alpha\,(7-10\cos 2\alpha+\cos 4\alpha)}{\left(1+2\sin^2\alpha\right)^2}d\alpha\wedge D\psi'\wedge \mathrm{Re}\,\boldsymbol{\Omega}' - \frac{6\cos^4\alpha}{(1+2\sin^2\alpha)^2}d\alpha\wedge\boldsymbol{\eta}'\wedge\mathrm{Re}\,\boldsymbol{\Omega}' + \frac{6\sin\alpha\cos^3\alpha}{1+2\sin^2\alpha}D\psi'\wedge\boldsymbol{\eta}'\wedge\mathrm{Im}\,\boldsymbol{\Omega}'\Bigg] + \frac{g^{-2}}{2\sqrt{3}}\left[\frac{2\sin\alpha\cos^3\alpha}{1+2\sin^2\alpha}\,\bar{F}\wedge\mathrm{Re}\,\boldsymbol{\Omega}' + \cos\alpha\,\bar{*}\,\bar{F}\wedge\Big(\cos\alpha\,\boldsymbol{J}' - \sin\alpha\,d\alpha\wedge\boldsymbol{\eta}'\Big)\right].$$

The G-structures route

For $\bar{A} = 0$, the CPW solution is an $\mathcal{N} = 2$ AdS_4 background.

$\Rightarrow$ included in the GMPS classification [M.Gabella, D.Martelli, A.Passias, J.Sparks, '12]

GMPS considered M-theory backgrounds

$$d\hat{s}^2_{11} = e^{2\Delta}\left(ds^2(\text{AdS}_4) + ds^2_7\right) , \qquad \hat{F}_{(4)} = m\,\text{vol}(\text{AdS}_4) + F_{(4)} ,$$

admitting two solutions of Killing's spinor equation, $\delta_\epsilon \Psi_M = 0$.

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Out of the spinor bilinears,

$$ds^2_7 = ds^2_{\mathrm{SU}(2)} + E_1^2 + E_2^2 + E_3^2 \,,$$

$$F_{(4)} = \frac{1}{\|\xi\|} E_1 \wedge d\left(e^{3\Delta}\sqrt{1-\|\xi\|^2} J_1\right) - m\frac{\sqrt{1-\|\xi\|^2}}{\|\xi\|} J_1 \wedge E_2 \wedge E_3 \,,$$

with $E_1 = \frac{1}{4}\|\xi\|(d\psi + \mathcal{A})$ dual to the Reeb vector field, and the SU(2)-structure governed by torsion conditions.

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Gauging the Reeb direction, $E_1 \mapsto \tilde{E}_1 = \frac{1}{4}\|\xi\|(d\psi + \mathcal{A} - g\bar{A})$, consider [J.Gauntlett, O.Varela, '07]:

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From Bianchi and EoM for $\hat{F}_{(4)}$,

$$\alpha = -\frac{1}{4} e^{3\Delta}\sqrt{1 - \|\xi\|^2}\, J_1 \ , \qquad \beta = -\frac{1}{4} e^{3\Delta}(J_3 - \|\xi\| E_2 \wedge E_3) \, .$$

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Consistent with Einstein,

$$\hat{\mathcal{E}}_{ab} = 0 , \qquad \hat{\mathcal{E}}_{\alpha 8} \sim \nabla_\gamma \bar{F}_\alpha{}^\gamma , \qquad \hat{\mathcal{E}}_{\alpha\beta} = \mathcal{E}_{\alpha\beta} ,$$

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and susy

$$\delta_\epsilon \Psi_m = 0 , \qquad \delta_\epsilon \Psi_\mu = \delta\psi^i_\mu \otimes e^{\Delta/2}\chi_i .$$

Conclusions

Today we showed:

- Some systematics about the “nesting” of $D = 4$ theories within $\mathcal{N} = 8$ SO(8)-gauged supergravity.
- A new $\mathcal{N} = 2$ minimal supergravity whose embedding is not driven by group-theoretical considerations.
- More instances of the GV conjecture for any $\mathcal{N} = 2$ AdS_4 background.


Many thanks for your attention!!

Details on SU(3) $\hookrightarrow$ SO(8)

SU(3) commutes with SU(1,1)$\times$ SU(2,1) within $E_{7(7)}$

$$\text{Scalar coset} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SU}(2,1)}{\text{SU}(2)\times\text{U}(1)}$$

For the *restricted* tensor hierarchy, use the SU(3)-invariant tensors within $\mathbb{R}^8 = \mathbb{R}^6 \times \mathbb{R}^2$:

$$\mathcal{A}^{ij} = A^1 J^{(6)ij} \,, \qquad \mathcal{A}^{ab} = A^0 \epsilon^{ab} \,,$$

$$\tilde{\mathcal{A}}_{ij} = \frac{1}{3}\tilde{A}_1 J_{(6)ij} \,, \qquad \tilde{\mathcal{A}}_{ab} = \tilde{A}_0 \epsilon_{ab} \,,$$

$$\mathcal{B}_i{}^j = -\frac{1}{12} B_a{}^a \delta_i^j + \frac{1}{3} B^2 J^{(6)}{}_i{}^j \,, \qquad \mathcal{B}_a{}^b = \frac{1}{2} B_a{}^b - \frac{1}{2} B^0 \epsilon_a{}^b \,,$$

$$\mathcal{C}^{ij} = C^1 \delta^{ij} \,, \qquad \mathcal{C}^{ab} = C^{ab} \,,$$

Geometry of S^7

$S^1 \hookrightarrow S^7 \to D^5$ perspective:

$$\mu^i = \cos\alpha\tilde{\mu}^i\ , \qquad \mu^7 = \sin\alpha\cos\psi\ , \qquad \mu^8 = \sin\alpha\sin\psi$$

S^5 equipped with natural SE_5 structure:

$$\boldsymbol{\eta}^{(5)} = J^{(6)}_{ij}\tilde{\mu}^i d\tilde{\mu}^j\ , \quad \boldsymbol{J}^{(5)} = \frac{1}{2}J^{(6)}_{ij}d\tilde{\mu}^i\wedge d\tilde{\mu}^j\ , \quad \boldsymbol{\Omega}^{(5)} = \frac{1}{2}\Omega^{(6)}_{ijk}\tilde{\mu}^i d\tilde{\mu}^j\wedge d\tilde{\mu}^k$$

$$\Downarrow$$

$$d\boldsymbol{\eta}^{(5)} = 2\boldsymbol{J}^{(5)}\ , \qquad d\boldsymbol{\Omega}^{(5)} = 3i\boldsymbol{\eta}^{(5)}\wedge\boldsymbol{\Omega}^{(5)}\ ,$$

$$\boldsymbol{J}^{(5)}\wedge\boldsymbol{\Omega}^{(5)} = 0\ , \qquad \frac{1}{2}\boldsymbol{\eta}^{(5)}\wedge\boldsymbol{J}^{(5)}\wedge\boldsymbol{J}^{(5)} = \frac{1}{4}\boldsymbol{\eta}^{(5)}\wedge\boldsymbol{\Omega}^{(5)}\wedge\boldsymbol{\Omega}^{(5)} = \mathrm{vol}(S^5)$$

So that

$$ds^2(S^5) = ds^2(\mathbb{CP}^2) + (\boldsymbol{\eta}^{(5)})^2 = ds^2(\mathbb{CP}^2)) + (d\tau + \sigma)^2$$

For CPW, $\psi = \psi'$, $\tau = \tau' - \frac{1}{3}\psi'$ and $\Omega' = e^{i(\psi+\frac{\pi}{4})}\Omega^{(5)}$.